Discussion

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I am pleased to offer a comment on this very interesting article by an author who is always in the forefront of the research on empirical financial models. This article presents data analysis that estabishes the stylized facts about stock market volatility around market crashes. He concludes that volatility is high during periods of stock market decline and that it gradually returns to more normal levels. In the case of 1987, the peak was higher than usual and the decline was more rapid.

The article uses 28,000 daily observations but does not really estimate a usable model; instead, it explores the data by estimating highly overparameterized models that reveal important features of the data. I suggest that this be considered an exploratory investigation and that in the face of more parsimonious models, rather interesting and somewhat different conclusions are revealed.

The basic model estimated by Schwert is a 22-order autoregression of daily returns with a heteroskedastic error standard deviation which is itself assumed to be a 22-order autoregression in the absolute errors. Even with 28,000 observations, there is apparently a lot of noise in the coefficients. To allow for a risk premium, the mean is related to the variance, and in this case it is therefore related to 22 lagged absolute residuals. This part of the model uses 66 parameters. An alternative model is a first-order generalized autoregressive conditionally heteroskedastic model with variance influencing the mean [GARCH(1,1)-m], with a first-order moving average to correct for nonsynchronous trading as used in Engle, Lilien, and Robins (1987), French, Schwert, and Stambaugh (1987), or Chou (1988), following the earlier work of Engle (1982). This requires only four coefficients! In the context of the parsimonious model, the parameter regulating the risk–return trade-off can be interpreted as the median agent's taste for risk or his coefficient of relative-risk aversion. One naturally asks whether this parameter is constant over time, and we then recognize that the Schwert parameterization cannot answer the question.

Chou, Engle, and Kane (1989) tested the price of volatility for stability over time. By estimating rolling regressions and also using

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a Kalman filter/time-varying parameter model, it appears that it is not. The parameter is high for the mid-1950s through the early 1970s and low in the 1980s and in the depression and war years. These movements are associated with macroeconomic variables such as inflation and aggregate profits. They interpret this as a constant covariance with a market which contains assets other than equities that have unobservable returns.

In a very interesting extension, Schwert then tests whether the level of past residuals (as well as their absolute value) can predict volatility. The answer is found to be "yes"; falling equity values predict greater volatility, presumably resulting from a leverage effect. The appeal to leverage is rather heuristic since it should then be the level of prices rather than the change that affects volatility; nevertheless, the finding is interesting and reproducible. The statistical approach is to introduce another 22 lags of the levels of the residuals into the standard deviation equation but not into the mean equation. It is surprising to me that the predicted standard deviations are nowhere negative.

To compare his models with the autoregressive conditionally heteroskedastic (ARCH) models, I reexamined the widely used daily Center for Research in Security Prices (CRSP) value-weighted stock return index from July 3, 1962 to December 31, 1985. Schwert relates the standard deviation to the absolute residuals, hence squares of both sides relate the conditional variance of returns to squares, cross products, and levels of residuals. I approximate this as a geometric distributed lag of absolute residuals to an unknown but constant power but allow asymmetric effects. Letting y be the daily percentage return, the estimate is

$$\begin{aligned} y_t &= \underset{(0.013)}{0.0169} + \underset{(0.031)}{0.0768 h_t} + \underset{(0.013)}{0.235\epsilon_{t-1}} + \epsilon_t \\ h_t &= \underset{(0.0009)}{0.002} + \underset{(0.005)}{0.080|\epsilon_{t-1}|^{1.70*}} - \underset{(0.003)}{0.047\epsilon_{t-1}} + \underset{(0.005)}{0.917 h_{t-1}} \end{aligned}$$

The estimated coefficient of relative-risk aversion is 7.68. The model reveals that a geometric decay of the absolute residuals to the 1.7 power plus the same geometric decay of the negative of the residuals is the best predictor of the variance. The exponent is statistically significantly less than 2, the value typically used in ARCH applications, but not close to 1. The level of the residual is highly significant and negative as found by Schwert with his data and sample period. If the exponent were, in fact, 2, the second and third terms could be rewritten as

* Standard error = 0.075.

$$0.080(\epsilon_{t-1} - 0.29)^2$$

by completing the square; there is no nonnegativity problem with such a specification as long as the derived intercept remains positive. The return which minimizes the next period variance is now +0.29 percent rather than 0, which reflects the "leverage effect." The coefficients on the absolute error and the lagged h sum to 0.997, which indicates high persistence of the shocks. If the exponent were precisely 2, then the decay rate of volatility shocks on the conditional variance is given by this sum. The process appears integrated in variance as in Engle and Bollerslev (1986) in contrast to Schwert's estimates (although not his prose) where the sum of the coefficients is substantially less than unity.

In a related analysis, the article computes implied volatilities from at-the-money index options and compares these with other estimates of volatility. Around the crash of '87 the option measure went up the least and fell the soonest. Since the volatilities are different, it may be that the options were mispriced. Theoretically, however, the option prices need not be given by the Black–Scholes formula when volatility is stochastic. A more consistent approach is given in Engle and Mustafa (1989), who find the ARCH model for the underlying security that most closely approximates the full set of options prices in the market at any particular instant. This "implied" ARCH model is typically quite similar to the one estimated from historical data. However, after the crash of '87 the "implied" ARCH model shows a much lower persistence of volatility than before. The options market perceived that the volatility would not last as long as in previous episodes, and this in fact was a correct perception judging from Schwert's article as well as my own. Thus a parametric treatment of the options market reinforces Schwert's conclusion but gives more precise predictions.

Thus, in summary, estimation of a parsimonious stochastic volatility model, such as a variation of an ARCH model, allows tests for parameter stability, better estimates of the persistence of volatility shocks, estimation of the parameter regulating the risk and return trade-off, quantification of the "leverage or asymmetry effect," and comparison with options market prices.

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