Empirical Evidence on the Conditional Relation between Higher-Order Systematic Co-moments and Security Returns

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This paper contributes to the debate on whether systematic higher-order co-moments are able to explain cross-sectional security returns. We argue that a possible reason for inconclusive evidence in the previous studies might be the failure to accommodate market movements in the analysis. We estimate two risk premiums for each systematic variance, systematic skewness, and systematic kurtosis corresponding to the up market (positive excess market returns) and the down market (negative excess market returns). The results show that in the up market, the beta risk premium is positive and the co-skewness risk premium has the opposite sign to the up market skewness. In the down market, the beta risk premium is negative and the co-skewness risk premium has the same sign as the up market skewness. The systematic co-kurtosis does not appear to be priced.

Introduction

Since the seminal paper by Markowitz (1959), the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) has become an important tool in finance for assessment of cost of capital, portfolio performance and diversification, valuing investments, and choosing portfolio strategy, among others. The CAPM

* We thank the referees for their helpful comments and suggestions that improved the presentation of this paper. The comments of the participants at the 2002 Econometric Society Australasian Meeting, Brisbane, Australia on an earlier version of this paper are acknowledged. The usual disclaimer applies.

0747–5535/02/1600 & 1300/0121/$2.50

relates the expected rate of return of an individual security with a measure of its systematic risk. To test the validity of the CAPM, researchers test the security market line given as: $E(R_i) = R_f + \beta_{im}\{E(R_m) - R_f\}$ where R_i, R_f and R_m are return on risky asset i, risk-free asset, and market portfolio, respectively and β_{im} is a measure of systematic risk defined as the ratio of $Cov(R_i, R_m)$ and $Var(R_m)$. The security market line suggests an overall positive relationship between beta and expected returns. Studies, however, provide weak empirical evidence[1] on this relationship. See, for example, Fama and French (1992), He and Ng (1994), Davis (1994), and Miles and Timmermann (1996).

Pettengill, Sundaram, and Mathur (1995) observe that the studies of beta and cross-sectional returns that use realized return as a proxy for the expected may have produced bias results due to aggregation of positive and negative market excess return periods. Pettengill, Sundaram, and Mathur argue that when the market return in excess of the risk-free rate is negative, an inverse relationship between the beta and portfolio returns should exist and test for a systematic conditional relationship between the realized portfolio returns and the beta. Their empirical investigation of U.S. data reveals a positive slope on beta in the up market and a negative relationship in the down market. An analysis of the unconditional and the systematic conditional relationship between returns and beta on the Brussels Stock Exchange reveals that unconditional betas are unable to explain the cross-sectional observed returns, where as the conditional model does (Crombez and Vander Vennet, 2000). Friend and Westerfield (1980) examine beta and co-skewness in the up- and down-markets and report that while beta is significant in both markets and its signs are consistent with the CAPM theory, the co-skewness is significant only in the up-market.

To date, we believe, no study has adopted the Pettengill, Sundaram, and Mathur approach to investigate the extended CAPM with higher-order co-moments (systematic variance, systematic skewness and systematic kurtosis) and for high frequency data. We investigate the CAPM with higher-order co-moments using a sample of securities listed in the Australian Stock Exchange and with daily data. As it is clear from stylized facts that the skewness and kurtosis of the returns distribution become prominent in the high frequency data, our study is more likely to uncover any unconditional relationship between return and higher moments.

[1] In empirical studies tests of the CAPM is usually performed in two stages. First, the beta is obtained by estimating the empirical version of the security market line given as: $R_{it} - R_{ft} = b_i(R_{mt} - R_{ft}) + \square_{it}$. b_i is an estimate of $\square_{im}$. The estimated $\square_{im}$ is used as the explanatory variable (in the following equation: $R_{it} = \square_0 + \square_1 b_i + u_{it}$) that relates cross-sectional returns to estimated beta. The coefficient $\square_1$ is the market price of beta (risk premium) that has to be positive and significant in order to support the validity of the CAPM.

Higher-Order Pricing Models

In this section, we include two versions of the four-moment CAPM and specify the cross-sectional model reflecting the relationship between the asset returns and higher-order co-moments conditioned on market movements.

Four-Moment CAPM

The following are two versions of the four-moment CAPM, in which it is assumed that only the risks measured by systematic variance, systematic skewness, and systematic kurtosis are priced.

Kraus and Litzenberger (1976) version:

$$E(R_i) - R_f = \alpha_1\beta_{im} + \alpha_2\gamma_{im} + \alpha_3\theta_{im} \tag{1}$$

where R_f and R_i are returns on the risk-free asset and risky asset i, respectively,

$$\beta_{im} = \frac{E\left[\left(R_{it} - E(R_i)\right)\left(R_{mt} - E(R_m)\right)\right]}{E\left(R_{mt} - E(R_m)\right)^2} = \text{beta}, \tag{2}$$

$$\gamma_{im} = \frac{E\left[\left(R_{it} - E(R_i)\right)\left(R_{mt} - E(R_m)\right)^2\right]}{E\left(R_{mt} - E(R_m)\right)^3} = \text{co-skewness}, \tag{3}$$

$$\theta_{im} = \frac{E\left[\left(R_{it} - E(R_i)\right)\left(R_{mt} - E(R_m)\right)^3\right]}{E\left(R_{mt} - E(R_m)\right)^4} = \text{co-kurtosis}. \tag{4}$$

Due to the desirable properties of the utility function, we expect the market price of beta reduction by one unit to be α_1, which is expected to be positive as in the conventional CAPM. The market price of co-skewness is α_2, which is expected to have the opposite sign to the skewness of the market return distribution. The market price of co-kurtosis is α_3, which is an additional measure of degree of dispersion in returns and is expected to be positive.

Sears and Wei (1985) version:

$$E(R_i) - R_f = (c_1\beta_{im} + c_2\gamma_{im} + c_3\theta_{im})(E(R_m) - R_f)\,. \tag{5}$$

A derivation of equations (1) and (5) is available in Hwang and Satchell (1999).

Four-Moment Conditional Model

When testing the significance of parameters in empirically testable forms of equation (1),[2] previous studies use realized returns in lieu of expected returns. Pettengill, Sundaram, and Mathur argue that when the market return is lower than the risk-free rate and the realized returns for high beta portfolios are lower than the real-

[2] The co-moments are usually calculated with equations (2) through (4). They are then proxied for higher-order co-moments and used as explanatory variables in the following cross-sectional model: $R_{it} = \gamma_0 + \gamma_1\hat{\beta}_{im} + \gamma_2\hat{\gamma}_{im} + \gamma_3\hat{\theta}_{im} + u_{it}$ to test the validity of the CAPM with higher-order co-moments.

ized returns for low beta portfolios an inverse relationship between the beta and returns could reasonably be inferred during the time periods when the realized market return is less than the risk-free return. In what follows, these ideas are extended to the CAPM with higher-order co-moments. We argue that using the empirical value of market return could lead to bias results because the relationship between co-moments and realized returns may differ from that of the relationship between co-moments and expected return.

The four-moment CAPM version given by equation (1) does not provide a direct relationship between security co-moments and security returns when the realized market return is less than the risk-free rate. Equating[3] the coefficients of the co-moments in equations (1) and (5), we obtain $\alpha_1 = c_1(E(R_m) - R_f)$, $\alpha_2 = c_2(E(R_m) - R_f)$, and $\alpha_3 = c_3(E(R_m) - R_f)$. These relationships suggest that the sign of the parameters of equation (1) could be affected, and, consequently, the tests of the unconditional asset pricing model when realized returns instead of expected returns are used (Sears and Wei, 1985). When the market return is lower than the risk-free rate, we hypothesize an inverse relationship between security returns and higher-order co-moments and suggest a systematic conditional relationship between the security return and the higher-order co-moments given as:

$$\begin{aligned} R_{it} &= \delta_{0t} + \delta_{1t}\kappa\beta_{im} + \delta_{2t}(1-\kappa)\beta_{im} + \delta_{3t}\kappa\gamma_{im} + \\ &\delta_{4t}(1-\kappa)\gamma_{im} + \delta_{5t}\kappa\theta_{im} + \delta_{6t}(1-\kappa)\theta_{im} + \varepsilon_{it} \end{aligned} \tag{6}$$

where $\kappa = 1$ when $(R_{mt} - R_{ft}) > 0$ and $\kappa = 0$ when $(R_{mt} - R_{ft}) < 0$. We refer to equation (6) as the four-moment conditional model.[4]

The sign expected for the coefficients of equation (6) can be inferred from the hypothesized inverse relationship and the signs expected for α_1, α_2, and α_3 in the unconditional model given in equation (1). That is, we expect $\overline{\delta}_1 > 0$, $\overline{\delta}_2 < 0$, $\overline{\delta}_3 > 0$ and $\overline{\delta}_4 < 0$ when up market return distribution is negatively skewed, $\overline{\delta}_3 < 0$ and $\overline{\delta}_4 > 0$ when up market return distribution is positively skewed, $\overline{\delta}_5 > 0$ and $\overline{\delta}_6 < 0$. $\overline{\delta}_1$, $\overline{\delta}_3$, and $\overline{\delta}_5$ are the average risk premium corresponding to systematic variance, systematic skewness and systematic kurtosis respectively in the up market and $\overline{\delta}_2$, $\overline{\delta}_4$, and $\overline{\delta}_6$ are the average risk premium corresponding to systematic variance, systematic skewness and systematic kurtosis respectively in the down market. We test the existence of a conditional relationship by examining the coefficients of the cross-sectional regression model in equation (6).

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[3] Equations (1) and (5) may be compared under the assumption that equation (1) holds for the market portfolio as well.

[4] The model given in equation (6) can be reduced to reflect the two-moment- (put $\delta_{jt} = 0$ for $j = 3,\ldots,6$) and three-moment- (put $\delta_{jt} = 0$ for $j = 5,6$) conditional model and the conditional model with beta and theta (put $\delta_{jt} = 0$ for $j = 3,4$).

Hypotheses of Interest

In order to see if there is supportive empirical evidence of a conditional relationship between expected return and higher-order co-moments, the following pairs of hypotheses are tested.

Test for a systematic conditional relationship between beta and realized returns: $\{H_0 : \overline{\delta}_1 = 0, H_1 : \overline{\delta}_1 > 0\}$ and $\{H_0 : \overline{\delta}_2 = 0, H_1 : \overline{\delta}_2 < 0\}$. If the null hypotheses in both are rejected, then a systematic conditional relationship between beta and realized return is supported.

A test for a systematic conditional relationship between gamma and realized returns when the up market return distribution is positively skewed is: $\{H_0 : \overline{\delta}_3 = 0, H_1 : \overline{\delta}_3 < 0\}$ and $\{H_0 : \overline{\delta}_4 = 0, H_1 : \overline{\delta}_4 > 0\}$. If the null hypotheses in both are rejected, then a systematic conditional relationship between gamma and realized return is supported.

A test for a systematic conditional relationship between gamma and realized returns when the up market return distribution is negatively skewed is: $\{H_0 : \overline{\delta}_3 = 0, H_1 : \overline{\delta}_3 > 0\}$ and $\{H_0 : \overline{\delta}_4 = 0, H_1 : \overline{\delta}_4 < 0\}$. If the null hypotheses in both are rejected, then a systematic conditional relationship between gamma and realized return is supported.

A test for a systematic conditional relationship between theta and realized returns is: $\{H_0 : \overline{\delta}_5 = 0, H_1 : \overline{\delta}_5 > 0\}$ and $\{H_0 : \overline{\delta}_6 = 0, H_1 : \overline{\delta}_6 < 0\}$. If the null hypotheses in both are rejected, then a systematic conditional relationship between theta and realized return is supported.

Assuming that the market movements (up or down) do not have asymmetric effects on risk premiums, we can obtain the symmetric models, which are (6) with $\delta_{1t} = \delta_{2t}$, $\delta_{3t} = \delta_{4t}$ and $\delta_{5t} = \delta_{6t}$. We estimate the symmetric and asymmetric models and compare the results.

Methodology

Portfolio Formation

The data set consists of daily[5] returns of 128 securities. The return series spans 3920 days. We begin with estimating beta, gamma, and theta for each security for the first 632 days (2.5 years) of the sample period according to the formulae given in equations (2) through (4). Based on these estimates the 128 securities are assigned to eight portfolios.[6] First, the securities are ranked based on their estimated betas, and

[5] When daily returns are used, a reduction in measurement error from beta estimates may be achieved. In Pogue and Conway's (1972) study of 90 mutual funds, as reported by Modigliani and Pogue (1974), a shift from monthly to daily returns reduced the average standard error of the estimated beta values approximately 75 percent.

[6] Portfolios generally are designed to minimize measurement error while maintaining considerable cross-sectional variation in portfolio betas, gammas and thetas (Fama and MacBeth, 1973). When grouping however, there will be a loss of information in the tests

they are divided into two groups. Securities in each group are again ranked according to gamma estimates and divided further into two subgroups. Securities in each subgroup are again divided into two subgroups after ranking the securities according to theta estimates. Consequently, eight subgroups each comprising 16 securities are formed with securities in a subgroup forming a portfolio.

Two-Stage Estimation Procedure

Stage-I: Portfolio Estimation Using Time Series Data

The 632-day period that follows the portfolio formation period is used to estimate the portfolio beta, gamma, and theta. As has been argued, the use of a non-overlapping period for estimating the portfolio beta, gamma, and theta would minimize the errors-in-variables problem. Using the 632 daily securities and market returns, the betas, gammas, and thetas are again computed via equations (2) through (4). A portfolio's beta, gamma, and theta are then obtained by averaging the betas, gammas, and thetas of the 16 individual securities that are assigned to that portfolio in the previous phase.

Stage-II: Estimation of Cross-Sectional Relationship between Returns and Higher-Order Co-moments and Hypothesis Testing

Over the 126 days (0.5 year) that follow the sample period used in estimation of time series models in Stage-I, the daily returns are computed for each portfolio. These daily portfolio returns are then regressed on the beta, gamma, and theta estimates obtained in Stage-I[7], according to the relationship specified in equation (6). The above procedure will uncover possible nonstationarities of the regression coefficients—risk premiums—within the 126-day period. The portfolio formation and the two-stage estimation procedure are repeated[8] after omitting the first-126 days from the original data series. This process is rolled over through the entire sample period, thereby allowing for further nonstationarities in the regression coefficients in Stage-II. This method enables 21 repetitions of the procedure spanning 2646 days.

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conducted in the second phase. Nevertheless, tests based on portfolio returns will be more effective than tests based on security returns (Modigliani and Pogue, 1974).

[7] It is assumed that portfolio beta, gamma and theta estimated in Stage-I proxy beta, gamma and theta of Stage-II.

[8] The regrouping of assets by repeating the three-stage process results in portfolios having stable beta, gamma and theta over time even when the characteristics of individual assets change over time (Kraus and Litzenberger, 1976).

Data

The data[9] series consists of 128 heavily traded securities listed in the Australian Stock Exchange and used in the construction of the Australian All Ordinaries Price Index. The sample spans the period 02 January 1985 through 30 June 2000, yielding a total number of 3920 observations, excluding holidays and weekends. Two proxies for market index are used: one is the equally weighted average of the prices of all the 128 stocks, which we refer to as *composite index*, and the other is the Australian All Ordinaries Price Index (*all ords*), which is the value-weighted average of 500 largest Australian companies based on market capitalization. Four proxies for the risk-free return are used: the 90-day, the 180-day bank-accepted bill rates, and the five-year and the ten-year Treasury bond rates.

As will be seen later, several pricing models are analyzed. The models differ by the number and/or the type of co-moments included in them. The construction of portfolios is based on security co-moments, and, therefore, the number of portfolios formed varies depending on the pricing model estimated. In order to provide typical characteristics of these portfolio distributions over the entire sample period, however, we construct 16 portfolios based on security betas only. All 128 securities are first ranked in descending order according to their betas and then 16 portfolios are formed: portfolio 1 is an equally weighted average of the first eight securities, portfolio 2 is the average of the next eight securities, and so on. The summary statistics of portfolios and market return distributions over the entire sample period are given in Table 1. It is clear from Table 1 that for portfolios, the minimum ranges from –7.8 percent to –39.1 percent while the maximum varies between 4.5 percent and 44 percent. As expected, high beta portfolios tend to have wider ranges between minimum and maximum. Excess kurtosis can be as high as 140.7 with the minimum being –1.2. Excluding the portfolios consisting of low beta securities would reduce the excess kurtosis interval to –1.2 to 75.9. The skewness ranges from –5.5 to 3.4 with four portfolios having positive skewness while the other twelve negative skewness.

The distributions of the two market indices appear to differ in the tails. They both have negative skewness and very high excess kurtosis with all ords having numerically lower value of the latter.

Empirical Results and Analysis

Estimation of Unconditional Risk-Return Relationship

First, we examine the unconditional relationship of portfolio return and co-moments: that is, symmetric models over the entire sample period. In this model, it is

[9] The data used are daily closing prices adjusted for share capitalization changes such as share splits and bonus issues. Data were obtained from the Core Research Database of the Securities Industry Research Centre of Australasia. Continuously compounded returns are calculated from the daily closing prices.

Table 1—Summary Statistics of Portfolio Return Distributions Based on the 3920 Observations in the Study Period

Item	Mean	Max	Min	SD	Skewness	Excess Kurtosis
Portfolio 1	0.0106	27.6072	-39.1399	1.7696	-0.6740	8.1035
Portfolio 2	0.0219	28.6624	-35.8522	2.0376	-0.0448	0.1283
Portfolio 3	0.0112	31.0057	-29.9952	2.0716	-0.0813	0.3586
Portfolio 4	0.0138	8.3127	-33.4042	1.1978	-3.3610	75.9250
Portfolio 5	0.0298	37.2071	-26.3932	2.0405	0.0700	-0.2762
Portfolio 6	0.0276	19.2242	-24.3248	1.4406	-0.4140	6.2190
Portfolio 7	0.0125	19.3193	-21.1739	1.1923	-0.7341	25.2797
Portfolio 8	0.0141	44.1651	-23.8296	1.7585	0.6062	10.0909
Portfolio 9	0.0199	16.9588	-18.1925	1.3500	-0.2286	5.5744
Portfolio 10	0.0141	9.3188	-16.0245	1.5782	-0.0910	-1.2202
Portfolio 11	0.0108	8.9449	-23.3255	1.2613	-1.3196	16.4429
Portfolio 12	0.0254	24.3079	-24.4780	1.3406	0.2234	31.0144
Portfolio 13	0.0210	7.5202	-7.8442	0.7821	-1.3010	39.8060
Portfolio 14	0.0170	8.6934	-10.7848	0.7634	-1.8787	108.0084
Portfolio 15	0.0096	18.3856	-11.0805	0.8905	3.3553	122.4609
Portfolio 16	0.0236	5.8698	-14.6947	0.7766	-5.5398	140.7268
Composite Index	0.0235	4.4691	-17.1663	0.6787	-5.6505	118.3724
All Ords	0.0382	6.0666	-28.7585	1.0187	-6.4200	168.0749

assumed that the excess market returns have symmetric effects on the model parameters, which are risk premiums. The portfolio formation and the two-stage estimation procedure are rolled over 21 times, obtaining 2646 estimates for the market risk premiums for beta, gamma, and theta risks. For comparison purpose, we investigate four pricing models:

(i) The four-moment CAPM

Model A: $$R_{it} = \alpha_0 + \alpha_1\beta_{im} + \alpha_2\gamma_{im} + \alpha_3\theta_{im} + \varepsilon_{it}\,, \tag{7}$$

(ii) The three-moment CAPM

Model B: $$R_{it} = \alpha_0 + \alpha_1\beta_{im} + \alpha_2\gamma_{im} + \varepsilon_{it}\,, \tag{8}$$

(iii) The CAPM with beta and co-kurtosis

Model C: $$R_{it} = \alpha_0 + \alpha_1\beta_{im} + \alpha_3\theta_{im} + \varepsilon_{it} \tag{9}$$

and

(iv) The conventional two-moment CAPM

Model D: $$R_{it} = \alpha_0 + \alpha_1\beta_{im} + \varepsilon_{it}\,. \tag{10}$$

The results[10] reported in Table 2 reveal that none of the risk premiums in all four pricing models is significant. We argue that this observed weak result is due to the assumption that up and down markets have symmetric affects on the risk premiums.

[10] When estimating models B, C, and D, 16 portfolios are constructed, each containing eight securities.

Table 2—Estimates of Risk Premium in Unconditional Pricing Models

Coefficient	Estimate	t-value
Model A: $R_{it} = \alpha_0 + \alpha_1\beta_{im} + \alpha_2\gamma_{im} + \alpha_3\theta_{im} + \varepsilon_{it}$		
$\bar{\hat{\alpha}}_0$	0.0231	1.1323
$\bar{\hat{\alpha}}_1$	-0.0888	-1.2980
$\bar{\hat{\alpha}}_2$	0.1295	0.7331
$\bar{\hat{\alpha}}_3$	-0.0525	-0.3231
Model B: $R_{it} = \alpha_0 + \alpha_1\beta_{im} + \alpha_2\gamma_{im} + \varepsilon_{it}$		
$\bar{\hat{\alpha}}_0$	0.0355	2.6574*
$\bar{\hat{\alpha}}_1$	-0.0453	-1.5662
$\bar{\hat{\alpha}}_2$	0.0235	1.1165
Model C: $R_{it} = \alpha_0 + \alpha_1\beta_{im} + \alpha_3\theta_{im} + \varepsilon_{it}$		
$\bar{\hat{\alpha}}_0$	0.0308	2.4761**
$\bar{\hat{\alpha}}_1$	-0.0595	-1.6920
$\bar{\hat{\alpha}}_3$	0.0450	1.5621
Model D: $R_{it} = \alpha_0 + \alpha_1\beta_{im} + \varepsilon_{it}$		
$\bar{\hat{\alpha}}_0$	0.0271	2.1082**
$\bar{\hat{\alpha}}_1$	-0.0135	-0.8282

* Significant at the 1 percent level and ** significant at the 5 percent level

Estimation of Conditional Risk-Return Relationship

Second, we estimate the risk premiums of four conditional pricing (asymmetric) models:

(i) Four-moment conditional model

Model A^c:
$$R_{it} = \delta_{0t} + \delta_{1t}\kappa\beta_{im} + \delta_{2t}(1-\kappa)\beta_{im} + \delta_{3t}\kappa\gamma_{im} + \delta_{4t}(1-\kappa)\gamma_{im} + \delta_{5t}\kappa\theta_{im} + \delta_{6t}(1-\kappa)\theta_{im} + \varepsilon_{it}, \tag{11}$$

(ii) Three-moment conditional model

Model B^c:
$$R_{it} = \delta_{0t} + \delta_{1t}\kappa\beta_{im} + \delta_{2t}(1-\kappa)\beta_{im} + \delta_{3t}\kappa\gamma_{im} + \delta_{4t}(1-\kappa)\gamma_{im} + \varepsilon_{it}, \tag{12}$$

(iii) Conditional model with beta and co-kurtosis

Model C^c:
$$R_{it} = \delta_{0t} + \delta_{1t}\kappa\beta_{im} + \delta_{2t}(1-\kappa)\beta_{im} + \delta_{5t}\kappa\theta_{im} + \delta_{6t}(1-\kappa)\theta_{im} + \varepsilon_{it} \tag{13}$$

and

(iv) Two-moment conditional model

Model D^c:
$$R_{it} = \delta_{0t} + \delta_{1t}\kappa\beta_{im} + \delta_{2t}(1-\kappa)\beta_{im} + \varepsilon_{it}. \tag{14}$$

In these conditional models, excess market returns are assumed to have asymmetric effects on the model parameters, depending on whether the excess market return is positive or negative. The proxies used for the market index and risk-free rate are the composite index and the 90-day bill rate, respectively. The excess market returns are computed as the difference between composite index returns and the 90-day bill rates. Of the 2464 days in the testing period, 1371 days (51.8 percent)[11] are up market days. The models are estimated using the OLS, and the results[12] are reported in Table 3.

Table 3—Estimates of Risk Premium in Conditional Pricing Models Using Composite Return

Model	Up Market				Down Market			
	δ_0	δ_1	δ_3	δ_5	δ_0	δ_2	δ_4	δ_6
Model A[c]								
Estimate	0.049	0.549	-0.199	0.009	-0.005	-0.774	0.482	-0.119
t-value	1.774	5.882*	-0.822	0.042	-0.181	-7.978*	1.866	-0.499
Model B[c]								
Estimate	0.096	0.399	-0.099	-	-0.029	-0.524	0.156	-
t-value	4.995*	10.33*	-3.444*	-	-1.590	-13.39*	5.193*	-
Model C[c]								
Estimate	0.074	0.419	-	-0.096	-0.016	-0.575	-	0.196
t-value	4.300*	9.008*	-	-2.561**	-0.872	-11.73*	-	4.476*
Model D[c]								
Estimate	0.091	0.296	-	-	-0.042	-0.347	-	-
t-value	5.051*	13.93*	-	-	-2.316*	-16.26*	-	-

* Significant at the 1 percent level and ** significant at the 5 percent level

It is clear in Tables 2 and 3 that the models in which up and down markets are allowed to have distinct effects on risk premiums uncover a significant relationship between high moment risks and asset returns with strong evidence, while the unconditional models would not uncover such a relationship.

The results in Table 3 reveal that the up market and the down market movements have significant systematic asymmetric effects on beta risk premium. In all pricing models the premium for beta risk (i) in the up market is significant and positive and (ii) in the down market the beta risk premium is significant and negative, as predicted. The explanatory power of the two-moment conditional model is slightly higher in the down market (with average adjusted R-square $\left(\overline{R}^2\right)$ 12.3 percent) than in the up market (where $\overline{R}^2$ is 11.4 percent). This pattern is observed in 12 of the 21 repetitions of the estimation procedure in which $\overline{R}^2$ varies from 7.7 percent to 18.6 percent in the up market and from 8.9 percent to 17.5 percent in the down market.

[11] The Pettengill, Sundaram, and Mathur study that used monthly US data from January 1926 through December 1990 reports that 57.6 percent of the months correspond to up market days. The Faff (2001) study that used monthly Australian data over the period 1974 to 1995 reports that 54.2 percent of the months provide positive excess market return.

[12] We also used the 180-day bill rate and five-year and ten year bond rates as proxies for the risk-free rate and found that the results and conclusions are largely unchanged.

Further, the results of estimation of Model B^c reveal that co-skewness risk in addition to the beta risk in the regression of portfolio returns is significant in both the up and down markets. The skewness of the market returns in the up market and down market period is 2.7687 and -4.7581, respectively. Being opposite to the sign of the market skewness in the up market and in the down market, the signs of the co-skewness risk premiums in the three-moment conditional model are consistent with what was expected. The explanatory power of this three-moment conditional model is similar $\left(\overline{R}^2 = 12.7 \text{ percent}\right)$ in the up and down markets. The average adjusted R-square in the 21 repetitions, however, varies from 5.3 percent to 19.2 percent in the up market and from 6.8 percent to 25.6 percent in the down market. Although co-skewness as an explanatory variable in the four-moment model (Model A^c) is not significant, the market price of co-skewness has the correct sign postulated in the up and down markets. In this model $\overline{R}^2$ in the up (35.2 percent) and down (31.6 percent) market are higher than those observed with the other models.

The market risk premia associated with co-kurtosis, though significant in Model C^c, have signs contrary to that postulated, while those of Model A^c have the expected signs but are not significant. The explanatory power of Model C^c is similar to Model B^c.

Sensitivity Analysis

This section examines the sensitivity of the results reported in the previous section to alternative proxies for the market portfolio, up- and down-market definition, time interval captured in the testing period, and inclusion of a proxy for unsystematic risk as an explanatory variable in the conditional model.

Choice of Market Portfolio

The results[13] reported in Table 4 of the analysis with all ords as a proxy for the market index and the 90-day bill rate for risk-free rate support the conditional relationship only in Model D^c. It appears that when all ords is used[14] the conditional relationship is not supported in the models that include higher-order co-moments. Even though the data does not support Model B^c when all ords is used, the market price of co-skewness has the correct sign. All ords is a broader market index compared to composite index, and this could be a reason for the observed differences in the results with the two indices.

[13] The choice of the risk-free rate does not affect our conclusions.

[14] When all ords is used, the breakdown of the total test period of 2646 days into up (51.5 percent) and down market days is only slightly different from that of when the composite index is used.

Table 4—Estimates of Risk Premium in Conditional Pricing Models Using Australian All Ordinaries Price Index Return

Model	Up Market				Down Market			
	δ_0	δ_1	δ_3	δ_5	δ_0	δ_2	δ_4	δ_6
Model A[c]								
Estimate	0.054	0.499	-0.351	0.369	-0.020	-0.666	0.812	-0.735
t-value	1.807	1.952	-0.622	0.633	-0.592	-2.030**	1.247	-1.038
Model B[c]								
Estimate	0.063	0.542	-0.047	-	0.015	-0.666	0.055	-
t-value	3.571	12.525*	-1.275	-	0.753	-13.79*	1.315	-
Model C[c]								
Estimate	0.047	0.535	-	-0.030	0.015	-0.757	-	0.139
t-value	2.844	4.368*	-	-0.237	0.754	-6.182*	-	1.131
Model D[c]								
Estimate	0.039	0.515	-	-	0.026	-0.611	-	-
t-value	2.619	19.385*	-	-	1.552	-22.74*	-	-

* Significant at the 1 percent level and ** significant at the 5 percent level

Up and Down Market Definition

We repeat the analysis without reference to a risk-free rate. To do this we define the up and down market time periods as the periods in which the market return is positive and negative, respectively. Then of the 2646 days in the testing period, 1385 (52.3 per cnt) days belonged to the up market period in which the skew of the market return distribution is positive. The estimates of the conditional pricing models with composite index are reported in Table 5. The results obtained here are consistent[15] with the results in Table 3 where a positive market return in excess of the risk-free return is defined as an up market.

Testing Period

We observed that only the two- and three-moment conditional models are evidently supported. In this section we estimate and test risk premia of these two models separately in seven subperiods by partitioning[16] the full testing period into intervals of length 378 days (1.5 years).

The results of the two-moment conditional model, reported in Table 6, reveal that when excess market returns are positive (negative), a significant positive (negative) relationship exists between beta and returns for each subperiod suggesting that the results are consistent in all subperiods.

[15] Kim and Zumwalt (1979) in their analysis of security return based on two-beta model divided the market returns into up- and down-markets using three alternative cut-off levels: average monthly market return, average risk-free rate and zero. They reported that the different cut-off levels produced virtually identical results.

[16] Numerous studies have tested the stability of parameters in asset pricing models mainly to investigate whether notable shocks to the system may change historical relationships. Usually, the dates on which the shocks occur are selected as breakpoints that partition the study period. See for example, Prakash, Moncarz, and Anderson (2001).

Table 5—Estimates of Risk Premium in Conditional Pricing Models with Up Market and Down Market Defined as $R_{mt} > 0$ and $R_{mt} \leq 0$, Respectively, and Using Composite Index Return

Model	Up Market				Down Market			
	δ_0	δ_1	δ_3	δ_5	δ_0	δ_2	δ_4	δ_6
Model A[c]								
Estimate	0.051	0.562	-0.193	-0.013	-0.008	-0.804	0.484	-0.096
t-value	1.831	6.063*	-0.806	-0.057	-0.259	-8.246*	1.854	-0.401
Model B[c]								
Estimate	0.092	0.399	-0.098	-	-0.027	-0.533	0.157	-
t-value	4.833*	10.373*	-3.402*	-	-1.455	-13.59*	5.163*	-
Model C[c]								
Estimate	0.0767	0.418	-	-0.098	-0.018	-0.584	-	0.202
t-value	4.385*	9.028*	-	-2.630*	-1.030	-11.81*	-	4.574*
Model D[c]								
Estimate	0.089	0.297	-	-	-0.040	-0.355	-	-
t-value	4.932*	14.03*	-	-	-2.209**	-16.63*	-	-

* Significant at the 1 percent level and ** significant at the 5 percent level

Table 6—Estimates of Risk Premium in Conditional Two-Moment CAPM Using Composite Index Return in Sub-sample Periods

Testing Period		Up Market		Down Market	
		δ_0	δ_1	δ_0	δ_2
Full Period	Days	1371		1275	
01/1990 – 06/2000	Estimate	0.0913	0.2963	-0.0419	-0.3466
	t-value	5.0509*	13.927*	-2.3155**	-16.262*
Subperiod 1	Days	176		202	
01/1990 – 06/1991	Estimate	0.0531	0.2876	-0.0930	-0.2547
	t-value	1.2041	5.7030*	-2.3354**	-5.5615*
Subperiod 2	Days	185		193	
07/1991 – 12/1992	Estimate	0.1475	0.2246	0.0229	-0.3364
	t-value	2.4982**	3.2093*	0.5853	-5.0031*
Subperiod 3	Days	205		173	
01/1993 – 06/1994	Estimate	0.1261	0.3872	0.0075	-0.4344
	t-value	2.9151*	6.6485*	0.1315	-6.6297*
Subperiod 4	Days	199		179	
07/1994 – 12/1995	Estimate	0.1731	0.1653	-0.1222	-0.2131
	t-value	3.9291*	3.7160*	0.4515	-5.3739*
Subperiod 5	Days	227		151	
01/1996 – 06/1997	Estimate	0.1933	0.1694	-0.1043	-0.2707
	t-value	6.2599*	4.9403*	-2.5156**	-6.6378*
Subperiod 6	Days	188		190	
07/1997 – 12/1998	Estimate	0.0156	0.4236	-0.0633	-0.5287
	t-value	0.2445	5.7135*	-0.9010	-7.4522*
Subperiod 7	Days	191		187	
01/1999 – 06/2000	Estimate	-0.0970	0.4380	0.0496	-0.3793
	t-value	-2.1120**	7.7817*	1.2252	-8.1484*

* Significant at the 1 percent level and ** significant at the 5 percent level.

The results of the analysis of the three-moment conditional model in the subperiods are reported in Table 7. The results reveal that when excess market returns are positive (negative), a significant positive (negative) relationship exists between beta and returns for each subperiod. This suggests consistency of premium estimates in the up and down markets for the beta risk when gamma risk is also considered in the pricing model.

The tests of gamma risk premium estimates in the up and down markets reveal that in subperiods 1, 2, 3, and 7 a systematic conditional relationship between gamma risk and realized return is not supported. The gamma risk premium estimates appear to support a systematic conditional relationship in the latter part of the testing period, namely in subperiods 4 through 6. Nevertheless, the sign of estimated means of the gamma risk premium is consistent in the subperiods where in the up market, the mean is negative and in the down market, the mean is positive.

Table 7—Estimates of Risk Premium in Conditional Three-Moment CAPM Using Composite Index Return in Sub-sample Periods

Testing		Up Market			Down Market		
Period		δ_0	δ_1	δ_3	δ_0	δ_2	δ_4
Full Period	Days		1371			1275	
01/1990 –	Estimate	0.096	0.399	-0.099	-0.029	-0.524	0.156
06/2000	t-value	4.995*	10.33*	-3.444*	-1.590	-13.39*	5.193*
Subperiod 1	Days		176			202	
01/1990 –	Estimate	0.073	0.477	-0.165	-0.119	-0.522	0.259
06/1991	t-value	1.178	2.529**	-1.023	-2.077**	-3.097*	1.806
Subperiod 2	Days		185			193	
07/1991 –	Estimate	0.205	0.204	-0.029	-0.017	-0.439	0.106
12/1992	t-value	3.163*	1.669	-0.309	-0.393	-3.672*	1.327
Subperiod 3	Days		205			173	
01/1993 –	Estimate	0.099	0.449	-0.041	0.045	-0.522	0.066
06/1994	t-value	2.761*	7.181*	-1.047	1.045	-7.90*	1.725
Subperiod 4	Days		199			179	
07/1994 –	Estimate	0.111	0.340	-0.135	-0.047	-0.464	0.216
12/1995	t-value	2.128**	3.898*	-2.752*	-1.257	-8.059*	6.042*
Subperiod 5	Days		227			151	
01/1996 –	Estimate	0.167	0.256	-0.064	-0.063	-0.459	0.153
06/1997	t-value	5.493*	5.497*	-3.161*	-1.663	-9.203*	4.453*
Subperiod 6	Days		188			190	
07/1997 –	Estimate	0.036	0.595	-0.172	-0.047	-0.704	0.144
12/1998	t-value	0.556	6.075*	-3.079*	-0.701	-7.800*	2.504**
Subperiod 7	Days		191			187	
01/1999 –	Estimate	-0.038	0.502	-0.106	0.048	-0.537	0.138
06/2000	t-value	-0.897	5.886*	-1.526	1.194	-7.079*	2.039*

* Significant at the 1 percent level and ** significant at the 5 percent level

Unsystematic Risk

Some studies reveal that unsystematic risk is priced, thereby casting doubt on the validity of the two-moment CAPM. In this study, we examine conditional models and therefore investigate whether the unsystematic risk is priced in the conditional models. Toward this end, we consider the variance of the residuals obtained in the estimation of the following portfolio return-generating process:

$$R_{pt} = \alpha_p + \beta_p R_{mt} + \gamma_p R_{mt}^2 + \theta_p R_{mt}^3 + \varepsilon_{pt} \tag{15}$$

as a proxy for the unsystematic risk. Here, the portfolio beta, gamma and theta in Stage-I of the two-stage estimation procedure is estimated with the return-generating process given in equation (15) instead of using the formulae (2) through (4). To test whether unsystematic risk is priced the conditional model (11) is enhanced to

$$\begin{aligned} R_{pt} = {} & \delta_{0t} + \delta_{1t}\kappa\beta_p + \delta_{2t}(1-\kappa)\beta_p + \delta_{3t}\kappa\gamma_p + \delta_{4t}(1-\kappa)\gamma_p \\ & + \delta_{5t}\kappa\theta_p + \delta_{6t}(1-\kappa)\theta_p + \delta_7\kappa \mathrm{Var}(\varepsilon_p) + \delta_8(1-\kappa)\mathrm{Var}(\varepsilon_p) + U_{pt} \end{aligned} \tag{16}$$

by adding the variance of the residuals estimated in equation (15). We refer to equation (16) as the extended Model A^c.

The results of (16) and the other three enhanced conditional models that correspond to equations (12) through (14) are reported in Table 8. In all four models there is no evidence to reject the null hypothesis $\overline{\delta}_7 = 0$ in favor of the alternative $\overline{\delta}_7 \neq 0$ at the 5 percent level. The results support the two- and three-moment conditional models as did in the models (11) through (14) without the error variance term.

Conclusions

The debate about the ability of systematic co-moments to explain the cross-sectional returns continues. This paper contributes to this debate. Consistent with previous studies, we find that unconditional models fail to explain the cross-sectional variation in the observed returns. When the sample period is split into two subperiods depending on whether the market return in excess of the risk-free rate is positive or negative, the co-variance and co-skewness are priced. In the up market, the market returns distribution is positively skewed. In the down market, it is negatively skewed. The signs of beta and co-skewness risk premium are consistent with what was expected: (i) the positive beta risk premium and the negative co-skewness risk premium in the up market and (ii) the negative beta risk premium and the positive co-skewness risk premium in the down market. The systematic co-kurtosis does not appear to be priced.

We considered Australian data where the market return distribution in the sample period is negatively skewed. Many emerging markets were found to have positive skewness, while most of the developed markets have negative skewness. (See, for example, Aggarwal, Inclan and Leal, 1999.) Thus, an extension of this study to emerging markets might give further insights into how the conditional pricing models would work under different economic conditions. Further, our analysis considers only a small sample of portfolios, and stronger evidence may be found with larger number of portfolios. The issue of adopting the results of conditional higher-order pricing models to test risk-return trade off is not addressed in this paper, and it would be a topic for future research.

Table 8—Estimates of Risk Premium in Extended Conditional Pricing Models Using Composite Return

	Up Market					Down Market				
Model	δ_0	δ_1	δ_3	δ_5	δ_7	δ_0	δ_2	δ_4	δ_6	δ_8
Extended Model A^c: $R_{pt} = \delta_{0t} + \delta_{1t}\kappa\beta_p + \delta_{2t}(1 - \kappa)\beta_p + \delta_{3t}\kappa\gamma_p + \delta_{4t}(1 - \kappa)\gamma_p + \delta_{5t}\kappa\theta_p + \delta_{6t}(1 - \kappa)\theta_p + \delta_7\kappa Var(\varepsilon_p) + \delta_8(1 - \kappa)Var(\varepsilon_p) + U_{pt}$										
Estimate	0.076	0.611	-0.242	-0.011	-9.4x10-5	0.001	-0.773	0.385	-0.012	-4.3x10-5
t-value	2.187**	7.628*	-1.030	-0.054	-1.684	0.020	-7.680*	1.469	-0.049	-0.732
Extended Model B^c: $R_{pt} = \delta_{0t} + \delta_{1t}\kappa\beta_p + \delta_{2t}(1 - \kappa)\beta_p + \delta_{3t}\kappa\theta_p + \delta_{4t}(1 - \kappa)\gamma_p + \delta_{7t}\kappa Var(\varepsilon_p) + \delta_8(1 - \kappa)Var(\varepsilon_p) + U_{pt}$										
Estimate	0.103	0.411	-0.110	-	-8.7x10-6	-0.025	-0.512	0.143	-	-4.3x10-6
t-value	4.320*	10.206*	-3.653*	-	-0.645	-1.085	-12.684*	4.620*	-	-0.311
Extended Model C^c: $= \delta_{0t} + \delta_{1t}\kappa\beta_p + \delta_{2t}(1 - \kappa)\beta_p + \delta_{5t}\kappa\theta_p + \delta_{6t}(1 - \kappa)\theta_p + \delta_{7t}\kappa Var(\varepsilon_p) + \delta_8(1 - \kappa)Var(\varepsilon_p) + U_{pt}$										
Estimate	0.070	0.298	-	0.008	6.9x10-6	-0.027	-0.339	-	0.015	-2.1x10-6
t-value	3.656*	13.604*	-	0.583	0.377	-1.168	-13.393*	-	1.001	-1.042
Extended Model D^c: $R_{pt} = \delta_{0t} + \delta_{1t}\kappa\beta_p + \delta_{2t}(1 - \kappa)\beta_p + \delta_{7t}\kappa Var(\varepsilon_p) + \delta_8(1 - \kappa)Var(\varepsilon_p) + U_{pt}$										
Estimate	0.096	0.299	-	-	-8.1x10-6	-0.037	-0.3510	-	-	2.4x10-6
t-value	4.766*	13.789*	-	-	-0.537	-1.757	-16.062*	-	-	0.130

* Significant at the 1 percent level and ** significant at the 5 percent level

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