Seasonal and Non-Seasonal Long Memory in the US Interest Rate and the Monetary Aggregates

Juncal Cuñado
Universidad de Navarra

Luis A. Gil-Alana
Universidad de Navarra

Fernando Pérez de Gracia*
Universidad de Navarra

In this article we examine the US interest rate along with the monetary aggregates M1, M2, and M3 by means of fractional integration techniques. We use a procedure established by Robinson (1994) that permits us to simultaneously test the degrees of integration at both the zero and the seasonal frequencies. The tests have standard null and local limit distributions, and several Monte Carlo experiments conducted across the paper show that the tests perform relatively well even with small samples. The results show that the root at the long-run or zero frequency plays a much more important role than do the seasonal ones. Thus, the order of integration at the zero frequency seems to be equal to or higher than one in all cases, while the one corresponding to the seasonal frequencies is around 0.25.

Introduction

Many macroeconomic time series contain important seasonal components. It is a common belief that modelers need to pay specific attention to the nature of seasonality rather than essentially to ignore it. The concept of seasonality is seldom defined rigorously. It seems clear that any definition of seasonality must include something such as a systematic intra-year movement, though the relevant question is how sys-

* The authors gratefully acknowledge the comments and suggestions from the editor and the anonymous referee. Financial support from the Ministerio de Ciencia y Tecnologia (SEC2002-01839, Spain) is also acknowledged.

0747-5535/06/1400 & 1500/00031/$2.50
Copyright 2006 University of Nebraska—Lincoln

tematic such movement is. (See, e.g., Thomas and Wallis, 1972; Granger, 1978; Hylleberg, 1986, 1992, etc.).

Traditionally, seasonal fluctuations have been considered as a nuisance that obscure the more important components of the series (presumably the growth and the cyclical components, e.g., Burns and Mitchell, 1946), and seasonal adjustment procedures have been implemented to eliminate seasonality. Thus, most of the empirical work on economic variables has been based on seasonally adjusted data, and little attention has been paid to the role of seasonality. In a number of papers, seasonal fluctuations have been found to play a significant role in accounting for most of the variation in many macroeconomic time series. Contributors to this view include Ghysels (1988), Barsky and Miron (1989), Faig (1989), Beaulieu and Miron (1992), Braun and Evans (1995), Lee and Siklos (1997), Bohl (2000), etc. The first two authors observe that seasonal adjustment might lead to mistaken inferences about economic relationships between time series data. Seasonal fluctuations have been found to be economically significant and an important source of variation in economic time series. For example, Bohl (2000) finds a stable long-run relationship between money demand (M2) and output only when using seasonally unadjusted data. Similarly, Lee and Siklos (1997) find evidence of some feedback from output to money when using seasonally unadjusted data and, consequently, suggest that researchers should use raw data instead of seasonally adjusted data for inference and forecasting purposes.

Nevertheless, there is little consensus on how seasonality should be treated in empirical research on economic modeling. While Barsky and Miron (1989) use seasonal dummy variables, Faig (1989) and Beaulieu and Miron (1993) emphasize the problem of using this adjustment procedure when the observed seasonality is generated by an integrated process. The use of a filter $(1 - L^s)$ to a series with a seasonal component is justified when the series is integrated at the zero and the seasonal frequencies. The application of the seasonal difference operator may produce serious misspecifications if unit roots are absent at some or at all of the seasonal frequencies. Several papers (Porter-Hudak, 1990; Franses and Ooms, 1997; Arteche and Robinson, 2000; Gil-Alana and Robinson, 2001; Gil-Alana, 2002) analyze the usefulness of seasonal fractional models in contrast to the procedure of seasonal (integer) differencing of the series. In most of these papers, they concentrate on the seasonal structure and pay no attention to the other possible (trend) structures underlying the series. Clearly, the polynomial $(1 - L^s)$ can be decomposed into $(1 – L) (1 + L + L^2 + \dots + L^{s-1})$, i.e., separating the roots at zero and the seasonal frequencies. The polynomial at zero may be important by itself, and first (or fractional) differences may be required before proceeding to the examination of the seasonal part.

In this paper we analyze the monetary aggregates M1, M2, and M3 as well as the interest rates for the US using monthly data based on fractional integration techniques. An extensive work has been done in modeling monetary aggregates in a variety of contexts: first, as a preliminary step before estimating some macroeconomic relationships (as the money demand function); second, the analysis of these series received great attention because they were significant variables used by the US Federal Reserve as targets of monetary policy; moreover, the impact on some real variables such as output has been an important issue in monetary economics. Thus, a number of papers have tried to model these variables taking into account their seasonal components. (See, e.g., Moore et al., 1981; Porter-Hudak, 1990; Lee and Siklos, 1997; Bohl, 2000, etc.). Lee and Siklos (1997) and Bohl (2000) use a classical method developed by Hylleberg et al. (1990) for testing seasonal unit roots, while Porter-Hudak (1990) uses seasonal fractional integration based on semiparametric techniques. Our methodology is closer to the latter paper in the sense that we use fractional models.

We present a procedure that has several distinguishing features compared with Porter-Hudak's (1990) method. First, we do not only concentrate on the seasonal frequencies but also on the long-run one. Thus, we simultaneously test for the presence of roots at the zero and the seasonal frequencies. The method used in this paper is parametric, has standard null and local limit distributions, and permits us to consider other approaches for testing unit and fractional seasonal roots, (e.g., Hylleberg et al., 1990, or its extension to monthly data in Beaulieu and Miron, 1993; Porter-Hudak, 1990; or the airline model) as particular cases to be tested under the null.

The Tests of Robinson (1994) in the Context of Seasonal Data

Let us assume that y_t is the time series we observe from t = 1, 2, … , T, and consider the following model:

$$(1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2}\,y_t \;=\; u_t, \qquad t \;=\; 1,2,... \tag{1}$$

for given real numbers d_1 and d_2, where L and L^{12} are respectively the lag and the seasonal (monthly) lag operators ($Lx_t = x_{t-1}$; $L^{12}x_t = x_{t-12}$), and where u_t is an I(0) process defined, for the purpose of the present paper, as a covariance stationary process with spectral density function that is positive and finite at any frequency on the interval $[0, \pi]$. We consider the null hypothesis:

$$H_o:\; \theta \;\equiv\; (\theta_1; \theta_2)' \;=\; (0,0)' \;\equiv\; 0, \tag{2}$$

against the alternative $\theta \neq 0$ in equation (1). Thus, under H_o (2), y_t in equation (1) follows a fractionally integrated process with orders of integration, d_1, corresponding to the long-run or zero frequency, and d_2, corresponding to the seasonal structure. This type of specification permits us to consider special cases of interest. For example, if $d_1 = 1$ and $d_2 = 0$, we test for a single unit root at the zero frequency (i.e., as in Dickey and Fuller, 1979; Phillips, 1987; etc.) and, if d_1 is a real number, a fractional model of the form advocated by Diebold and Rudebusch (1989) and others. Similarly, if $d_1 = 0$ and $d_2 = 1$, we test for seasonal unit roots (e.g., Beaulieu and Miron,

1993) and allowing d_2 to be a real number, we test for seasonal fractional models as in Gil-Alana (1999). We simultaneously can consider, however, the two polynomials and, if $d_1 = 1$ and $d_2 = 1$, we are in the classical multiplicative airline model.

Robinson (1994) proposed a Lagrange Multiplier (LM) method for testing H_o (2) against H_a: $\theta \neq 0$ in (1). Specifically, the test statistic is given by:

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a}, \tag{3}$$

where

$$\hat{a} = (\hat{a}_1; \hat{a}_2)', \qquad \hat{a}_i = \frac{-2\pi}{T} \sum_j^* \psi_i(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j);$$

$\hat{A}$ is a (2×2) matrix with $(r,s)^{th}$ element:

$$\hat{a}_{rs} = \frac{2}{T}\left(\sum_j^* \psi_r(\lambda_j)\psi_s(\lambda_j) - \sum_j^* \psi_r(\lambda_j)\hat{\varepsilon}(\lambda_j)' x \left(\sum_j^* \hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)' \right)^{-1} x \sum_j^* \hat{\varepsilon}(\lambda_j)\psi_s(\lambda_j) \right)$$

$$\psi_1(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|;$$

$$\psi_2(\lambda_j) = \psi_1(\lambda_j) + \log\left(2\cos\frac{\lambda_j}{2}\right) + \log\left|2\cos\lambda_j\right| + \log\left|2\left(\cos\lambda_j - \cos\frac{\pi}{3}\right)\right| +$$

$$\log\left|2\left(\cos\lambda - \cos\frac{2\pi}{3}\right)\right| + \log\left|2\left(\cos\lambda - \cos\frac{\pi}{6}\right)\right| + \log\left|2\left(\cos\lambda - \cos\frac{5\pi}{6}\right)\right|;$$

$$\hat{\sigma}^2 = \frac{2\pi}{T}\sum_j^* g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau}\log g(\lambda_j; \hat{\tau}); \qquad \lambda_j = \frac{2\pi j}{T}.$$

The function g above is known and is obtained from the spectral density function of u_t,

$$f(\lambda; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau)$$

evaluated at $\hat{\tau} = \arg\min_{\tau \in T} \sigma^2(\tau)$. Note that these tests are purely parametric and, therefore, they require specific modelling assumptions to be made regarding the short memory specification of u_t. Thus, if u_t is white noise, then, $g \equiv 1$, and if u_t is an AR process of form $\phi(L)u_t = \varepsilon_t$, $g = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ. $I(\lambda_j)$ is the periodogram of $\hat{u}_t$, where

$$\hat{u}_t = (1 - L)^{d_1}(1 - L^{12})^{d_2} y_t,$$

and the summation on * in the above expressions are over $\lambda \in M$ where $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_l - \lambda_1, \rho_l + \lambda_1), l = 1, 2, \ldots, s\}$ such that ρ_l, $l = 1, 2, \ldots, s < \infty$ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$.

Based on the null hypothesis (2), Robinson (1994) showed that, under certain regularity conditions,[1]

$$\hat{R} \to_d \chi_2^2 \quad \text{as} \quad T \to \infty, \tag{4}$$

and also, the Pitman efficiency property of the tests, that against local alternatives of form: H_a: $\theta = \delta T^{-1/2}$ for $\delta \neq 0$, $\hat{R}$ has an asymptotic distribution given by $\chi_2^2(\nu)$, with a non-centrality parameter, ν, that is optimal under Gaussianity of u_t.

Other versions of the tests of Robinson (1994) that study separately the orders of integration at each of the components of the series have been applied in a number of papers. Thus, for example, Gil-Alana and Robinson (1997) consider the case of roots occurring at the long-run or zero frequency in several US macroeconomic historical annual time series. Similarly, Gil-Alana (1999) and Gil-Alana and Robinson (2001) look at the cases of unit (and fractional) roots with monthly and quarterly data. Here, we consider another version of Robinson's (1994) testing procedure that allows us to simultaneously test the orders of integration at each of these components. Given the lack of empirical work in this direction, an empirical study based on fractionally-seasonally-based tests seems overdue.

A Finite-Sample Experiment

This section examines the finite-sample behaviour of the version of the tests of Robinson (1994) described previously by means of Monte Carlo simulations. We generate Gaussian series generated by the routines GASDEV and RAN3 of Press et al. (1986), with 10,000 replications of each case.

The null model consists of the classical multiplicative airline model,

$$(1 - L)(1 - L^{12})y_t = u_t, \quad t = 1, 2, \ldots, \tag{5}$$

with u_t correctly assumed to be white noise. The alternatives are, in Table 1, fractional of form as in equation (1), with d_1 and $d_2 = 1$, and θ_1, θ_2 = -0.50, (0.25), 0.50. In Table 2, the alternatives are autoregressive (AR) of the form:

$$(1 - \rho_1 L)\,(1 - \rho_2 L^{12})\,x_t = u_t, \tag{6}$$

with ρ_1, ρ_2 = 0.50, (0.25), 1.50. Thus, the rejection frequencies corresponding to $\theta_1 = \theta_2 = 0$ in equation (1), (with d_1, $d_2 = 1$), or $\rho_1 = \rho_2 = 1$ in equation (6) will indicate the sizes of the tests. The nominal sizes are 5 percent and 1 percent and the sample sizes are 120, 240, and 360.

---

[1] These conditions are mild regarding technical assumptions that are satisfied by model (1).

Table 1—Rejection Frequencies of $\hat{R}$ in Equation (3) Against Fractional Alternatives

$$H_o: \theta = (\theta_1; \theta_2)' = 0 \text{ in } (1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2} y_t = u_t, \quad t = 1,2,...$$

$$H_1: \theta = (\theta_1; \theta_2)' \neq 0 \text{ in } (1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2} y_t = u_t, \quad t = 1,2,...$$

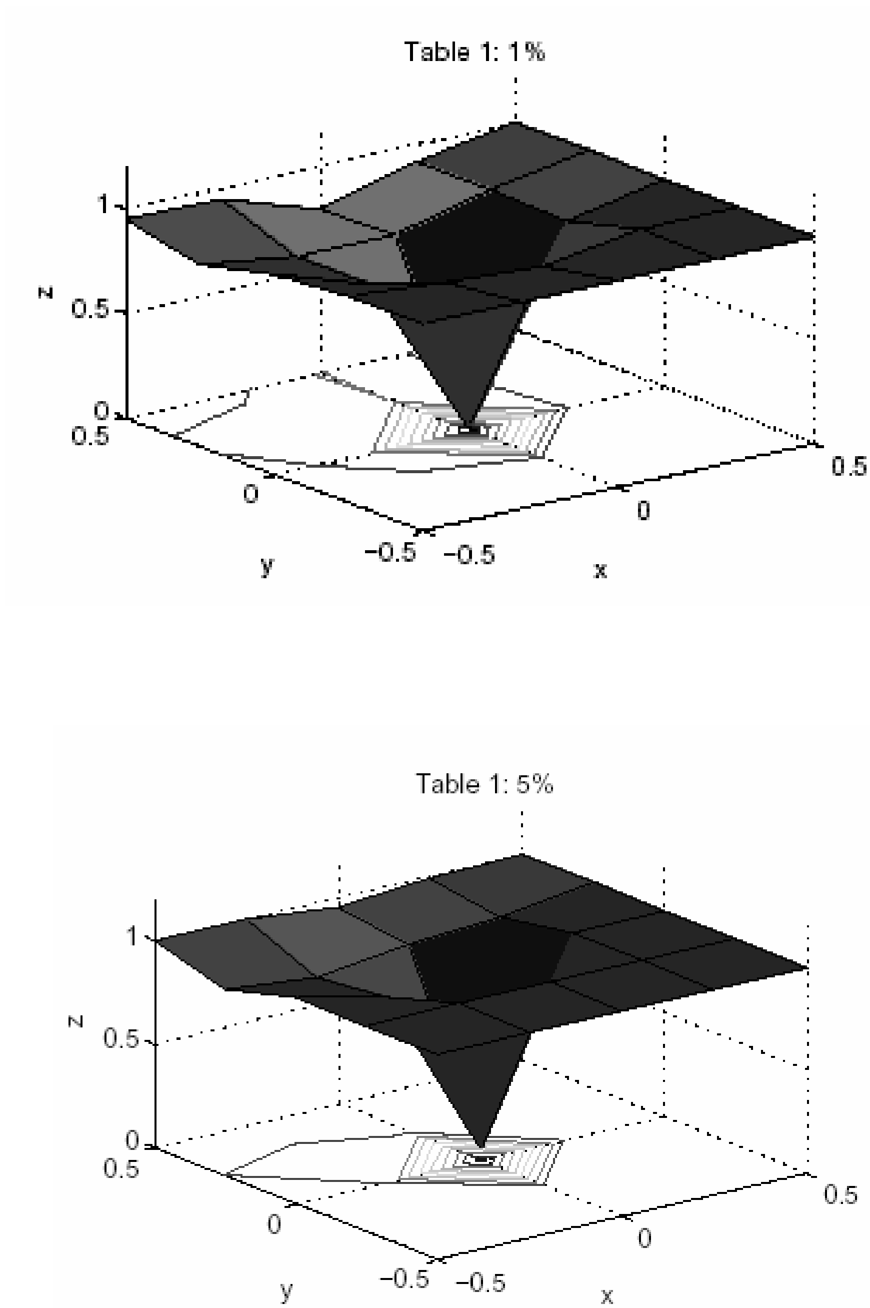
		T = 120		T = 240		T = 360	
θ_1	θ_2	5%	1%	5%	1%	5%	1%
-0.50	-0.50	0.998	0.922	1.000	0.999	1.000	1.000
-0.50	-0.25	0.952	0.543	0.999	0.927	1.000	0.999
-0.50	0.00	0.553	0.066	0.887	0.166	1.000	0.966
-0.50	0.25	0.051	0.001	0.682	0.173	0.897	0.873
-0.50	0.50	0.006	0.001	0.889	0.554	1.000	0.954
-0.25	-0.50	0.992	0.815	0.999	0.995	1.000	0.999
-0.25	-0.25	0.836	0.297	0.961	0.555	1.000	0.955
-0.25	0.00	0.247	0.011	0.196	0.106	0.896	0.806
-0.25	0.25	0.004	0.002	0.054	0.102	0.854	0.802
-0.25	0.50	0.001	0.002	0.597	0.230	0.997	0.943
0.00	-0.50	0.987	0.732	0.999	0.979	1.000	0.999
0.00	-0.25	0.724	0.176	0.807	0.227	1.000	0.931
0.00	0.00	**0.115**	**0.030**	**0.076**	**0.016**	**0.054**	**0.011**
0.00	0.25	0.006	0.001	0.109	0.078	0.911	0.801
0.00	0.50	0.002	0.002	0.210	0.049	0.950	0.793
0.25	-0.50	0.997	0.927	1.000	0.999	1.000	1.000
0.25	-0.25	0.947	0.690	0.996	0.942	1.000	1.000
0.25	0.00	0.690	0.368	0.888	0.702	1.000	0.899
0.25	0.25	0.387	0.207	0.775	0.626	0.934	0.912
0.25	0.50	0.283	0.176	0.823	0.700	0.989	0.909
0.50	-0.50	1.000	0.998	1.000	1.000	1.000	1.000
0.50	-0.25	0.999	0.993	1.000	1.000	1.000	1.000
0.50	0.00	0.995	0.980	1.000	1.000	1.000	1.000
0.50	0.25	0.980	0.941	0.999	0.999	1.000	1.000
0.50	0.50	0.936	0.883	0.999	0.997	1.000	1.000

In bold: Non-rejection values of the null hypothesis at the 95 percent significance level

Across Tables 1 and 2 we report the rejection probabilities corresponding to the test statistic given by $\hat{R}$ in equation (3). We observe that the sizes are too large in all cases, although as we increase the sample size they tend to improve and approximate to their nominal values. Thus, for example, if α = 5 percent, the size is 11.5 percent with T = 120. Increasing T to 240 observations, however, it becomes 7.6 percent. If T = 360, it is 5.4 percent. Starting with the fractional alternatives (in Table 1), we see that if T = 120, the rejection probabilities are low in some cases (e.g., $\theta_1 = -0.25$ with θ_2 = 0.25, 0.50, or $\theta_1 = 0$ with θ_2 = 0.25, 0.50). Increasing T, however, they also increase. If T = 360, the rejection frequencies are higher than 0.9 in practically all cases.

Table 2 displays results of the same experiment but based on AR alternatives. As we should expect, the rejection probabilities are now lower than in the previous case, given that the tests of Robinson (1994) are specifically designed against fractional-type alternatives. Increasing the sample size, the rejection probabilities also increase. They are relatively large if T = 360. Moreover, the efficiency property of

Figure 1—3-D Plots for the Rejection Frequencies across Table 1



the tests seems to assert itself in view of the large values in the rejection probabilities in case of fractional departures close to the null, if T = 360. We also display, at the end of the paper, 3-D plots of the two tables for a shorter grid of (θ_1, θ_2)-values, with T = 360. In particular, we try with θ_1 and θ_2 = -0.50, -0.40, … , 0.40 and 0.50, and the results were qualitatively the same as those presented across Tables 1 and 2. (See Figures 1 and 2 with a 3-D plots for the rejection frequencies across Tables 1 and 2.)

Table 2—Rejection Frequencies of $\hat{R}$ in Equation (3) Against Autoregressive Alternatives

$$H_o: \theta = (\theta_1; \theta_2)' = 0 \text{ in } (1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2} y_t = u_t, \quad t = 1,2,...$$

$$H_1: \theta = (\theta_1; \theta_2)' \neq 0 \text{ in } (1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2} y_t = u_t, \quad t = 1,2,...$$

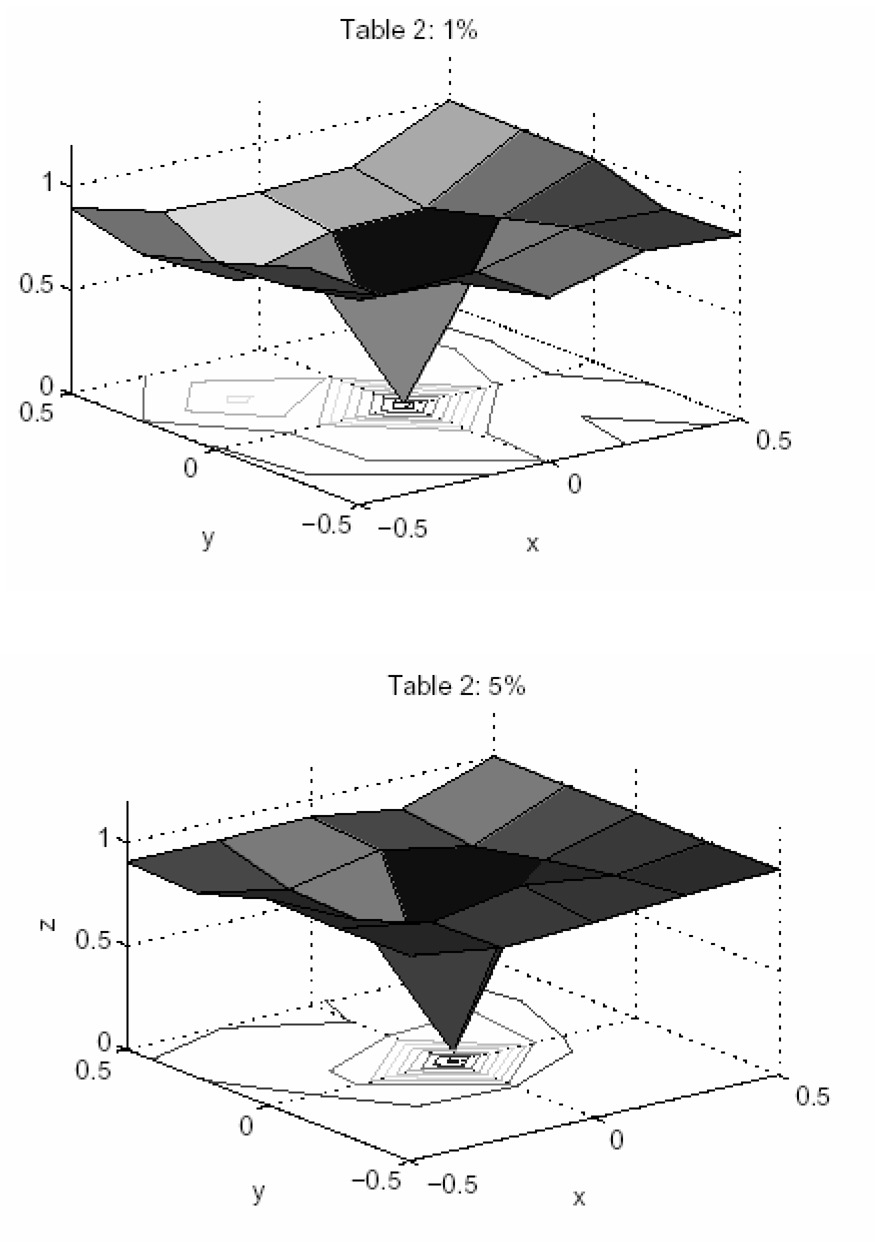
		T = 120		T = 240		T = 360	
ρ_1	ρ_2	5%	1%	5%	1%	5%	1%
-0.50	-0.50	0.983	0.687	0.999	0.987	1.000	0.999
-0.50	-0.25	0.774	0.221	0.991	0.871	1.000	0.928
-0.50	0.00	0.421	0.032	0.813	0.154	0.998	0.909
-0.50	0.25	0.007	0.001	0.654	0.158	0.887	0.802
-0.50	0.50	0.048	0.001	0.881	0.522	0.908	0.897
-0.25	-0.50	0.940	0.501	0.965	0.980	0.934	0.983
-0.25	-0.25	0.573	0.095	0.903	0.531	0.922	0.776
-0.25	0.00	0.199	0.007	0.191	0.099	0.795	0.771
-0.25	0.25	0.001	0.002	0.016	0.084	0.807	0.577
-0.25	0.50	0.021	0.002	0.590	0.177	0.906	0.775
0.00	-0.50	0.908	0.412	0.991	0.932	0.955	0.801
0.00	-0.25	0.454	0.051	0.754	0.201	0.997	0.789
0.00	0.00	**0.115**	**0.030**	**0.076**	**0.016**	**0.054**	**0.011**
0.00	0.25	0.001	0.001	0.093	0.071	0.891	0.707
0.00	0.50	0.029	0.011	0.160	0.036	0.912	0.766
0.25	-0.50	1.000	1.000	0.954	0.904	0.977	0.923
0.25	-0.25	1.000	1.000	0.916	0.900	0.934	0.896
0.25	0.00	1.000	1.000	0.834	0.755	0.877	0.808
0.25	0.25	1.000	1.000	0.711	0.612	0.799	0.714
0.25	0.50	1.000	1.000	0.799	0.703	0.845	0.776
0.50	-0.50	1.000	1.000	0.964	0.879	0.993	0.890
0.50	-0.25	1.000	1.000	0.966	0.960	0.990	0.873
0.50	0.00	1.000	1.000	0.966	0.963	0.997	0.991
0.50	0.25	1.000	1.000	0.968	0.965	0.991	0.993
050	0.50	1.000	1.000	0.969	0.967	0.996	0.994

In bold: Non-rejection values of the null hypothesis at the 95 percent significance level

An Empirical Application

The data for the monetary aggregates have been obtained from the US Federal Reserve Board, Monthly Historical Money Stock Tables and cover the period 1959m1-2002m4. M1, M2, and M3 are not seasonally adjusted (billions of dollars). We used the following definitions of monetary aggregates:

Figure 2—3-D Plots for the Rejection Frequencies across Table 2



M1 = Currency + travelers checks + demand deposits + checkable deposits,
M2 = M1+ small time deposits + money market demand deposit accounts + retail money market accounts + overnight repurchase agreements + overnight Eurodollars,
M3 = M2 + large time deposits + term-repurchase agreements + term Eurodollars + institutional money market funds.[2]

The nominal interest rate is the rate on three-month T-bills, covering the time period 1941m1-2002m6. It has been obtained from the Board of Governors.[3]

Figure 3 contains plots of the original series along with their corresponding correlograms and periodograms. We see that all of them have a nonstationary appearance; this is substantiated by the correlograms (with values decaying very slowly) and the periodograms (with large values around the smallest frequency).

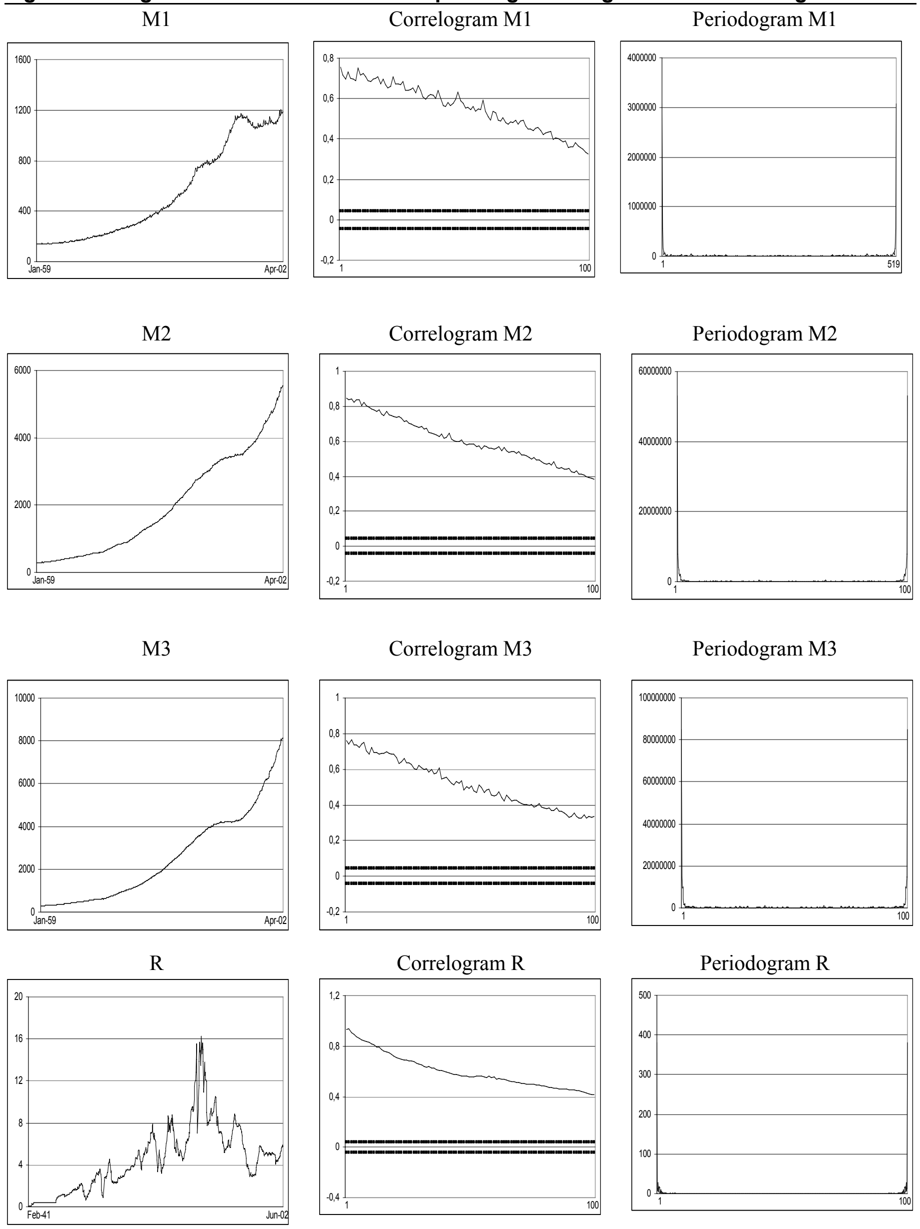
Figures 4 and 5 contain similar plots to Figure 3, but are based on the first differenced data (in Figure 4) and on first and monthly first differences (in Figure 5). The plots of the first differences clearly show the existence of a seasonal pattern. Taking also into account seasonal differences, the series might now be overdifferenced. Therefore, it might be of interest to deeper examine the degrees of integration of both the zero and the seasonal frequencies.

Denoting any of the series by y_t, we employ throughout the model in equation (1), testing H_o (2) for values d_1, d_2 = 0.50, (0.25), 1.50, thus also including tests for a unit root exclusively at the long-run or zero frequency ($d_1 = 1$, $d_2 = 0$); tests for seasonal unit roots ($d_1 = 0$, $d_2 = 1$); unit and seasonal unit roots ($d_1 = d_2 = 1$); as well as other fractionally integrated possibilities. We initially assume that u_t is white noise, (Table 3), though we also include weakly parametrically autocorrelated disturbances. In particular, we also employ AR(1) and seasonally monthly AR(1) processes for u_t, in Tables 4 and 5, respectively.

The test statistic reported across Tables 3 through 5 is the one given by $\hat{R}$ in equation (3). Instead of presenting the results for all series and all values of d_1 and d_2, we report those cases where we find at least one non-rejection value for each series. Starting with the case of white noise disturbances (Table 3), we observe that for M1, there are five non-rejection values corresponding to (d_1, d_2) = (1, 0), (1, 0.25), (1.25, 0.25), (1.50, 0.25), and (1.75, 0.25). Although there are some quantitative differences across the different models, they are consistent with the values of d_1 and d_2. The procedure employed in this paper does not directly estimate the fractional differencing parameters, but simply computes diagnostic departures from the null. The null hypothesis is always rejected for values of d_1 smaller than 1 and for values of d_2

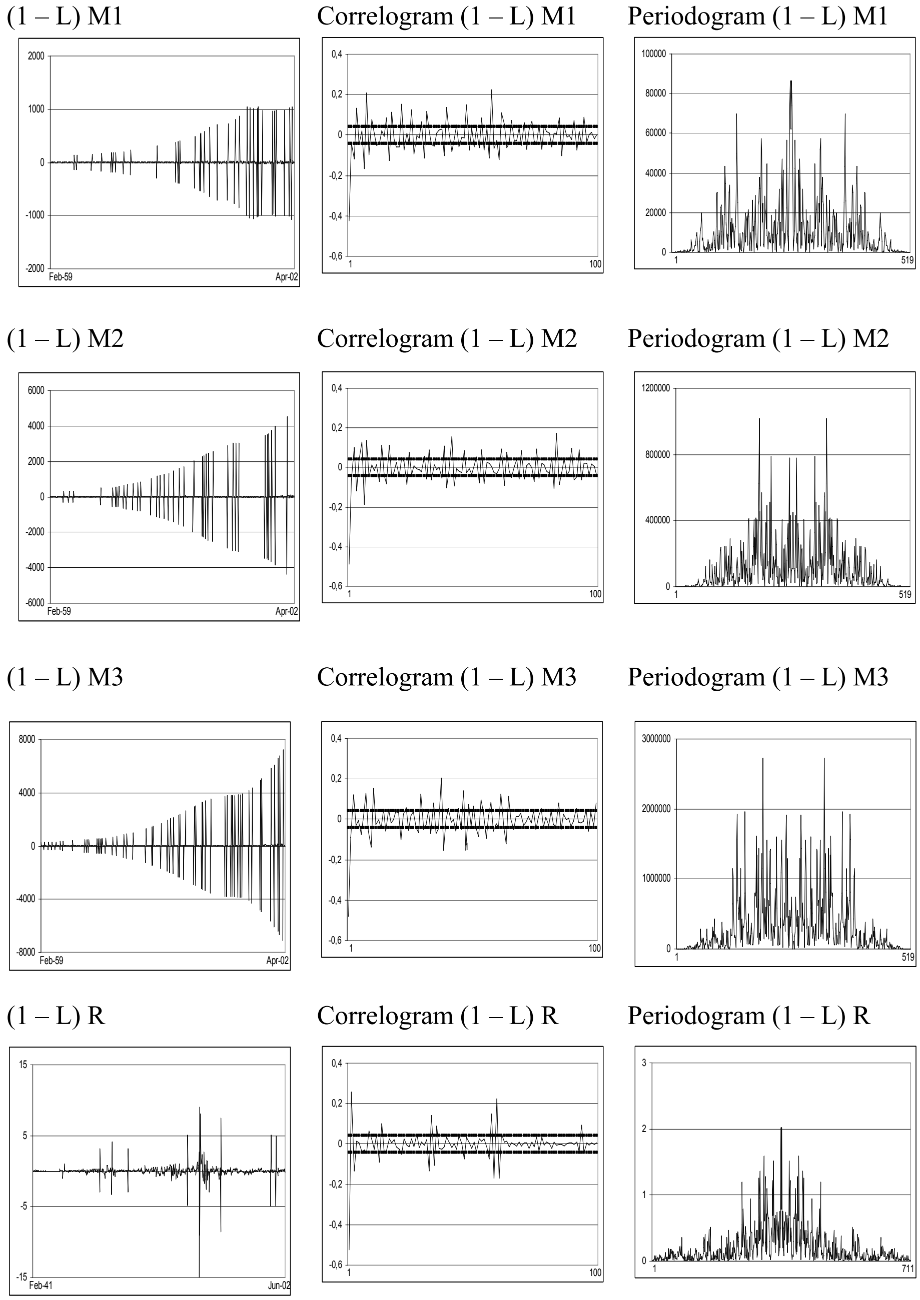
[2] We produce results based on the same definitions of monetary aggregates as in Porter-Hudak (1990).
[3] Here we produce results based on the same definition of interest rate as in Beaulieu and Miron (1993).

Figure 3—Original Series with their Corresponding Correlograms and Periodograms



The large sample standard errors under the null hypothesis of no autocorrelation is 1/√T or roughly 0.043 for the monetary aggregates and 0.037 for the interest rate

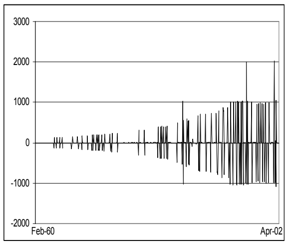
Figure 4—First Differenced Series with Corresponding Correlograms and Periodograms



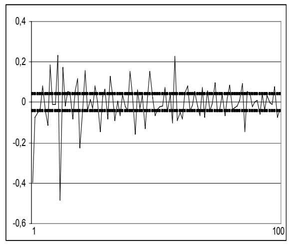
The large sample standard errors under the null hypothesis of no autocorrelation is 1/√T or roughly 0.043 for the monetary aggregates and 0.037 for the interest rate

Figure 5—First and Monthly Differenced Series with Correlograms and Periodograms

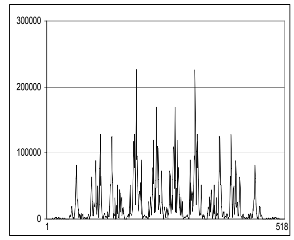
$(1 - L)(1 - L^{12})$ M1



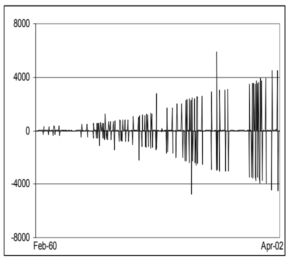
Correlogram $(1–L)(1–L^{12})$ M1



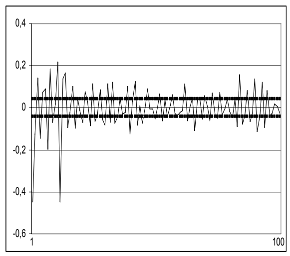
Periodogram $(1–L)(1–L^{12})$ M1



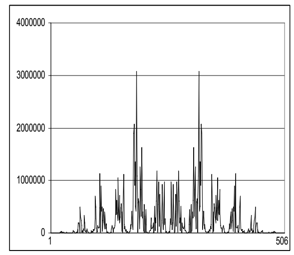
$(1 - L)(1 - L^{12})$ M2



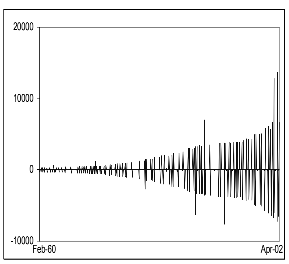
Correlogram $(1–L)(1–L^{12})$ M2



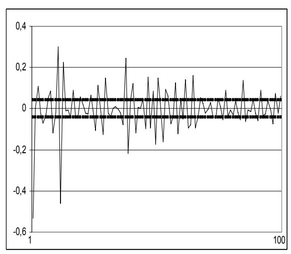
Periodogram $(1–L)(1–L^{12})$ M2



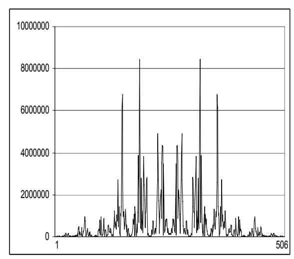
$(1 - L)(1 - L^{12})$ M3



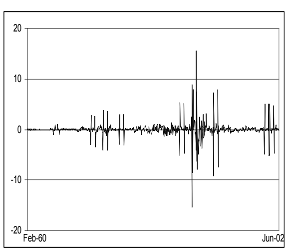
Correlogram $(1–L)(1–L^{12})$ M3



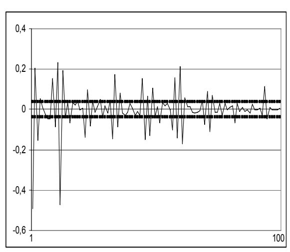
Periodogram $(1–L)(1–L^{12})$ M3



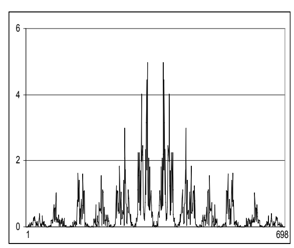
$(1 - L)(1 - L^{12})$ R



Correlogram $(1–L)(1–L^{12})$ R



Periodogram $(1–L)(1–L^{12})$ R



The large sample standard errors under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.043 for the monetary aggregates and 0.037 for the interest rate

Table 3—Testing H_o (2) in Equation (1) with White Noise Disturbances

$$H_o: \theta = (\theta_1; \theta_2)' = 0 \text{ in } (1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2} y_t = u_t, \quad t = 1, 2, \ldots$$

$$H_1: \theta = (\theta_1; \theta_2)' \neq 0 \text{ in } (1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2} y_t = u_t, \quad t = 1, 2, \ldots$$

d_1	d_2	M1	M2	M3	R
1.00	0.00	**5.630**	28.441	179.562	6.594
1.00	0.25	**3.772**	14.037	83.825	19.938
1.25	0.00	9.942	6.812	11.140	**3.685**
1.25	0.25	**1.863**	**0.635**	**3.171**	17.924
1.25	0.50	10.678	8.818	7.610	30.299
1.50	0.00	14.454	11.724	**3.595**	6.782
1.50	0.25	**3.041**	**1.960**	**1.993**	21.920
1.50	0.50	10.090	8.722	9.958	33.941
1.75	0.00	18.315	16.479	6.897	12.069
1.75	0.25	**4.580**	**4.022**	**5.120**	27.414
1.75	0.50	10.164	9.100	12.641	38.732
2.00	0.25	6.456	6.426	8.244	32.230
2.00	0.50	10.838	9.864	14.914	42.813

In bold: Non-rejection values of the null hypothesis at the 95 percent significance level

higher than 0.25. Moreover, in all cases the order of integration at the long-run or zero frequency appears higher than the one corresponding to the seasonal frequencies. If we concentrate on the other monetary aggregates (M2 and M3), the non-rejection values are (1.25, 0.25), (1.50, 0.25), and (1.75, 0.25) for both aggregates along with (1.50, 0) for M3. Thus, the order of integration at the long-run frequency seems to be higher than 1, while the degree of persistence of the seasonal component is around 0.25 in practically all cases. Finally, the results for the US interest rate show a single non-rejection case corresponding to $(d_1, d_2) = (1.25, 0)$. In conclusion, the long-run or zero frequency seems to play a much more important role than the seasonal ones, at least for white noise disturbances. While the trending component appears nonstationary, the seasonal structure is stationary but with a long memory behaviour.

The significance of the above results may be due to the un-accounted for I(0) autocorrelation in u_t. In Tables 4 and 5, we allow for a weakly autocorrelated structure on the disturbances. In Table 4, we assume that u_t follows an AR(1) process. Table 5 reports results based on seasonal (monthly) AR(1) disturbances of the form:

$$u_t = \tau u_{t-12} + \varepsilon_t, \quad t = 1, 2, \ldots..$$

Higher seasonal and non-seasonal AR orders also were tried, and the results are similar to those reported across these two tables. Starting with the non-seasonal case, (Table 4), we observe few non-rejection cases; all of them form a proper subset of the non-rejection values obtained in Table 3. For M1, the values are (1, 0), (1.25, 0.25), and (1.50, 0.25). The last two values are the only non-rejections for M2.

For M3, H_o cannot be rejected for (1.50, 0) and (1.50, 0.25). For the interest rate, the null is rejected in all cases at the 95 percent significance level, though (d_1, d_2) = (1.25, 0) is not rejected at the 90 percent level. Similarly for the seasonal case (Table 5), few non-rejection values are obtained: (1, 0) for M1; (1.25, 0.25) for M1 and M2; (1.50, 0) for M3; and (1.50, 0.25) for M2 and M3. Similar to the case of white noise disturbances, the order of integration at the long-run or zero frequency plays a much more important role than the seasonal frequencies; the order of integration at zero is 1 or higher than 1, while it is 0 or 0.25 for the seasonal monthly structure.

Table 4—Testing H_o (2) in Equation (1) with AR(1) Disturbances

$$H_o: \theta = (\theta_1; \theta_2)' = 0 \textbf{ in } (1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2} y_t = u_t, \quad t = 1,2,...$$
$$H_1: \theta = (\theta_1; \theta_2)' \neq 0 \textbf{ in } (1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2} y_t = u_t, \quad t = 1,2,...$$

d_1	d_2	M1	M2	M3	R
1.00	0.00	**5.779**	68.404	1704.333	24.734
1.00	0.25	7.763	51.858	609.847	37.510
1.25	0.00	14.781	7.914	20.597	6.505
1.25	0.25	**2.247**	**0.652**	8.903	20.591
1.25	0.50	10.698	8.892	8.528	32.223
1.50	0.00	211.865	16.789	**4.675**	17.159
1.50	0.25	**5.176**	**3.549**	**2.595**	29.787
1.50	0.50	11.395	9.892	11.381	40.489
1.75	0.25	8.088	7.384	8.154	38.304

In bold: Non-rejection values of the null hypothesis at the 95 percent significance level

Table 5—Testing H_o (2) in Equation (1) with Seasonally Monthly AR(1) Disturbances

$$H_o: \theta = (\theta_1; \theta_2)' = 0 \textbf{ in } (1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2} y_t = u_t, \quad t = 1,2,...$$
$$H_1: \theta = (\theta_1; \theta_2)' \neq 0 \textbf{ in } (1-L)^{d_1+\theta_1}(1-L^{12})^{d_2+\theta_2} y_t = u_t, \quad t = 1,2,...$$

d_1	d_2	M1	M2	M3	R
1.00	0.00	**5.750**	37.971	256.989	14.309
1.00	0.25	7.578	40.786	218.948	26.069
1.25	0.00	11.998	7.614	15.265	6.180
1.25	0.25	**2.407**	**0.658**	8.684	20.304
1.25	0.50	10.682	8.898	8.214	31.904
1.50	0.00	17.650	15.168	**4.598**	16.358
1.50	0.25	6.673	**4.712**	**2.896**	27.175
1.50	0.50	11.588	9.923	11.040	37.235

In bold: Non-rejection values of the null hypothesis at the 95 percent significance level

Concluding Comments

In this article we have examined the monthly structure of several US monetary aggregates along with the interest rate. Instead of using classical approaches based on unit-root or seasonal unit-root tests (e.g., Dickey and Fuller, 1979; Phillips, 1987; Kwiatkowski et al., 1992; Hylleberg et al., 1990; Beaulieu and Miron, 1993; etc.), we have used a methodology based on fractional integration. Seasonal fractional models have been empirically studied in Porter-Hudak (1990), Gil-Alana and Robinson (2001), etc., but these studies concentrate on the seasonal structure, not other components of the series. Here, we have used a version of a procedure of Robinson (1994) that permits us to simultaneously test the orders of integration at both, the zero and the seasonal frequencies. The tests have standard null and local limit distributions, and several Monte Carlo experiments conducted across the paper show that the tests perform relatively well even with small samples. The results based on the monetary aggregates show that the root at the long run or zero frequency plays a much more important role than the one corresponding to the seasonal frequencies. The order of integration at zero seems to be equal to or higher than 1, while the one at the seasonal component is around 0.25. For the interest rate, however, the seasonal root does not seem to be important, and only the one at zero (with d_1 around 1.25) should be considered.

These results are in line with those of Beaulieu and Miron (1993) who reject unit roots at most frequencies in a number of macroeconomic series in the US (including interest rate and monetary aggregates), although they find evidence of unit root at frequency zero. The results are also consistent with the paper by Porter-Hudak (1990), who compares the fractionally differenced model with the airline model for characterizing the monetary aggregates M1, M2, and M3 and find evidence in favor of the first model. All this evidence calls into question the practice of seasonal (integer) differencing when analyzing seasonal macroeconomic time series.

References

1. Arteche, J., and P.M. Robinson, "Inference in Seasonal and Cyclical Long Memory Processes," *Journal of Time Series Analysis*, 21 (2000), pp. 1-25.

2. Barsky, R.B., and J.A. Miron, "The Seasonal Cycle and the Business Cycle," *Journal of Political Economy*, 97 (1989), pp. 503-534.

3. Beaulieu, J.J. and J.A. Miron, "A Cross Country Comparison of Seasonal Cycles and Business Cycles," *The Economic Journal*, 102 (1992), pp. 702-788.

4. Beaulieu, J.J. and J.A. Miron, "Seasonal Unit Roots in Aggregate U.S. Data," *Journal of Econometrics*, 55 (1993), pp. 305-328.

5. Bohl, M.T., "Nonstationary Stochastic Seasonality and the German M2 Money Demand Function," *European Economic Review*, 44 (2000), pp. 61-70.

6. Braun, R.A., and C.L. Evans, "Seasonality and Equilibrium Business Cycle Theories," *Journal of Economics, Dynamics and Control*, 19 (1995), pp. 503-531.

7. Burns, A.F., and W.C. Mitchell, *Measuring Business Cycles* (New York: National Bureau of Economic Research, 1946).

8. Dickey, D.A., and W.A. Fuller, "Distributions for the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association*, 74 (1979), pp. 427-431.

9. Diebold, F.X., and G. Rudebusch, "Long Memory and Persistence in Aggregate Output," *Journal of Monetary Economics*, 24 (1989), pp. 189-209.

10. Faig, M., "Seasonal Fluctuations and the Demand for Money," *Quarterly Journal of Economics*, 99 (1989), pp. 857-861.

11. Franses, P.H., and M. Ooms, “A Periodic Long-Memory Model for Quarterly UK Inflation,” *International Journal of Forecasting*, 13 (1997), pp. 117-126.

12. Gil-Alana, L.A., “Testing Fractional Integration with Monthly Data,” *Economic Modelling*, 16 (1999), pp. 613-629.

13. Gil-Alana, L.A., “Seasonal Long Memory in the Aggregate Output,” *Economics Letters*, 74 (2002), pp. 333-337.

14. Gil-Alana, L.A., and P.M. Robinson, “Testing of Unit Roots and Other Nonstationary Hypotheses in Macroeconomic Time Series,” *Journal of Econometrics*, 80 (1997), pp. 241-268.

15. Gil-Alana, L.A., and P.M. Robinson, “Testing Seasonal Fractional Integration in the UK and Japanese Consumption and Income,” *Journal of Applied Econometrics*, 16 (2001), pp. 95-114.

16. Granger, C.W.J., “Seasonality: Causation, Interpretation and Implication in Seasonal Analysis of Economic Time Series,” *Proceeding of the Conference of the Seasonal Analysis of Economic Time Series* (Washington, D.C.: US Department of Commerce, Bureau of Census, 1978).

17. Ghysels, E., “A Study toward a Dynamic Theory of Seasonality for Economic Time Series,” *Journal of the American Statistical Association*, 83 (1988), pp. 168-172.

18. Hylleberg, S., *Seasonality in Regression* (New York: Academic Press, 1986).

19. Hylleberg, S., *Modelling Seasonality, Advanced Texts in Econometrics* (Oxford: Oxford University Press, 1992).

20. Hylleberg, S., R.F. Engle, C.W.J. Granger and B.S. Yoo, “Seasonal Integration and Cointegration,” *Journal of Econometrics*, 44 (1990), pp. 215-238.

21. Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin, “Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root,” *Journal of Econometrics*, 54 (1992), pp. 159-178.

22. Lee, H.S., and P.L. Siklos, “The Role of Seasonality in Economic Time Series. Reinterpreting Money-Output Causality in U.S. Data,” *International Journal of Forecasting*, 13 (1997), pp. 381-391.

23. Moore, G., G.E.P. Box, B.K. Hyman, J.A. Stephenson and A. Zellner, *Seasonal Adjustment of the Monetary Aggregate* (Washington, D.C.: Federal Reserve System Board of Governors, 1981).

24. Porter-Hudak, S., “An Application of the Seasonal Fractionally Differenced Model to the Monetary Aggregates,” *Journal of the American Statistical Association*, 85 (1990), pp. 338-344.

25. Phillips, P.C.B., “Time Series Regression with a Unit Root,” *Econometrica*, 55 (1987), pp. 277-301.

26. Press, W.H., B.P. Flannery, S.A. Teukolsky and W.T. Wetterling, *Numerical Recipes: The Art of Scientific Computing* (Cambridge: Cambridge University Press, 1986).

27. Robinson, P.M., “Efficient Tests of Nonstationary Hypotheses,” *Journal of the American Statistical Association*, 89 (1994), pp. 1420-1437.

28. Thomas, J.J., and K.F. Wallis, “Seasonal Variation in Regression Analysis,” *Journal of the Royal Statistical Association*, Series A, 134 (1972), pp. 57-72.

Copyright of Quarterly Journal of Business & Economics is the property of College of Business Administration/Nebraska and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.