An Econometric Investigation of the Volatility and Market Efficiency of the U.S. Small Cap 600 Stock Index

Godwin Onyeaso*
Concordia College/University System

Michael Rogers
Albany State University

Using the framework of econometric models, this paper investigates three research questions and finds positive answers. First, is the volatility of Small Cap 600 predictable? Second, does the volatility of Small Cap 600 exhibit the same empirical regularities reported in the literature about the behavior of other stock prices? Finally, can Small Cap 600 pass a test of market efficiency? These research results will benefit the following entities: (1) economic agents investing in Small Cap 600, (2) business professors interested in Small Cap 600 for teaching and research, (3) policy makers who are watching the stock market volatilities because the volatilities of the stock markets dampen investors' incentive to invest, among other consequences.

Introduction

There are two constants in business–change and risk–that are manifest in financial asset volatility. Financial assets volatility is now predictable due to the groundbreaking work of Engle (1982) which gave birth to ARCH models capable of predicting the hitherto unpredictable heteroskedastic residuals from the mean equation (Engle, 1991). Consequently, a critical question emerges: Is empirical study of financial asset volatility important? If so, which entities will perceive it as important?

The study of financial assets volatility is important to academics, policy makers, and financial market participants for several reasons. First, prediction of financial assets volatility is critical to economic agents because it helps them make rational portfolio risk diversification, risk reduction, and management decisions. Volatility is critically important to economic agents because it represents a measure of risk exposure in their investments. Second, a volatile stock market is an unstable stock market.

* We thank S&P management for the free release of data and the editor and referees for helpful comments. Financial support from Dr. Jenkins, president of Concordia College, Selma, Alabama, is acknowledged.

0747-5535/04/1400 & 1500/000139/$2.50

And an unstable stock market is a serious concern of policy makers because instability of the stock market impacts the U.S. economy negatively (Greenspan, 1997). A recent proclamation states that when markets are perceived as highly volatile the perceived volatility "may act as a potential barrier to investing" (Poshakwale and Murinde, 2001, p. 445). Pindyck (1984) argues that the fall of stock prices in the U.S. in 1970s can be explained by increases in risk premium associated with a rise in stock volatility.

Third, the negative impact of stock market volatility has attracted the attention of many scholars. For example, Garner (1990) found that the stock market crash in 1987 brought a reduction in the consumer spending in the U.S. Similarly, Maskus (1990) found that foreign exchange market volatility impacts trade. Fourth, from a theoretical perspective, volatility occupies a center stage in pricing of derivative securities. For example, key strands of the Black-Scholes formula indicate that the pricing of a U.S. call option is a function of volatility. Therefore, in practical terms, option markets can be characterized as a market where economic agents trade volatility. Finally, stock return forecasting is in a sense volatility forecasting, and this is a huge industry in the U.S. where econometric models of volatility forecasting are diverse. In this respect, some scholars (e.g., Yu, 2002) believe that empirical research in modeling volatility may in fact result in the reduction of confidence intervals in forecasting time-varying confidence intervals of stock return series. If this outcome is tenable, the science of forecast accuracy shall improve. In conclusion, the preceding review clearly indicates that stock market volatility is a worthwhile topic of practical and academic interests.

Consequently, more theories are emerging about the importance of financial assets volatility, and this time from industry practitioners. In particular, investment and finance industry analysts postulate that economic agents perceive small capitalization stock indices to be less volatile than large capitalization stock indices.[1] For example, *The InvestMentor* (June 16, 1997) discusses this phenomenon and concludes that it is an unresolved myth. Among all small capitalization stock indices, Small Cap (SC) 600 is particularly popular, according to industry observers. This small cap stock index is owned and managed by the *Standard & Poor's Corporation* who believes that SC 600 is an important member of the universe of small caps. Because SC 600 is a subset of the entire population of small cap indices, those who invest in SC 600 are therefore captives to the same unresolved myth stated above. Therefore, we argue that the volatility of SC 600 beckons for empirical scrutiny. First, it is plausible that the absence of empirical research about the behavior of SC 600 may have contributed to this unresolved myth in investors' perceptions. Hence, there is a need for empirical research to inform investors about the underlying statis-

[1] Standard & Poor's started collecting SC 600 stock index data in January 1995. In order to have a larger sample, non-trading (public holidays) are replaced by the average of trading day values to increase the sample size.

tical behavior of SC 600. Second, it is important to examine whether the statistical characterizations of SC 600 in terms of its volatility are different from observed regularities of stock prices in general. We also believe that such empirical investigation will be a necessary prelude to subsequent research into the perceptual myth of investors. To the best of our knowledge, no research has done this. We conclude that this research void is a gap in the current knowledge of volatility dynamics and prediction. To this end, at least three research questions spring to mind: (1) Are the volatilities of SC 600 predictable? (2) Do the volatilities of SC 600 exhibit the same empirical regularities in the behavior of other stock prices? (3) Can SC 600 pass the strict form test of market efficiency?

The extant financial econometric literature classifies these empirical regularities into two broad, related categories: (1) asymmetric or leverage effect and, (2) leptokurtosis or fat-tail distribution. While we do not present a detailed discussion of these, excellent reviews and discussion are available in the literature (Bera and Higgins, 1993; Pagan, 1996; Bollerslev et al., 1994). First, the distribution of stock prices is highly asymmetric. Negative returns (bad news) are followed by larger increases in stock price volatility than positive returns (good news) of the same size. The landmark study by Black (1976) reports this phenomenon, which is now commonly dubbed the *leverage effect*. In a nutshell, the leverage hypothesis asserts that as stock prices fall, the debt-to-equity ratio of firms affected by the stock price fall will tend to rise. Consequently, rising debt-to-equity ratio will increase the volatility of returns to equity holders. Corroborating Black (1976), Christie (1982) and Schwert (1989) report the same phenomenon. Black indicates, however, that financial leverage effect alone cannot empirically explain the magnitude of the asymmetry he observed. In that spirit, some scholars reason that the leverage effect (asymmetry) may stem from the feedback from volatility to stock price as changes in volatility instigate changes in risk premiums (Campbell and Hentschel, 1992; Wu, 2001; French et al., 1987).

The second empirical regularity relates to leptokurtosis or fat-tailed distribution of stock prices. That is, the kurtosis of stock price distribution is greater than that of a Gaussian distribution. The seminal studies in this area are those done by Mandelbrot (1963) and Fama (1965); both report fat-tails of stock price distributions. This phenomenon makes U.S. government policy makers uneasy (Greenspan, 1997, p. 54.); it makes researchers uneasy (Bera and Higgins, 1993, p. 335); and it poses a challenge to econometricians and statisticians (McAleer and Oxley, 2002). Even though the leptokurtosis can be marginally reduced when stock returns series are normalized, the problem of how to reduce the kurtosis to the level of a normal distribution framework still poses a challenge to researchers.

Finally, in general, financial asset returns may indicate zero autocorrelation even though their squared values typically indicate serial dependence, thereby suggesting the presence of nonlinear dependencies in the lagged values of the returns—so-called volatility clustering. Volatility clustering (or temporal variations) is a significant factor in the failure of the empirical distributions of the return series to follow the Gaussian distribution (Fornari and Mele, 1996). Equivalently, the empirical distribution of financial asset returns exhibits non-Gaussian distribution characteristics such as leptokurtosis as well as negative and positive skewness. Even though these empirical regularities are reported in studies focusing on large stock indexes such as the S&P 500 (Schwert, 1989; Bollerslev et al., 1988), the extent the same empirical regularities are prevalent in SC 600 index is not known. Our paper attempts to answer the following testable hypotheses stated in the null

1. Hypothesis 1: The volatility of SC 600 is not predictable.
2. Hypothesis 2: SC 600 does not exhibit the same empirical regularities observed in the behavior of other stock prices.
3. Hypothesis 3: SC 600 cannot pass the strict form test of market efficiency.

Data and Preliminary Empirical Results

We gather data on daily closing prices of the Small Cap (SC) 600 stock price index from January 3, 1995 to August 19, 2002. (The sample size is dictated by data availability, and data are generously supplied by *Standard & Poor's Corporation)*. We follow other scholars (e.g., Engle and Patton, 2001; Park, 2002) to transform the return series into their log-differences computed as $\log(p_t) - \log(p_t(-1))$, t = 1, 2, 3, …, T, yielding 1989 trading days. This procedure affords two advantages. First, it eliminates the possible dependence of changes in stock price index on the price level of the index. Second, the change in the log of the stock price index yields continuously compounded series. We graph the series for visual evaluation, and we observe extreme values, temporal clustering, and fat-tail (leptokurtosis), as shown in Figure 1 to 3 below. The dynamism of volatility clustering is visually evident in Figures 2 and 3. The descriptive statistics in Table 1 tell the same tale. The mean of the series is extremely small, close to zero (0.0004), and the unconditional standard deviation as measure of variation is quite small (0.01). This finding suggests the absence of non-synchronous (thin) trading during the sample period. Ruling out non-synchronous trading, the observed small variation may be due to some form of market inefficiency.

The series is negatively skewed (-0.26) with excess kurtosis more than twice the kurtosis for a Gaussian distribution. In sum, the series is highly non-normal (asymmetric) as confirmed by Jarque-Bera[2] test for normality. In other words, the null of normality is strongly rejected, as the evidence in Table 1 suggests. Finally, the preceding empirical results corroborate numerous studies on stock price behavior.

[2] The B-J test statistic is $T[\text{skewness}^2/6 + (\text{kurtosis} - 3)^2/24]$.

Figure 1—Graph of the Series

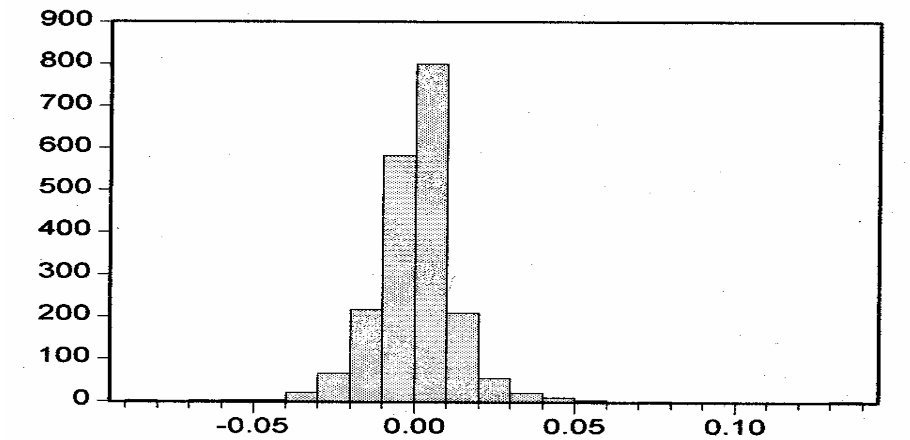


Figure 2—Conditional Volatility

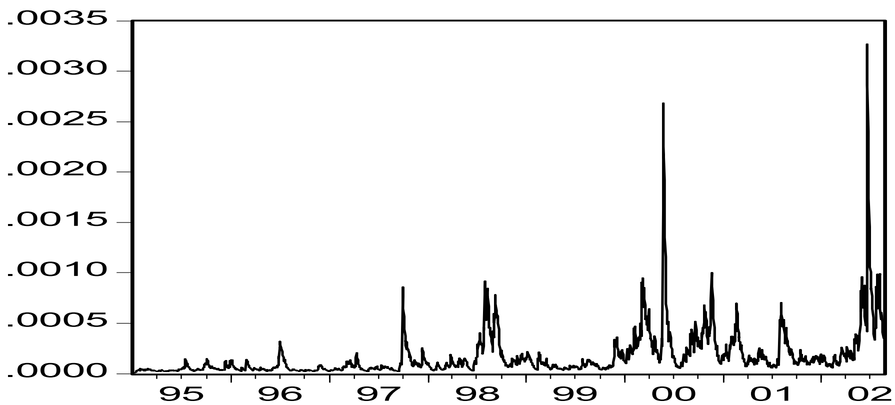
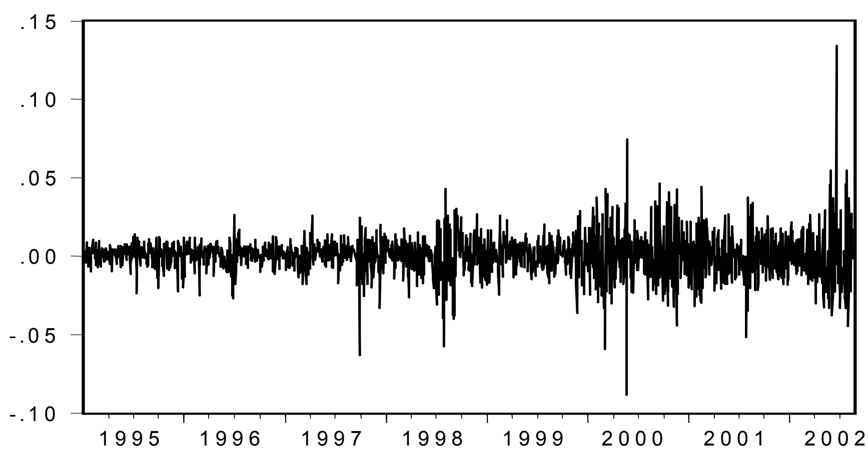


Figure 3—The Trajectory of Volatility of Returns



Therefore, hypothesis 2 is rejected. Equivalently, as our hypotheses are stated in the null, a rejection of the null means that SC 600 exhibits the same observed regularities as other stock prices and stock price indexes. Finally, even though this preliminary empirical evidence gives justification for ARCH modeling for our data set, we nonetheless provide additional justification for ARCH modeling following the suggestions by Engle and Ng (1991).

Table 1—Descriptive Statistics

Series	RS
Sample	1/04/1995 to 8/19/2002
Observations	1989
Mean	0.000403
Median	0.000863
Maximum	0.134584
Minimum	-0.088757
Standard Deviation	0.012470
Skewness	0.424272
Kurtosis	13.41976
Jarque-Bera	9057.531
Probability	0.000000

Additional Justification for ARCH Modeling

Both the empirical literature on ARCH modeling procedures (Bollerslev et al., 1994) and recent reviews of ARCH models (Bera and Higgin, 1993; Bollerslev et al., 1988) provide support indicating that ARCH modeling is appropriate for the present paper. For example, Bera and Higgins (1993, p. 318) declare that “leptokurtosis in the unconditional distribution is a characteristic of conditional heteroskedasticity data.” This declaration by Bera and Higgins points to the evidence shown in Figure 1 and Table 1 above. Second, stock index returns are notoriously known for positive autocorrelation at high frequencies (Lo and MacKinlay, 1988; Cutler et al., 1991; Fama, 1965) which includes daily frequencies for the present paper. The data for the present paper satisfy this condition. Third, one of the empirical regularities discussed above about stock return distributions is autocorrelation in the raw series and their squares. Autocorrelation in the squares of the raw series is indicative of volatility clustering (temporal variation) in the heteroskedasitic second moment of the return series. It is common practice to take these features as evidence in support of the fact that an ARCH model will fit the data set of interest.

To address the preceding concerns related to autocorrelation, we test for autocorrelation in the raw returns and their squares. We reject the null of no autocorrelation in both the raw returns and their squares using Ljung-Box (L-B) Q-statistics. We compute Ljung-Box Q-statistics for 36 lags (we report 10 lags) for both raw returns and their squares to test for linear and nonlinear dependencies, respectively. We rejected the null of no linear dependencies in the returns and no nonlinear dependencies in their squares. The results are shown in Table 2 below. All the lags are significant, and the squares are evidently larger. Again, linear dependencies may be due to some form of market imperfections, as non-synchronous trading is ruled because of the unconditional standard deviation discussed above. Furthermore, nonlinear dependencies are usually attributed to the presence of autoregressive conditional heteroskedasticity (i.e., ARCH) suggesting that ARCH type modeling is necessary (Nelson, 1991). Finally, the clustering present in the squared returns suggests that an ARCH type parameterization will approximate the structure of the heteroskedastic second moment, and that is exactly what ARCH models are designed to accomplish (Engle, 2002).

Table 2—Sample Autocorrelation

	1	2	3	4	5	6	7	8	9	10
LG	0.086	-0.047	0.027	0.024	0.011	0.030	0.012	0.036	-0.004	-0.045
	(14.9)	(19.3)	(20.7)	(21.9)	(22.1)	(23.8)	(24.2)	(26.7)	(26.8)	(30.8)
LG2	0.112	0.082	0.098	0.094	0.057	0.075	0.057	0.070	0.011	0.056
	(29.5)	(42.8)	(61.9)	(79.4)	(85.8)	(97.2)	(104)	(113)	(140)	(146)

LG and LG2 are lags of the raw returns and their squares, respectively. All are significant at P = 0.000 level or better. (.) are Ljung-Box Q-statistics values

Finally, some scholars propose that a statistical test should first confirm the presence of an ARCH effect in the series rather than impose an ARCH type model on the data (Engle and Ng, 1991; Breusch and Pagan, 1979). We shall call this approach *ex-ante test* for ARCH effect. To this end, we use a framework proposed by Breusch and Pagan (1979) and discussed in Wooldridge (2003). Specifically,

$$RS_t = C + RS_{t-1} + U_t \tag{1}$$

where RS_t is the raw returns, C is a constant, RS_{t-1} is a one day lag of the raw returns and U_t is the error of the OLS framework. The results are in panel A in Table 3. Next, our purpose is collect U_t and fit the following regression

$$\hat{u}_t^2 = c + RS_{t-1} + \hat{e}_t \tag{2}$$

where $\hat{u}_t^2$ is the square of the residual from equation (1) and is regressed on a constant and one lag of raw returns. The results are in panel B in Table 3. Finally, we fit

$$\hat{u}_t^2 = c + \hat{u}_{t-1}^2 + \hat{e}_t \tag{3}$$

where $\hat{u}_t^2$, one period lag of ($\hat{u}_{t-1}^2$) and C are as defined in equation (2) above. We report the results in panel C in Table 3.

Drawing insights from Wooldridge (2003) to interpret the outcome of Breusch and Pagan (B-P) tests in Table 3, the results are striking in key respects. First, the t-statistic (-4.7) on the lagged return in panel B suggests strong evidence of heteroskedasticity in the returns series. Second, the negative coefficient (-0.005) can be interpreted as follows. The volatility of SC 600 is higher when the previous return is

Table 3—Results of Ex-Ante of ARCH Effect Proposed by Breusch and Pagan (1979)

Panel A: Model (1)				
Variable	Coefficient	T-Statistics	P-Value	
C	0.0003 (0.0002)	1.33	0.18	
RS_{t-1}	0.09 (0.022)	3.9	0.0001	
Panel B: Model (2)				
Variable	Coefficient	T-Statistics	P-Value	
C	0.0001 (1.23E-05)	12.7	0.0000	
RS_{t-1}	-0.005 (0.0009)	-4.7	0.0000	
Panel C: Model (3)				
Variable	Coefficient	T-Statistics	P-Value	
C	0.0001 (1.27E-05)	10.4	0.0000	
$\hat{u}_t^2$	0.15 (0.0222)	6.6	0.0000	
Panel D: Model (4)				
Variable	Coefficient	$\hat{\rho}_t$ t	P-Value	R^2
$\hat{u}_t^2$	0.0047 (0.022)	0.21	0.83	0.0000

(.) denotes standard errors, the coefficient in panel D is $\hat{\rho}$ coefficient (autocorrelation coefficient) and t $\hat{\rho}$ is the autocorrelation coefficient t-statistic

low, and vice versa (cf. Wooldridge, 2003, p. 415). Therefore, this finding corroborates a part of the reported regularities about the volatility of stock price returns discussed in previous sections of the present paper (cf. Wooldridge, 2003, p. 415). Third, this finding corroborates abundant studies in the finance literature indicating that the expected value of stock returns is not a function of past return values but a function of the variance of past returns. Equivalently, in making their investment decisions, rational economic agents would evaluate the variance of returns in their investment decisions and not the expected (mean) value of the returns. The variance of returns is far more a critical factor in investment decisions than are the expected (mean) returns.

Even though these results are interesting in their own right, our main purpose is the *ex-ante* test for an ARCH effect. To this end, we turn to panel C in Table 3. The t-statistic (t = 6.6) on one period lag of the error indicates an ARCH effect (cf. Wooldridge, 2003, p. 417). Finally, following Wooldridge (2003) we use the preceding framework to test the market efficiency of SC 600 stock index by regressing $\hat{u}_t$ on $\hat{u}_{t-1}$ as stated in equation (3) above. The results are reported in panel D in Table 3. The efficient market hypothesis (EMH) interpretation of this result stems from the finding that the OLS residuals squared are autocorrelated, pointing to heteroskedasticity of the second moment. But the OLS residuals (not squared) are not autocorrelated. These results suggest that an investment strategy based on historic information in the returns series is worthless. In other words, this is the strict form of the EMH test (Fama, 1965) in the sense that information impounded in past stock prices is useless in predicting current and future prices for profit exploitations.

Finally, a second way to test for an ARCH effect is to fit an ARCH type model on the data of interest and test whether there is any remaining ARCH effect in the model estimated. We shall call this approach the *ex-post* test for the presence of ARCH effect. This is the most common approach in the extant financial econometric literature where the Lagrange multiplier (LM) test statistic has become the work-horse (e.g., Bollerslev et al., 1994). These results are reported under the ARCH models presented below.

Although we have shown evidence justifying ARCH models for the present paper, we cannot proceed without filtering the autocorrelation reported in Table 2. Autocorrelation renders stationary series non-stationary, as demonstrated by Bera and Higgins (1993, p. 234). Typically, a moving average of order one [i.e., MA (1)] has been found adequate to purge autocorrelations of this magnitude (cf. Alles and Murray, 2001). Hence, MA (1) is fit on the raw returns in the framework of model (4). That is,

$$R_t = S_t + \delta X_{t-1} \tag{4}$$

Next, let $\hat{\varepsilon}$ be an estimate of the deviations of the raw returns from a MA (1) of expected (mean) return. This quantity is input into the ARCH models discussed below.

ARCH Models

Because ARCH models are a family of models, we experiment and find that a generalized ARCH (i.e., GARCH) is the best parsimonious model characterizing the data-generating process of SC 600 for the following reasons. First, a GARCH model is an infinite order ARCH model (Bollerslev, 1986). Second, a GARCH model is an ARMA model (Engle, 2002) belonging to model (4) above. Finally, our experiment[3] suggested ARMA (0,1)-GARCH (1,1) model as a lower order of the higher order type shown in equations (5) to (7) below, (cf. Bollerslev, 1986). That is:

$$r_t = \mu + \sum_{i=1}^{p} a_i r_{t-1} + \sum_{i=1}^{q} b_i \varepsilon_{t-1} + \varepsilon_t \tag{5}$$

$$\varepsilon_t = z_t h_t \approx N(0,1) \tag{6}$$

$$h_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^{q} \beta_i h_{t-1}^2 \tag{7}$$

where β_i, a_i, b_i, ω, a_i, and μ are real constants. It typically is assumed that the mean process in equation (5) is linear and the disturbances are innovations following the Gaussian distribution. Alternative formulations of the conditional variance function

---

[3] Absence of serial correlation in the standardized residuals squared obviates the need to specify a higher order ARCH model. For instance, GARCH (1,2) yields insignificant results with a drop in likelihood ratio value, and BIC confirms this.

in equation (5) exist. Under such alternative formulations, subset restrictions on the parameters of the general structure define special cases as well as ensure finite variance and stationarity (c.f., Bollerslev, 1986). In model (7), q is the number of lagged conditional variances, and p is the number of past sample variances (squared random component of return) (Engle, 1982; Bollerslev, 1986). The hallmark of this symmetric GARCH model is that it incorporates heteroscedasticity into the estimation of volatility and does so parsimoniously. The model is well defined, however, if the coefficients of the infinite autoregressive structure are all non-negative and the roots of the moving average polynomial of the squared innovations lie outside the unit circle. The restriction of the parameter sum, $\Psi = \Sigma_i\alpha_i + \Sigma_i\beta_i < 1$, is necessary to: (1) quantify the degree of volatility persistence from a shock, (2) ensure the stability and covariance stationarity of the error process, and (3) ensure a necessary condition for the existence of a finite unconditional variance. And, the half-life[4] of the shock persistence is $1/2L = [-\mathrm{In}(2)/\mathrm{In}(\Psi)]$. Finally, we estimate a lower order GARCH model in equations (8) through (10); the results are reported in Table 4.

Table 4—ARCH (0,1)-GARCH (1,1) Results

$$rs_t = x_t + \theta x_{t-1} + \varepsilon_t \tag{8}$$

$$\text{where } \varepsilon_t|\Omega_{t-1} \approx N(0,h_t^2) \tag{9}$$

$$\varepsilon_t = \rho_t - \rho\varepsilon_{t-1}$$

$$h_t^2 = a_0 + \alpha_{t-1} + \beta h_{t-1}^2 \tag{10}$$

Mean Equation					Variance Equation				
x	t	θ	t.	a_0	t	α	t	β	t
0.001	5.2	0.16	6.1	1.9E-06	2.7	0.16	6.5	0.836	39.2
[0.000]		[0.026]		[6.9E-07]		[0.025]		[0.021]	
(0.000)		(0.023)		(3.24E-07)		(0.012)		(0.011)	

t is t-ratio; all are significant at $p < 0.01$. [.] and (.) are Bollerslev-Wooldridge robust standard error and covariances and non-robust standard errors, respectively

In the framework of the ARMA(0,1)-GARCH (1.1) model in equations (5) through (7), the mean of the stock index return is a linear function of its time-varying variance, (h_t). Assuming that the error (ε_t) is serially correlated and follows a MA(1) process, the variance (h_t or volatility) is conditioned on the information set Ω impounded in time t-1. Specifically, Ω impounds past conditional variance (volatility) and past squared error terms. Because in Table 4 all the variances are positive, and the constraint $\Psi = \Sigma_i\alpha_i + \Sigma_i\beta_i < 1$ is satisfied, the model has second

[4] This is half of the discrete time it will take the conditional variance to return to the unconditional mean from where it deviated; this also is relevant to the mean-revision concept.

order stationarity (Bollerslev, 1986). The significance of α and β confirm the presence of ARCH and GARCH effects, respectively. There is high volatility persistence indicated by the following argument: $\Psi = \Sigma_i\alpha_i + \Sigma_i\beta_i < 1 = 0.996$. Also, mean-revision is indicated in that a less-than-unity inequality constraint is almost satisfied. These results are among the empirical regularities puzzling to financial econometricians (Bera and Higgins, 1993, p.342).

This is evidence to reject hypothesis 2. Finally, the half-life of volatility persistence is $1/2L = [-\text{In}(2)/\text{In}(\Psi)] = 69$ days. We now turn to the caveats of the standard GARCH model discussed thus far.

The genesis of the caveats inherent in standard GARCH is the measurement of variance in finance as expected squared deviations from a typical location, the first moment. Consequently, a mathematically plausible way to approximate the trajectory of time-varying variance on current conditional variance is to enlist a linear combination of a constant, past conditional variance, past lagged squared errors—and that is symmetric GARCH model (Bollerslev, 1986). The squaring of past errors to circumvent negative variances imposes a symmetric structure with the serious implication that past negative and positive shocks affect current volatility equally. In other words, a symmetric GARCH model fails to capture the leverage effect. Second, a symmetric GARCH model is basically a quadratic specification in the sense implied by the squaring done. It follows that if the shock impact on current returns is more than a quadratic magnitude, the symmetric GARCH model fails. Finally, the extent to which these caveats are manifest in the return-generating mechanism for a particular data set reflects the extent to which inferences and conclusions based on symmetric GARCH models are flawed. In other words, GARCH models with asymmetric capabilities are needed. In response to this need, Engle and Ng (1993) demonstrate that Glosten, Jagannathan, and Runkle (1993) TGARCH model is the best parsimonious asymmetric GARCH model available. Therefore, we first present a brief discussion of the Glosten, Jagannathan, and Runkle model. Then we employ it for the purposes of the present paper.

The appeal of asymmetric GARCH models stems from their ability to capture asymmetries in volatility. The Glosten, Jagannathan, and Runkle model may be explained as follows. Consider extending model (10) above by adding an indicator variable, D, such that Dt-1 < 0 if the first lag of the error from the mean equation is negative and zero otherwise. This yields a regime-switching model with zero as the threshold in

$$h_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j-1}^{q} \beta_j h_{t-j}^2 + \gamma \varepsilon_t^2 d_{t-1} \tag{11}$$

TGARCH has interesting properties: it permits positive (negative) shocks (ε_t^2) to have a greater impact on subsequent volatility if $\gamma > 0$ (< 0). Notice that good news (increase in asset price) is captured by α alone while bad news while bad news (decrease in asset price) is captured by the sum of $\alpha + \gamma$. If $\gamma > 0$ the leverage effect exists, while bad news (decrease in asset price) is captured by the sum of $\alpha + \gamma$. If $\gamma \neq 0$, the news impact is asymmetric. The vector $(\varepsilon^2, \varepsilon^2)$ of shocks can follow an ARMA process as the GARCH model when the $\{\varepsilon\}$ sequence follows an ARMA process. The results are detailed in Table 5.

Table 5—The TGARCH Test Results

$$rs_t = x_t + \theta x_{t-1} + \varepsilon_t$$

$$\text{where } \varepsilon_t|\Omega_{t-1} \approx N(0,h_t^2)$$

$$\varepsilon_t = \rho_t - \rho\varepsilon_{t-1}$$

$$h_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j-1}^{q} \beta_j h_{t-j}^2 + \gamma\varepsilon_t^2 d_{t-1}$$

	Coefficient	Std. Error		T-Ratio
Panel A: Mean Equation				
X	0.0006	[0.0002]	(0.0006)	2.9
θ	0.174	0.0275	(0.0220)	6.3
Panel B: Variance Equation				
ω	2.06E-06	[7.17E-07]	(2.77E-07)	2.8
α	0.057	[0.030]	(0.0130)	1.93
γ	0.189	[0.044]	(0.0208)	4.3
β	0.845	[0.010]	(0.0104)	80.88

All t-ratios are significant at 1% level or better. [.] and (.) are Bollerslev and Wooldridge robust standard errors and covariance, and ordinary non-robust standard errors, respectively. All reported estimated values are those based on Bollerslev and Wooldridge estimations

The results of the Glosten, Jagannathan, and Runkle's TGARCH model detailed in Table 5 are striking. First, the ARCH term (α) is significant but less than unity, suggesting that shocks to volatility are not explosive. Second, the coefficient for the non-linear dummy is significant and positive. This indicates that: (a) the leverage effect exists, and (b) the news impact is asymmetric, or equivalently, positive innovations have a lower effect on volatility than their negative counterparts. Finally, the GARCH term (β) is significant with a strong effect. Importantly, again, to address the violation of normality assumption, we report two types of stand error: (a) the ordinary standard errors which are not consistent and efficient given the violation of Gaussian distribution assumption, and (b) the Bollerslev-Wooldridge robust standard error and covariance which are consistent and efficient when the normality assumption is violated. One may ask whether these models correctly specified. We present a battery of diagnostic tests to assess model specifications.

A Battery of Diagnostic Tests for Model Specifications

If a model is correctly specified, the residuals must be white noise (i.e., the standardized residuals must have zero mean and unit variance). Recall that in Table 1 above, we rejected the null hypothesis of no autocorrelation in the levels and squares of the raw series. This autocorrelation should not manifest in the standardized residuals and their squares if and only if the models are correctly specified. Specifically, let $\hat{\varepsilon}_t$ and $\hat{h}_t$ be the estimates of the error and conditional variance, respectively, we employ Ljung-Box Q statistics to access the adequacy of the model by examining the standardized (normalized) residuals ($\varepsilon_t / h_t^{1/2}$), and standardized squared residuals $(\varepsilon_t / h_t^{1/2})^2$.

Table 6—Diagnostic Tests for Specifications, Standard GARCH and TGARCH Models

	Standard GARCH				TGARCH			
		AC	PAC	Q		AC	PAC	Q
$(\varepsilon_t/h_t^{1/2})$	L-B (10)	-0.011	-0.011	3.5	L-B (10)	-0.01	-0.011	4.1
$(\varepsilon_t/h_t^{1/2})^2$	L-B (10)	-0.028	-0.028	4.3	L-B (10)	-0.029	-0.030	5.0
Skewness	-0.369					-0.280		
Kurtosis	5.804	(13.96)**	6.048	(14.00)**				

L-B (10) is Ljung-Box statistics for autocorrelation (AC) and partial autocorrelation (PAC) functions at order ten. All values are significant at p=0.000. (.)** is the value for non-standardized residuals for the sample kurtosis

The coefficients on the kurtosis for both models reported in Table 6 are about fifty percent greater than the kurtosis for Gaussian distribution, even though these tests statistics indicate model adequacy. Second, in the framework of Table 6 above, model misspecification concerns arise if the sample autocorrelation (AC) and partial autocorrelation (PAC) coefficients exceed twice their asymptotic standard errors (ASEs) computed as $2/(T)^{1/2} = 0.044$. No value of AC and PAC in Table 6 is close to this value. Third, Hsiel (1989) asserts that if the conditional variance is correctly specified, the kurtosis in the standardized residual should be less than the kurtosis in the non-standardized residual. This proposition is satisfied as shown in Table 6 because the kurtosis for the non-standardized residual is more than twice the values for their standardized counterparts. Fourth, we employ a specification test proposed by Pagan and Sabau (1987) in a proprietary unpublished paper reported by Kearn and Pagan (1993). We note that, among others, Alles and Murray (2001, p. 140) enlisted this test as "a diagnostic test." Specifically, Kearn and Pagan propose squaring the residuals from the mean equation and regressing that on a constant and the conditional variance (h_t^2) to conduct their tests as follows:

$$\varepsilon_t^2 = c + bh_t^2 + e \tag{9}$$

Conceptually, model (9) examines how much of the variations in unobserved actual volatility (proxied by ε_t^2) can be explained by the conditional variance. Because, at least theoretically, both the regressand and the regressor are the same in the framework of ARCH models, the slope on equation (9) should ideally be equal to unity, with zero intercept. Then, the fit of the model can be assessed via R^2. The results are detailed in Table 7.

Table 7—Kearn and Pagan Diagnostic Test Results

	Standard GARCH	t-ratio	TGARCH	t-ratio
C	6.6E-05 (1.5E-05)	4.5	2.8E-05 (1.5E-05)	1.84
b-coefficient	0.523 (0.051)	10.2	0.774 (0.06)	13.31
R^2	0.5		0.82	

(.) = the standard error. The significant level is p = 0.000 or better for both models

The Kearn-Pagan (K-P) test results in Table 7 indicate that the theoretical expectations are confirmed by the data. First, the intercepts (denoted as C) are insignificantly different from zero. Second, the slope coefficients are significantly positive and large. Third, all the coefficients are statistical significant at less than five percent level with standard errors insignificantly different from zero. Fourth, the explanatory powers of the model based on R^2 are encouraging, and it is interesting to notice that the TGARCH has a larger R^2 than the standard GARCH. This result should not be a surprise in that the theoretical expectation is that TGARCH should have better capability to capture asymmetries in the data than does the standard GARCH. In other words, the difference in the results reported for both models is an indirect confirmation of the relative asymptotic superior performance of Glosten et al. (1993) model in capturing asymmetries in volatility as reported by Engle and Ng (1993). Equivalently, the differences in the results offer indirect evidence reflecting the extent to which the standard GARCH model fails to map the asymmetries in the data. Finally, among others, Diebold (1986) proposes that if a GARCH model is correctly specified, there should be no ARCH effects remaining in the levels and squares of the standardized residuals for the mean and the variance equations, respectively. This test is a Lagrange multiplier test which is asymptotically the same as $T^* R^2$where T is the sample size and R^2 is the familiar coefficient of determination. Equivalently, this test is a chi-square test with K degrees of freedom. The result for the standard GARCH model is (computed as $T^* R^2$) 0.109 (p = 0.74), and that for the TGARCH model it is ($T^* R^2$) 0.043 (p = 0.834). Therefore, these insignificant p-values indicate that the null hypothesis that ARCH effects remain in the estimation is rejected. In conclusion, overall, our battery of diagnostic tests suggests that the models are correctly specified.[5]

[5] They are by far better than those reported by Kearn and Pagan (1993, p. 171).

Conclusions

This paper discusses the importance of stock price volatility dynamics and prediction to three entities: academics, investors, and policy makers. Evidence based on citations from the extant financial econometrics literature indicates that volatility is an important concept to these entities. Even though volatility of large cap stock indices has been given much empirical attention, the volatility of small cap stock indices has received almost no attention. This paper identifies at least three empirical questions to be investigated using small cap (SC) 600 as the research setting.

These testable hypotheses are the main focus of the paper. Hypothesis 1 is a test of the proposition that the volatility of SC 600 is not predictable. This hypothesis is rejected based on evidence from standard GARCH and TGARCH models indicating that the volatility of small cap 600 can be predicted in the same way that the volatility of other stock prices are predicted. Hypothesis 2 is a test of the proposition that SC 600 does not exhibit the same empirical regularities observed in the behavior of other stock prices. The results of this paper show that SC 600 exhibits the same underlying statistical properties in terms of observed empirical regularities governing the empirical distributions of stock prices in general.

Finally, hypothesis 3 is a test of the proposition that SC 600 cannot pass the strict form test of market efficiency. This hypothesis is rejected, suggesting that SC 600 passed this form of efficient market hypothesis (EMH) test. Our results may be seen as the beginning of further investigations in the behavior of other small capitalization stock indices, especially with respect to the EMH test. In particular, our findings invite more research efforts toward a deeper empirical investigation of the unresolved myth in investors' perceptions.

References

1. Alles, Lakshman, and Louis Murray, "An Examination of Returns and Volatility Patterns on the Irish Equity Market," *Applied Financial Economics*, 11, no. 2 (2001), pp. 137-146.

2. Bera, A., and M. Higgins, "ARCH Models: Properties, Estimation and Testing," *Journal of Economic Surveys*, 7, no. 4 (1993), pp. 305-362.

3. Black, F. (1976) Studies in stock volatility changes, in *Proceedings of the 1976 Meetings of the Business and Economics Statistics Section*, 177-181.

4. Bollerslev, T., "Generalized Autoregressive Conditional Heteroscedasticity," *Journal of Econometrics*, 31 (1986), pp. 307-327.

5. Bollerslev, T., R. Engle, and D. Nelson, "ARCH Models," in R. Engle and D. McFadden (eds.), *Handbook of Econometrics*, Vol. 4 (North Holland, Amsterdam, 1994).

6. Bollerslev, T., R.F. Engle, and J.M. Wooldridge, "A Capital Asset Pricing Model with Time Varying Variance," *Journal of Political Economy*, 96 (1998), pp. 116-131.

7. Breusch, T.S., and A.R. Pagan, "A Simple Test for Heteroskedasticity and Random Coefficient Variations," *Econometrica*, 50 (1979), pp. 987-1007.

8. Campbell, J.Y., and L. Hentschel, "No News is Good News: an Asymmetric Model of Changing Volatility in Stock Returns," *Journal of Financial Economics*, 31 (1992), pp. 281-318.

9. Christie, A., “The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects,” *Journal of Financial Economics,* 10 (1982), pp. 407-432.

10. Cutler, D.M., J.M. Poterb, and L.H. Summer, “Speculative Dynamics,” *Review of Economic Studies*, 58 (1991), pp. 529-546.

11. Diebold, F., “Temporal Aggregation of ARCH Processes and the Distribution of Asset Returns,” Special Studies Paper 200, Federal Reserve Board, Division of Research and Statistics, Washington, D.C. (1986).

12. Engle, R., “Introduction,” in *ARCH Selected Reading: Advanced Texts in Econometrics* (Oxford University Press, 2002).

13. Engle, R., and V. Ng, “Measuring and Testing the Impact of News on Volatility,” University of California, San Diego (1991).

14. Engle, R., “Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U.K. Inflation,” *Econometrica,* 50 (1982), pp. 987-1008.

15. Engle, R., and V. Ng, “Measuring and Testing the Impact of News on Volatility,” *Journal of Finance*, 48 (1993), pp. 1749-1778.

16. Engle, R.F., and J. Patton, “What Good is a Volatility Model?” NYU Stern School of Business (2001).

17. Fama, E., “The Behavior of Stock Market Prices,” *Journal of Business*, 38 (1965), pp. 34-105.

18. Fornari, F., and A. Mele, “Modeling the Changing Asymmetry of Conditional Variances,” *Economics Letters*, 50 (1996), pp. 197-203.

19. French, K., G. Schwert, and R. Stambaugh, “Expected Stock Returns and Volatility,” *Journal of Financial Economics*, 19 (1987), pp. 3-29.

20. Garner, C.A., “Has the Stock Market Crash Reduced Consumer Spending?” *Financial Market Volatility and the Economy,* Federal Reserve Bank of Kansas City (1990).

21. Glosten, L., R. Jagannathan, and D. Runkle, “Relationship between the Expected Value and Volatility of the Nominal Excess Returns on Stocks,” *Journal of Finance*, 48 (1993), pp. 1779-1802.

22. Greenspan, Alan, “Maintaining Financial Stability in a Global Economy,” *Symposium Proceedings*, The Federal Reserve Bank of Kansas City (1997).

23. Hsieh, D., “Modeling Heteroskedasticity in the Daily Foreign Exchange Rates,” *Journal of Business and Statistics*, 7 (1989), pp. 307-317.

24. Kearn, P., and A.R. Pagan, “Australian Stock Market Volatility: 1875-1987,” *The Economic Record*, 69, no. 205 (1993), pp. 163-178.

25. Lo, A.W., and A.C. MacKinlay, “Stock Market Prices do not Follow Random Walk: Evidence from a Simple Specification Test,” *Review of Financial Studies*, 1 (1988), pp. 41-66.

26. Mandelbrot, B., “The Variation of Certain Speculative Prices,” *Journal of Business*, 36 (1963), pp. 394-419

27. Maskus, K.E., “Exchange Rate Risk and U.S. Trade: A Sectoral Analysis,” *Financial Market Volatility and the Economy,* Federal Reserve Bank of Kansas City (1990).

28. McAleer, Michael, and Les Oxley, “The Econometrics of Financial Series,” *Journal of Economic Surveys,* 16, no. 3 (2002), pp. 237-243.

29. Nelson, D.B. “Conditional Heteroscedasticity in Asset Returns: A New Approach,” *Econometrica*, 59 (1991), pp. 347-370

30. Pagan, A., “The Econometrics of Financial Markets,” *Journal of Empirical Finance*, 3 (1996), pp. 15-102.

31. Pindyck, R., “Risk, Inflation, and the Stock Market,” *American Economic Review,* 76 (1984), pp. 335-351.

32. Poshakwale, S., and V. Murinde, “Modeling the Volatility in East European Emerging Markets: Evidence on Hungary and Poland,” *Applied Financial Economics*, 11 (2001), pp. 445-456.

33. Schwert, G.W., “Why Does Stock Market Volatility Change over Time?” *Journal of Finance*, 54 (1989), pp. 1115-1151

34. Wooldridge, Jeffery M., *Introductory Econometrics: A Modern Approach* (Mason, Ohio: South-Western Publishing, 2003).

35. Wu, G., “The Determinants of Asymmetric Volatility,” *Review of Financial Studies*, 14 (2001), pp. 837-859.

36. Yu, Jun, “Forecasting Volatility in the New Zealand Stock Market,” *Applied Financial Economics*, 12 (2002), pp. 193-202