Empirical Evidence of Nonlinearity and Chaos in the Returns of American Depository Receipts

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This paper examines the time series behavior of the returns of American depository receipts, ADRs. We postulate that the unique characteristics of these financial instruments make the ADR returns exhibit time series properties that are different from that of the U.S. equity market as a whole. Variance ratio tests of linear dependence reject the null hypothesis of random walk for the ADR returns. Tests based on the correlation dimension and the BDS statistic indicate the presence of nonlinearity in the ADR data. The BDS statistic applied to the standardized residuals of the EGARCH model rejects the null hypothesis that the data are independently and identically distributed, suggesting that conditional heteroskedasticity is not the cause of nonlinear structure in the data. On the other hand, tests of chaos, based on the locally weighted regression indicate that ADR returns exhibit chaotic behavior. This finding differs from previous research, which has failed to report evidence of chaos in the time series of American stock returns. Important contributions of this paper are the findings of statistically significant evidence of nonlinearity and low deterministic chaotic behavior in ADR returns. These results are important because knowing that returns of ADRs exhibit chaotic behavior can help us to understand this sector of the market better and find ways of predicting returns. In this respect our results also have practical implications, because they suggest that pricing forecasting models for ADR returns should include some nonlinear terms.

Purpose and Motivation of the Research

The purpose of this paper is to examine the time series behavior of ADR returns. Specifically, we are looking for linear and nonlinear structures and chaos. The analysis of the behavior over time of ADR returns is important for several reasons. The finding of a nonlinear structure would open the possibility for nonlinear pricing forecasting models that could better explain certain aspects of the time series behavior of

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ADRs. Chaotic behavior is interesting because it potentially can explain fluctuations in the economy and financial markets, which appear to be random processes. Moreover, low complexity chaotic processes are more relevant in economics and finance because they have short-term predictability.

The empirical evidence in favor of chaotic dynamic in equity returns has been generally weak (Hsieh, 1991). An obvious question is if the evidence of the presence of deterministic chaos is weak for equity in general, why should one expect ADR returns to behave differently? First, we postulate that ADRs behave differently from the rest of the market because they are unique financial instruments with some unique characteristics. Second, Brock and Sayers (1988) point out that one of the reasons obscuring evidence of chaos is the use of aggregate data that can hide nonlinearity at the micro level. In this paper we avoid this problem by employing data for a small sector of the capital market.

ADRs are unique financial instruments for several reasons. First, they are traded in the U.S. markets but represent ownership of foreign securities. ADRs have been around for a long time, but they have become popular in the U.S. only recently when an organized and liquid market was developed for the primary issue of foreign securities (Rosenthal, 1983; Kim et al., 2000). In addition, the increase in globalization of capital markets, the intensification of privatization of public enterprises around the globe, and the growing need of American investors for international portfolio diversification, also have contributed to the rapid expansion of the ADR market (Bin, Morris, and Chen, 2003).

Second, ADRs are quoted in U.S. dollars; therefore the price of an ADR reflects not only changes in the value in local currency of the underlying foreign security, but also changes in the exchange rate of the local currency against the U.S. dollar. Kim et al. (2000) suggest that the role of the exchange rate has been growing in recent years. In effect, the role of exchange rates in the return of foreign portfolio investing cannot be ignored. Bodart and Reding (2001) document that exchange rate fluctuations have a significant effect on the conditional distribution of stock returns, and Bailey, Chan, and Chung (2000) find that the devaluation of the Mexican peso had a negative significant tequila effect on the prices of Latin American ADRs.

Third, the price of ADRs may be affected by U.S. market conditions. American investors may evaluate the systematic risk of ADRs with reference to the U.S. capital market rather than with reference to the domestic market of the underlying security.

Fourth, ADR owners can convert the ADRs into foreign currency-denominated underlying shares subject to cancellation and conversion fees. By the same token, holders of the underlying shares can convert them in ADRs. Therefore, investors can earn a risk-free profit if the price differential between the price of the ADR and the price in dollar of the underlying security is sufficiently large to cover the transaction costs.

Fifth, ADRs and their underlying shares are expected to be perfect substitutes, and no arbitrage opportunities should be present. Studies by Miller and Morey (1996) and Karolyi and Stulz (1996) indicate that ADRs do not provide any arbitrage opportunities. ADR and the underlying security may not be perfect substitutes for each other, however, for several reasons. First, they trade at different trading hours in different markets (Rosenthal, 1983). For instance, there is little, if any, simultaneous trading among European and Asian markets and the American market. After the European markets close and before the Asian markets open, the ADRs are being traded in the U.S. market. Second, the type of exchange rate system (fixed-exchange rate system or flexible exchange rate system) and the level of restrictions on foreign investments (such as double taxation of dividends and limitations in rights offerings) also could be factors that explain the differences in the distributions of returns of ADRs and their underlying assets (Rabinovitch, Silva, and Susmel, 2003). In short, ADRs have some similarities to the underlying shares, but international capital market imperfections could result in some differential pricing vis-à-vis the underlying shares.

We postulate that because of their unique characteristics ADRs might exhibit a time series behavior that is different from that of the U.S. market as a whole. Specifically, we postulate that the time series of ADR returns exhibits strong nonlinear dependence and chaotic behavior. Our hypothesis contrasts with the weak empirical evidence in favor of chaos reported in previous research for American stock returns (Brock and Sayers, 1988; Scheinkman and LeBaron, 1989; Hsieh, 1991).

An Overview of ADRs

American depository receipts, ADRs, are surrogate securities used by non-U.S. firms to access the American capital markets. In other words, ADRs are a form of equity-based financing for non-American multinational firms facing a small home capital market. An ADR is a certificate, denominated in shares, representing proof of ownership of foreign securities on deposit with a large non-U.S. bank affiliated with a major U.S. financial institution, such as Morgan Guaranty or Irving Trust (Stonehill and Dullum, 1982). One ADR can represent one or several underlying foreign shares. Once the shares are on deposit, the American bank arranges for a primary issue of U.S. dollar-denominated ADRs. These ADRs are usually convertible into denominated shares in the underlying foreign currency. After the primary issue, the ADRs can be traded in the U.S. secondary markets of the New York Stock Exchange, American Stock Exchange, and Nasdaq. ADRs are subject to the SEC regulations, and the holders are entitled to full protection under U.S. laws and enjoy most of the privileges of stock ownership accorded in the U.S.

The major advantages of holding ADRs rather than the underlying foreign stocks are liquidity, service, and cost (Harr et al., 1990). Because ADRs commonly are traded in the U.S. security markets, they may have a higher degree of liquidity than the underlying foreign shares. The U.S. bank servicing the ADR provides several services, such as keeping the investor informed of any corporate activities affecting the value of the ADR. Also, transactional and custodial costs on foreign stocks can be reduced by using ADRs.

It is well known that diversification reduces risk. Empirical evidence has shown that even greater benefits from diversification can be obtained by diversifying across countries (Solnik, 1974). The international investment objectives of American investors cannot be met by holding shares of U.S. multinational firms, because the stock prices of these multinationals behave much like the stock prices of American domestic firms. Also, for many American individual and institutional investors it is not practical to directly invest in foreign stocks. A major advantage of ADRs is, precisely, to serve as an alternative vehicle for international portfolio diversification. They represent an easy means of acquiring foreign stocks.

Not only investors benefit from ADRs. There are advantages for the issuing foreign firms as well. First, the foreign firm is evaluated in the U.S. market, which is more efficient than the small, thin, local capital market. Second, the ADR is an avenue for expansion. In effect, many foreign multinationals have started operations in the U.S. following a successful ADR issue. Finally, ADRs can lower the cost of capital of the multinational firm. It is argued that a firm can achieve a lower cost of equity capital through ADR issues by circumventing home country laws and regulations (Haar et al., 1990). By raising money internationally, a firm can not only lower the cost of equity capital but also reduce the cost of debt, because it is able to operate at a lower debt-to-equity ratio. Therefore, ADR issues can lower the overall cost of capital of the non-U.S. multinational firm.

Because ADRs are convertible into the underlying shares and vice versa, there might be some opportunities for risk-free arbitrage between the ADRs and the underlying shares. Rosenthal (1983) reports, however that ADR prices are fairly consistent with weak-form efficiency. Also, Kato et al. (1991) and Wahab et al. (1992) find that, after transaction costs, few profitable arbitrage opportunities exist between the market for ADRs and the markets for underlying securities, implying that these markets appear to be efficient. Kim et al. (2000), however, report that ADRs appear to underreact to innovations in the underlying security and exchange rate, but they overreact to innovations in the U.S. market. That is, there is also some empirical evidence against the efficiency of the ADR market.

Methodology

We test first for linear dependence by means of the variance ratio of Lo and MacKinlay (1988). Following, we test for nonlinearity with diagnostic tests based on the correlation dimension, CD, of Grassberger and Procaccia (1983) and the BDS statistic of Brock, Dechert, and Scheinkman (1987). We also test for heteroskedasticity and low deterministic chaos in the data. In testing for heteroskedasticity, we employ the EGARCH model (Engle, 1982; Hsieh, 1991). The test of chaos is based on the locally weighted regression, LWR (Diebold and Nason, 1990; LeBaron, 1988).

The Variance Ratio Test

The variance ratio of Lo and MacKinlay (1988) is a test of random walk. Basically, if the returns of a time series Y_t are a pure random walk then, the variance of the q-differences grows proportionally with the difference q. The variance ratio is defined as:

$$VR(q) = \sigma^2(q)/\sigma^2(1)$$

Where $\sigma^2(q)$ is 1/q the variance of the q-differences and $\sigma^2(1)$ is the variance of the first differences (q = 1).

The formulas for calculating $\sigma^2(q)$ and $\sigma^2(1)$ are the following (Lo and MacKinlay, 1988):

$$\sigma^2(q) = 1/m\ \Sigma\ (Y_t - Y_{t-1} - q\mu)^2 \text{ from } t = q \text{ to } t = nq$$

$$\sigma^2(1) = 1/(nq - 1)\ \Sigma\ (Y_t - Y_{t-1} - \mu)^2 \text{ from } t = 1 \text{ to } t = nq$$

where:

m = q(nq - q + l) (1 q/nq) and

μ = l/nq(Y_{nq} - Y_0).

Y_{nq} and Y_0 are the last and first observations of the time series, respectively. The test statistic Z(q), robust to heteroskedasticity, is:

$$Z(q) = [VR(q) - 1]/[\phi(q)]^{0.5}$$

where:

$$\phi(q) = \Sigma\ [2(q - j)/q]^2\delta(j) \text{ from } j = 1 \text{ to } j = q\text{-}1 \text{ and}$$

$$\delta(j) = \Sigma(Y_t - Y_{t-1} - \mu)^2\ (Y_{t-j-1} - Y\text{-}j\text{-}l - \mu)^2/[\ \Sigma(Y_t - Y_{t-1} - \mu)^2]^2.$$

The Grassberger and Procaccia Test

Grassberger and Procaccia (1983) develop the notion of correlation dimension, CD. According to Brock and Sayers (1988) dimension is an indication of the number of nonlinear factors that describe the data. For example, a single point has zero-dimension. A line has one dimension. A solid has three dimensions. A white noise process and a purely random process have infinite dimension. A chaotic system has a positive but finite dimension.

The key in detecting nonlinearities with the Grassberger and Procaccia test is to observe the changes in the correlation dimension as we increase the embedding dimension. If the correlation dimension does not explode with the embedding dimension, then there is evidence of nonlinearity in the data.

The correlation dimension is developed in several steps:

Construct the n-histories of the filtered data in order to obtain the embedding dimension. An n-history is a point in n-dimensional space; n is called the embedding dimension. The n-histories are denoted as follows:

1-history: $x_t^1 = x_t$

2-history: x_t^2 (x_{t-1}, x_t)

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n-history: $x_t^n = (x_{t-n+1}, \ldots, x_t)$.

Grassberger and Procaccia (1983) and Swinney (1985) use a form of correlation integral to define the correlation dimension:

$$C_m(e) = \lim_{T \to \infty} \#\{(t,s), 0 < t, s, < T : \| x_t^m - x_x^m \| < e\}/T^2,$$

where $\| \;\|$ is the sup or max norm. The correlation integral is the fraction of pairs (x_s^m, x_t^m), which are close to each other, in the sense that:

$$\max_{i = 0, \ldots, m-1} \{|x_{s-i} - x_{t-i}|\} < e.$$

Calculate the slope of the graph of log $C_m(e)$ versus log ε for small values of e. More precisely, we want to calculate the following quantity:

$$V_m = \lim_{\varepsilon \to \infty} \log C_m(e)/\log e.$$

If V_m does not increase with m, the data are consistent with chaotic behavior.

The correlation dimension can be used to detect nonlinearity in a time series. In effect, as the correlation dimension is estimated for increasingly larger values of the embedding dimension, if nonlinearity is present, the correlation dimension estimates will stabilize at some value. If this stabilization does not occur, the system is considered high-dimensional or stochastic.

Theoretically $T \to \infty$, but it is actually determined by the number of observations available for the analysis. This places limits on possible values for e and m. For a given m, e cannot be too small because $C_m(e, T)$ will contain too few observations. Also, e cannot be too large because $C_m(e, T)$, will contain too many observations. Barnett and Choi (1989) suggest selecting a small value for e, without allowing it to reach zero, in order to eliminate noise in the data. Hsieh (1989) defines e in terms of multiples of the series standard deviation. These multiples are 1.50, 1.25, 1.00, 0.75, and 0.50.

In this paper we use the program "Chaos Data Analyzer," developed by Sprott and Rowlands (1995). The program automatically chooses the optimal e and leaves two parameters under user control: the embedding dimension D and the parameter n, which is the number of sample intervals over which each pair of points is followed before a new pair is chosen. Essentially, if n is too large, the trajectories get too far apart. If n is too small, the calculation becomes too slow. We report results for embedding dimensions 2 through 10 and n of 1, 2, 4, and 8.

The correlation dimension of Grassberger-Procaccia is essentially a graphical procedure and not a statistical test and has the problem of small sample bias when it is applied to economic and financial data (Scheinkman and LeBaron (1989). Ramsey and Yuan (1989) show that the slope of the Grassberger-Procaccia plots is biased downward in small data sets.

The Brock, Dechert, and Scheinkman BDS Statistics

Brock, Dechert, and Scheinkman (1987) develop the BDS statistical test. Contrary to other tests, the BDS statistic does not try to determine if the data are stochastic or chaotic. In effect, the null hypothesis is that the data are independently and identically distributed, IID, or random walk. Brock, Dechert, and Scheinkman have shown that tests based on their statistic have a higher power for testing for stochastic or chaotic independence than other statistical techniques.

Brock, Dechert, and Scheinkman (1987) demonstrate that under the null hypothesis (x_t) is IID with a nondegenerate density F (also Hsieh, 1989), $C_m(e, T) \rightarrow C_1(e)^m$ with probability equal to unity, as $T \rightarrow \infty$, for any fixed m and e. In addition, they show that $T^{1/2}[C_m(e, T) - C_1(e, T)^m]$ has a normal limiting distribution with zero mean and variance equal to

$$\sigma_m^2(e) = 4\left[K^m + 2\sum_{j=1}^{m-1} K^{m-j}C^{2j} + (m-1)^2 C^{2m} - m^2 KC^{2m-2}\right],$$

where:

$$C = C(e) = \int[F(z+e) - F(z-e)]dF(z),$$

$$K = K(e) = \int\int[F(z+e) - F(z-e)]^2 dF(z).$$

$C_i(e, T)$ is a consistent estimate of C(e), and

$$K(e, t) = \frac{6}{T_m(T_m - 1)(T_{m-2})} \sum_{t<s<r} I_e(s_t, x_s) I_e(x_s, x_r)$$

is a consistent estimate of K(e). Therefore, $\sigma_m(e)$ can be estimated by $\sigma_m(e, T)$, which uses $C_1(e, T)$ and $K(e, T)$ instead of C(e) and K(e). The BDS statistic has a standard normal limiting distribution and is calculated by

$$W_m(e, T) = T^{1/2}\left\lfloor C_m(e, T) - C_1(e, T)^m \right\rfloor / \sigma_m(e, T).$$

Brock, Dechert, and Scheinkman show that, under the null of IID, $W_m \rightarrow N(0, 1)$, as $T \rightarrow \infty$. If the residuals from the estimated linear (nonlinear) model are actually IID, the BDS statistic should be asymptotically N (0, 1). Large values of the BDS statistic indicate the presence of nonlinearity in the data.

It is important to keep in mind that the rejection of the null of IID by the BDS statistic does not necessarily mean the time series exhibits a low complexity chaotic behavior. In effect, rejection of IID can be consistent with any of the following four types of non-IID behavior: linear dependence, nonstationarity, nonlinear stochastic processes (ARCH-types models), and chaos (nonlinear deterministic process). The linear dependence can be easily ruled out by running the BDS test in the filtered data. Nonstationarity can also be ruled out because it can be eliminated by differencing the data. Additional tests are needed to discriminate between nonlinearities due to stochastic behavior and nonlinearities due to the existence of chaotic behavior.

Testing for Conditional Heteroskedasticity

There is growing evidence that stock return volatility is not only time-varying (French, Schwert, and Stambaugh, 1987), but also predictable (Schwert and Seguin, 1990). Our objective here is to investigate whether the conditional heteroskedasticity captured by ARCH-type models is responsible for nonlinearity in the ADR returns.

Following Engle (1982) and Hsieh (1991), we proceed to consider whether ADR returns are nonlinear in variance. Consider the equation: $X_t = g\,(x_{t-1}, \ldots ,)\varepsilon_t$. This is a general model of conditional heteroskedasticity, which includes ARCH-type models as special cases. Let us take the absolute values of the above equation:

$$|x_t| = |g(x_{t-1}, \ldots)|\ |\varepsilon_t|$$

If g ( ) is differentiable, a Taylor series expansion would yield the result that $|x_t|$ depends on $|x_{t-1}|$. Thus, if we compute the autocorrelation of the absolute valued data, the finding of correlation of $|x_t|$ with $|x_{t-1}|$ is evidence of conditional heteroskedasticity.

ARCH and GARCH impose restrictions on the signs of the parameters to guarantee that estimated variances are positive, which creates numerical problems associated with constrained optimization. The EGARCH model does not impose these restrictions. In addition, EGARCH can accommodate conditional skewness (Hsieh, 1991). Because of these nice statistical properties we have decided to use the following EGARCH model in determining whether ARCH-type models capture the nonlinear dependence of ADR returns:

$$x_t \sim N(0,\ \sigma_t^2)$$

$$\log \sigma_t^2 = \phi_0 + \phi \left| x_{t-1} / \sigma_{t-1} \right| + \Psi \log \sigma_{t-1}^2 + \gamma\, x_{t-1} / \sigma_{t-1}\,.$$

If the EGARCH model is correctly specified, then the standardized residuals $Z_t = x_t/\sigma_t$, should be IID in large samples. In other words, the BDS statistic can be applied to the standardized residuals to test if EGARCH captures all nonlinearities present in the ADR returns.

Testing for Low Deterministic Chaotic Behavior

Chaos is a nonlinear deterministic process that looks random (Hsieh, 1991 and 1993). In chaotic growth models the economy follows dynamic processes that are self-generating and never die down. This is different from the Box-Jenkins times-series models, under which the economy is in equilibrium but constantly is perturbed by external shocks. Chaotic dynamics is nonlinear. This is appealing because nonlinear models can generate rich types of behavior.

Chaos theory provides a set of diagnostic tools to distinguish underlying structures that appear random or unpredictable under traditional analysis (but are nonlinear deterministic processes) and underlying structures that contain stochastic components. Thus, low-level chaotic behavior is differentiated from random behavior by the presence of identifiable nonlinear structures.

Chaos theory has been applied to economic data. Brock and Sayers (1988) test for nonlinearities in quarterly data on U.S. real gross national product, GNP, and U.S. real gross private domestic investment, GPDI. These authors calculate the correlation dimension, CD, of Grassberger-Procaccia (1983) and conclude that more data are needed to establish that U.S. real GNP and real GPDI are generated by a low dimension chaotic deterministic dynamical system. They also observe that their results should not discourage new attempts to find evidence of significant nonlinearities and chaos in economic data, mainly because they use aggregate data that can hide nonlinearities at the micro level. The Brock, Dechert, and Scheinkman (1987), BDS, statistic has been used to detect nonlinear dependence in the time series of employment, unemployment, industrial production, and pig iron production.

Empirical nonlinear science has enjoyed not only a boom in economics, but also in finance, with numerous applications to stock returns. The evidence supporting the existence of low level deterministic chaos in equity returns is generally weak. Scheinkman and LeBaron (1989) estimate the correlation dimension for 1226 weekly observations on the CRSP value-weighted U.S. stock returns index starting in the early 1960s. They arrive at a correlation dimension of 6. In general, they find evidence of nonlinear dependence for the weekly value-weighted CRSP index but not for the daily value-weighted CRSP index. Their results suggest that a substantial part of the variation of weekly returns is coming from nonlinearities as opposed to randomness. This result is not compatible with the random walk theory, which predicts that the returns are generated by independent and identically distributed, IID, random variables (Granger and Morgenstern, 1963; and Fama, 1970). They conclude that nonlinearities may play an important role in explaining financial asset returns.

The test of low deterministic chaos employed in this paper is one type of nonparametric regression known as the locally weighted regression, LWR (Diebold

and Nason, 1990; LeBaron, 1988). The LWR can be briefly described as follows: suppose the time series data are generated according to:
$X_{t+1} = f(x_t)$.

We have observed x, x_{t-1}, … , and would like to forecast x_{t+1}. Locally weighted regression uses the k nearest neighbors of x_t and a scheme that gives more weight to closer observations and less weight to farther observations. Several parameters must be selected: (a) the number of nearest neighbors to use (we try 10 percent of all observations, up to 90 percent, increasing in steps of 10 percent); (b) the number of lags of x_t to include as arguments of the unknown function f(x) (we use lags 1 through 5); (c) the weighting scheme (we use the tricubic scheme proposed by Cleveland and Devlin, 1988). Following Hsieh (1991, 1993), if returns are governed by low complexity chaos, we should be able to use LWR to forecast returns better than with simple methods, such as the random walk model.

Data

The data set consists of a portfolio of daily returns, including dividends, of ADRs traded on the NYSE, AMEX, and Nasdaq. The data are extracted from the Center for Research in Security Prices (CRSP) daily tape of the University of Chicago and cover the period from January 1, 1984 to December 31, 1998. The number of ADRs in the sample varies from 52 in 1984 to 440 in 1998.

Table 1—Summary Statistics of Daily Raw Returns of the ADR Portfolio

	ADR Portfolio
Number of Observations	3915
Mean	0.00073
Standard Deviation	0.00842
Skewness	-1.40866
Kurtosis	23.92729

Analysis of Empirical Findings

In this section we analyze the empirical results of the several tests of linearity, nonlinearity, and chaos employed in the paper

Table 2—Variance Ratio Test for the Daily Raw Returns of the ADR Portfolio

q-Differences	Variance Ratio: VR(q)	Test Statistic: Z(q)
5	2.0875	8.67*
10	2.3401	734*
20	2.7868	797*
60	3.2582	7.67*
120	3.0471	5.98*
240	2.5000	3.75*
480	1.9524	1.98*

The null hypothesis is that the VR(q) is equal to one (the time series follows a random walk)
An * indicates that the variance ratio is statistically significantly different from one at the 5 percent significance level (rejection of the null hypothesis of random walk)

Table 2 reports the results of the variance ratio test. Variance ratios are reported for the intervals q = 5, 10, 20, 60, 120, 240, and 480 days. It can be observed that the null hypothesis of random walk is rejected for all values of q. In effect, all the variance ratios are statistically significantly larger than one. Variance ratios larger than one imply that the variances grow more than proportionally with time. Because the Z-statistic is robust to heteroskedasticity, we can safely conclude that the rejection of the random walk is due to autocorrelation in the returns. That is, the variance ratio test strongly rejects the null hypothesis of random walk, suggesting the presence of a linear trend in the ADR raw returns.

We remove any linear dependence by filtering the data. We fit an autoregressive model to the transformed series and compute the correlation dimension, CD, of Grassberger and Procaccia and the BDS statistic of Brock, Dechert, and Scheinkman on the residuals of the filtered data. The results of the CD and the DBS for the filtered data are reported in Table 3, Panels A and B, respectively. It can be observed that the correlation dimension increases with the embedding dimension but does not explode. Thus, the correlation dimension indicates the presence of nonlinearity in the ADR returns. In addition, the more powerful BDS statistics also strongly reject the null of IID for the ADR portfolio.

Table 3—Daily Filtered Returns Correlation Dimensions and the BDS Statistics at Embedding Dimensions 2 through 10 and N Equaling 1, 2, 4, and 8 for the ADR Portfolio

Panel A: Correlation Dimensions

	Embedding Dimension								
n	2	3	4	5	6	7	8	9	10
1	1.969	2.513	3.111	3.679	3.917	4.270	4.764	5.211	5.613
2	1.935	2.584	3.060	3.644	3963	4.602	4.814	5.193	5.554
4	2.010	2.482	2.921	3.585	4.053	4.290	4.563	5.043	5.507
8	1.922	2.632	2.916	3.457	4.005	4.388	4.857	5.185	5.516

Panel B: BDS Statistics

	Embedding Dimension								
n	2	3	4	5	6	7	8	9	10
1	1.767	4.227	5.982	6.148	5.274	4.325	3.509	2.763	2.101
2	3.086	5.208	6.074	6.167	5.372	4.541	3.601	2.793	2.121
4	1.853	4.029	5.647	6.107	5.511	4.518	3.578	2.801	2.145
8	2.793	5.461	6.290	6.409	5.770	4.766	3.843	2.972	2.248

Because our data set covers a relatively long period of time, it would be difficult to argue that the behavior of ADR stock prices has remained unchanged during this extended time period. Changes in economic fundamentals, changes in volatility of financial markets, etc., can cause nonstationary in the price time series. Nonstationary in prices does not necessarily imply nonstationarity in returns. In effect, the augmented Dickey and Fuller (1979) tests (not reported here for the sake of space) indicate that the ADR daily return time series are stationary. Therefore, we can rule out nonstationarity as the cause of rejection of IID returns.

Nonlinearity can be stochastic or chaotic. The autocorrelation coefficients and the Q-statistics, reported in Table 4, indicate the presence of heteroskedasticity in the time series of ADR returns. Therefore, we must perform additional tests in the data in order to discriminate between stochastic nonlinearity (i.e., nonlinearity due to heteroskedasticity) and chaotic nonlinearity (nonlinearity due to chaos).

Table 4—Autocorrelation Coefficients and Q-statistics, Absolute Value of Daily Raw Returns of the ADR Portfolio

Lag	Q	Prob > Q
1	295.56	0.0000
2	403.04	0.0000
3	464.74	0.0000
4	563.13	0.0000
5	624.30	0.0000
6	723.83	0.0000
7	842.15	0.0000
8	918.33	0.0000
9	961.55	0.0000
10	990.00	0.0000

In a correctly specified EGARCH model the standardized residuals will be IID. Table 5 presents the CD and BDS statistic results when applied to the standardized residuals from the EGARCH model. Again, the CDs do not explode with increments in the embedding dimension, and the BDS statistic rejects the null of IID. That is, the CD and the BDS detect the presence of nonlinearity in the data, even after controlling for heteroskedasticity. These results allow us to conclude that conditional heteroskedasticity is not the cause of nonlinearity in the time series returns of the ADR portfolio.

Table 5—EGARCH Standardized Residual Correlation Dimensions and BDS Statistics, at Embedding Dimensions 2 through 10 and n Equaling 1, 2, 4, and 8, for the Daily Filtered Returns of the ADR Portfolio

Panel A: Correlation Dimensions

	Embedding Dimension								
n	2	3	4	5	6	7	8	9	10
1	2.076	3.065	3.874	4.548	5.113	5.527	5.923	6.294	6.280
2	2.054	3.048	3.864	4.534	5.120	5.546	5.891	6.249	6.238
4	2.068	3.062	3.875	4.532	5.149	5.566	5.896	6.304	6.250
8	2.074	3.056	3.873	4.529	5.137	5.550	5.943	6.323	6.330

Panel B: BDS Statistics

	Embedding Dimension								
n	2	3	4	5	6	7	8	9	10
1	3.640	8.414	12.638	15.336	16.491	16.465	15.609	14.300	12.819
2	3.608	8.303	12.415	15.076	16.284	16.281	15.475	14.221	12.778
4	3.569	8.219	12.228	14.944	16.192	16.277	15.528	14.295	12.849
8	3.725	8.650	13.015	15.870	17.208	17.237	16.413	15.104	13.588

Having ruling out heteroskedasticity as the cause of nonlinearity in ADR returns, we proceed to test for chaos by means of the locally weighted regression, LWR (Diebold and Nason, 1990; LeBaron, 1988). As we indicated in the methodology section, if stock returns are governed by low complexity chaotic behavior, we should be able to use LWR to forecast returns better than the random walk model (Hsieh, 1991). We measured forecastability by the mean squared errors of the LWR. Table 6 indicates that the mean squared errors obtained from the LWR are smaller than those of the random walk. Thus, the results from the locally weighted regression suggest that the existence of nonlinearity in the time series of ADR returns is due to the presence of low deterministic chaos. This is an important result because most previous research has failed to report empirical evidence that stock returns exhibit chaotic behavior.

In this paper we do not conduct test of nonlinearity and chaos for the American stock market as a whole, because such testing has already been done and reported elsewhere (Hsieh, 1991, 1993; Brock and Sayers, 1988; Scheinkman and LeBaron, 1989). Given that previous research has reported only weak evidence of nonlinearity and low deterministic chaos for the U.S. market as a whole, our findings suggest that the ADR returns may have a time series behavior that differs from that of the market in general. We argue that this different behavior is due to the unique characteristics of ADRs.

Table 6—Root Mean Squared Forecast Errors for the Daily Filtered Returns of the ADRs Portfolio

Fraction	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
.10	0.0082915	0.0084263	0.0082945	0.0084199	0.0084304
.20	0.0083073	0.0084298	0.0083106	0.0084316	0.0084427
.30	0.0083132	0.0084330	0.0083182	0.0084367	0.0084464
.40	0.0083198	0.0084347	0.0083255	0.0084421	0.0084475
.50	0.0083233	0.0084356	0.0083314	0.00844448	0.0084496
.60	0.0083263	0.0084375	0.0083379	0.0084452	0.0084507
.70	0.0083279	0.0084394	0.0083419	0.0084456	0.0084517
.80	0.0083304	0.0084398	0.0083447	0.0084456	0.0084509
.90	0.0083335	0.0084407	0.0083484	0.0084461	0.0084519
Random Walk	0.0112487	0.0112487	0.0112487	0.0112487	0.0112487

Errors are obtained by running locally weighted regressions with tricubic weights, lags 1 through 5, and the number of nearest neighbors equaling the fractions 0.1 through 0.9 of the data. The root mean squared forecast error of the random walk model is also provided

Summary and Conclusions

The objective of this paper is to find predictable patterns in the time series of ADR returns. The motivation of our research is that the unique characteristics of ADRs may make this sector exhibit a time series behavior that is different from that of the U.S. capital market as a whole. We empirically investigate linear and nonlinear trends and low deterministic chaos in the time series of a portfolio of ADR returns. The data correspond to daily returns covering the January 1, 1984 to December 31, 1998 time period.

Our methodology consists of

- A variance ratio test of linear dependence;
- Tests of nonlinearity based on the correlation dimension of Grassberger and Procaccia and the BDS statistic of Brock, Dechert and Scheinkman;
- The EGARCH model test of heteroskedasticity; and,
- The locally weighted regression test of chaos.

The results of the variance ratio test suggest that the time series of ADR returns exhibit linear trends.

The correlation dimension and the BDS statistic indicate the presence of nonlinearity in the filtered data.

Because nonlinearity in the data can be due to either stochastic or chaotic behavior, we also run tests to discriminate between these two processes. We provide empirical evidence that ADR stock return volatility is time varying. In order to investigate whether conditional heteroskedasticity is responsible for the nonlinearity in ADR returns, we apply the BDS statistic to the standardized residuals of the EGARCH model. The BDS statistic rejects the null of IID, indicating that conditional heteroskedasticity is not the cause of the presence of nonlinearity in the time series returns of the ADR portfolios.

In testing for chaotic behavior we employ the nonparametric regression known as locally weighted regression (LWR). If stock returns are governed by low complexity chaos, we should be able to use the LWR to forecast returns better than the random walk model. We measure forecastability by the mean squared errors of the LWR. Our results indicate that the mean squared errors obtained from the LWR are smaller than those of the random walk. Thus, the locally weighted regression suggests that the existence of nonlinearity in the ADR returns is due to the presence of low deterministic chaos. This finding differs from previous research, which has failed to report empirical evidence that the time series of stock returns exhibit low complexity chaotic behavior.

We claim the following important contributions of this paper to the literature of empirical analysis of time series and prediction of stock returns. First, the findings of statistically significant evidence of non-linearity and low deterministic chaotic behavior in the ADR returns suggest that the time series of ADRs returns behaves differently from the U.S. equity market as a whole. This can be due to the unique characteristics of these financial instruments. Second, these results are important because knowing that returns of some sectors of the capital market exhibit chaotic behavior can help us to understand the market better and find ways of predicting returns. Third, our results have important practical implications because they suggest that pricing forecasting models for ADRs should include some nonlinear terms.

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