Testing for Asymmetry in the Relationship between the Malaysian Business Cycle and the Stock Market

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This paper investigates whether the systematic asymmetric behavior of Malaysian industrial production can be explained by the stock market. We consider threshold models to capture the asymmetric relationship between monthly Malaysian industrial production and Kuala Lumpur Composite Stock Index (KLCI) returns. We test a range of null hypotheses of equality restrictions against inequality constraints and the composite null hypothesis involving steepness in business cycles. The results show that negative returns have steeper effects on the business cycle than do positive returns. Incorporating these findings into modeling the relationship between industrial production and the stock market will improve the prediction of business cycles, particularly their downturns. Because Malaysia is a major trading partner in the Asia-Pacific region and because of the growing trend toward trade liberalization and globalization, the findings are useful to many economists and investment decision-makers around the world. The methodology used in this paper can be utilized for testing asymmetry in a wider context.

Introduction

Following the work of Mitchell (1927) and Keynes (1936), many economists have argued that major cyclical variables such as the unemployment rate, production, and interest rates display systematic asymmetric behavior over various phases of the

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business cycle (Neftci, 1984; Falk, 1986; Hamilton, 1989; Rotham, 1991; and Sichel, 1993). We adopt the following definition of the business cycle:

> Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises. A cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals that merge into the expansion face of the next cycle. This sequence of changes is recurrent but not periodic. In duration, business cycles vary from more than one year to ten to twelve years.

See Burns and Mitchell (1946) for details.

The business cycle hypothesis of Mitchell (1927) and Keynes (1936) has two prongs: economic expansions are longer but less sharp than downturns. Many researchers have stressed the significance of this issue for both theoretical and empirical work over the years. The asymmetric nature of the business cycle cannot be represented adequately by any member of the wide class of standard linear models—this suggests a need to develop a nonlinear model.

This study investigates the relationship between the stock market and the business cycle in Malaysia. Tamin (1988) and Osada (1995) have observed that the Malaysian business cycle in industrial production has an asymmetric property. Therefore, we attempt to investigate whether the stock market can explain the observed systematic asymmetric business cycle. Motivated by Cover (1992), we consider threshold models suggested by Tong (1990) and Terasvirta (1990) to capture the asymmetric nature of the business cycle. We test a range of null hypotheses of equality restrictions against inequality constraints and the composite null hypothesis of steepness in the business cycle. Numerous studies have found that the stock market is an indicator of future economic activity. An increase in the stock market return is an indication of an increase in industrial production growth rates, which also reflects the positive relationship between these two series. We utilize this stylized fact and one-sided F statistics for testing hypotheses involving alternatives with inequality constraints. The familiar two-sided F statistic may not be appropriate when it is known that production and the stock market are positively related. The power of the test can be improved by taking account of the one-sided nature of the testing problem (Silvapulle and Silvapulle, 1995; Wolak, 1987). Although these one-sided F tests have been available for some time, they are not widely used in economics and finance.

Tamin (1988) found that Malaysian industrial production decreased more sharply than it increased, exhibiting asymmetric behavior. It is important to detect such asymmetry in economic time series for several reasons. If asymmetric behavior is systematic, then models generating such behavior endogenously need to be developed. The models of economic series that generate sharp drops during contractions followed by gradual movements during expansions will have better predictive power. Otherwise, one would expect the fit to deteriorate considerably, particularly around turning points. Such models also would improve forecasting of major economic variables such as unemployment rates, production, and interest rates. We test the composite null hypothesis of the asymmetric relationship between industrial production and the stock market. If the null is not rejected, the model taking account of such a relationship improves the prediction of business cycles, particularly the downturns. Testing for such a null hypothesis to our knowledge is new in applied economics and finance.

Sichel (1993) and others found two types of asymmetries in the business cycles of GDP, industrial production, and unemployment series: steepness and deepness. According to Sichel's (1993) evidence, steepness exists in a cycle when a moderate slope during expansions is followed by a relatively steep and negative slope during contractions. Deepness asymmetry occurs when the range from the mean to the peak is not equal to the range from the mean to the trough. In other words, troughs can be deeper than peaks. See McQueen and Thorley (1993) and Beaudry and Koop (1993) for more detailed definitions of asymmetry.

We use the stock market to explain asymmetry in the business cycle for several reasons. It is well known that expectations of future economic conditions have an important influence on the stock market, and, hence, the stock market long has been recognized as a predictor of the business cycle. Moore (1983) notes that since 1873 stock prices have led the business cycle at 18 of the 23 peaks and at 17 of the 23 troughs. Overwhelming evidence in the literature suggests that the stock market is the best single leading indicator of the business cycle (Fama, 1981).

The Models, Hypotheses, and the Test Statistics

In this section we specify the models reflecting the relationship between business cycles and the stock market. We include threshold models and several hypotheses concerning equality constraints and steepness in the business cycle. We also introduce test statistics when there are inequality constraints.

The Models

We first consider the conventional autoregressive model,

$$(1)\quad GP_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j GP_{t-j} + \sum_{i=1}^{q} \beta_i R_{t-k} + e_t \qquad t = 1,2,\dots,n$$

where:

- GP = The first differenced real industrial production series in logs as it is I(1). [See Table 1];
- R_t = The real stock return which is I(0);
- p and q = The number of lags of GP_t and R_t, respectively;
- e_t = Assumed to be normal and serially uncorrelated;
- n = The sample size.

The variables in the model are all assumed to be stationary. Appropriate values of p and q are chosen using the Akaike (1973, 1978) and Schwartz's Bayesian information criteria. A crucial limitation of equation (1) is that it does not capture the asymmetric nature of the relationship between GP_t and R_t. In order to improve equation (1), we decompose R_t into two components:

$$RPOS_t = \begin{cases} R_t & \text{if } R_t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Table 1—Application of Unit Root Tests

	Test Statistics					
	ADF			KPSS		
Series	$\ell=3$	$\ell=6$	$\ell=9$	$\ell=3$	$\ell=6$	$\ell=9$
IP	-1.78	-1.42	-1.41	0.63	0.59	0.52
GP	-5.88	-5.95	-5.82	0.61	0.38	0.30
KLCI	-2.16	-2.28	-2.34	1.09	0.81	0.67
R	-6.00	-5.98	-6.12	0.32	0.18	0.12

The 5 percent critical value of the ADF and KPSS are -2.82 and 0.43 respectively without time trend, and ℓ is the number of lags included to whiten the noise

and

$$RNEG_t = \begin{cases} R_t & \text{if } R_t \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

To capture the asymmetric relationship between GP and R, model (1) can be generalized as

$$(2)\quad GP_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j GP_{t-j} + \sum_{i=1}^{q} (a_i RPOS_{t-i} + b_i RNEG_{t-i}) + e_t .$$

In equation (2), we define RPOS and RNEG based on whether the stock return is above or below zero. According to Tong's (1990) terminology, the threshold parameter is zero. In setting the threshold parameter at zero we have assumed that returns below zero are bad news, indicating a future downturn in economic activity. Now, the aim is to determine the threshold parameter—the actual return level—below which is bad news, using the Akaike and Schwartz information criteria. Assuming such a parameter value is r, we define high return (RH) and low return (RL) as,

$$RH_t = \begin{cases} R_t & \text{if } R_t \geq r \\ 0 & \text{otherwise} \end{cases}$$

and

$$RL_t = \begin{cases} R_t & \text{if } R_t \leq r \\ 0 & \text{otherwise} \end{cases}$$

Now, model (2) may be modified as

$$(3)\quad GP_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j GP_{t-j} + \sum_{i=1}^{q} (a_i RH_{t-i} + b_i RL_{t-i}) + e_t .$$

This model has sufficient flexibility to accommodate asymmetry and to allow the effect of R on GP to be zero for $|R| < r$ for some r.

The Hypotheses

To test whether stock returns have any effects on industrial production, the null and alternative hypotheses take the form

H_0: $\beta_i = 0$ for $i = 1, 2, \ldots, q$

(4) and

H_0: $\beta_i \neq 0$ for $i = 1, 2, \ldots, q$

We can test the standard H_0 against H_1 using the usual F-ratio, which asymptotically has a chi-squared distribution with q degrees of freedom.

In accordance with the widely held view, we assume that an increase in stock returns is an indication of an upturn in future economic activity and hence an increase in industrial production growth rates. If equation (2) is adopted, then to test whether RPOS has any effects on GP, the null and alternative hypotheses may be stated as

$H_0^+ : a_i = 0$ for $i = 1, 2, \ldots, q$

(5) and

$H_1^+ : a_i \geq 0$ for all i and $a_i > 0$ at least for one i,

respectively. Similarly, to test whether RNEG has any effects on GP, the null and alternative hypotheses may be formulated as

$H_0^- : b_i = 0$ for $i = 1, 2, \ldots, q$

(6) and

$H_1^- : b_i \geq 0$ for all i and $b_i > 0$ at least for one i,

respectively. If equation (3) is adopted instead of equation (2), the hypotheses in equations (5) and (6) respectively are still valid formulations for testing whether RH and RL have any effects on GP.

We test whether steepness in GP can be explained by stock market returns by testing whether negative returns have steeper effects than positive returns on GP, which is equivalent to testing

$H_{a0}: b_i \geq a_i$ for $i = 1, 2, \ldots, q$

(7) against

$H_{a1}: b_i \ngeq a_i$ for $i = 1, 2, \ldots, q$

in equation (2) or equation (3). Let $b_i = a_i + \delta_i$. Substituting this in equation (2) we obtain

$$GP_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j GP_{t-j} + \sum_{i=1}^{q} (a_i RPOS_{t-i} + (a_i + \delta_i) RNEG_{t-i}) + e_t$$

(8)

$$GP_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j GP_{t-j} + \sum_{i=1}^{q} (a_i (RPOS_{t-i} + RNEG_{t-i}) + \delta_i RNEG_{t-i}) + e_t \,.$$

Similar substitution in equation (3) yields

(9) $$GP_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j GP_{t-j} + \sum_{i=1}^{q} (a_i (RH_{t-i} + RL_{t-i}) + \delta_i RL_{t-i}) + e_t \,.$$

In models (8) and (9), the hypotheses in (7) are equivalent to H_{00}: $\delta_i \geq 0$ and H_{11}: $\delta_i \ngeq 0$, respectively.

The Test Statistics

Let X be the set of all regressors and β be the set of all parameters in equation (2). We can restate the null and alternative hypotheses defined in equations (5) and (6) as H_0: $R\beta = 0$ and H_1: $R\beta \geq 0$, respectively, with the restriction matrix R appropriately defined.

Consider the likelihood ratio (LR), defined as

$$(10)\ LR = -2(\ell n\tilde{L} - \ell n\hat{L})$$

where $\tilde{L}$ and $\hat{L}$ are the maximum values of the likelihood function under the null and alternative hypotheses, respectively. Because H_1^+ is a one-sided alternative, the LR statistic for testing H_o^+ against H_1^+ asymptotically has a weighted average of chi-squared distributions, known as a chi-bar squared distribution. [See Gourieroux (1982) for details.] Further, the F-statistic for testing these hypotheses has a weighted average of F distributions.

If the LR statistic for testing hypotheses in equation (5) is computed as c; then the asymptotic p-value is given by

$$(11)\ \text{p-value} = \text{pr}[LR \geq c] = \sum_{k=1}^{q} \text{pr}[\chi_k^2 \geq c]w(p, k, \Omega)$$

where Ω is the covariance matrix defined as $R(X'X)^{-1}R'$ and $w(q, k, \Omega)$, $k = 1, \dots, q$ are some nonnegative weights computed approximately by simulation as mentioned in Wolak (1987, p. 786). The weight $w(q, k, \Omega)$ has the following interpretation. Let $Z \sim N(0,\Omega)$, and $\tilde{Z}$ be the point in the positive quadrant $\{z \in R^q: z_1, \dots, z_q \geq 0\}$ that is closest to Z in the following sense:

$$(Z-\tilde{Z})'\,\Omega^{-1}(Z-\tilde{Z}) = \text{Min}\{(Z-z)'\Omega^{-1}(Z-z): z_1, z_2, \dots, z_q \geq 0\}$$

$w(q, k, \Omega)$ is the probability that $\tilde{Z}$ has exactly k positive components. A simple way of computing $w(q, k, \Omega)$ as suggested by Wolak (1987) is to take 1000 draws on Z that has a multinormal distribution with mean zero and covariance matrix Ω. For each of these draws, compute $\tilde{Z}$, which is the projection of Z onto the positive quadrant, and then approximate $w(q, k, \Omega)$ by the proportion of times $\tilde{Z}$ has exactly k positive components. Previous simulation studies show that although these computed weights are not exact, they are close.

For testing the steepness hypothesis H_{00}: $\delta_i \geq 0$ against H_{11}: $\delta_i \ngeq 0$ for some i, the parameter space under H_{00} can be defined as $\theta_H = \{\beta \mid R\beta \geq 0\}$. The null is a composite hypothesis.

The p-value of the likelihood ratio (LR) statistic is computed as

$$(12)\ p-\text{value} = \sup_{\beta\in\theta_H} \text{pr}\{LR \geq c\} = \text{pr}\{LR \geq c\}\big|_{\beta}$$

The point $\beta = 0$ is the least favorable null value of β. The p-value defined in equation (12) can be computed using an expression similar to equation (11). See the appendix for the details and for the robust estimates and the related tests involving inequality constraints.

Time Series Properties of Data

The monthly industrial production (IP) series measured in 1985 constant prices was collected from the Malaysian Department of Statistics, and the Kuala Lumpur Composite Stock Index (KLCI) series was collected from the DATA STREAM database for the period September 1974 to December 1997. Hence $n = 280$. These two variables were used in logarithms.

Using the augmented Dickey and Fuller (1979) test (ADF) and Kwiatkowski, Phillips, Schmidt, and Shin's (1992) test (KPSS hereafter), we examine the time series properties of the IP, GP, KLCI, and R series. Where $GP = IP_t - IP_{t-1}$, real return series $R = NR - \pi$, $NR = KLCI_t - KLCI_{t-1}$, π is inflation rate, defined as $\pi = CPI_t - CPI_{t-1}$, and CPI is the Consumer Price Index series in logs. The results indicate that the IP and KLCI are I(1) and GP and R is I(0); see Table 1 for the results. GP is considered as the dependent variable in all models, eliminating the stochastic trend.

Empirical Analysis and the Results

The conventional autoregressive model (1) is estimated as a benchmark for models with asymmetries. By definition equation (1) is the symmetric response model because GP is forced to have identical effects on both positive and negative stock returns. To conduct Granger causality testing, the key element for the identification of the bivariate model, such as equation (1), is the appropriate number of lag terms included in the model. A widely used method of lag length selection is the Akaike (1973, 1978) information criterion (AIC), which is based on maximizing the log likelihood of a model. For linear autoregressive models, this is equivalent to minimizing $\{n\,\ell og(RSS) + 2k\}$ where RSS is the residual sum of squares and k is the number of regressors. We also use Schwartz's Bayesian information criterion (SBIC), which is computed as $\{n\,\ell og(RSS) + k\ell og(n)\}$.

Model (1) is estimated by OLS for all 576 combinations of lag lengths p and q between 1 and 24. We observe that the AIC values generally increase for lag lengths $p > 7$ and $q > 4$. Therefore, we consider the lags between 1 and 7. The results for $p = 6$ and 7 and $q = 2, 3, 4$ are reported in Table 2. Based on AIC and SBIC, the optimum value of (p, q) is selected as (6, 3) for which the F-ratio is also highly significant at the 1 percent level, indicating that stock returns cause GP. A limitation of model (1) is that the positive and negative returns are forced to have equal effects on industrial production.

We consider the threshold model (2) with the threshold parameter 0. The one- and two-sided F-ratios corresponding to various values of (p, q) and for positive and negative returns are computed and are reported in Table 3. The alternative hypothesis for the one-sided testing problem is that the coefficients are all nonnegative, based on the prevalent view that there is a positive relationship between past stock returns and current industrial production. The results provide convincing evidence that negative returns have a significant effect on business cycles; this evidence is not strong for positive returns. The two-sided F-test fails to detect the relationship between positive returns and the production growth rate in some cases, whereas the one-sided F-test does detect it. Further, the values of AIC and SBIC of model (1) are larger than those of model (2) with an optimum lag (6, 4), which shows the superiority of model (2).

Table 2—Industrial Production Responses to the Stock Market in Symmetric Equation (1): Testing Restrictions that $\gamma_1 = ... = \gamma_q = 0$

	Real Stock Return		
Lag Lengths (p, q)	F – Statistic	SBC	AIC
(6, 2)	7.91	319.06	300.90
(6, 3)	6.12	315.07	298.99
(6, 4)	5.18	317.60	299.88
(7, 2)	8.12	316.93	306.61
(7, 3)	5.28	317.08	307.18
(7, 4)	6.13	319.52	308.10

Note:

1. Both AIC and SBIC information criteria identify that optimum (p, q) is (6, 3)
2. The AIC is computed as {nℓog(RSS) + 2k} while the SBC is computed as {nℓog(RSS) + kℓog(n)}

Table 3—Industrial Production Responses to Positive and Negative Stock Returns in Equation (2)

	RPOS		RNEG	
Lag length (p, q)	Two-Sided F-Test	One-Sided F-Test	Two-Sided F-Test	One-Sided F-Test
(6, 2)	1.68	1.78	5.82	5.82
	(0.19)	(0.07)	(0.00)	(0.00)
(6, 3)	2.46	2.48	7.12	7.12
	(0.07)	(0.04)	(0.00)	(0.00)
(6, 4)	3.98	4.60	7.99	7.99
	(0.00)	(0.00)	(0.00)	(0.00)
(7, 2)	1.99	1.99	6.87	6.87
	(0.14)	(0.0)	(0.00)	(0.00)
(7, 3)	2.28	2.28	4.99	4.99
	(0.08)	(0.05)	(0.00)	(0.00)
(8, 4)	3.58	3.51	5.28	5.28
	(0.01)	(0.0)	(0.00)	(0.00)

Note:

1. Figures in parentheses are the p-values. The two-sided test statistic has an F distribution with (q, v) degrees of freedoms where v is the error degrees of freedom. The one-sided test statistic has a weighted average of F-distributions
2. The AIC and SBIC select the optimum values of (p, q) as (6, 4). The corresponding values of AIC and SBIC are 314.09 and 298.12, respectively

To estimate the threshold parameter r in equation (3), the model is estimated for lag lengths varying from 1 to 7. For each combination of p and q, we define RH and RL with the threshold parameter varying from -0.1 to 0.1 with an increase of 0.001; the results are reported in Table 4. The optimal threshold parameter r in equation (3) is computed as 0.034 and the corresponding lag length (p, q) is (6, 4), which is the same as for model (2). The values of AIC and SBIC of model (3), however, are only marginally smaller than those of model (2) with the optimum lag (6, 4). These results reinforce the previous findings for model (2). The results with model (3) are not reported but are available from the authors.

Table 4—Testing for Steepness Hypothesis in Equations (2) and (3)

	$H_0 : b_i \geq a_i$ $H_1 : b_i \ngeq a_i$	
Lag length (p, q)	Model (2) F-ratio	Model (3) F- ratio
(6, 2)	0.96	0.89
	(0.27)	(0.34)
(6, 3)	1.01	0.99
	(0.30)	(0.43)
(6, 4)	0.81	0.78
	(0.41)	(0.45)
(7, 2)	1.12	1.10
	(0.19)	(0.27)
(7, 3)	0.97	0.92
	(0.53)	(0.58)
(7,4)	1.46	1.28
	(0.09)	(0.25)

Note:

1. The p-values were computed using equation (12) with different sets of weights; see the appendix for more details
2. See footnote (a) for Table 3

We test the composite hypothesis that $b_i \geq a_i$ for $i = 1, \ldots, q$, reflecting the negative returns having steeper effects on industrial production than the positive returns in model (2). The results are reported in Table 4. The same hypothesis relating RH and RL to the industrial production in equation (3) is also tested. The computed LR statistics are insignificant in all cases; thus, there is overwhelming evidence to support the steepness null hypothesis in both models. Because the residuals of the models are not normally distributed, Silvapulle's (1992a, b) one-sided tests that are based on the M-estimator and are robust to nonnormal errors are applied. (See the appendix for details on the one-sided tests.) We find that although the computed robust test statistics are much higher than the reported ones, conclusions are largely unchanged. These results are not reported but available from the authors.

Conclusion

It has been argued that major cyclical variables such as the unemployment rate and production display systematic asymmetric behavior over various phases of the business cycle. This paper investigates whether this observed asymmetric behavior in Malaysian industrial production can be explained by the stock market. We consider threshold models developed by Tong (1990) and Terasvirta (1990) to capture the asymmetric feature of the relationship between monthly Malaysian industrial production and Kuala Lumpur Composite Stock Index (KLCI) returns. The data series used in this study covers the period September 1974 to December 1997. Using one-sided testing procedures we test a range of null hypotheses of equality restrictions against inequality constraints and the composite null hypothesis involving steepness in business cycles. The results show that negative returns have significant effects on industrial production. Moreover, tests of the composite null hypothesis support the claim that negative returns have steeper effects on the business cycle than do positive returns. Incorporating these findings in a model will give it better predictive power for business cycles, particularly downturns.

Because Malaysia is a major trading partner of other ASEAN countries and a number of developed countries, the results are useful to local as well as foreign economic and business decision-makers. Further, although the methodology used in the paper is for testing the asymmetric relationship between the Malaysian business cycle and the stock market, it can be used to test for asymmetric relationships among variables in a wider context.

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Appendix: Tests Involving Inequality Constraints

Let us write the linear model as $Y = 1\alpha + X\beta + E$, where $\mathrm{cov}(E) = \sigma^2 I$, and each column of X is centered by subtracting the sample mean. Let $(\hat{\beta}_L, \hat{\sigma}_L)$ denote the OLS estimates. In M-estimation, we estimate (α, β) by minimizing a loss function of the form $\Sigma\rho\{(y_t - \alpha - x_t\,\beta)/\tilde{\sigma}\}$, where $\tilde{\sigma}$ is a suitable estimate of scale for the errors. For example, $\tilde{\sigma}$ could be the OLS estimate $\hat{\sigma}_L$ or a more robust estimate of scale such as $1.483\mathrm{med}\{|\hat{\varepsilon}|\}$ where the $\{|\hat{\varepsilon}|\}$s are the OLS residuals; the latter is much less sensitive to large values of $|\hat{\varepsilon}|$ and is preferred to $\hat{\sigma}_L$. If $\rho(x) = x^2$ then the M-estimate of β is the OLS estimate. In robust estimation, we choose $\rho(x)$ so that its tails are not as steep as that of x^2. The robust estimator that is widely used corresponds to $\rho(x) = x^2$ for $0 \le x \le 1.5$, $\rho(x) = 3|x| - 9$ for $x \ge 1.5$, and $\rho(x) = \rho(-x)$ for $x \le 0$. The M-estimate that we use is precisely this, with $\tilde{\sigma}$ being the one given by the so-called Huber's Proposal II that we denote $\hat{\sigma}_M$. (See Huber, 1981.) We denote this M-estimator of β as $\hat{\beta}_M$.

The technical details are given in Silvapulle (1992a, b). For large samples, we have $(\hat{\beta}_M - \beta) \approx N(0, v^2\,(X^TX)^{-1})$ and $(\hat{\beta}_L - \beta) \approx N(0, \sigma^2\,(X^TX)^{-1})$ for some ν and σ, respectively. Let $(\hat{\beta}, \hat{\sigma})$ denote either $(\hat{\beta}_L, \hat{\sigma}_L)$ or $(\hat{\beta}_M, \hat{\sigma}_M)$; thus $(\hat{\beta} - \beta) \approx N(0, \lambda^2\,(X^TX)^{-1})$, where $\lambda = \nu$ or σ according to whether $\hat{\beta}$ is $\hat{\beta}_M$ or $\hat{\beta}_L$. The OLS estimate is a particular M-estimate corresponding to $(\rho(x), \tilde{\sigma}) = (x^2, \hat{\sigma}_L)$.

Let H_{a0} and H_{a1} denote the null and alternative hypotheses and assume that they involve only the β parameters. A Wald-type statistic for testing H_{a0} against H_{a1} is

(13) $RW = \min_{H_{a0}}\{(\hat{\beta}-b)^T \hat{V}^{-1}(\beta-b) - \min_{H_{a1}}\{(\hat{\beta}-b)^T \hat{V}^{-1}(\beta-b)\}$

where $\hat{V}$ is an estimate of cov($\hat{\beta}$). For the M- and least squares estimation procedures, $\hat{V}$ may be taken as $\hat{v}^2(X^TX)^{-1})$ and $\hat{\sigma}^2(X^TX)^{-1}$ respectively. For a given testing problem, the large sample null distributions of RW, and hence the critical values, are the same for the OLS- and M-estimation-based procedures.

Consider the test of H_0: $R\beta = 0$ against H_1: $R\beta \geq 0$. The large sample distribution of RW under the null hypothesis is given by

$$pr(RW \geq c|R\beta = 0) \cong \sum_{i=1}^{q} \omega\{q, i, R(X^TX)^{-1}R^T\}pr(\chi_i^2 \geq c),$$

for some nonnegative quantities $\omega\{q, i, R(X^TX)^{-1}R^T\}$, $i = 1, \ldots, q$; these quantities can be computed approximately by simulation or other methods as explained in Walk (1987) and Shapiro (1988).

A statistic for testing H_1: $R\beta \geq 0$ against H_3: $R\beta \ngeq 0$ is

$$S = \min\{(\hat{\beta}-b)^T \hat{V}^{-1}(\hat{\beta}-b) : R\beta \geq 0\},$$

where $\hat{V}$ is the same in equation (13). The null distribution of S depends on the particular value of β in the null parameter space. Therefore, the p-value is defined as $\sup\{pr(S \geq c\,|\beta): R\beta \geq 0\}$, where c is the sample value of S. In general statistical models such a supremum is difficult to evaluate. Because our model is linear, however, the supremum is achieved at any value of β such that $R\beta = 0$. Therefore, we have p-value = $\sup\{pr(S \geq c\,|\beta): R\beta \geq 0\} = pr(S \geq c\,|R\beta = 0\}$. It can be shown that, for large samples,

(14) $$pr(S \geq c \mid R\beta = 0) \cong \sum_{i=1}^{q} w\{q, q-i, R(X^TX)R^T\}pr(\chi_i^2 \geq c).$$

Consequently, if the sample value of S is c, then the right side in equation (14) is a large sample p-value for testing H_0: $R\beta \geq 0$ against H_1: $R\beta \ngeq 0$.