The Dynamic Interaction between Equity Prices and Supply Shocks

Jakob B Madsen*
University of Copenhagen

This paper develops a theory of medium term stock price movements under slow adjustment in the labor market relative to the stock market and perfect foresight in the stock market. The model seeks to explain the slow movements in real stock prices that have been observed in the industrialized countries in the post-war period. The empirical estimates indicate that changes in factor shares are important determinants of stock returns.

Introduction

Real stock prices have been fluctuating significantly around an upward trend on frequencies of approximately 25-years duration in most of the industrialized countries over the past century (Siegel, 2002; Shiller, 2001). Although no systematic investigation of forces that may have been responsible for these large swings have been undertaken, large swings in stock prices often have been associated with technological revolutions (Hobijn and Jovanovic, 2001; Madsen and Davis, 2006; Shiller, 2001).

In the models of Hobijn and Jovanovic (2001) and Madsen and Davis (2006), stock prices of incumbents are influenced adversely by a technological revolution due to creative destruction, whereas entrants experience no decrease in stock prices as they benefit from the higher productivity of the new technology. Due to a long time lag between the innovation and the entrance of new firms in the Hobijn and Jovanovic (2001) model, technological revolutions will result in large swings in aggregate stock prices. Madsen and Davis (2006) argue that technological revolutions tend to generate large swings in stock prices because the stock market focuses on the incorrect earnings indicators and uses the incorrect valuation model.

This paper argues that swings in the distribution of income between capital and labor, like technological revolutions, may be influential for the large low frequency

*Excellent research assistance from John Gould is gratefully acknowledged. Helpful comments and suggestions from seminar participants at University of Copenhagen, Odense University, University of Western Australia, and University of Konstantz, Nic Groenewold, and two referees are gratefully acknowledged.

0747-5535/06/1400 & 1500/0003/$2.50

swings in stock prices. The leading idea is that profits may be low over a prolonged period because wage-push factors (such as union militancy and supply shocks that bring real wages in excess of workers' marginal productivity) result in distribution of income in favor of labor and, consequently, depress stock markets. Conversely, stock markets are buoyant in periods of cooperation between unions and firms and when beneficial supply shocks bring labor's share of income below its long-run equilibrium.

A macroeconomic framework for analyzing the dynamic interaction between the stock and the labor markets is established. The model extends Blanchard's (1981) model of the interaction between stock prices and output by allowing for the influence of the supply side. The supply side plays the dual role of determining the distribution of income between wages and profits and of ensuring that profits are neutral to supply shocks in the long run.

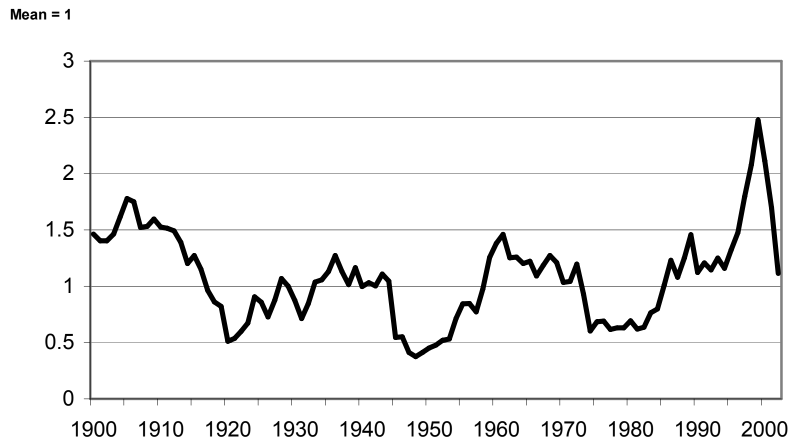
The supply side is incorporated into the model using the augmented Phillips curve framework. The model shows that supply shocks are influential for movements in earnings in the short run and in the medium term, and that demand shocks only play a role for fluctuations in stock prices on business cycle frequencies. The model incorporates into stock prices the empirical regularity that earnings per unit of capital are neutral to shocks in the long run. An adverse supply shock that pushes wages in excess of the full employment equilibrium simultaneously increases unemployment and squeezes the firm's cash flow. Higher unemployment reduces the wage growth rate, which in turn increases cash flow until the natural rate of unemployment is re-established. Due to sluggish adjustment in the labor market, the model embodies into stock prices that the reduction in earnings is temporary and therefore leads to a substantially smaller reduction in stock prices than would be predicted by the myopic Gordon growth model.

Low Frequency Movements in Stock Prices

Using data for 11 industrialized countries over the period from 1900 to 2002 this section shows informally how stock prices are influenced by capital's share of total income and the innovative activity. Here, capital's share in total income is defined as nominal GDP minus compensation to employees divided by GDP, and labor's share is compensation to employees divided by GDP. To preserve space, only the weighted averages of all eleven countries are shown. Formal testing with disaggregated data is undertaken in the empirical section below. The data sources are detailed in an appendix.

First consider the accumulated stock index, which embodies stock prices and dividends reinvested in the stock index, deflated by consumer prices, and denominated in domestic currency. The real accumulated index will be termed *stock prices*. The trends in the stock are removed by least squares regressions of the log of the weighted average of stock prices on a linear time trend. The stock market indexes have been chosen to have the broadest possible economic coverage. Countries are weighted by their USD GDP in 1950 at purchasing power parity.

Figure 1—Real Accumulated Index, Deviation from Trend



Notes. The following 11 countries are included in the index: Canada, the US, Japan, Australia, Denmark, France, Germany, Italy, the Netherlands, Sweden, and the UK. Accumulated stock prices are deflated with consumer prices, normalized to a mean of one for each country, and weighted by 1950 USD GDP in purchasing power parity units

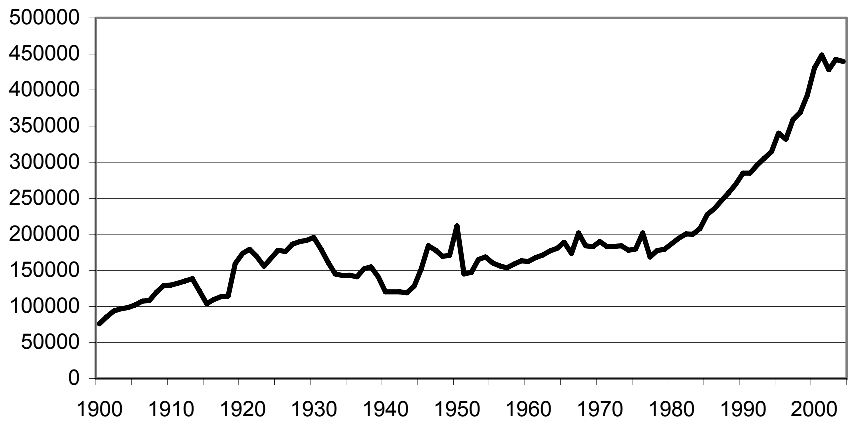
The figure shows that the stock market has undergone four distinct low-frequency swings during the last century that peaked around 1905, 1936, 1961, and 1999 and reached troughs in 1920, in the late 1940s, and in 1974. Although little attention has been given in the literature to explaining these large swings, a few economists have tried to explain low-frequency movements in stock prices in terms of technological revolutions. Hobijn and Jovanovic (2001) and Jovanovic and Rousseau (2002), for instance, attribute the stock market booms in the 1920s and the 1990s to the appearance of new companies associated with the technological revolutions in 1890-1930 and 1971-2001. The declining stock markets 1906-1920 and 1974-1983 prior to the stock market booms, conversely, according to Hobijn and Jovanovic (2001) and Jovanovic and Rousseau (2002), were caused by the creative destruction of the value of the incumbent's capital stock.

Similarly, Madsen and Davis (2006) and Perez (2002) relate the low-frequency fluctuations in stock prices to technological revolutions, but, in contrast to Hobijn and Jovanovic (2001) and Jovanovic and Rousseau (2002), argue that technology-induced stock markets booms often are triggered by unrealistic earnings expectations among investors and, consequently, result in declining stock markets when the earnings expectations are not met.

To give a visual impression of the history of the innovative activity, Figure 2 shows the number of patent applications by residents. The number of patent applications is a recognized measure of innovative activity, as argued by Griliches (1990), among others. Furthermore, patents are available over a much longer historical timespan than other measures of innovative activity such as R&D expenditures, which are first available from 1953 for the US and much later for other countries.

Figure 2 identifies two waves of high innovative activity: namely, the period from 1900 to the 1930s and the period from 1980 to 2000. The declining trend from 1930 was first reversed in the beginning of the 1980s. Allowing for the possibility that the post-WWI surge in the number of patents applied for was probably a catch-up from the low level of innovative activity during the war, the innovative activity was kept on a high level over the period from 1910 to 1930. In this period the deviation of stock prices from their trend was not consistently high, and the stock markets experienced a major setback over the period from 1907 to 1920 and only partly recovered during the 1920s. The recent surge in innovative activity appears to be associated with the stock market boom experienced in the 1980s and in the 1990s. These informal observations suggest that technological innovations may be of some importance for stock prices, but that, at the same time, most of the low-frequency fluctuations in stock prices remain unaccounted for by innovative activity.

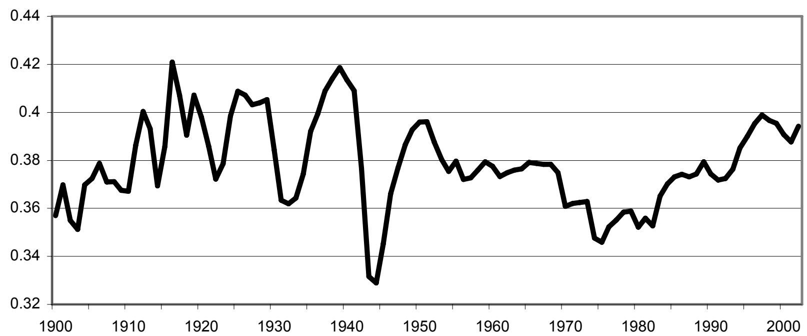
Figure 2—Patent Applications by Residents



Notes. The numbers are calculated as the sum of the number of patents applied for the 11 countries considered in Figure 1.

Turning to factor shares, Figure 3 shows the weighted share of total income going to capital in the eleven countries considered in this section, where 1950 GDP at purchasing power parities is used as a weight. The factor shares are adjusted by the wage income of the self-employed, as detailed in the data appendix, because all income of the self-employed is accounted for as capital income in national accounts, although remuneration for their labor input should be accounted for as compensation to labor. The figure shows that over the past century capital's share has fluctuated around a constant level, which is consistent with the predictions of the model in the next section and the neoclassical model of economic growth.

Figure 3—Share of Income Going to Capital



The share of income going to capital fluctuated substantially before the end of WWII. Major declines in capital's income share can be identified during WWI and its immediate aftermath, during the Great Depression, and at the end of WWII. The decline in capital's share around the world wars was associated with labor market unrest and increasing union membership (Goerke and Madsen, 2004). The decline in the share of income going to capital during the Depression was due to the combination of the largest world economic setback in modern history and pro-cyclicality of earnings, as discussed in Fama (1981) and Blanchard (1981). The setbacks in the share of income going to capital in these three periods were associated with poor performance in the stock market (Figure 1). On a low-frequency basis, however, there is no clear relationship between share of income going to capital and stock prices before WWII. The long decline in de-trended stock prices over the period from 1907 to 1920 and the subsequent increase cannot be clearly identified in the data for capital's income share.

Compared to the pre-WWII period there is a much closer relationship between the low-frequency movements in capital's income share and the deviation of stock prices from their trend in the postwar period. The relatively large share of income going to capital over the periods 1950-1970 and 1983-2002 are associated with strong stock markets over almost the same periods (namely, 1950-1973 and 1983-

2002). These periods are known to be periods of consensus in the labor market and, consequently, exhibit low strike activity (Goerke and Madsen, 2004).

Overall, the informal evidence in this section suggests that there is some relationship between innovative activity and stock prices in the postwar period, but that the relationship is relatively weak prior to WWII. The relationship between share of income going to capital and stock prices is tight in the postwar period, but not particularly strong prior to the end of WWII, which may reflect the poor quality of the income accounts data prior to WWII and that other factors were influential for stock prices in that period. The high inflation during, and in the immediate aftermath of, the world wars appears to have contributed to declining stock markets in these periods.

A Model of the Interaction between the Stock and the Labor Markets

In this section a model of the supply side is incorporated into a dynamic model of stock prices in order to examine how supply shocks and unemployment interact with the stock market. The idea underlying the model is that total income is distributed between labor and capital, which is determined in the labor market. Because the share of income going to capital is influential for earnings per share, it follows that stock prices are influenced by the conditions in the labor market. That labor's share of income is determined in the labor market has a long history in the macroeconomics of unemployment, in which labor's income share is positively related to the tightness of the labor market, supply shocks, and union activity among other variables. (See, for instance, Bruno and Sachs, 1985, and Blanchard, 1997.) The first part of this section incorporates factor shares into the optimization problem of the individual firm. The supply side, which determines factor shares, is subsequently incorporated into the model.

Consider the firm's objective function:

$$\max \Pi = (1-\tau) \int_{t=0}^{\infty} e^{-rt}[P_tF(K_t,L_t), - W_tL_t - P_tI_t - P_tC(I_t)]dt$$

st.

$$\dot{K}_t = I_t$$

where Π is nominal profits, F is aggregate value-added output, K is capital services, I is net investment, W is the wage rate, L is labor services, r is a constant required return to equity, C(I) is the adjustment cost of investment as a positive function of investment, τ is the corporate tax rate, and P is the value added price-deflator, which for simplicity is the same for investment and output. A dot over a variable indicates a time-derivative. The depreciation rate is set to zero for simplicity and does not affect the principal results.

The current-value Hamiltonian of this optimization problem is given by:

$$H = (1 - \tau)[P_tF(K_t,L_t), - W_tL_t - P_tI_t - P_tC(I_t)] + Q_t[I_t - \dot{K}_t], \tag{1}$$

where Q is the shadow price of capital, which is the incremental value of the firm it acquires and additional unit of capital. In other words, Q is the profits derived from one additional unit of capital discounted by the required returns to equity and, therefore, represents the value of a stock. The shadow price is often referred to as *Tobin's q*. Under some regularity conditions Hayashi (1982) has shown that that the Tobin's average and the marginal Q's are identical (the so-called Hayashi's theorem). Thus, if Hayashi's theorem is satisfied, it follows that share prices are identical to the price of new shares from which the proceeds are used to finance an additional unit of capital. In equation (1) the firm maximizes its after-tax cash-flow each period under the capital stock constraint. The cash-flow is given by turnover minus cost to labor, investment, and the costs associated with investment such as installation costs, costs of reorganization, and costs of changing business practices.

Equation (1) yields the first order conditions as follows, where the time-subscripts are omitted for simplicity:

$$(1 - \tau)MP_K(1 + 1/\eta) = rQ_t - \dot{Q}_t, \tag{2}$$

$MP_L(1 + 1/\eta) = W/P$

where η is the price elasticity of demand facing the firm, MP_K is the marginal productivity of capital, and MP_L is the marginal productivity of labor. Equation (2) says that earnings per stock are equal to the sum of dividends and capital gains.

From the condition of profit maximization, $\eta = P/(P - MC)$, where MC is marginal cost, equation (2) can be written as:

$$\dot{Q}_t = rQ_t - (1 - \tau)MP_K(2 - MC/P). \tag{3}$$

To find an exact expression for the marginal productivity of capital and marginal costs, the following Cobb-Douglas production function is assumed:

$Y_t = AL_t^{\alpha}K_t^{\beta}$

where Y is value added output, A is a constant, L is labor inputs, K is capital stock, and α and β are output elasticities of labor and capital. The marginal productivity of capital equals $\beta(Y/K)$. Furthermore, the marginal costs can be further decomposed into

$$MC_t = W_tMP_{t,L}^{-1} = W_t\alpha^{-1}(L/Y)_t,$$

where W is the wage rate and MP_L is the marginal productivity of labor. Substituting into equation (3) yields:

$$\dot{Q}_t = rQ_t - (1-\tau)\beta\left(\frac{Y_t}{K_t}\right)(2-\alpha^{-1}S_{L,t})\ . \tag{4}$$

where S^L is the share of income going to labor, $S_L = W \cdot L/Y \cdot P$. This equation applies for the representative individual firm and therefore also holds on an aggregated level. Given that no distinction is made between expected and unexpected values, demand shocks do not play any role in equation (4). This restriction is relaxed below.

Rewriting equation (4) shows that the expected stock returns equal to capital gain per share plus earnings per shares:

$$r_t = \frac{\dot{Q}_t}{Q_t} + \frac{(1-\tau)\beta(Y_t / K_t)(2-\alpha^{-1}S_{/k,t})}{Q_t}\ .$$

Earnings, in turn, depend on the corporate tax rate, the marginal productivity of capital, and the share of income going to labor. Factor shares are the most influential determinants of movements in earnings because Y/K is relatively constant over time, as shown below.

To identify which factors determine the share of income going to labor, consider the following standard augmented Phillips curve:

$$\Delta \ln W_t = \beta_1\, \pi_t^{va} + \beta_2\, \Delta \ln MP_{L,t} - \beta_3(U_t - \phi_t) + \Delta \ln Z_t v', \tag{5}$$

where π^{va} is the expected inflation rate measured by the value added price-deflator, MP_L is the expected marginal productivity of labor, U is the rate of unemployment, ϕ is the natural rate of unemployment, Z is a vector of wage push variables, β_1-β_3 are constants, and ν is a vector of constants.[1] The natural rate of unemployment is the benchmark rate of unemployment below which inflation starts increasing. The Z-variables consist of direct and indirect taxes, relative commodity prices, relative food prices, union wage pushiness, unemployment benefits, and other wage push factors. (See, for instance, Madsen, 1998.)

[1] Blanchflower and Oswald (1994) argue that the labor market obeys the wage equation and not the Phillips curve. Blanchard and Katz (1999), among others, argue that the Phillips curve applies for some labor markets and the wage curve for others. The wage curve-Phillips curve controversy has not been resolved. Incorporating the wage curve into the framework here yields the same principal results as the ones based on the Phillips curve that the income share of labor is a mean-reverting process if Z is a mean-reverting process. Because there is strong evidence that labor's income share is mean-reverting, the results are insensitive to the choice of labor market behavior.

Under the Cobb-Douglas technology assumption, the marginal productivity of labor is given by $\alpha(Y/L)$. Imposing long-run price and productivity neutrality, $\beta_1 = \beta_2 = 1$, equation (5) reduces to:[2]

$$\Delta \ln S_t^L = \beta_3(U_t - \phi_t) + \Delta \ln Z_t v'. \qquad (6)$$

This equation shows that labor's income share is intimately related to the deviation of unemployment from the natural rate and supply shocks. Unemployment above its natural rate reduces the growth in excess wages until unemployment is brought to its equilibrium. An unemployment rate above the natural rate has been observed in most OECD countries in the 1980s and 1990s and has contributed to the decrease in labor's income share in the same period (Bruno and Sachs, 1985). By contrast, a rate of unemployment below the natural rate in the second half of the 1960s and the first half of the 1970s contributed to the increase in labor's income share over the same period. Supply shocks will change the wedge between labor costs and the value-added price-deflator and consequently influence firms' earnings as discussed above.

To find the dynamic path of mark-ups following a supply shock, unemployment is assumed to be a positive function of excess wages and a negative function of demand shocks, which is a standard assumption in modern theories of unemployment (Madsen, 1998). Excess wages are measured by the real wage gap, W^x, following Bruno and Sachs (1985). Denoting the full employment share of labor in total income, $\overline{S}^L$, which is assumed to be constant, and the wage gap is defined as $W^x = (S^L / \overline{S}^L - 1)$, which is the wage in excess of the full employment equilibrium wage (Bruno and Sachs, 1985). In other words the real wage gap measures real wages in excess of the marginal productivity of labor at full employment. Inserting this equation into equation (6) yields the following first-order differential equation:

$$\Delta \ln(W_t^x + 1) = -U(W_t^x, M_t, Q_t, \phi_t) + \Delta \ln Z_{t-1} v', \qquad (7)$$

where demand shocks are decomposed into real stock prices, Q, and other demand shocks, M. Real stock prices affect demand due to wealth effects in consumption and through investment by influencing the marginal value of fixed capital stock relative to its replacement cost and hence investment (Tobin's q). Following the natural rate hypothesis, demand shocks affect unemployment due to expectational errors. A demand shock drives a wedge between expected and actual wage and price inflation and hence affects employment. Supply shocks are assumed to influence wages with a one-period lag due to nominal rigidity in the labor market.

Equation (7) shows that a one-off adverse supply shock increases the real wage gap with a one-period lag and hence increases labor's factor share by reducing the monopoly profits. The effect on factor shares is only temporary. Assuming that the

---

[2] These assumptions are standard in the natural rate framework. If they are not satisfied, then a unique natural rate of unemployment does not exist.

labor market is initially in its long-run equilibrium, a supply shock brings unemployment above its natural rate and leads to subsequent reductions in the real wage gap until labor market equilibrium is restored. The question is what the optimal stock market reaction to changes in labor shares is. We now turn to this issue.

To find the dynamic path of the wage gap and stock prices, equations (4) and (7), define two first order differential equations as follows, where equation (4) is written in compact form for simplicity:

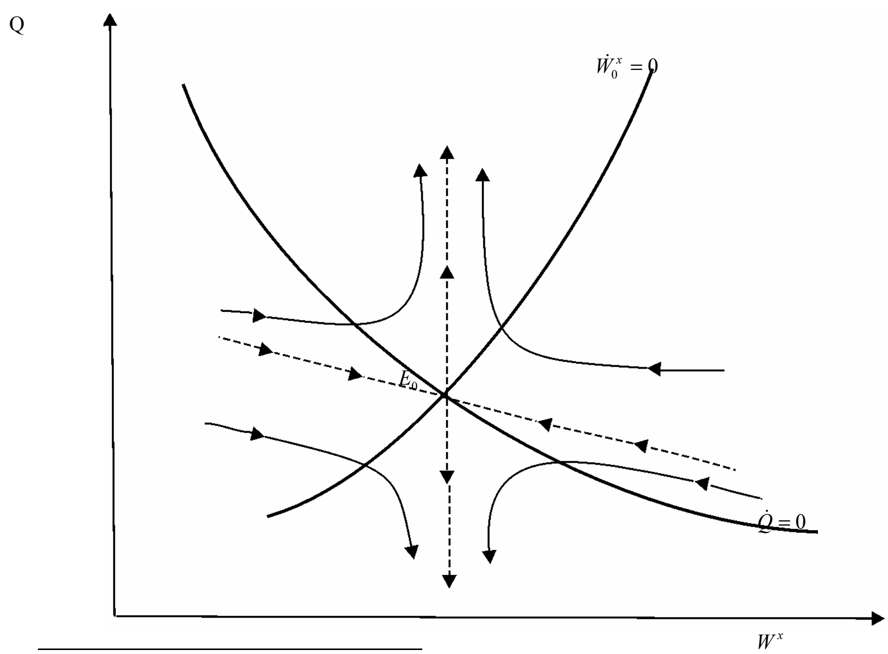
$$\dot{W}_t^x = -U(W_t^x, Q_t, M_t, \phi_t) + \dot{Z}_{t-1} v' \tag{8}$$

$$\dot{Q}_t = Q_t r - \Psi(W_t^x, \tau_t, Y_t / K_t, M_t), \tag{9}$$

where ψ is earnings. The equation system has four unknowns, where Q is the jump variable and W^x is the variable that gradually adjusts toward equilibrium.

Demand shocks affect stock prices positively due to price expectational errors following standard macroeconomic models. An unanticipated demand shock increases the price mark-up over marginal cost because prices are unexpectedly higher, whereas marginal costs remain constant. Furthermore, due to adjustment costs, firms cannot adjust the capital stock instantaneously and therefore tend to always have excess capacity except at cyclical peaks.[3]

Figure 4—Dynamics of Q and W^x



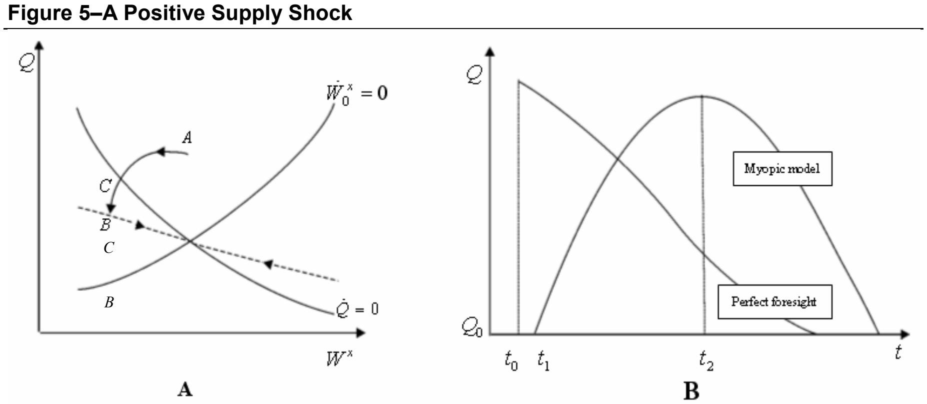
[3] A more formal incorporation of demand shocks is not included, as demand shocks are not the focus of the paper.

The phase diagram in Figure 4 illustrates the dynamics of equations (8) and (9). In steady state equilibrium, $\dot{Q}_t = \dot{W}_t^x = 0$. In the labor market the steady state equilibrium, $\dot{W}_t^x = 0$, is depicted by a positive relationship between real stock prices and the wage gap. An increase in stock prices, for example, brings the rate of unemployment below the natural rate, and the wage gap has to increase to restore the equilibrium in the labor market. The optimal dynamic path for the system lies on the saddle point stable manifold shown by the dashed line, where W^x and Q converge to a stable equilibrium (point E_0 in Figure 4).[4]

Supply Shocks and the Dynamics of Stock Prices and the Wage Gap

Suppose that the economy is initially in equilibrium and then is hit by an unanticipated positive supply shock such as a decrease in the income tax rate, lower real oil prices, or a reduction in union activity that is perceived to be permanent by the stock market. A perfect foresight stock market will not only take into account that profits will increase shortly after the tax cut has been announced, but also that W^x will revert to its initial equilibrium in the long run. When the economy is hit by the positive supply shock, Q jumps from its initial equilibrium to the point A to capitalize on expected temporarily higher profits. The economy then moves gradually toward the point B as the supply shock materializes in lower labor costs. The effects on the wage gap of the positive supply shock are borne at point B. At the point B there is excess demand for labor because the lower wage gap has pulled unemployment below its natural rate; thus leading to a growing wage gap. This process continues until the economy is at its initial equilibrium where unemployment equals its natural rate.

Figure 5–A Positive Supply Shock

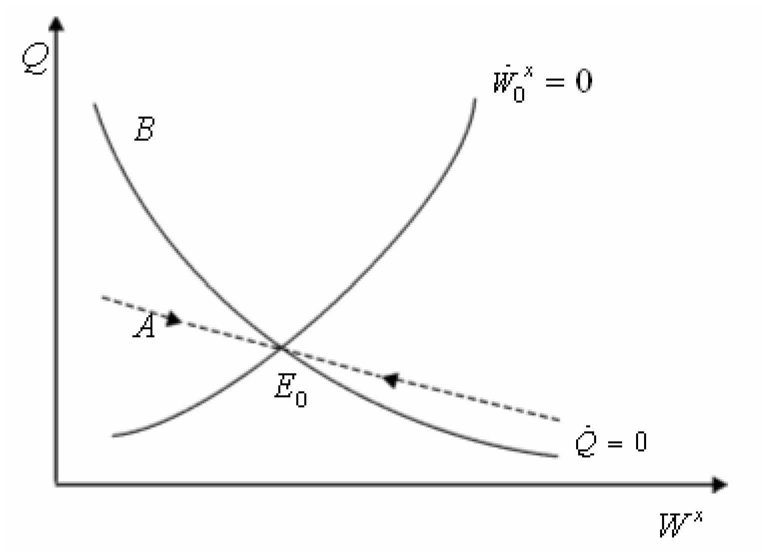


[4] This path is stable, which is demonstrated in the mathematical appendix.

The rational stock market incurs a capital loss in the adjustment path A-B and back to its steady state equilibrium. The capital loss is counterbalanced by earnings that are higher than in their long-run equilibrium, however, so that a constant required rate of return to shares is maintained. The speed of adjustment toward equilibrium depends predominantly on the speed of adjustment of wages to unemployment and innovations in wage push factors.

The dynamic path of the myopic stock market, where stock prices reflect current earnings, is different from the perfect foresight market in that stock prices first increase as the supply shock materializes in a lower wage gap and therefore higher earnings. The myopic market will always be on the $\dot{Q}_t = 0$ line. Stock prices therefore increase slowly to the point C. At the point C unemployment is below the natural rate and the wage gap increases until labor market equilibrium is established at its steady state equilibrium.

Figure 6—A Series of Adverse Supply Shocks



The time-profiles of stock prices, which are displayed in section B in Figure 5, show different paths for the myopic and the perfect foresight markets. Stock prices jump in the perfect foresight market when the income tax cut is announced at time t_0 and decline slowly thereafter, thus suggesting overshooting in the stock market. Due to wage stickiness, wages will start declining with a delay from t_1 and stock prices will first start increasing in the myopic market from t_1 to t_2 following increasing profits. From t_2 onwards, earnings decrease because the effects on wages of an excess demand for labor outweigh the tax reduction effects. It is noteworthy that Q moves in opposite directions in the myopic and the rational stock markets over the time period from t_1 to t_2. At the time when earnings start increasing, stock prices in the perfect foresight market start decreasing because the market has already incorporated the positive earning prospects into stock prices.

An empirically more relevant case is the stock market response to a series of supply shocks as shown in Figure 6. Historically supply shocks have tended to cluster in certain periods, such as the adverse supply shocks following the two world wars and the period from the late 1960s to the mid 1970s, during which the wage gap was driven up for several years, as shown by Bruno and Sachs (1985) and as can be seen from Figure 3. Conversely, the positive supply shocks in the 1980s and 1990s and a rate of unemployment that was well above the natural rate have driven the share of income going to profits well above its long-run equilibrium. The effects of the series of positive supply shocks are depicted in Figure 6. Assuming that the recent decline in W^x has reached its minimum at the point A in Figure 6, the perfect foresight stock prices are almost at the same level as in the long-run equilibrium in E_0 because the slope of the stable manifold is almost horizontal. By contrast the myopic market is always on the $\dot{Q}_t = 0$ line and has reached the point B at the time when the wage gap reached its minimum. Clearly, a myopic stock market reacts substantially more positively to a decrease in the wage gap than does a perfect foresight market.

In summary, this section has shown that stock prices are intimately related to factor shares, which in turn are driven by supply shocks and the deviation of unemployment from its natural rate. Furthermore, it was shown that factor shares are slow to revert to their long run equilibrium due to sluggish adjustment of the labor market toward its long-run equilibrium. This explains why the adverse supply shocks in the post war periods and from the mid 1960s to the mid 1970s simultaneously increased unemployment and depressed stock markets for prolonged periods. An important result from the analysis is that stock holders need not be myopic to react positively to a positive supply shock, even if they know that profits are temporarily higher, because the present value of expected earnings has increased.

Empirical Estimates

The model in the previous section predicts 1) that labor's income share is influential for stock prices; 2) that factor shares are determined in the labor market and therefore that supply shocks and unemployment are the primary sources of changes in factor shares; and 3) that adverse supply shocks affect stock prices negatively on impact but subsequently lead to increasing stock prices because factor shares converge to a constant level in the long run. These implications are tested in this section using data for 16 industrialized countries over the period from 1953 to 1999, where the countries are listed in the notes to Table 1 below. Allowing for leads, lags, and first difference, the estimation period is reduced to 1955-1998.

First, the importance of factor shares for stock prices and the dynamic adjustment of stock prices to disequilibrium in the labor market are examined. In order to do that, the model presented in the previous section needs to be transformed to an estimation equation. In steady state the stock price is given by the following expression:

$$Q = (1-\tau)\beta\left(\frac{Y}{K}\right)\left(2 - \frac{1}{\alpha}S_L\right)r^{-1} = (1-\tau)\beta\left(\frac{Y}{K}\right)\psi r^{-1}\,, \qquad (10)$$

where $\psi = (1-\alpha^{-1}S_L)$ and is referred to below as labor shares.

Because equation (10) is a steady state relationship, it does not allow for the temporary influences on stock prices of supply shocks, technology shocks, and disequilibria in the labor market. To allow for the influence on stock prices of these factors, the log-first differences of equation (10) are specified to allow for sluggish adjustment of stock prices to changes in to labor's income shares, the Y/K ratio, and disequilibrium in the labor market:

$$\begin{aligned}\Delta\ln Q_{it} = {} & a_0 + a_1\Delta\ln(Y/K)_{it} + a_2\Delta\ln(Y/K)^{e}_{i,t+1} + a_3\Delta\psi_{it} + a_4\Delta\ln\psi^{e}_{i,t+1} \\ & + a_5\Delta\ln(1 - \tau_{it}) + a_6\Delta\ln Pat_{it} + a_7\Delta\ln Pat^{e}_{i,t+1} + a_8\Delta\ln r^{B}_{it} \\ & + a_9\Delta\ln r^{B,e}_{i,t+1} + a_{10}\ln\left(S^{L}_{i,t-1}/\overline{S}^{L}_{i,t-1}\right) + \varepsilon_{1,it}\,. \qquad (11)\end{aligned}$$

where the subscripts i and t refer to country i and time t; the superscript e stands for expected value conditional on information available to the stock markets at period t; a_1 to a_{10} are fixed coefficients; Pat is the number of patent applications by domestic residents; r^B is the nominal interest rate on long-term government bonds minus the rate of consumer price inflation; and ε is a stochastic error term. The accumulated stock index, Q, is deflated by consumer prices.

The model is augmented with patents to allow for the effects of technology innovations on stock prices. Technological innovations have not been explicitly incorporated into the model in this paper. This is done formally in Madsen and Davis (2006) and Hobijn and Jovanovic (2001). The output elasticity of labor, α, is set to 0.65, which is approximately equal to the estimates of Coe and Helpman (1995). The average α for the countries that are included in the estimates in this paper and estimated by Coe and Helpman (1995) is 0.656, and their estimates show that there is little variation across countries. The Y/K ratio is measured as the economy-wide real GNP divided by the non-residential capital stock, and $\overline{S}^{L}_{it}$ is measured as the least squares deviation of the log of labor's income share from a deterministic time-trend for each individual country. The capital stock is based on the perpetual inventory method using data over a century.

The term $\left(S^{L}_{i,t-1}/\overline{S}^{L}_{i,t-1}\right)$ allows for adjustment of stock prices to the disequilibrium in the factor market, where $\ln\left(S^{L}_{i,t-1}/\overline{S}^{L}_{i,t-1}\right)$ is computed as the residual from regressing the log of S^L on a time-trend for each individual country. A generalized instrumental variable estimator that allows for the correlation of error terms across countries and correlation over time is used to gain efficiency.[5]

The results of estimating the unrestricted and the restricted models of equation (11) are shown in the left side of Table 1. The general-to-specific model reduction procedure is applied in the restricted model at the one-percentage significance level. Instruments are used for expectational variables, and the instruments are listed in the notes to Table 1. The estimates suggest that factor shares, patents, the expected real interest rate, and labor market disequilibrium, as represented by the deviation of factor shares from their long-run equilibrium, are statistically significant determinants of stock prices. The estimated coefficient of the period $(Y/K)_t$ ratio is negative and therefore is restricted to zero in the restricted model. Its negativity could be due to reverse causation running from real stock prices to the Y/K ratio. The increasing stock prices in the 1980s and 1990s, for instance, reduced the cost of capital and hence induced an expansion in the fixed capital investment to such an extent that the Y/K ratio was reduced at the same time as real stock prices increased.

In both the restricted and unrestricted models, the sum of the estimated coefficients of labor shares is approximately 4, which suggests that factor shares are potentially important for stock prices given that labor shares, ψ, typically move 20 percent within a 20 to 30 year cycle. Using the point estimates of 4, the approximately 15 percent increase in capital's factor share in the 16 countries on average over the period from the mid 1970s to the late 1990s contributed to a 60 percent increase in stock prices over the same period; thus the increasing capital share is an important influence behind the stock market run-up in the last two decades of the last century. The contribution of patents to the stock market run-up in the 1980s and the 1990s has been less important. The number of patent applications increased 150 percent in the 16 countries considered here over the period from 1980 to 1999. Using the point estimate of 0.08 from the restricted model, patents have contributed to a 13 percent increase in the stock prices over the same period. Using the sum of the estimated coefficients of patents in the last column of Table 1 of 0.39, however, the 150 percent increase in patent applications over the period from 1980 to 1999 has contributed to a 58.5 percent increase in the accumulated stock index, which sug-

[5] More specifically, the following variance-covariance structure is assumed:

$$E\{\varepsilon_{it}^2\} = \sigma_i^2,\ i = 1, 2, \ldots N,$$
$$E\{\varepsilon_{it}, \varepsilon_{jt}\} = \sigma_{ij},\ i \neq j,$$

where σ_i^2 = the variance of the disturbance terms for country i = 1, 2,... N; σ_{ij} = the covariance of the disturbance terms across countries i and j; and ε is the disturbance term. The variance σ_i^2 is assumed to be constant over time but to vary across countries and the error terms are assumed to be mutually correlated across countries, σ_{ij}, as random shocks are likely to impact on all countries at the same time. σ_i^2 and σ_{ij}, are estimated using the feasible generalized least squares method described in Greene (2000, Ch. 15).

gests that patents may be more important for stock prices than indicated by the estimates of equation (11).

The estimated coefficient of the deviation of labor's income share from its long-run equilibrium is significantly positive, which is consistent with the predictions of the perfect foresight model in the previous section. The stock market adjusts stock prices downward if factor shares are in favor of capital and vice versa. The stock market, therefore, incorporates into stock prices the fact that factor shares will converge toward a constant mean in the long run.

To investigate the supply factors that determine stock prices via factor shares, equation (6) is substituted into equation (11), which yields the following stochastic approximation:

$$\Delta \ln Q_{it} = b_0 + b_1 \Delta \ln(Y/K)_{it} + b_2 \Delta \ln(Y/K)^{e}_{i,t+1} + b_3 \Delta \ln Str_{it} + b_4 \Delta \ln t^{d}_{it}$$
$$+ b_5 \Delta \ln Tot_{it} + b_6 (U_{it} - \phi_{it}) + b_7 \Delta \ln Pat_{it} + b_8 \Delta \ln Pat^{e}_{i,t+1} + b_9 \Delta r^{B}_{it}$$
$$+ a_{10} \Delta r^{B,e}_{i,t+1} + b_{11} \ln\left(S^{L}_{i,t-1} / \overline{S}^{L}_{i,t-1}\right) + \varepsilon_{2,it}\,. \qquad (12)$$

Table 1—Parameter Estimates of Equations (11) and (12)

	Equation (11)			Equation (12)	
	Unrestricted	Restricted		Unrestricted	Restricted
$\Delta \ln(Y/K)_t$	-1.52 (7.07)		$\Delta \ln(Y/K)_t$	0.45 (1.34)	
$\Delta \ln(Y/K)^e_{t+1}$	0.56 (1.01)		$\Delta \ln(Y/K)^e_{t+1}$	-3.67 (5.39)	
$\Delta\ln\psi_t$	0.55 (7.44)	0.72 (9.46)	$\Delta\ln Pat_t$	0.18 (4.37)	0.18 (5.27)
$\Delta \ln \psi^e_{t+1}$	3.50 (10.1)	3.09 (9.55)	$\Delta \ln Pat^e_{t+1}$	0.21 (2.75)	0.21 (3.22)
$\Delta\ln Pat_t$	0.13 (3.35)	0.08 (4.24)	$\Delta\ln Tot_t$	-0.81 (5.62)	-0.53 (6.79)
$\Delta \ln Pat^e_{t+1}$	0.11 (1.48)		$\Delta \ln t^d_t$	-1.02 (3.43)	-1.32 (4.58)
$\Delta \ln(1-\tau)_t$	0.35 (2.54)		$(U_t - \phi_t)$	1.32 (5.19)	1.91 (7.81)
$\Delta \ln r^B_t$	2.04 (2.10)		$\Delta\ln Str_t$	0.00 (0.01)	
$\Delta \ln r^{B,e}_{t+1}$	-8.01 (4.77)	-5.70 (6.31)	$\Delta \ln r^B_t$	-0.12 (0.13)	
$\ln(S^L_{i,t-1}/\overline{S}^L_{i,t-1})$	0.13 (3.13)	0.13 (3.19)	$\Delta \ln r^{B,e}_{t+1}$	-1.41 (0.82)	
Constant	0.03 (2.91)	0.03 (2.62)	$\ln(S^L_{i,t-1}/\overline{S}^L_{i,t-1})$	0.02 (0.44)	
			Constant	-0.07 (4.56)	-0.01 (0.50)
R^2(mom)	0.27	0.20	R^2(mom)	0.20	0.16
DW	1.96	1.84	DW	1.93	1.87

Notes: Absolute t-statistics are given in parentheses. R^2(mom) = Buse's raw moment R-squared. DW = Durbin-Watson test for first order serial correlation in fixed effect panel data models. The following variables in first differences are used as instruments for time t+1 variables: Lagged dependent variable, one period lag of the real interest rate, unlagged values and one period lag of the log of real GNP, a one period lag of the log of real stock prices, and the log of consumer prices. The following 16 countries are included in the data sample: Canada, the US, Japan, Australia, Austria, Belgium, Denmark, France, Germany, Italy, the Netherlands, Norway, Spain, Sweden, Switzerland, and the UK. Estimation period: 1955 to 1998

where str are strikes, t^d is the direct tax rate, and Tot represents terms of trade. Strikes are measured as number of days lost in strikes and lockouts per worker, the direct tax rate is measured as total direct taxes divided by nominal GNP, and Tot is measured as consumer prices divided by the value-added price-deflator and therefore encompasses indirect taxes and terms-of-trade shocks that create a wedge between the prices facing consumers and producers. Increasing oil prices, for instance, increase consumer prices more than does the value-added price-deflator and hence push wages in excess of their equilibrium and squeeze profits. Similarly, if unemployment is pushed below its natural rate, then wages also will be pushed in excess of equilibrium. The deviation of unemployment from its natural rate is estimated as the residual from regressing unemployment on a linear time trend for each individual country.[6]

The results of estimating the restricted and unrestricted versions of equation (12) are shown in the right side of Table 1. The sum of the estimated coefficients of the real interest rates and the output-capital ratio are opposite to what one should expect, and these variables consequently are deleted in the restricted estimates. The estimated coefficient of strikes is statistically insignificant, which is not unusual in empirical applications, and the insignificance probably reflects that the strike activity is highly volatile and is often an outcome of several years' tension between employees and employers. In contrast to the estimates of equation (11), the estimated coefficient of the deviation of labor's income share from its long-run equilibrium is insignificant at the 1 percentage level.

Apart from strikes, the wage push factors are highly significant and have the expected signs. This implies that oil price shocks and innovations to direct and indirect taxes are important for stock prices because they affect labor's share in total income. Similarly, the deviation of the level of unemployment from its natural rate is influential for the growth in real stock prices. Because unemployment often stays from its natural rate equilibrium for prolonged periods, stock markets are similarly affected by the labor market disequilibrium for long periods.

Overall, the estimates indicate that movements in labor's share of total income are potentially important for stock returns. It cannot be excluded, however, that there is a two-way causation between factor shares and stock returns or that a third factor, such as technology, has simultaneously influenced both stock prices and factor shares. The recent technological revolution may have been labor saving and therefore may have shifted the labor demand schedule to the left and consequently reduced labor's income share and simultaneously increased the marginal productivity of capital. Unfortunately, the literature has not been successful in tests of this mechanism. Blanchard (1997) examined whether shifts in the labor demand functions in many industrialized countries in the 1980s and 1990s were due to shifts in rents or due to a labor saving technological bias. Blanchard's (1997) tests were inconclusive, reflecting the difficulties associated with estimates of production functions.

Comparing the estimation results of equations (11) and (12) in Table 1 may provide insight into the role of factor shares and technological innovations for stock returns. The significant increase in the estimated coefficients of patents when supply shocks are included in the estimates (instead of labor's income share) suggests some interesting interaction between labor's share and technological innovations. These estimates open the possibility that the reduction in labor's income share since the

[6] It is questionable whether the natural rate follows a linear trend. The estimates, however, are robust to different specifications of the natural rate (such as a constant and a second order polynomial).

beginning of the 1980s may have been, in part, technology-induced. The estimates of equation (11), therefore, may have overplayed the role of supply shocks and underplayed the role of technology shocks in the 1980s and in the 1990s. From the empirical estimates in this section it only can be concluded that although labor rents and technological innovations have both played an important role in the stock-market run-up in the 1980s and 1990s, their individual contributions cannot be exactly determined by the empirical estimates undertaken in this paper.

Conclusion

Real accumulated stock prices have shown large low-frequency fluctuations around an upward trend in the industrialized countries in the past century. This paper has argued that labor's income share, which is determined in the labor market, has been an important factor explaining the movements in stock prices around this upward trend. A simple macroeconomic model was established to show how fluctuations in the stock market can be generated as responses to supply shocks. In the long run, however, factor shares will revert to their initial equilibrium and stock prices will return to their long-run equilibrium trend.

The model has important implications for the predictability of real stock prices. The model implies that the deviations of real stock prices from their long run trend and dividend yield are predictors of stock prices in a rational stock market. The empirical literature often has found that the dividend-price ratio predicts growth in stock prices. (See, for instance, Fama and French, 1989.) The model in this paper suggests that this is a rational response among shareholders. A positive supply shock, for example, increases stock prices on impact and reduces the dividend-price ratio as stock prices jump. Dividends remain approximately unaltered because managers know that the higher earnings are temporary. Because stock prices will decline after the initial impact, it follows that the dividend-price ratio will be a predictor of the growth in stock prices.

Using data for 16 industrialized countries over the period from 1953 to 1999, the empirical estimates show that factor shares, supply shocks, and technology innovations are potential important macroeconomic variables driving stock returns. Hence, the increase in the share of income going to capital in the 1980s and the 1990s and the information and communication technological revolution may have been important forces behind the stock market run-up in the 1980s and 1990s.

References

1. Blanchard, Olivier J., "Output, the Stock Market, and Interest Rates," *American Economic Review*, 71, no.1 (March 1981), pp. 132-143.

2. Blanchard, Olivier, and Lawrence F Katz, "Wage Dynamics: Reconciling Theory and Evidence," *American Economic Review, Papers and Proceedings*, 89, no. 2, (May 1999), pp. 69-74.

3. Blanchflower, David G., and Andrew J. Oswald, *The Wage Curve* (Cambridge, Mass: MIT Press, 1994).

4. Bruno, Michael, and Jeffrey D. Sachs, *Economics of Worldwide Stagflation* (Cambridge, MA: Harvard University Press, 1985).

5. Campbell, John Y., and Robert J. Shiller, "Stock Prices, Earnings, and Expected Dividends," *Journal of Finance*, 43, no. 3 (July 1988), pp. 661-676.

6. Chan-Lee, James H., and Helen Sutch, "Profits and Rates of Return," *OECD Economic Studies*, no. 5 (Autumn 1985), pp. 127-167.

7. Fama, Eugene F., and Kenneth R. French, "Business Conditions and the Expected Returns on Stocks and Bonds," *Journal of Financial Economics*, 25, no. 1 (November 1989), pp. 23-49.

8. Goerke, Laszlo, and Jakob B. Madsen, "Labor Disputes in the Twentieth Century: An International Comparison and Evaluation of Theories," *Homo Oeconomicus*, 20 (2003), pp. 391-421.

9. Greene, William H., *Econometric Analysis*, 4th Edition (Upper Saddle River, NJ: Prentice Hall International, 2000).

10. Griliches, Zvi, "Patent Statistics as Economic Indicators: A Survey," *Journal of Economic Literature*, 28, no. 4 (December 1990), pp.1661-1707.

11. Hayashi, Fumio, "Tobin's Marginal q and Average q: A Neoclassical Interpretation," *Econometrica*, 50, no. 1 (January 1982), pp. 213-224.

12. Hobijn, Bart, and Boyan Jovanovic, "The Information-Technology Revolution and the Stock Market: Evidence," *American Economic Review*, 91, no. 5 (December 2001), pp. 1203-1220.

13. International Monetary Fund, *World Economic Outlook* (Washington: IMF, May 2000).

14. Jovanovic, Boyan, and Peter L Rousseau, "Stock Markets in the New Economy," in Chong En Bai and Chi-Wa Yuen, *Technology and the New Economy* (Cambridge, MA: MIT Press, 2002), pp. 9-48.

15. Madsen, Jakob B., "General Equilibrium Macroeconomic Models of Unemployment: Can They Explain the Unemployment Path in the OECD?" *Economic Journal*, 108, no. 448 (May 1998), pp. 850-867.

16. Madsen, Jakob B., and E. Philip Davis, "Equity Prices, Productivity Growth and the New Economy," *Economic Journal* (2006 - forthcoming).

17. Perez, Charlota, *Technological Revolutions and Financial Capital* (Cheltenham: Edward Elgar, 2002).

18. Siegel, Jeremy J., *Stocks for the Long Run* (New York: McGraw Hill 2002).

Data Appendix

- **Capital stock of equipment and non-residential structures:** The perpetual inventory method is used with the following depreciation rates. 17.6 percent for machinery and equipment, and 3 percent for non-residential buildings and structures. The stock of capital is set equal to the average investment over the first six years initially and multiplied by 5.68 for machinery and equipment and 33.6 for non-residential structures and construction, reflecting depreciating rates of 17.6 percent and 3 percent respectively. The post-1950 data are from OECD, *National Accounts, Vol. II,* Paris, (NA). Before 1950 the following sources are used for the countries at which historical data are available.
  - Canada: 1870-1900: Both types of investment are assumed to follow total non-residential investment in nominal prices deflated by the CPI. 1901-1925: 5-year average disaggregated into 1-year intervals using total non-residential investment deflated by CPI. Source: F.H. Leacy (ed.), 1983, *Historical Statistics of Canada*, Statistics Canada: Ottawa.
  - United States: 1870-1900: Department of Commerce, 1975, *Historical Statistics of the United States: Colonial Times to 1970*, Bureau of the Census: Washington DC. Only 10-year averages are available in the two decades 1870-1878 and 1879-1888. The data are uniformly distributed within the 10-year intervals. 1901-1949. T. Liesner, "One Hundred Years of Economic Statistics," *The Economist*: Oxford.
  - Japan: 1885-1949: K. Ohkawa, M. Shinchara and L. Meissner, 1979, *Patterns of Japanese Economic Development: A Quantitative Appraisal*, Yale University Press: New Haven. Total non-residential investment is used for the period 1940-1949. 1860-1885. Investment in equipment and non-residential buildings, respectively, for Sweden (see source below), using the average over the period 1885-1890 as scaling factor.
  - Australia: 1863-1902: C. Clark, 1970, "Net Capital Stock," *Economic Record*, pp. 449-466. 1903-1950: M.W. Butlin, 1977, *A Preliminary Annual Database 1900/01 to 1973/74*, Research Discussion Paper 7701, Reserve Bank of Australia: Sydney.
  - Denmark: 1870-1950: K. Bjerke and Nils Ussing, 1958, *Studier Over Danmarks Nationalprodukt 1870-1950*, G.E.C. Gads Forlag: København.
  - France: 1856-1895: Total investment deflated by industry prices. E. Chadeau, 1989, *l'Economie Nationale Aux XIX et XX Siecles*, Paris: Presses de l'Ecole normale Superieure. 1896-1914 and 1921-1938. J.J. Carre, P. Dubois, and E. Malinvaud, 1975, *French Economic Growth*, Stanford: Stanford University Press. 1914-1921 and 1939-1949. Crude steel production adjusted. Liesner *op. cit.*

- Germany: W. Kirner, 1968, *Zeitreihen fur das Anlagevermogen der Wirtschaftsbereiche in der Bundesreplublik Deutschland*, Deutsches Institut fur Wirtschaftsforschnung, Duncker & Humbolt: Berlin. The data are adjusted for war damage in the source. Non-residential buildings and structures: 1850-1949. The following categories are added together: Land und Forstwirtschaft, Energiewirtschaft, Bergbau, Grundstoff- und Productionguter-industrie, Investeringsguterindustrie, Verbrauchenguterindustrie, Nahrings- und Genussmittel-industrie, Industrie Kleinbetr. und Handwerk, Baugewerbe, Handel, Eisenbahnen, Schifffahrt, Ubringer Verkehr, Nachr. ubermittlg, Kreditintitutionen und Vers. gew., Wohnungsvermietung, Sonst. Dienstleist., Strassen und Brukken, Wasser strassen und Hafen, and Ubrige staatl. Bereiche. Machinery and equipment: 1926-1949. The same categories are added as for investment in non-residential buildings and structures. 1870-1925: Scaled investment in machinery and equipment for Denmark, using the average over the period 1926-1930 as scaling factor.
- Italy: Instituto Centrale di Statistica, 1976, *Statistiche Storiche Dell'Italia 1861-1975*. Residential building investment is included in investment in buildings. Only 10-year averages are available before 1945. The data are uniformly distributed within the 10-year intervals.
- Netherlands: C.A. van Bochove and T.A. Huitker, 1987, *Main National Accounting Series, 1900-1986*, Central Bureau of Statistics, the Netherlands.
- Norway: Statistisk Sentralbyraa, 1968, *Nasjonalregnskap*, Oslo. 1865-1930: The investment data are derived from capital stock and official depreciation rates using the following formulae for buildings and equipment: $I_t^{eq} = K_t^{eq} - K_{t-1}^{eq}(1-0.15)$ and $I_t^{st} = K_t^{st} - K_{t-1}^{st}(1-0.02)$. 1930-1949: The data are interpolated from 1940 to 1945 using the algorithm which is suggested by V. Gomez and A. Maravall, 1994, "Estimation Prediction and Interpolation for Nonstationary Series with the Kalman Filter," *Journal of the American Statistical Association*, 89, 611-624. The general investment price deflator is used to adjust the pre-1940 data which are in 1938 prices, whereas the post-1945 data are in 1955 prices.
- Sweden: 1861-1949. O. Krantz and C.A. Nilsson, 1975, *Swedish National Product 1861-1970*, C.W.K. Gleerup. Investment in buildings includes residential investment.
- UK: 1856-1900: B.R. Mitchell, 1962, *Abstract in British Historical Statistics*, Cambridge University Press: Cambridge. The data are measured in nominal terms. The data are deflated by the general investment price deflator. 1901-1949: T. Liesner, *op. cit*.

- **Patent applications**: World Intellectual Property Organization, *Industrial Property Statistics*, Geneva.

- **Corporate taxes**: OECD, *Revenue Statistics of OECD Member Countries*, and UN, *Yearbook of National Accounts Statistics*, New York.

- **GDP at purchasing power parities**: A. Maddison, 1995, *Monitoring the World Economy 1820-1992*, Development Centre, OECD. Updated from NA.

- **Consumer prices**: IMF, *International Financial Statistics* (IFS).

- **Interest rates**: IFS.

- **Accumulated stock prices**:
  - Canada. 1900-1924: Dimson, Elroy, Paul Marsh and Mike Staunton, 2000, *The Millennium Book, A Century of Investment Returns*, London Business School, ABN, AMRO, 1924-98: Panjer, Harry H and Keith P Sharpe, 1999, "Report on Economic Statistics 1924-1998," Canadian Institute of Actuaries.
  - USA. Jack W Wilson, 2002, "An Analysis of the S&P 500 Index and Cowles's Extensions: Price Indexes and Stock Returns, 1870-1999," *Journal of Business*, 75, 505-533.
  - Japan. Global Financial Data.
  - Australia. All Ordinary Index, Sydney Stock Exchange, collected for the Reserve Bank of Australia.
  - Austria. Global Financial Data.
  - Belgium. Global Financial Data.
  - Denmark. 1900-1924: Hansen, K, 1976, "Om Afkastet af Danske Aktier i Tiden 1900-1974," Optrykt i Nordisk Fjerfabriks Jubilæumsskrift, Nordisk Fjerfabrik, Aktieselskab, 1924-99: Claus Parum, "Aktieindeks, aktieafkast og risikopræmier, Manuscript, 98-7, Institute of Finance, Copenhagen Business School, Claus Parum, "Estimation af Realkreditobligationsafkast i Danmark i Perioden 1925-1998," *Finans/Invest* 1999/7, Claus Parum, "Historisk Afkast af Aktier og Obligationer i Danmark," *Finans/Invest* 1999/3.
  - France. Global Financial Data.
  - Italy. Panetta, Fabio and Robert Violi, 1999, "Is there an Equity Premium Puzzle in Italy? A look at Asset Returns, Consumption and Financial Structure Data over the Last Century," Termi di Discussione 353, Bank of Italy (data received by personal correspondence with Fabio Panetta).

- Germany. Gregor Gielen, 1994, *Konnen Aktienkurse Noch Steigen?*, Wiesbaden: Gabler Verlag. The index is multiplied by 10 before 1948 to account for monetary reform that changed the old Reismark with a new DEM at the ratio of 1:10.
- Netherlands. Eichholtz, Piet, Kees Koedijk and Roger Otten, 2000, "De Eeuw van Het Aandeel," *Economisch Statistische Berichten*, January (data received by personal correspondence with Piet Eichholtz and Roger Otten).
- Norway. 1960-1970: Roger G Ibbotsen, Richard C Carr and Anthony W Robinsen, 1982, "International Equity and Bond Returns," *Financial Analysts Journal*, 5-27.
- Spain. The data are kindly provided by Santiago Fernández Valbuena, Universidad Complutense. Madrid.
- Sweden. Per Frennberg and Bjørn Hansson, 1992, "Computation of a Monthly Index for Swedish Stock Returns: 1919-1989," *Scandinavian Economic History Review*, XL, No 1, 3-27 (data received by personal correspondence with Bjørn Hansson).
- Switzerland. H Ritzmann-Blickenstorfer, 1996, *Historical Statistics of Switzerland*, Chronos: Zurich.
- United Kingdom. 1900-1913: Richard S Grossman, 2002, "New Indices of British Equity Prices, 1870-1913," *Journal of Economic History*, 62 (1), 121-146. 1914-1999: Barclays Capital, 2001, "Equity Guild Study," Barclays Capital.
- USA, Austria, Belgium, New Zealand, Finland, Germany, and Ireland. Global Financial Data.

- **Strikes**: Days lost in strikes and lockouts. ILO, *Yearbook*, Geneva (YB).

- **Employment**: YB.

- **Direct tax rates**: Income tax revenue divided by nominal GDP. NA.

- **Unemployment rates**: YB.

- **Labor's income share**: Compensation to employees divided by nominal income. Payments to self-employed are imputed in the post-1950 data by adding to compensation to employees the number of self-employed (OECD, *Labor Force Statistics*, Paris, LFS) multiplied by the ratio of compensation to employees and number of employees.

- **Total employment**: After 1950. LFS. Before 1950 the following sources are used. The algorithm which is suggested by V. Gomez and A. Maravall, 1994, *op. cit.*, is used to interpolate between the benchmark years as indicated for the individual countries.
    - Canada. 1921-1959. F. H. Leacy (ed.), 1983, *op. cit.* 1913. A. Maddison, 1991, *Dynamic Forces in Capitalist Development*, Oxford University Press: Oxford.
    - US. 1900- 1949. T. Liesner, *op. cit.*
    - Japan. K. Ohkawa, M. Shinohara, and L. Meissner, 1979, *op. cit.*
    - Australia. M.W. Butlin, 1977, *op. cit.*
    - Denmark. 1870-1949. S.A. Hansen, *Økonomisk Vækst I Danmark*, Akademisk Forlag: København.
    - France. ILO, *Yearbook of Labor Statistics* and C. Clark, 1957, *The Conditions of Economic Progress*, Macmillan: London.
    - Germany. 1900-1914, 1924-1940, and 1949. W.G. Hoffmann, F. Grumbach, and H. Hesse, 1965, *Das Wachstum der Deutschen Wirtschaft seit der mitte des 19. Jahrhunderts*, Springer-Verlag: Berlin.
    - Italy. Clark, 1957, *op. cit.*
    - Netherlands. 1800-1913: J.-P. Smits, E. Horlings, and J.L. van Zanden, 2000, *Dutch GNP and its Components, 1800-1913*, Groningen, http://www.eco.rug.nl/ggdc/PUB/dutchgnp.pdf. The general investment deflator is used for separate items. 1913-60: Central Bureau voor de Statistiek, 2001, *Tweehondred Jaar Statistiek in Tijdreeksen, 1800-1999*, Centraal Bureau voor de Statistiek, Voorburg.
    - Sweden. O. Johansson, 1967, *The Gross Domestic Product of Sweden and its Composition 1861-1955*, Almqvist and Wiksell: Stockholm. UK. Clark, 1957, *op. cit.*

- **Economy-wide nominal GDP**: After 1950. NA. Before 1960 the following sources are used. Real GDP multiplied by GDP-deflators from the following sources.
    - Canada. M.C. Urquhart, 1988, "Canadian Economic Growth 1870-1980," Queens University Discussion Paper No 734.
    - USA. 1870-1929: N.S. Balke and R.J. Gordon, 1986, *The American Business Cycle: Continuity and Change*, Chicago: University of Chicago Press. 1929-1960 *Survey of Current Business* August 1998: "GDP and Other Major NIPA Series 1927-97."
    - Japan. K. Ohkawa *et al.,* 1979, *op cit.*
    - Australia. 1863-1939. W. Vamplew, 1987, *Australian Historical Statistics*, Broadway, N.S.W: Fairfax, Syme & Weldon Associates. 1940-1960. Liesner, *op. cit.*

- Denmark. S.A. Hansen, 1976, *op. cit.*
- France. P Villa, 1993, *Une Analyse Macroéconomique De La France Au XX^e Siècle*, Paris: CNRS Editions, and M. Lévy-Leboyer and F. Bourguignon, 1985, *The French Economy in the Nineteenth Century*, Cambridge: Cambridge University Press.
- Germany. T. Liesner, 1989, *op. cit. The Economist* and interpolated using CPI over the periods 1914-1924 and 1939-1949.
- Italy. C. Bardini, A. Carreras, and P. Lains, 1995, "The National Accounts for Italy, Spain and Portugal," *Scandinavian Economic History Review* XLII, 115-146.
- Netherlands. Central Bureau voor de Statistiek, 2001, *op cit*.
- Sweden. O. Johansson, 1967, *op. cit*.
- UK. C.H. Feinstein, 1976, *Statistical tables of national income, expenditure and output of the U.K. 1855-1965*, Cambridge: Cambridge University Press.

- **Compensation to employees**:
  - Canada. Leacy, 1983, *op. cit*.
  - USA. T. Liesner, *op. cit*.
  - Japan. Ohkawa *et al.*, 1979, *op. cit*.
  - Australia. Glenn Withers, Tony Endres and Len Perry, 1985, "Australian Historical Statistics: Labor Statistics, "Australian National University, Source Papers in Economic History, No. 7.
  - Denmark. H.C. Johansen, 1985, *Dansk Historisk Statistik, 1814-1980,* København: Gyldendal.
  - France. Table F.4, T. Liesner, *op. cit.* Include the non-agricultural sector.
  - Germany. Table 122, Hoffmann, 1965, *op. cit*.
  - Netherlands. Central Bureau voor de Statistiek, 2001, *op. cit*.
  - Sweden. Karl G. Jungenfelt, 1966, *Lonandelen och den Ekonomiska Utvecklingen*, Almquvist & Wiksell: Stockholm.
  - UK. Table 1, C.H. Feinstein, 1976, *op. cit*. Includes all sectors in the economy.

Mathematical Appendix

This appendix shows stability and the solution path to the simultaneous first-order differential system.

Stability

Equations (13) and (14) are linearized as follows:

$$\dot{Q} = \lambda_0 + rQ + \lambda_1 W^x - \lambda_2 M$$
$$\dot{W}^x = \alpha_0 - \alpha_1 W^x + \alpha_2 Q + \alpha_3 \dot{Z} + \alpha_4 M$$

where λ_i are constants for all $i = 0, 1, 2$ and α_i are constants for all $i = 0,.., 4$. The system can be compactly written as

$$\begin{pmatrix} \dot{Q} \\ \dot{W}^x \end{pmatrix} = \begin{pmatrix} \lambda_0 \\ \alpha_0 \end{pmatrix} + \begin{pmatrix} r & \lambda_1 \\ \alpha_2 & -\alpha_1 \end{pmatrix} \begin{pmatrix} Q \\ W^x \end{pmatrix} + \begin{pmatrix} 0 \\ \alpha_3 \end{pmatrix} \dot{Z} + \begin{pmatrix} -\lambda_2 \\ \alpha_4 \end{pmatrix} M . \qquad \text{(A1)}$$

A long-run solution to this system, where $\dot{Q} = \dot{W} = \dot{Z} = M = 0$ is given by

$$\begin{pmatrix} r & \lambda_1 \\ \alpha_2 & -\alpha_1 \end{pmatrix} \begin{pmatrix} Q \\ W^x \end{pmatrix} = \begin{pmatrix} -\lambda_0 \\ -\alpha_0 \end{pmatrix}$$

and solving for Q and W yields

$$\begin{pmatrix} \overline{Q} \\ \overline{W} \end{pmatrix} = \frac{1}{r\alpha_1 + \lambda_1 \alpha_2} \begin{pmatrix} -\lambda_0 \alpha_1 - \lambda_1 \alpha_0 \\ -\lambda_0 \alpha_2 + r\alpha_0 \end{pmatrix}.$$

Solution Path

Let

$$X = \begin{pmatrix} Q - \overline{Q} \\ W^x - \overline{W}^x \end{pmatrix} \dot{X} = \begin{pmatrix} \dot{Q} \\ \dot{W}^x \end{pmatrix} \text{ and } A = \begin{pmatrix} r & \lambda_1 \\ \alpha_2 & -\alpha_1 \end{pmatrix}.$$

Then the complementary solution to this system of ordinary differential equations is the solution to the system: $\dot{X} = AX$. Its complementary solution has the form

$$X = c_1 \xi_1 e^{\mu_1 t} + c_2 \xi_2 e^{\mu_2 t}$$

where c_1, c_2 are constants subject to some initial conditions; μ_1, μ_2 are the eigenvalues of A; and ξ_1, ξ_2 are the eigenvectors associated with the eigenvalues.

By definition, $A\xi = \mu\xi$; hence $| A - \mu I | = 0$, which implies that

$$\begin{vmatrix} r - \mu & \lambda_1 \\ \alpha_2 & -\alpha_1 - \mu \end{vmatrix} = 0,$$

or

$$-\mu^2+(r-\alpha_1)\mu+(\lambda_1\alpha_2+r\alpha_1)=0$$

which yield the two roots as follows

$$\mu_1=\frac{(r-\alpha_1)+\sqrt{(r-\alpha_1)^2+4(\lambda_1\alpha_2+r\alpha_1)}}{2}$$
$$\mu_2=\frac{(r-\alpha_1)-\sqrt{(r-\alpha_1)^2+4(\lambda_1\alpha_2+r\alpha_1)}}{2}$$

or

$$\mu_1=\frac{(r-\alpha_1)+\sqrt{(r+\alpha_1)^2+4\lambda_1\alpha_2}}{2} \text{ and } \mu_2=\frac{(r-\alpha_1)-\sqrt{(r+\alpha_1)^2+4\lambda_1\alpha_2}}{2}.$$

Assuming that the $(r-\alpha_1)>0$, then we have $\mu_1>0>\mu_2$. Therefore, the steady-state is a saddle point. Hence, only (W^x,Q) that lies on ξ_2 will be attracted toward the steady state, and any other point on the (W^x,Q)-plane, diverges.

The eigenvectors associated with μ_1 and μ_2 are given by:

$$\xi_1=(\frac{(r+\alpha_1)-\sqrt{\alpha_1^2+4\alpha_2\lambda+2\alpha_1 r+r^2}}{2\alpha_2},1)$$

$$\xi_2=(\frac{(r+\alpha_1)+\sqrt{\alpha_1^2+4\alpha_2\lambda+2\alpha_1 r+r^2}}{2\alpha_2},1).$$

The initial value must satisfy $c_1=0$ in order to reach the steady state. Any initial value that fails to satisfy this condition will drift away from the steady state as time heads toward infinity. Therefore, the steady path is

$$c_1\begin{pmatrix}\dfrac{(r+\alpha_1)+\sqrt{\alpha_1^2+4\alpha_2\lambda+2\alpha_1 r+r^2}}{2\alpha_2}\\ \\ 1\end{pmatrix}$$

where $c_1\in\Re$. It also can be expressed as a linear equation on the (W^x,Q)-plane as

$$Q-\overline{Q}=\frac{(r+\alpha_1)+\sqrt{\alpha_1^2+4\alpha_2\lambda+2\alpha_1 r+r^2}}{2\alpha_2}(W^x-\overline{W}^x).$$