On the Gompertz Process and New Product Sales: Some Further Results from Cointegration Analysis

Ali F. Darrat*
Louisiana Tech University

This paper reevaluates Franses' (1994) recent cointegration and error-correction analysis of the Gompertz process for new product sales. I find that Franses' results in support of the Gompertz process and its implied cointegration relation questionable due to his use of inappropriate statistical methods. Stronger and more recent testing procedures reject Franses' inferences and yield no compelling evidence of cointegration in the data. Results from out-of-sample forecasts provide further evidence bolstering the central conclusion of the paper.

Introduction

A common diffusion model for estimating new product sales is the Gompertz process whereby the growth rate of sales first increases, reaches an inflection or peak point, and then decreases over time until it settles at a finite saturation level. (See Mahajan and Muller, 1979; Meade, 1984; and Lilien *et al.*, 1992.) In an insightful paper, Franses (1994) uses Dutch data to investigate whether new car sales follow a Gompertz-type pattern in the context of a cointegrated trivariate system. As Franses demonstrates, the Gompertz model presupposes the use of stationary variables whose stochastic properties (mean, variance, and covariance) are time-invariant. In addition, the validity of the Gompertz process in new product sales also requires the presence of a cointegrating (equilibrium) relationship binding the variables of the model.

Cointegrating analysis has become an important area of research in many business fields, and the growing numbers of marketing studies show the keen interest in the subject of cointegration. While several marketing researchers employ the cointegration approach, particularly in the advertising/sales relationship (e.g., Baghestani, 1992; Zanias, 1994), a quick review of the literature suggests that Franses' (1994) study represents the first attempt to use cointegration and error-correction analysis in the area

* I wish to thank G. Brown, S. Dwyer, H. Mesak, and an anonymous referee for many helpful comments and suggestions. The usual disclaimer applies.

0747-5535/00/1600/0049/$2.00

of new product introductions. Nevertheless, Franses' pioneering application contains some methodological errors that need to be rectified in order to minimize their repetition in future marketing research. Correcting for these errors, the empirical results I obtain from the same Dutch data are at variance with his conclusions. I should emphasize at the outset that Franses' theoretical attempt to model the Gompertz process within a cointegrated error-correction framework is provocative and not disputed here. Rather, my main purpose is to show that Franses' *empirical* results are doubtful, for they seem predicated on a fragile statistical basis.

Stationary Test Results

Because most economic (and marketing) variables are often nonstationary, Franses examines the stationarity of the three main variables in the model, each expressed in natural logarithms. The variables are new car sales, x; the relative price of petrol, p; and consumers' income proxied by real gross domestic product, y. Franses uses the Dickey-Fuller test (DF), reporting that all three variables are nonstationary in levels.[1] Without any prior testing, Franses then presumes that first-differences will induce stationarity in each of three variables. He proceeds to test for cointegration among the levels of the variables using the Engle-Granger (1987) two-step procedure. Franses estimates a cointegrating equation using the levels of the three variables and then applies the Dickey-Fuller test to the residuals. Compared to the critical value tabulated in Engle and Yoo (1987), Franses contends that the calculated statistic (= -4.296) is sufficiently large to reject the null of no cointegration at the 5 percent level of significance. Franses' apparent finding of significant cointegration among the three marketing variables supports the Gompertz process and leads him to estimate an error-correction model in first-differences (i.e., $\Delta_1 x_t = a_0 + a_1 \Delta_1 p_t + a_2 \Delta_1 y_t + a_3 EC_{t-1} + e_t$, where EC is the error-correction term distilled from the alleged cointegrating relation).

Franses employs the Dickey-Fuller procedure to test for nonstationarity of the variables and also for their cointegratedness. Yet, it is well-known that this test is inappropriate due to the typical presence of significant autocorrelation problems. Therefore, researchers often augment the Dickey-Fuller testing equation with some lagged values of the dependent variable to mitigate the autocorrelation problem. This modification yields the augmented Dickey-Fuller (ADF) test, which is a better alternative to test for nonstationarity in time series. (See, for example, Baillie and Selover, 1987; Zanias, 1994; and Harris, 1995.)[2] Therefore, I use the augmented Dickey-Fuller test to examine nonstationarity of the three variables. Because the ADF test is potentially sensitive to the particular selection of the lag truncation, I choose the proper lag length for each variable by the Akaike information criterion (AIC). Besides

1 Franses reports the following DF test statistics for the levels of the variables: -1.61 for x, -.29 for p, and -0.81 for y. Compared to the 5 percent critical value of -3.45, the statistics are too small to reject the null hypothesis of nonstationarity for each variable.

2 Although Franses discusses the ADF test in his appendix, he does not apply it to the data.

the ADF test, the recent Monte Carlo evidence of Pantula *et al.* (1994) supports the weighted-symmetric, (WS) test over many alternatives, including the ADF procedure. I apply both the ADF and the WS tests to check the sensitivity of the nonstationarity inferences to the testing procedures.

Table 1 assembles the results from the ADF and WS tests for each of the three variables expressed in levels, first-differences, and second-differences. To further check robustness, I perform the tests with and without time trends, but the results are essentially the same. Consistent with Franses, the results do not reject the null hypothesis of nonstationarity for all three variables if expressed in levels (Panel A in Table 1). In contrast to Franses' implicit conjecture, however, first-differencing the variables fails to convert *all* of them to stationary processes. While the new car sales variable (x) is stationary in first-differences, both relative prices (p) and real income (y) continue to be nonstationary in first-differences according to both tests (Panel B). Nonstationarity in relative prices and real income is eliminated only when both variables are transformed to second-differences (Panel C).

Table 1—Unit Root Test Results for the Univariate Time Series

	The ADF Test		The WS Test	
Variables	With Trend	Without Trend	With Trend	Without Trend
Panel A: In Levels				
Sales, x_t	-1.66 (2)	-2.06 (2)	-0.69 (2)	0.29 (2)
Price, p_t	-2.32 (4)	-2.36 (4)	-2.48 (2)	-2.34 (2)
Income, y_t	-1.28 (2)	-2.10 (2)	-0.76 (2)	-0.36 (4)
Panel B: In First-Differences (Δ_1)				
$\Delta_1 x_t$	-3.49 (2)**	-2.88 (2)**	-3.72 (2)**	-2.38 (3)
$\Delta_1 p_t$	-2.62 (2)	-2.72 (2)	-2.93 (2)	-2.86 (2)**
$\Delta_1 y_t$	-2.21 (2)	-1.64 (2)	-2.29 (2)	-1.91 (2)
Panel C: In Second-Differences (Δ_2)				
$\Delta_2 p_t$	-3.13 (3)*	-3.13 (2)**	-3.42 (2)**	-3.41 (2)**
$\Delta_2 y_t$	-3.47 (2)**	-2.88 (3)**	-3.21 (3)**	-3.11 (3)**

ADF = The augmented Dickey-Fuller test;
WS = The weighted symmetric test
The numbers in parentheses are the appropriate degree of augmentation (number of lags) determined by the Akaike information criterion (AIC)
** Rejection of the null hypothesis of nonstationarity (unit root) at the 5 percent level of significance
* Rejection at the 10 percent level

Cointegration and Forecasting Results

According to the above stationarity results, it appears that new car sales are stationary in first-differences but, and contrary to Franses' presumption, both relative prices and real income variables are stationary in second-differences. This finding has at least two critical implications: first, regression estimates using first-differences of all three variables such as those reported by Franses are potentially spurious due to the use of nonstationary variables (i.e., $\Delta_1 p$, and $\Delta_1 y$). Second, the above stationarity tests suggest that both p and y are integrated of a higher order [~I(2)] than that of x [~I(1)]. Thus, relative prices and real income cannot be cointegrated with new car sales (Granger, 1986), an inference at odds with Franses' central thesis.

One might object to the last implication (no cointegratedness among the three variables) on the ground that it is not based on formal testing. This objection seems reasonable particularly because recent research (e.g., Shoesmith, 1995) suggests the need to use formal statistical tests to confirm the absence of cointegration among variables having different orders of integration. The only testing prerequisite is for the variables to enter the cointegration test in their nonstationary forms (regardless of their integration order). Therefore, I formally test for the presence (or absence) of cointegration among new car sales (in levels) and relative prices and real income (both in first-differences).

Franses uses the Engle and Granger (EG) procedure to test for cointegration among the three variables.[3] But the literature shows that this testing procedure suffers from poor finite sample properties and also exhibits low empirical power (Kramers *et al.*, 1992; and Inders, 1993). Moreover, the EG method is a bivariate procedure, essentially unfit to test for cointegration in a multivariate context (Enders, 1995).[4]

An attractive alternative to the EG procedure, which Franses mentions but does not use, is the Johansen-Juselius (1990) efficient maximum likelihood method. The Johansen-Juselius (JJ) approach is based on the widely-accepted likelihood ratio principle and extensive Monte Carlo evidence, discussed in Cheung and Lai (1993) and Gonzalo (1994), shows that the approach is superior to many alternative methods. Another advantage of the JJ test, as Franses acknowledges, is that it does not require the assumption that the variables be weakly exogenous.[5] In addition, the JJ approach is particularly appropriate to test for cointegration in multivariate models with the potential for multiple cointegrating vectors.

Table 2 shows the cointegration test results from the JJ test. For completeness, I also report in the table the results from the EG test using the ADF and WS tests but with

[3]Besides the EG test, Franses also employs the approach of Boswijk (1992). Both the EG and the Boswijk methods are inappropriate for multivariate models.

[4]In his application of the EG test, Franses uses the critical values tabulated by Engle and Yoo (1987). Yet Davidson and MacKinnon (1993) and Harris (1995) demonstrate that these critical values are inaccurate. They recommend instead the use of the critical values reported in MacKinnon (1991).

[5]My own results from the exogeneity test proposed by Engle and Hendry (1993) indicate that neither p nor y are weakly exogenous to x. This finding strengthens the case for using the JJ approach. For space limitations, I do not report the results from the exogeneity test here, but they are available upon request.

MacKinnon's (1991) correct critical values.[6] To minimize biases in the EG results from the normalization problem, I follow Miller (1991) and choose to normalize on the variable that maximizes adjusted R-squares in the alternative cointegrating regressions. The statistical evidence in Table 2, Panel B, from the JJ efficient test (both in its λ-trace and λ-maximum eigenvalue versions) is consistent with my earlier (informal) inference, as it reveals no significant cointegration among the three variables. For the null hypothesis of no cointegration (r = 0), the λ-trace test statistic is only 16.84, comfortably below the 95 percent critical value of 42.44. Likewise, the λ-maximal eigenvalue test statistic of 11.33 is also much smaller than the 95 percent critical value of 25.54.[7] The EG procedure (with MacKinnon's correct critical values) suggests an identical verdict whereby both the ADF and the WS tests indicate no cointegration at any conventional level of significance (Table 2, Panel A).

Table 2—Cointegration Test Results

Panel A: Engle-Granger Approach

Appropriate Cointegrating Regression	The ADF Test	The WS Test
$x_t = f(\Delta_1 p_t, \Delta_1 y_t)$	-1.92 (2)	-1.11 (2)
	[0.64]	[0.96]

Panel B: The Johansen-Juselius Approach (Cointegrating Vector: x_t, $\Delta_1 p_t$, $\Delta_1 y_t$)

	The λ_{trace} Test			The λ_{max} Test		
Null Hypothesis	Alternative Hypothesis	Statistics	C. V. 95 Percent	Alternative Hypothesis	Statistics	C. V. 95 Percent
$r = 0$	$r \geq 1$	16.84	42.44	$r = 1$	11.33	25.54
$r \leq 1$	$r \geq 2$	5.51	25.32	$r = 2$	4.10	18.96
$r \leq 2$	$r = 3$	1.41	12.25	$r = 3$	1.41	12.25

See notes to Table 1. The values in brackets beneath the test statistics of the EG test are the p-values of significance obtained from the TSP computer package using the response surface estimates of MacKinnon (1991). r in the Johansen-Juselius test denotes the number of cointegrating vectors. The critical values for the Johansen-Juselius tests come from Osterwald-Lenum (1992)

In sum, contrary to Franses' contention, a battery of stationarity tests, results from the EG test but with correct critical values, and results from the JJ efficient test all consistently deny the presence of any significant cointegration in the Dutch data. Such a finding is at odds with the Gompertz process, at least in the context of Franses' trivariate model comprising new car sales, relative prices of petrol, and real income.

Before concluding, it is important to note that one principal purpose for constructing single-equation models is to forecast beyond the period of estimation (Pindyck and Rubinfeld, 1998). As Zanias (1994) and Curry *et al.* (1995) argue, the quality of

[6] I should caution, however, that powerful cointegration tests require a relatively long span of data, as Hakkio and Rush (1991) argue. In keeping with Franses' posture, though, I assume that the 30-year period of Dutch data is sufficiently long for this purpose.

[7] All tests include a time trend, though excluding it yields qualitatively similar results. According to the AIC procedure, the tests use four annual lags in the underlying VARs. While higher lags seriously deplete degrees of freedom, shorter lags give similar results.

out-of-sample forecasts is perhaps the most effective metric for judging rival models. Thus, holding the last five years (1984-1988) of the data set for out-of-sample forecasting exercises, I re-estimate Franses' ECM equation (with the alleged cointegrating relationship, but with nonstationary variables; that is, $\Delta_1 x_t = a_0 + a_1\Delta_1 p_t + a_2\Delta_1 y_t + a_3\ EC_{t-1} + e_t$) and estimate my alternative model (without cointegration, but with stationary variables; that is, $\Delta_1 x_t = b_0 + b_1\Delta_2 p_t + b_2\Delta_2 y_t + u_t$) over the subperiod 1960-1983. I then use the resulting estimates from the two alternative models to generate forecasts of new car sales (x) successively (one-year-ahead) for each of the five years outside the sample period (1984-1988). I compare the quality of the out-of-sample forecasts from the two rival models using the root-mean-square-error (RMSE) as well as Theil's inequality coefficient (U). Judged by the ability to forecast outside the sample, the results show that my model dominates Franses' ECM, as it reduces the forecasting errors by a considerable margin. My model yields RMSE = 35.496 and U = 0.108, which are 13 percent less than Franses' RMSE = 40.992 and U = 0.124.

Summary and Conclusions

My main purpose in this paper is to reevaluate Franses' (1994) empirical claim that the Dutch data of new car sales follow a Gompertz growth pattern. As Franses observes, the Gompertz pattern requires the presence of a significant cointegrating (long-run) relationship binding new car sales with their underlying determinants (relative prices of petrol and real income). Based on his empirical results, Franses contends that cointegration exists in the Dutch data in support of the Gompertz model and concludes that an error-correction model is an adequate representation of the data.

Applying cointegration and error-correction techniques is relatively novel in the marketing literature and, as such, Franses' work is useful. I show that Franses' results do not hold under stronger and more appropriate statistical tests of stationarity and cointegration. Using these tests on the same data and model as Franses, I find no compelling evidence of cointegration in the Dutch data and, moreover, Franses' ECM results appear spurious due to nonstationarity. To shed further light on these modeling issues, I compute out-of-sample, one-period-ahead, forecasts of new car sales from Franses' ECM equation (with the alleged cointegrating relation, but with nonstationary variables) and compare his results with those obtained from my proposed model (without cointegration, but with stationary variables). The latter model significantly dominates Franses' ECM, lending additional support to my stance against cointegration and stationarity in Franses' model.

Empirical results are predicated on the particular statistical procedures used. As more advanced tests evolve, so do conclusions. Therefore, my inference against cointegration and the Gompertz growth pattern in the Dutch data by no means represents a final verdict. Using a similar testing strategy as above but with different explanatory variables such as the price of new cars or personal disposable income (Chow, 1967; Horski, 1990) may overturn the conclusion of no cointegration.

Moreover, as the work of Mesak (1996) seems to imply, lack of cointegration could instead be the result of omitting distribution from the tested model.[8]

References

1. Baghestani, H., "Cointegration Analysis of the Advertising-Sales Relationship," *Journal of Industrial Economics*, 39 (1992), pp. 671-681.

2. Baillie, R.T., and D.D. Selover, "Cointegration and Models of Exchange Rate Determination," *International Journal of Forecasting*, 3 (1987), pp. 43-51.

3. Bliemel, F., "Theil's Forecast Accuracy Coefficient: A Clarification," *Journal of Marketing Research*, 10 (1973), pp. 444-446.

4. Boswijk, H.P., "Cointegration, Identification and Exogeneity: Inference in Structural Error-Correction Methods," unpublished thesis (1992).

5. Cheung, Y.W., and K.S. Lai, "Finite-Sample Sizes of Johansen's Likelihood Ratio Tests for Cointegration," *Oxford Bulletin of Economics and Statistics*, 55 (1993), pp. 313-328.

6. Chow, G.C., "Technological Change and the Demand for Computers," *American Economic Review*, 57 (1967), pp. 1117-1130.

7. Curry, D.J., S. Divakar, S.K. Mathur, and C.H. Whitemaker, "BVAR as a Category Management Tool: An Illustration and Comparison with Alternative Techniques," *Journal of Forecasting*, 14 (NEED YEAR), pp. 181-199.

8. Darrat, A.F., "Wage Growth and the Inflationary Process: A Reexamination," *Southern Economic Journal*, 61 (1994), pp. 181-190.

9. Davidson, R., and J.G. MacKinnon, *Estimation and Inference in Econometrics* (New York: Oxford University Press, 1993).

10. Enders, W.S., *Applied Econometrics Time Series* (New York: John Wiley & Sons, Inc., 1995.)

11. Engle, R.F., and C.W.J. Granger, "Co-Integration and Error-Correction: Representation, Estimation, and Testing," *Econometrica*, 55 (1987), pp. 251-276.

12. Engle, R.F., and D.F. Hendry, "Testing Superexogeneity and Invariance in Regression Models," *Econometrica*, 48 (1993), pp. 283-306.

13. Engle, R.F., and B.S. Yoo, "Forecasting and Testing in Co-Integrated Systems," *Journal of Econometrics*, 35 (1987), pp. 143-159

14. Franses, P.H., "Modeling New Product Sales; An Application of Cointegration Analysis," *International Journal of Research in Marketing*, 11 (1994), pp. 41-502.

15. Gonzalo, J., "Five Alternative Methods of Estimating Long-Run Equilibrium Relationships," *Journal of Econometrics*, 60 (1994), pp. 203-233.

16. Granger, C.W.J., "Developments in the Study of Cointegrated Economic Variables," *Oxford Bulletin of Economics and Statistics*, 48 (1986), pp. 213-228.

17. Granger, C.W.J., "Some Recent Developments in the Concept of Causality," *Journal of Econometrics*, 39 (1988), pp. 199-211.

---

[8]As Granger (1988) theorizes, and Miller (1991) and Darrat (1994) empirically verify, the omission of important variables from a given model can be responsible for noncointegration findings. Also, the Johansen-Juselius test only rejects the presence of a *linear* long-run relationship between new car sales and the two explanatory variables. It is still possible that a significant cointegrating relationship exists but of a nonlinear type.

18. Hakkio, C.S., and M.Rush, "Cointegration: How Short Is the Long Run?" *Journal of International Money and Finance*, 10 (1991), pp. 571-581.

19. Harris, R., *Using Cointegration Analysis in Econometric Modelling* (London: Prentice Hall, 1995).

20. Horsky, D., "A Diffusion Model Incorporating Product Benefits, Price, Income, and Information," *Marketing Science*, 24 (1990), pp. 342-365.

21. Inders, B., "Estimating Long-Run Relationships in Econometrics: A Comparison of Different Approaches," *Journal of Econometrics*, 57 (1993), pp. 53-68.

22. Johansen, S., and K. Juselius, "Maximum-Likelihood Estimation and Inference on Cointegration—with Application to the Demand for Money," *Oxford Bulletin of Economics and Statistics*, 52 (1990), pp. 169-210.

23. Kramers, J.J.M., N.R. Ericsson, and J. Dolado, "The Power of Cointegration Tests," *Oxford Bulletin of Economics and Statistics*, 54 (1992), pp. 325-348.

24. Lilien, G.L., P. Kotler, and K.S. Moorthy, *Marketing Models* (Englewood Cliffs, New Jersey: Prentice Hall, 1992).

25. MacKinnon, J.G., "Critical Values for Cointegration Tests," in R.F.Engle and C.W.J.Granger (eds.), *Long-Run Economic Relationships* (New York: Oxford University Press, 1991).

26. Mahajan, V., and E. Muller, "Innovation Diffusion and New Product Growth Models in Marketing," *Journal of Marketing*, 43 (1979), pp. 55-68.

27. Meade, N., "The Use of Growth Curves in Forecasting Market Development—A Review and Appraisal," *Journal of Forecasting*, 3 (1984), pp. 429-451.

28. Mesak, H.I., "Incorporating Price, Advertising and Distribution in Diffusion Models of Innovation: Some Theoretical and Empirical Results," *Computers Operational Research*, 23 (1996), pp. 1007-1023.

29. Miller, S.M., "Monetary Dynamics: An Application of Cointegration and Error-Correction Modeling," *Journal of Money, Credit and Banking*, 23 (1991), pp. 139-154.

30. Osterwald-Lenum, M., "A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics," *Oxford Bulletin of Economics and Statistics*, 54 (1992), pp. 461-472.

31. Pantula, S.G., A. Gonzales-Farias, and W.A. Fuller, "A Comparison of Unit-Root Test Criteria," *Journal of Business and Economics Statistics*, 12 (1994), pp. 449-459.

32. Pindyck, R.S., and D.L. Rubinfeld, *Econometric Models and Economic Forecasts*, Fourth Edition (Boston: Irwin McGrath-Hill, 1998).

33. Shoesmith, G.L., "Multiple Cointegrating Vectors, Error-Correction, and Forecasting With Litterman's Model," *International Journal of Forecasting*, 11 (1995), pp. 557-567.

34. Theil, H., *Economic Forecasts and Policy* (Amsterdam: North Holland, 1965).

35. Theil, H., *Applied Economic Forecasts* (Amsterdam: North Holland, 1966).

36. Zanias, G.P., "The Long Run, Causality, and Forecasting in the Advertising-Sales Relationship," *Journal of Forecasting*, 13 (1994), pp. 601-610.