Tracking Error in the Dow Jones Industrial Average Versus Alternative Market Indices: New Evidence

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This study investigates whether changes in the Dow Jones Industrial Average (DJIA) adequately track changes in the U.S. stock market by comparing DJIA changes during a given time period with contemporaneous changes in alternative market indices. Earlier studies show that change in the DJIA is an unbiased predictor of change in capitalization of value-weighted portfolios of the Dow Jones Industrials (DJI) or the S&P 500. The authors observe that the DJIA exhibits a systematic tendency to overstate contemporaneous changes in market capitalization versus value-weighted portfolios of either the DJI or S&P 500. The authors also find that the DJIA exhibits significant and systematic tracking error as a measure of change in total wealth in the market.

Introduction

The Dow Jones Industrial Average (DJIA) is the most widely followed stock market index in the United States. Many people intuitively use the DJIA as an indicator for performance in the broad market. But does the DJIA track the market accurately?

The editors of *The Wall Street Journal* construct the DJIA by selecting a subset of 30 stocks to represent the entire U.S. stock market and then computing a price-weighted index of the subset. This approach to measuring the market has four potential flaws. First, a price-weighted index reflects changes in stock prices but not changes in market capitalization. In particular, each time a component stock splits, its weight in the DJIA decreases. A value-weighted (i.e., capitalization-weighted)

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index would overcome this potential weakness.[1] Second, the DJIA may include too few stocks to adequately represent the U.S. stock market. Over 8,000 common stocks actively trade in the U.S. today. The 30 stocks in the DJIA may be too small a sample. Third, the editors of *The Wall Street Journal* follow an informal and subjective approach to select the Dow Jones Industrials that may not yield the most representative sample. Finally, changes in the DJIA do not reflect the contribution that cash dividends make to total return.

The S&P 500 index addresses the first three potential flaws in the DJIA. The S&P 500 is a value-weighted index of 500 common stocks traded on U.S. markets and includes the 30 Dow Jones Industrials. In addition, the S&P 500 stocks currently make up roughly 80 percent of the total market capitalization in U.S. markets, while the Dow Jones Industrials account for about 27 percent as of the end of 1999. Thus, the specific details of the selection criteria applied by Standard & Poor are unlikely to have a significant effect on how well the index represents the market. For these reasons, we use the S&P 500 in our paper as a proxy for the U.S. stock market.

Recent studies show that the DJIA performs similarly to value-weighted indexes over the long run. Shoven and Sialm (2000) find that the DJIA is similar to a value-weighted version of the index as well as the S&P 500 over the period October 1928–December 1999. Both Shoven and Sialm (2000) and Clarke and Statman (2000) show that the chief flaw in the DJIA over the long run is that the index measures only capital appreciation, not total returns. (The DJIA shares this weakness with other widely reported indexes, e.g., the S&P 500, the Russell 3000, and the Wilshire 5000.)

We take a different approach. Our objective is to determine whether changes in the DJIA adequately track concurrent changes in the U.S. stock market. In other words, if the DJIA goes up or down one percent in a given time period, what does this tell us about change in the stock market for the same period? Do the returns of the DJIA and value-weighted market indices correlate? Do the returns distributions have similar variances and symmetry? Does the DJIA accurately track change in alternative value-weighted market portfolios? Is the DJIA an accurate measure of market performance?

Earlier studies report that the DJIA is an unbiased predictor of change in capitalization of value-weighted portfolios of the DJI and S&P 500. In this paper, we observe that the Dow's tracking error, as a measure of market performance, is both significant and systematic. We find the DJIA exhibits a systematic tendency to overstate and understate changes in market capitalization versus DJI and S&P value-weighted portfolios. When the market declines, the DJIA is more likely than not to exhibit a greater percentage decline; when the market rises, the DJIA is more likely

[1] In recent years, writers at *The Wall Street Journal* itself have recognized that an index weighted by market capitalization would better represent the stock market. For example, see Raghavan and Jeffrey (1996).

than not to exhibit a greater percentage rise. We also find that the magnitude of the corresponding change in value-weighted portfolios may differ from change in the DJIA as much as two to three percentage points. For example, if the DJIA decreases one percent, then capitalization of the market might actually have decreased as much as three or four percent (or—somewhat less likely—increased as much as one or two percent).

Finally, we confirm findings of earlier studies that the DJIA exhibits significant and systematic tracking error as a measure of change in total wealth in the market. Specifically, the DJIA tends to understate change in total wealth, because the index does not reflect cash dividends paid by its component stocks.

Data and Methodology

Our source for changes in the Dow Jones Industrials from 1980 through 1988 is the *Dow Jones Investor's Handbook* (1988). For information about changes in the Dow Jones Industrials after 1988, we rely on news stories from *The Wall Street Journal* published around the dates of the changes.[2]

We draw all other data from Standard & Poor's Research Insight (formerly Compustat PC). We collect monthly prices, monthly shares outstanding, monthly cash dividends, and monthly split adjustment factors to compute index weights, capitalization changes, and total returns. We examine Research Insight's monthly total return series for each stock to calculate a stock distribution adjustment factor (e.g., when American Express distributed 0.2 share of Lehman Brothers Holdings per share of common in May 1994). To estimate inflation, we use the monthly Consumers Price Index (All Urban). To estimate the monthly risk free rate of return, we divide the annualized one-month T-bill yield by 12.

We calculate two general types of indexes for January 1981 through December 1999: capital appreciation and wealth. The capital appreciation indexes reflect change in share prices of component stocks, while the wealth indexes reflect total return. We examine three different methods for weighting portfolios of the 30 Dow Jones Industrials (DJI). (For a summary of the formulas for all indexes discussed below, see Appendix A.)

The general formula for all indexes is

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[2] The composition of the DJIA changed three times in 1989–1999. Caterpillar, Walt Disney, and J.P. Morgan replaced Navistar International, USX Corp., and Primerica on May 6, 1991 (*The Wall Street Journal*, 28 May 1996, p. R30). Hewlett-Packard, Johnson & Johnson, Travelers Group, and Wal-Mart Stores replaced Texaco, Bethlehem Steel, Westinghouse Electric, and Woolworth on March 12, 1997 (*The Wall Street Journal*, 13 March 1997, p. C1). SBC Communications, Microsoft, Home Depot, and Intel replaced Chevron, Goodyear Tire, Sears Roebuck, and Union Carbide on November 1, 1999 (*Wall Street Journal Interactive Edition*, 27 October 1999).

$$V_t = V_{t-1}\left(\sum_{i \in S} w_{i,t}(1 + r_{i,t})\right), \quad (1)$$

where:

V_t = Index value at the end of period t;
V_{t-1} = Index value at the beginning of period t;
$r_{i,t}$ = Capital appreciation or total return (depending on the index) for stock i over period t;
S = The set of stocks that are or have been in the DJI during Jan. 1981–Dec. 1999; and
$w_{i,t}$ = The weight for stock i for period t.

To compute the DJI capital appreciation indexes, we let

$$r_{i,t} = \frac{A_{i,t}P_{i,t}}{P_{i,t-1}} - 1, \quad (2)$$

where:

$P_{i,t}$ = Share price of stock i at the end of period t;
$P_{i,t-1}$ = Share price at the beginning of period t; and
$A_{i,t}$ = Adjustment factor for stock splits or stock distributions that occur during period t.

The adjustment factor equals the split or distribution ratio. For example, $A_{i,t} = 2$ if a two-for-one split occurred during period t. Otherwise, the adjustment factor equals one. To compute the DJI wealth indexes, we let

$$r_{i,t} = \frac{A_{i,t}(P_{i,t} + D_{i,t})}{P_{i,t-1}} - 1, \quad (3)$$

where:

$D_{i,t}$ = Cash dividend per share if stock i goes ex-dividend during period t; and all other symbols are defined as above.

We examine three different methods for weighting portfolios of the 30 Dow Jones Industrials: value-weighted (i.e., capitalization-weighted), price-weighted, and equal-weighted.

For the value-weighted index,

$$w_{i,t} = \frac{I_{i,t-1}N_{i,t-1}P_{i,t-1}}{\sum_{j \in S} I_{j,t-1}N_{j,t-1}P_{j,t-1}}, \quad (4)$$

where:

$I_{i,t}$ = 1 if stock i is in the index at time t, and $I_{i,t} = 0$ otherwise; and
$N_{i,t}$ = Number of shares of common stock outstanding at time t.

(Note that t = end of period t, while t-1 = beginning of period t.) We construct the value-weighted index so that the weight of the stock is determined by its market capitalization relative to all the stocks in the index at the beginning of period t. For monthly returns, the index measures capital appreciation of a value-weighted portfo-

lio of the DJI purchased at the beginning of the month and held for the entire month. Investors trying to match this index would adjust their portfolios at the end of the month if composition of the index changes during the month or if a component company changes its market cap by issuing new shares or repurchasing outstanding shares.

For the price-weighted index,

$$w_{i,t} = \frac{I_{i,t-1}P_{i,t-1}}{\sum_{j \in S} I_{j,t-1}P_{j,t-1}}, \quad (5)$$

where:

$I_{i,t}$ = 1 if stock i is in the index at time t, and
$I_{i,t}$ = 0 otherwise.

We construct the price-weighted index so that the weight of the stock is determined by its share price relative to the total share prices of all stocks in the index at the beginning of period t. For monthly returns, the index measures capital appreciation of a price-weighted portfolio of the DJI purchased at the beginning of the month and held for the entire month. The price-weighted index measures capital appreciation of a portfolio in which the investor buys an equal number of shares of each stock in the index at the beginning of the month and holds these shares (and any stock splits or distributions) for the month. Investors trying to match this index would adjust their portfolios at the end of the month if composition of the index changes during the month or if a component company has a stock split or stock distribution that goes ex-distribution during the month.

The price-weighted index is similar to but not quite the same as the DJIA, because the composition of the DJIA usually changes during the month rather than at the end. Furthermore, *The Wall Street Journal* editors adjust the DJIA divisor for stock splits and distributions when the stock goes ex-split or ex-distribution. The monthly price-weighted index, however, implicitly assumes rebalancing occurs at the end of the month.

For the equal-weighted index,

$$w_{i,t} = \frac{I_{i,t-1}}{\sum_{j \in S} I_{j,t-1}} = \frac{I_{i,t-1}}{30}, \quad (6)$$

where:

$I_{i,t}$ = 1 if stock i is in the index at time t, and
$I_{i,t}$ = 0 otherwise.

The DJIA has 30 stocks since 1 October 1928, so $w_{i,t} = 1/30$ if stock i is in the index at time t and $w_{i,t} = 0$ otherwise. For monthly returns, the index measures capital appreciation of an equal-weighted portfolio of the DJI purchased at the beginning of the month and held for the entire month. The equal-weighted index measures capital appreciation of a portfolio in which the investor buys an equal dollar amount of each

stock in the index at the beginning of the month and holds these shares (and any stock splits or distributions) for the month. Investors trying to match this index would adjust their portfolios at the end of the month if composition of the index changes during the month or if the portfolio is not equally balanced at the end of the month.

We set all DJI wealth and capital appreciation indexes equal to 100 for December 1980. When we compare these indexes to the DJIA and S&P 500, we rescale the latter indexes to equal 100 at the end of December 1980.

Findings: Comparison of Indexes for Capital Appreciation and Wealth

Our three alternative capital appreciation indexes—price-weighted (PWCAP), value-weighted (VWCAP), and equal-weighted (EWCAP)—perform similar to the DJIA over January 1981–December 1999. This conclusion is consistent with the results reported by Clarke and Statman (2000) for the longer historical period, October 1928–December 1999. Each of the DJI capital appreciation indexes is highly correlated with the DJIA. The sample correlations are greater than 0.96 (Table 1). The DJIA should be most similar to the PWCAP index, because both are price-weighted. The correlation between monthly returns on the DJIA and the PWCAP is 0.999.

Means and variances for the alternative DJI indexes are similar to those of the DJIA (Table 1). Mean monthly nominal capital appreciation returns are within 0.04 percent of each other. Based on the t-values presented in Table 1, the differences between the mean returns on the DJIA and other DJI capital appreciation indexes are not significantly different from zero. F-tests for differences between variance of monthly returns are not significant for any pair of capital appreciation indexes. In particular, the F-tests for the difference between return variances on the DJIA and each of the other DJI capital appreciation indexes are not significant.

One potential weakness in the DJIA is that 30 stocks may be too few to adequately represent U.S. stock markets. To evaluate the effect of increasing the number of stocks in the index, we compare VWCAP (our value-weighted version of the DJIA) to the S&P 500 (a value-weighted index).

The two value-weighted indexes closely track each other and the DJIA over January 1981–December 1999. The mean monthly nominal capital appreciation returns on the VWCAP and S&P 500 are nearly the same, and neither is significantly different from the mean return on the DJIA (Table 1). Variance of monthly nominal capital appreciation returns on the VWCAP and S&P 500 also are similar, and neither is significantly different from the monthly return variance on the DJIA.

Clarke and Statman (2000) and Shoven and Sialm (2000) both report that the DJIA drastically understates the total return on the Dow Jones Industrials over the

Table 1—Monthly Returns on DJIA Versus Various Capital Appreciation and Wealth Indexes of the Dow Jones Industrials and the S&P 500 (Jan. 1981–Dec. 1999)

	DJIA	PWCAP TOT	VWCAP TOT	EWCAP TOT	S&P 500
Mean Monthly Nominal Returns Based on Capital Appreciation	1.19%	1.17%	1.15%	1.17%	1.14%
Variance of Monthly Nominal Returns Based on Capital Appreciation	18.91	18.58	17.10	20.61	18.24
Correlation Coefficient Between Returns on DJIA & Other Capital Appreciation Index	na	0.999	0.961	0.983	0.951
t-statistic for Difference Between Mean Returns on DJIA & Other Capital Appreciation Index	na	0.045	0.095	0.054	0.116
F-statistic for Difference Between Return Variances on DJIA & Other Capital Appreciation Index	na	1.018	1.106	0.918	1.037
Index Value on Dec. 99 (Dec 80 = 100) for DJIA vs. Total Wealth Indexes	1193	1926	2266	2116	2019
Mean Monthly Nominal Returns Based on Total Wealth	na	1.39%	1.46%	1.45%	1.42%
Mean Monthly Real Returns Based on Total Wealth	na	1.08%	1.15%	1.14%	1.10%

Notes: PWCAP/TOT is price-weighted index of DJ stocks whose returns are based either on capital appreciation (CAP) or total wealth (TOT). VWCAP/TOT is the value-weighted index of DJ stocks whose returns are based either on CAP or TOT. EWCAP/TOT is the equally weighted index of DJ stocks whose returns are based either on CAP or TOT. Results for capital appreciation indexes are reported in the first five rows. Results for total wealth indexes are reported in the last two rows. All returns are based on end-of-month index values for January 1981 through December 1999

modern history of the DJIA that began on October 1, 1928.[3] We confirm that this behavior continues over the most recent two decades, January 1981–December 1999.

The nominal wealth indexes for price-weighted (PWTOT), value-weighted (VWTOT), and equal-weighted (EWTOT) portfolios of the Dow Jones Industrials all rise much higher than the DJIA and perform similar to the nominal wealth index for

[3] Clarke and Statman (2000) do not use actual cash dividends in their calculations. Instead, they assume DJIA dividend yields are equal to dividend yields on the S&P 500. This appears to be a reasonable approximation. The total returns that they compute for the DJIA for 1980–1998 are about the same as the total returns that we compute for that period using actual cash dividends paid by the Dow Jones Industrial stocks.

the S&P 500 over January 1981–December 1999 (Table 1). The mean monthly nominal returns for the DJI total return indexes are close to the mean for the S&P 500 total return index. This also is true for the corresponding mean monthly real returns. Thus, a total return index for the Dow Jones Industrials—whether for price-weighted, value-weighted, or equal-weighted returns—is a significantly better measure of total return on the market than the DJIA itself.

Findings: Conditional Distributions of Value-Weighted Capital Appreciation Indexes

Despite the similarities in their sampling distributions and high correlations, the DJIA is not a perfect substitute for the value-weighted indexes. In this section we analyze the tracking error between the DJIA and either its value-weighted version or the S&P 500.

We perform a bootstrap analysis on the observed differentials between the monthly returns on the DJIA and the two value-weighted capital appreciation indexes, VWCAP and the S&P 500. We draw 1,000 bootstrap samples with replacement from the 228 monthly differentials for January 1981–December 1999.[4] The bootstrap histograms are presented in Figures 1 and 2.

Both figures suggest that deviations between the percentage change in the DJIA and the percentage changes in the VWCAP and S&P 500 are both frequent and substantial. In about one-third of the months, the differential between monthly return on the DJIA and VWCAP is greater than one percent in magnitude. In about 40 percent of the months, the differential between monthly return on the DJIA and S&P 500 is greater than one percent in magnitude. This lack of precision suggests that change in the DJIA does not tell us how the capitalization of the market has changed, either for the Dow Jones Industrials stocks or for the broader market, except that the differential probably is no more than three percent.

The bootstrap histograms on the differentials between returns on the DJIA and the two value-weighted total return indexes are presented in Figures 3 and 4. We expect the DJIA to have a lower rate of change than either value-weighted total returns index, because the DJIA does not include cash dividends. The reported frequency distributions in both figures support this contention. The differential is negative in more than half of the months: 64.5 percent of months for the differential between the returns on the DJIA and VWTOT; 57.0 percent of months for the differential between the DJIA return and the S&P 500 total return.

We also examine the distributions of the differentials between monthly return on the DJIA and monthly return on the value-weighted capital appreciation indexes, VWCAP and S&P 500, conditional on the percentage change in these indexes. These

[4] Efron and Tibshirani (1998) suggest that 1,000 bootstrap samples provide reasonable estimates.

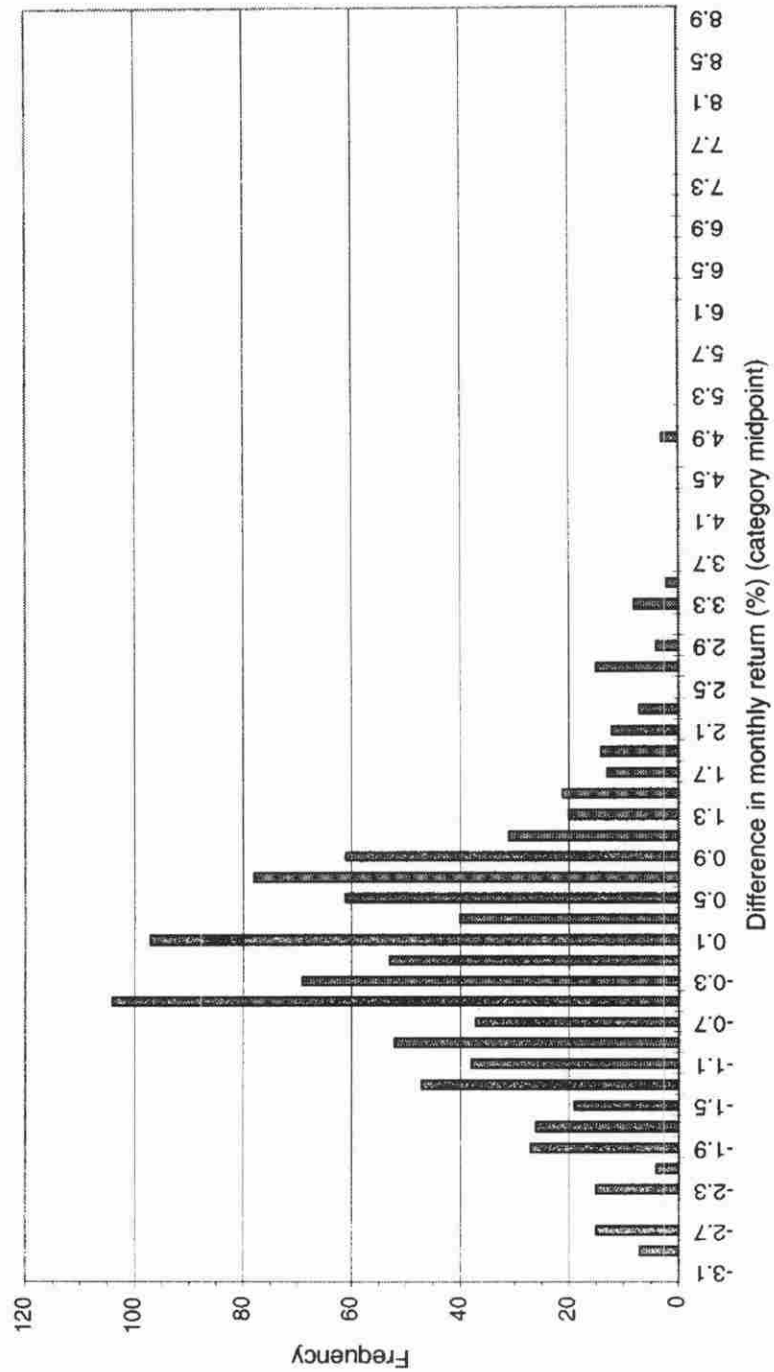

Figure 1—Bootstrap Histogram of Difference Between Return on DJIA (%) and Capital Appreciation on Value-Weighted Dow Jones Industrials (%) (1000 bootstrap samples from 1981-1999 monthly returns)

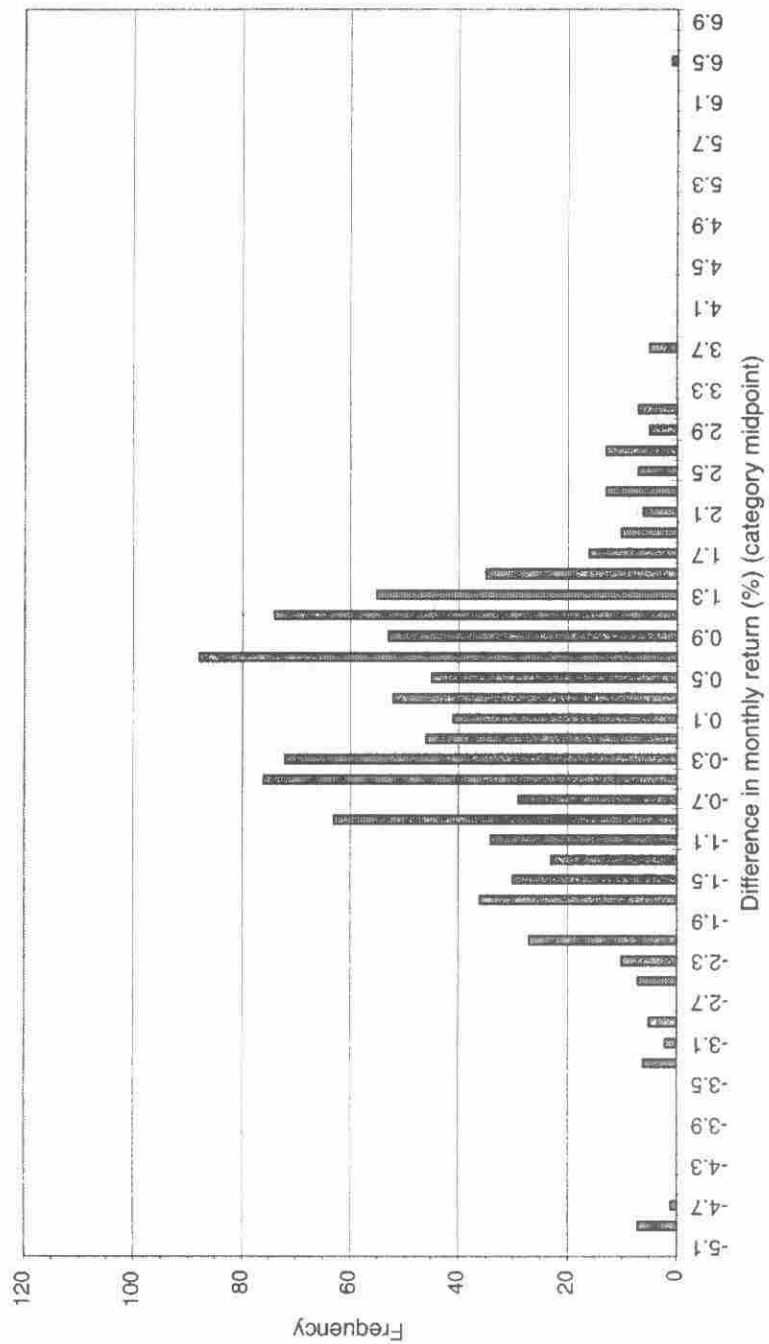

Figure 2—Bootstrap Histogram of Difference Between Return on DJIA (%) and S&P 500 (%) (1000 bootstrap samples from 1981-1999 monthly returns)

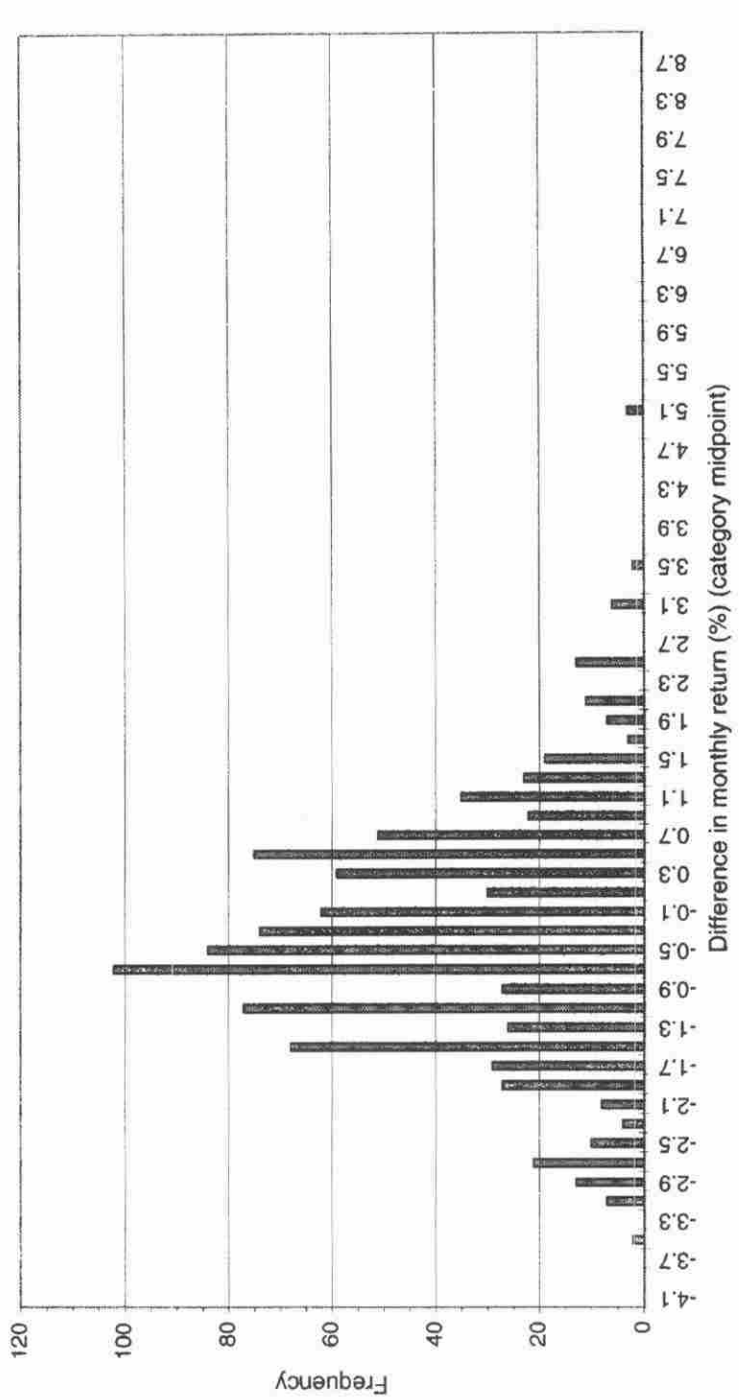

Figure 3—Bootstrap Histogram of Difference Between Return on the DJIA (%) and Nominal Total Return on Value-Weighted Dow Jones Industrials (%) (1000 bootstrap samples from 1981-1999 monthly returns)

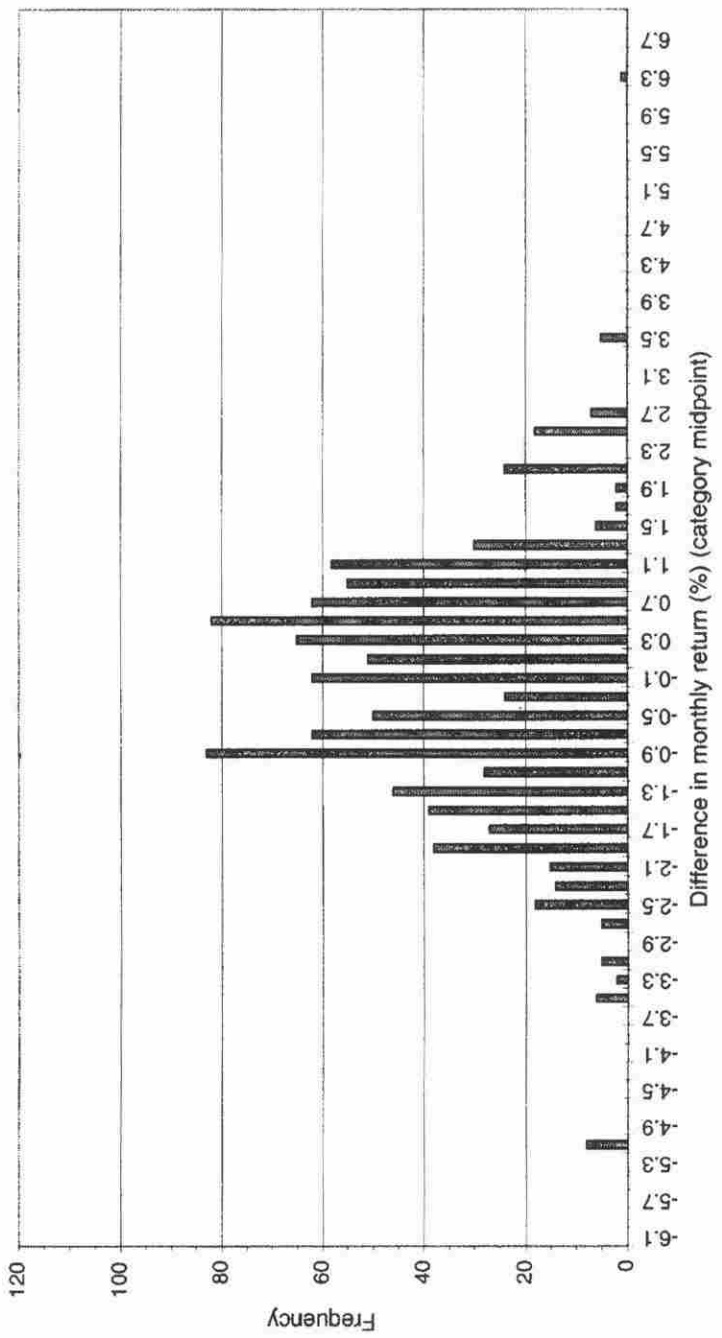

Figure 4—Bootstrap Histogram of Difference Between Monthly Return on the DJIA (%) and Monthly Nominal Total Return on S&P 500 (%) (1000 bootstrap samples from 1981-1999 monthly returns)

results are presented in Table 2. Panel A reports the results based on price appreciation, while Panel B reports results based on total returns. Panel A indicates that the return differentials are negatively skewed when return on the DJIA is negative but positively skewed when the return on the DJIA is positive. Based on chi-square tests, this pattern is statistically significant.

In panel B, we show that the percentage change in the DJIA tends to overstate the magnitude of the percentage change in VWTOT and wealth index for S&P 500. The differential with VWTOT is greater than one percent either way in about 40 percent of the months and is greater than two percent either way in about 9 percent of the months. The differential with the S&P 500 total return is greater than one percent either way in about 40 percent of the months and is greater than two percent either way in about 12 percent of the months.

The conditional distributions of the differentials between monthly return on the DJIA and monthly total return on the value-weighted indexes, VWTOT and the wealth index for the S&P 500, are skewed left. That is, the DJIA tends to underestimate change in wealth in the market. Chi-square statistics show that this pattern is statistically significant. As expected, the DJIA exhibits a lower rate of change compared to both value-weighted total returns indexes. This finding is consistent with prior studies [Clarke and Statman (2000) and Shoven and Sialm (2000)] and also with the bootstrap histograms presented in Figures 3 and 4.

Conclusions

Earlier work by Clarke and Statman (2000) and Shoven and Sialm (2000) shows that the DJIA performs reasonably well as a capital appreciation index for the U.S. stock market on average over the long run (1928–1999). We confirm that this pattern has continued over the last two decades. Furthermore, the method for weighting the stocks in the Dow Jones Industrials does not make an appreciable difference in this performance, on average.

We, however, show that a given monthly return on the DJIA is poor measure of change in the capitalization of the stock market for the same month; the DJIA exhibits significant tracking error. For monthly changes in market capitalization, we find that magnitude of the corresponding change in value-weighted portfolios of either the Dow Jones Industrials or the S&P 500 has differed from the DJIA by more than one percent about one-third of the time. In some months, this difference has been as large as two to three percentage points in either direction.

We also observe a systematic tendency of the DJIA to overstate change in capitalization of the market represented by value-weighted portfolios of either the Dow Jones Industrials or the S&P 500. Conditional distributions of the differentials between DJIA monthly return and monthly return on the value-weighted capital appreciation indexes are skewed left when return on the DJIA is negative but skewed right when the return on the DJIA is positive. This pattern is statistically significant.

Table 2—Degree to Which DJIA Over- or Underestimates the Market Represented by VW DJI and S&P 500 Indexes, Conditional on Monthly Return on the DJIA Index

Change in DJIA	DJIA **Under-estimates** VW Index Returns By > 1% (mos.)	DJIA Returns Within +/- 1% of VW Index Returns (mos.)	DJIA **Over-estimates** VW Index Returns By > 1% (mos.)	DJIA **Under-estimates** S&P 500 Returns By > 1% (mos.)	DJIA Returns Within +/- 1% of S&P 500 Returns (mos.)	DJIA **Over-estimates** S&P 500 Returns By > 1% (mos.)
Panel A: Returns Based on Capital Appreciation (228 monthly observations in Jan. 1981–Dec. 1999)						
< -1%	**16**	36	**5**	**16**	34	**7**
Between +/- 1%	**9**	35	**5**	**10**	24	**15**
> 1%	**16**	79	**27**	**17**	75	**30**
Total Obs.	41	150	37	43	133	52
Percent of Obs.	18.0%	65.8 %	16.2 %	18.9 %	58.3 %	22.8 %
Chi-Square Test		9.72**			8.12*	
Panel B: Returns Based on Total Wealth (228 monthly observations in Jan. 1981–Dec. 1999)						
< -1%	**28**	24	**5**	**17**	30	**10**
Between +/- 1%	**12**	33	**4**	**8**	35	**6**
> 1%	**24**	80	**18**	**33**	73	**16**
Total Obs.	64	137	27	58	138	32
Percent of Obs.	28.1 %	60.1 %	11.8 %	25.4 %	60.5 %	14.0 %
Chi-Square Test		20.89**			8.00*	

Notes: The chi-square statistic tests whether the proportion of over- and underestimations are equal, conditional on change in the DJIA; it has two degrees of freedom

* Significant at the 5% level

** Significant at the 1 percent level

Clarke and Statman (2000) and Shoven and Sialm (2000) also show that the DJIA grossly understates the total return on the U.S. stock market over the long run (1928–1999). We confirm that this pattern has continued over the last two decades. From December 1980 to December 1999, the DJIA has increased by a factor of 11.93, while the nominal value-weighted wealth indexes for the Dow Jones Industrials and the S&P 500 have risen by factors of 22.66 and 20.19, respectively.

We also show that the DJIA exhibits significant and systematic tracking error as a measure of change in total wealth in the market. The difference between the DJIA and the nominal wealth index for the S&P 500 is greater than one percent either way in about 40 percent of the months and is greater than two percent either way in about 12 percent of the months from 1981 to 1999. The DJIA also underestimates change

in the total return on the S&P 500 more often that it overestimates this change. This bias is significant.

Does change in the Dow tell us how the stock market performed last month? No, it does not. The DJIA tends to systematically overstate monthly change in capitalization of the market (either up or down) and tends to systematically underestimate change in total wealth in the market. Only over the long run does the DJIA accurately represent the market on average—but even then, only capital appreciation of the market.

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Appendix

Table 3—Composition of Value-, Price-, and Equal-Weighted Stock Indexes: A Summary

Generalized Index Value	$V_t = V_{t-1}\left(\sum_{i \in S} w_{i,t}(1 + r_{i,t})\right)$		
Return Calculation	Based on Price Appreciation $r_{i,t} = \frac{A_{i,t}(P_{i,t})}{P_{i,t-1}} - 1$		Based on Price Appreciation and Dividend Distribution $r_{i,t} = \frac{A_{i,t}(P_{i,t} + D_{i,t})}{P_{i,t-1}} - 1$
Weighting Factor	Value-Weighted Index $w_{i,t} = \frac{I_{i,t-1}N_{i,t-1}P_{i,t-1}}{\sum_{j \in S} I_{j,t-1}N_{j,t-1}P_{j,t-1}}$	Price-Weighted Index $w_{i,t} = \frac{I_{i,t-1}P_{i,t-1}}{\sum_{j \in S} I_{j,t-1}P_{j,t-1}}$	Equal-Weighted Index $w_{i,t} = \frac{I_{i,t-1}}{\sum_{j \in S} I_{j,t-1}} = \frac{I_{i,t-1}}{30}$

where:

- V = Index value
- r_i = Return on stock i during time period (t-1, t)
- w_i = Weighting factor for stock i during time period (t-1, t)
- S = Set of stocks that have been included in DJI during January 1981-December 1999
- A_i = Adjustment factor for stock i during time period (t-1, t)
- P_i = Price of stock i
- D_i = Dividend per share of stock i distributed during time period (t-1, t)
- I_i = One if stock i is in the index at time t, and zero otherwise
- N_i = Number of shares of stock i outstanding at time t