On Measuring the Response of Real GDP Growth to Changes in Inflation Volatility

Bradley K. Wilson*
University of Alabama at Birmingham

Sarah E. Culver
University of Alabama at Birmingham

For years a widely held belief by monetary policy practitioners has been that the volatility of inflation adversely affects the growth rate of an economy. The outcome of such a belief has been the willingness of policy makers to trade short-run output losses for progress toward price stability. Due to a void of persuasive empirical support for this belief, however, policy implementation to date has had to rest primarily on faith. In this study, we provide evidence that strongly supports the hypothesized negative relationship between output growth and inflation volatility. We estimate a bivariate EGARCH-M model of real GDP growth and inflation for the United States. Our results show that (1) inflation volatility, as measured by the conditional variance of inflation, robustly and significantly negatively affects the growth rate of real GDP; and (2) inflation volatility is significantly and positively related to its own conditional mean.

Introduction

For over 20 years, monetary policy makers and research economists alike have taken considerable interest in the relationship between economic growth and inflation uncertainty. Uncertainty about the future course of inflation stemming from monetary policy changes is widely believed to be detrimental to the growth of an economy. Friedman (1977) postulates (henceforth Friedman's hypothesis) that output growth is adversely affected by the volatility of inflation because volatility increases make it difficult for consumers to determine relative prices accurately. In addition, inflation volatility makes long-term contracts more costly and thus less attractive. In either case, economic efficiency is reduced, subsequently retarding economic growth.

To the monetary policy maker interested in fostering maximum long-term growth, Friedman's hypothesis has been construed as a mandate to trade short-run output losses

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for progress toward price stability over time. It follows that few economists would disagree with this view if said relationship exists. Yet after 20 years of empirical scrutiny, tests of the hypothesis have failed to yield a consensus. The evidence to date is ambiguous. There are as many studies finding no significant negative relationship between growth and inflation volatility as there are studies finding such a relationship [e.g., Logue and Sweeney (1981), Levine and Zervos (1993), Clark (1996), and Al-Marhubi (1998)]. Further evidence in this area is warranted.

The objective of this paper is to present additional evidence on the output-growth/inflation-volatility relationship. We improve on previous research by generating a more accurate measure of inflation uncertainty, thereby allowing a more robust test of Friedman's hypothesis. Our test reveals a statistically significant, negative influence of inflation uncertainty on real economic activity. Further, we find that inflation volatility rises in response to positive inflation surprises, yet falls in response to negative surprises. This finding provides additional empirical support for the contended positive relationship between inflation uncertainty and the level of inflation. Combined, these findings suggest that greater price stability is conducive to output growth, moving discussions of the merits of price stability goals forward.

The lack of empirical agreement on the topic is not surprising. Data realities and identification problems hobble the ability of researchers to pin down the links between inflation uncertainty and growth. One of the more popular approaches to such links is cross-country analysis. Its popularity can be attributed to its simplicity and insightful circumvention of the problems associated with measuring inflation uncertainty. Yet despite its attractiveness, it has not yielded solidarity. Adopting this approach, Logue and Sweeney (1981), Darrat and Lopez (1989), Al-Marhubi (1998), Levine and Zervos (1993), and Clark (1997) fail to find evidence of the relationship. Clark (1997) argues that quantifying the effects of uncertainty on growth with cross-country analysis should be discouraged, for such analysis is sensitive to both country sample and time period.

A parallel line of research has sought more powerful tests of Friedman's hypothesis. Its aim is to capture the fundamentally dynamic nature of the contended relationship. This research has focused on generating time series for the volatility of inflation, typically either by proxy or using survey data. This line of research subsequently allows time series inference of the hypothesis. Like their cross-country counterparts, however, the findings from such research have lacked commonality. For example, Katsimbris (1985) and Thornton (1988) find no significant relationship between inflation uncertainty and output growth, whereas the opposite is found in Mullineaux (1980), Hafer (1986), and Davis and Kanago (1996).

An often-cited criticism of time series investigations of Friedman's hypothesis is that they have not captured the dynamics of inflation uncertainty. For the proxy variable approach, this procedure has involved estimating time series models for the inflation rate and using the residuals to construct overlapping moving-average measures to proxy for the time-varying variance of inflation (heteroskedasticity). This procedure, however, suffers internal inconsistencies. The residuals used by this approach are obtained under the maintained hypothesis of homoskedasticity yet are used to construct proxies for heteroskedasticity, thus casting doubt on the quality of the proxies.

Survey-data analysis is fundamentally no different from cross-country analysis. The measure of inflation uncertainty generated by surveys measures the dispersion of point estimates of the inflation rate across individuals and does not necessarily capture the degree of uncertainty about future inflation rates.

In this paper, we continue to explore the time series links between inflation uncertainty and growth, but employ a more accurate measure of inflation volatility. Using U.S. per capita real GDP and CPI data, we find evidence indicating that both series possess autoregressive conditional heteroskedasticity (ARCH). Therefore, motivated by the works of Engle (1982, 1983), Engle *et al.* (1987), Bollerslev (1986, 1990), and Nelson (1991), we conduct our assessment of the relationship between output growth and inflation uncertainty employing a bivariate exponential generalized autoregressive conditional heteroskedasticity in mean (EGARCH-M) model. Generally, the ARCH class of time series models is a particularly appealing technique for testing Friedman's hypothesis, for it permits both a direct measurement of inflation volatility and straightforward assessment of the contended relationship. Given the large number of variants of this technique, our choice of ARCH class is based on two considerations. First, EGARCH-M circumvents the estimation problems encountered by other GARCH-M specifications (specifically, the restrictive stationarity conditions imposed on estimation). Second, EGARCH-M nests a simple test of the contended positive relationship between inflation volatility and the mean level of inflation, a relationship of practical importance in establishing price stability goals. In addition, this technique allows us to frame our investigation in a single-country framework, thus avoiding some of the robustness problems detailed in Levine and Renelt (1992), Levine and Zervos (1993), and Clark (1997). We are able to find a statistically negative relation between output growth and inflation volatility when examining the post-World War period of the U.S. (a low inflation country). Therefore, this technique may yield similar conclusions with higher inflation countries.

Data

We use quarterly, post-war U.S. CPI and per capital real (in 1990 dollars) GDP data spanning 1957:Q4 to 1996:Q4 to examine the relationship between inflation volatility and growth. The real GDP and population data were obtained from the International Financial Statistics.[1] The CPI data were obtained from the U.S. Bureau of Labor Statistics.[2] The series are all seasonally adjusted.

First, we implement preliminary diagnostic tests for time trends and unit roots in our series using the standard augmented Dickey Fuller (ADF) procedure. Because standard unit root tests possess low power when applied to short time spans, we also

[1] In the 1997 CD-ROM issue of the IFS, the IMF identification codes for U.S. real GDP and population are 11199B.RZF ... and 11199Z.ZF ..., respectively. The real GDP and population data used in this study also can be downloaded from

http://economagic.com/em-cgi/data.exe/beana/t025 and

http://economagic.com/em-cgi/data.exe/fedstl/pop:mzq+a, respectively.

[2] The CPI data provided by the U.S. Bureau of Labor Statistics can be downloaded from http://economagic.com/em-cgi/data.exe/blscu/cusr0000sa0:mzq+a.

employ the KPSS test for unit roots developed by Kwiatkowski *et al.* (1992). These results are reported in Table 1A. Neither series exhibits a time trend; both ADF F-test and t-tests fail to reject the null of no significant time trends. Both the ADF test and the KPSS test fail to identify a unit root for per capita real GDP growth. The ADF test rejects the null hypothesis of a unit root at the 5 percent level. The KPSS test fails to reject the null hypothesis of a unit root, and the KPSS test fails to reject the null of level stationarity. In the case of inflation, however, the two testing procedures produce different conclusions. The ADF test fails to reject the null hypothesis of a unit root, while the KPSS test fails to reject the null of level stationarity. The ADF testing procedure has low power against persistent alternatives; therefore, the results from this test are not surprising. However, the results from the KPSS test are particularly strong in light of Caner and Kilian (1999). Using Monte Carlo simulation, Caner and Kilian (1999) show that KPSS tests too often reject the null hypothesis of level stationarity in short time spans of data, data such as ours. Given these results and the findings of Culver and Papell (1997), Baillie *et al.* (1996), and others, we treat inflation as stationary.

Table 1A—Diagnostic Tests for the Presence of Time Trends and Unit Roots †

Panel A: Augmented Dickey Fuller Tests for a Time Trend[a]				
Statistics	log CPI	log RGDP		
t_τ	2.203	1.977		
F	4.853	3.907		
Panel B: Augmented Dickey Fuller Tests for a Unit Root[b]				
Statistics	log CPI	Δlog CPI	log RGDP	Δlog RGDP
t_α	-0.414	-2.651	-1.529	-5.03398*
Panel C: KPSS Tests for a Unit Root[c]				
Statistics	log CPI	Δlog CPI	log RGDP	Δlog RGDP
η_μ	1.258*	0.258	1.229*	0.219

† CPI denotes the quarterly U.S. Consumer Price Index; RGDP denotes the quarterly U.S. per capita real GDP; and Δ denotes the difference operator. For all testing procedures, the sample period evaluated is 1957:Q4 to 1996:Q4

* Significant at the 5 percent level

[a] (top) t_τ and F denote the augmented Dickey Fuller (ADF) t-statistic and F-statistic, respectively. Critical values are from Enders (1995). At the 5 percent level of significance, the critical values are 2.79 (t-test for a trend) and 6.49 (F-test for a trend and unit root). The null hypothesis of the t-test is that there is no trend, given there is a unit root. The null hypothesis of the F-test is the joint hypothesis of no trend and there is a unit root. Following the recursive t-statistic procedure suggested by Campbell and Perron (1991) to select lag lengths for ADF tests, we select a lag length of 6 in the case of log CPI, and 5 in the case of log RGDP

[b] (middle) t_α denotes the augmented Dickey Fuller (ADF) t-statistic based on the regression with a constant term. Critical values are from MacKinnon (1991). At the 5 percent level of significance, the critical value is –2.880. The null hypothesis of the t-test is that there is a unit root. Once again we follow the recursive t-statistic procedure suggested by Campbell and Perron (1991) to select the lag lengths used in the ADF tests. The maximum lag length considered in all tests was 8. The selected lag lengths are as follows: 6 in the case of log CPI; 5 in the case of Δlog CPI; 5 in the case of log RGDP; and 4 in the case of Δlog RGDP

[c] (bottom) η_μ denotes the KPSS test statistic based on the regression with a constant term. Asymptotic critical values are from Kwiatkowski *et al.* (1992). At the 5 percent level of significance, the critical value is 0.463. The null hypothesis of the KPSS test is level stationarity (no unit root). The lag truncation parameter for the KPSS test is based on $l = \text{int}[8(T/100)^{1/4}]$, where int denotes the integer part and T denotes the sample size

Given that the unit-root test results in Table 1A indicate that our output and price series are individually integrated of order 1, we test for cointegration between these variables. We present test results from both the Engle and Granger (1987) approach and the Johansen (1988, 1991) approach. Results from the Engle-Granger test are presented in Table 1B. We execute this test with and without a trend and report results for alternative lag lengths of the differenced residual. At all lag lengths, and with or without a trend, the Engle-Granger test fails to reject the null of no cointegration, thus indicating that U.S. per capital real GDP and CPI are not cointegrated.

Table 1B—Diagnostic Tests for Cointegration between U.S. CPI and Per Capita Real GDP †

Panel A: Engle-Granger Tests for Cointegration[a]

lags	t_α	t_{α^*}
0	0.458	0.451
1	0.054	0.047
2	0.069	0.061
3	-0.150	-0.158
4	-1.384	-1.394
5	-0.720	-0.729
6	-0.435	-0.444

Panel B: Johansen Test for Cointegration[b]

lags	λ_{max}	λ_{max^*}
6	24.880	18.386

† For both testing procedures, the test of cointegration is that between the logarithms of the U.S. CPI and U.S. per capita real GDP. The sample period evaluated is 1957:Q4 to 1996:Q4

* Significant at the 5 percent level

[a] (top) t_α and t_{α^*} denote the ADF t-statistics based on the regressions without a trend and with a trend, respectively. Critical values are from MacKinnon (1991). At the 5 percent level of significance, the critical values are −3.439 and −3.840 for tests leaving out a trend and tests including a trend, respectively. Lags denote the lag length used in each test. See Enders (1995) for a discussion of this testing procedure

[b] (bottom) λ_{max} and λ_{max^*} denote the maximal eigenvalue test statistics from the test including a constant and from the test including a constant and a trend, respectively. The test conducted is a test of the null hypothesis r = 0, where r denotes the rank of cointegration. Critical values are from Osterwald-Lenum (1992). At the 5 percent level of significance, the critical values are 39.37 and 42.48 for test including a constant and for tests including a constant and a trend, respectively. Lags denote the lag length used in the test of the undifferenced VAR. Enders (1995) suggests performing a series of likelihood ratio tests to determine the lag length. The likelihood ratio tests performed here chose a lag length equal to 6. The maximum lag length considered was 12

Also reported in Table 1B are the test statistics from the Johansen test. For details of this testing procedure and its application see Dickey *et al.* (1991), Johansen (1992), Hamilton (1994), Enders (1995), and Greene (1997). We report the maximal eigenvalue test statistics for both a restricted constant (no time trend) and an unrestricted constant (allowing for a time trend).[3] In both cases the test is conducted with a lag length equal to 6. The lag length is predetermined by a series of likelihood ratio tests.[4] Critical values for the test statistics are obtained from Osterwald-Lenum (1992). Consistent with the

3 A test statistic often reported with the maximal eigenvalue test statistic is the trace test statistic. Due to the relatively small sample employed in this paper, however, we do not perform the trace test.

4 For details regarding the choice of lag length used in Johansen's procedure, see Engers (1995).

results from the Engle-Granger test, we do not find evidence of cointegration; the Johansen test fails to reject the null of zero cointegrating vectors.

In addition to testing for cointegration, we also perform preliminary tests for the presence of autoregressive conditional heteroskedasticity (ARCH) in our series. In Table 1C asymptotic p-values from a set of Lagrange multiplier tests (LM tests) are presented. For details of this testing procedure, see Greene (1997). The LM tests are significant, which leads us to question the presence of ARCH in both our inflation and output growth series.

Table 1C—Preliminary Tests for ARCH in U.S. Inflation and Per Capita Real GDP Growth

Statistics	Δlog CPI	Δlog RGDP
ARCH(1)	0.000	0.000
ARCH(2)	0.000	0.000
ARCH(3)	0.000	0.000
ARCH(4)	0.000	0.000

ARCH denotes the LM test for autoregressive conditional heteroskedasticity. The number in parentheses is the degree of freedom of each test. CPI denotes the quarterly U.S. Consumer Price Index; RGDP denotes the quarterly U.S. per capita real GDP; and Δ denotes the difference operator. The asymptotic p-values are reported. The sample period evaluated is 1957:Q4 to 1996:Q4. In all tests, the number of lags used is the same as the number of lags used in the ADF unit root tests. For further details of this testing procedure see Greene (1997)

Bivariate EGARCH-M

As detailed by a substantial number of papers, for example, Engle (1982, 1983), Engle *et al.* (1987), Bollerslev (1986, 1990), and Nelson (1991), if a time series possesses heteroskedasticity, then its variance can be measured well by an ARCH class model. The analysis in this paper adopts this approach to assess the relationship between output growth and inflation uncertainty, measured as conditional heteroskedasticity.

Pioneered by Engle (1982), the ARCH class of time series models is a class of stochastic processes developed to account for several features of macroeconomic and financial data. First, it is often found that the data reject the restrictive assumption of homoskedasticity. The ARCH class, however, allows for heteroskedasticity by permitting the variance to depend on realizations of past variances and disturbances. Second, this class of models elegantly captures the commonly observed effect of clustering volatility (that is, the tendency for large forecast errors to be followed by large forecast errors and small errors to be followed by small errors). Last, the ARCH class allows for the leptokurticity typically exhibited by macroeconomic data.

To illustrate this class of models more formally, consider the following univariate generalized ARCH (GARCH) specification.

(1) $y_t = X_t'\beta + \varepsilon_t$,

(2) $\varepsilon_t = \sqrt{h_t} \cdot v_t$,

(3) $h_t = \kappa + \delta_1 h_{t-1} + \delta_2 h_{t-2} + \cdots + \delta_m h_{t-m} + \phi_1 \varepsilon_{t-1}^2 + \phi_2 \varepsilon_{t-2}^2 + \cdots + \phi_n \varepsilon_{t-n}^2$,

for v_t i.i.d. with zero mean and unit variance. Equation (1) represents a standard stochastic linear regression function with dependent variable γ_t, vector of explanatory variables X_t, parameter vector β, and a stochastic disturbance term ε_t with zero conditional mean and conditional variance described by equation (3). As exhibited in equation (3), the conditional variance of ε_t depends on past realizations of both itself and ε_t, thus permitting the explicit control of heteroskedasticity and capture of clustering volatility (The homoskedastic model is the special case when $\delta_i = \phi_i = 0$, for all i). The unconditional distribution of ε_t is not normal; the unconditional distribution can be leptokurtic. The conditional distribution of ε_t, however, is often assumed normal.

The GARCH model suffers a number of application drawbacks. For stability, the model requires that κ, δ_i, and ϕ_i in equation (3) be nonnegative. The requirement is not only restrictive in terms of estimation, but also rules out random oscillatory behavior in the h_t process. Another important limitation of the GARCH model is that it assumes only the magnitude (ε_{t-i}^2) and not the positivity or negativity of the disturbance term (ε_{t-i}) influences h_t. Yet dating back to Black (1976), the phenomenon of volatility rising in response to bad news and falling in response to good news has been observed in a number of time series.

To account for these drawbacks, Nelson (1991) proposes the following reformulation of equation (3):

$$(3')\quad \log h_t = \omega + \sum_{i=1}^{m} \theta_i \log h_{t-i} + \sum_{j=1}^{n} \lambda_j \{ |v_{t-j}| - E|v_{t-j}| + \xi v_{t-j} \},$$

where:

$v_t = \varepsilon_t / \sqrt{h_t}$ from equation (2); and

$E|v_t| = \sqrt{2/\pi}$ when v_t is distributed standard normal.

Equations (1) and (2) with (3') are commonly referred to as exponential GARCH or EGARCH. An attractive feature of this model is that it explicitly controls for possible asymmetric relationships between volatility and news. Such a relationship is captured in equation (3') by the parameter ξ. If $\xi = 0$, then a positive surprise $(v_{t-j} > 0)$ has the same effect on volatility as a negative surprise of the same magnitude. If $\xi > 0$, then positive surprises increase volatility. If $-1 < \xi < 0$, then a positive surprise increases volatility less than a negative surprise. When $\xi < -1$, positive surprises reduce volatility. In addition to this asymmetric feature of EGARCH, because the variance (h_t) is expressed in terms of logs, the model does not impose any restrictions on any of the parameters in equation (3') prior to estimation. This makes estimation simpler and allows more flexibility in the specification of dynamic models for the variance.

In this paper, we follow Nelson (1991) and specify a bivariate exponential GARCH-in-mean model, or EGARCH-M, to assess the contended relationship between output growth and inflation uncertainty.[5] Expansion of univariate EGARCH models to multivariate frameworks dictates specifying whole covariance matrices that change over time. Generally, such specification will involve the sacrifice of a large number of degrees of freedom with no guarantee of numerical tractability. To avoid this, we adopt Bollerslev's (1990) specification and assume conditional correlations to be constant so

[5] Introduced by Engle, Lilien, and Robins (1987), the "M" in EGARCH-M is a term accounting for possible relationships between the mean of a series and its own conditional volatility.

that all time variations in conditional covariance terms are due only to changes in the conditional variances.[6]

Given the unit root, cointegration, and ARCH test results reported in Tables 1A, 1B, and 1C, the model is as follows:

$$\text{(4a)}\quad \Delta p_t = \alpha_0 + \sum_{i=1}^{m} \alpha_{1i} \Delta p_{t-i} + \sum_{j=1}^{n} \alpha_{2j} \Delta q_{t-j} + \gamma_{11} h_{11,t}^{.5} + \gamma_{12} h_{22,t}^{.5} + \varepsilon_{1,t},$$

$$\text{(4b)}\quad \Delta q_t = \psi_0 + \sum_{i=1}^{r} \psi_{1i} \Delta q_{t-i} + \sum_{j=1}^{s} \psi_{2j} \Delta p_{t-j} + \gamma_{21} h_{22,t}^{.5} + \gamma_{22} h_{11,t}^{.5} + \varepsilon_{2,t},$$

$$\text{(5a)}\quad \varepsilon_{1,t} = \sqrt{h_{11,t}} \cdot v_{1,t},$$

$$\text{(5b)}\quad \varepsilon_{2,t} = \sqrt{h_{22,t}} \cdot v_{2,t},$$

$$\text{(6a)}\quad \log h_{11,t} = \omega_1 + \sum_{j=1}^{k} \theta_{1j} \log h_{11,t-j} + \sum_{i=1}^{d} \lambda_{1i} \{|v_{1,t-i}| - E|v_{1,t-i}| + \xi_1 v_{1,t-i}\},$$

$$\text{(6b)}\quad \log h_{22,t} = \omega_2 + \sum_{j=1}^{f} \theta_{2j} \log h_{22,t-j} + \sum_{i=1}^{l} \lambda_{2i} \{|v_{2,t-i}| - E|v_{2,t-i}| + \xi_2 v_{2,t-i}\},$$

$$\text{(6c)}\quad h_{12,t} = \rho\sqrt{h_{11,t} \cdot h_{22,t}},$$

where:

Δ = The difference operator;
p_t = The logarithm of the CPI;
q_t = The logarithm of per capita real GDP;
$v_{i,t}$ = i.i.d. with zero mean and unit variance;
$h_{ii,t}$ = $\mathrm{var}_{t-1}(\varepsilon_{i,t})$; $h_{12,t} = \mathrm{cov}_{t-1}(\varepsilon_{1,t}, \varepsilon_{2,t})$; and
$E_{t-1}(\varepsilon_{i,t})$ = 0 for i = 1, 2.

Equations (4a) and (4b) describe changes in means. Equations (6a), (6b), and (6c) specify time variations in the conditional second moments. Equations (6a) and (6b) describe changes in the conditional variances, while equation (6c), consistent with constant correlation specification of Bollerslev *et al.* (1988) and Bollerslev (1990), describes changes in the conditional covariance. Equations (4a) and (4b) are basically a simple vector autoregression (VAR), modified from the standard to account for heteroskedastic errors. Generally, $(\varepsilon_{1,t}, \varepsilon_{2,t})$ is assumed to be distributed bivariately normal or Student-t. The normality assumption is adopted in this study.

As in equation (3'), possible asymmetric relationships between volatility and news are captured in the above model by ξ_i in equations (6a) and (6b), for i = 1, 2. Moreover, the statistical significance and sign of ξ_1 form a simple and straightforward test of the contended relationship between uncertainty and the level of inflation. If $\xi_1 > 0$, then, as postulated by Okun (1971) and Friedman (1977), increases in the level of inflation brought by positive surprises will lead to increases in inflation volatility. As for the contended relationship between output growth and inflation uncertainty, assessing this relationship amounts to checking the sign and statistical significance of γ_{22} in equation

6 Such a constant correlation specification has been applied successfully by Bollerslev (1990) and Baillie and Bollerslev (1990a, 1990b).

(4b). If this parameter is negative, then, as postulated by Friedman (1977), inflation uncertainty adversely affects economic growth.

Results

Estimation in this paper is performed by maximum likelihood (ML) using the Berndt, Hall, Hall, and Hausman (1974) algorithm. Likelihood ratio tests first are conducted to determine the lag structures of equations (4a) and (4b) and equations (6a) and (6b).[7] After allowing for the construction of lags, the time period evaluated is 1959:Q4–1996:Q4. The results are presented in Table 2 with asymptotic t-values reported in parentheses.[8] The absence of lagged volatility terms (log $h_{ii,t-j}$) in both conditional variance equations indicates that neither U.S. inflation nor U.S. per capita real GDP growth exhibit significant GARCH effects. For per capita real GDP growth, inconsistent with our preliminary findings, the null hypothesis of homoskedasticity cannot be rejected. We fail to reject the hypothesis that $\theta_{2j} = \lambda_{2i} = 0$ for all i and j in equation (6b). For inflation, however, homoskedasticity is soundly rejected, as identified by the strong EGARCH effects in the conditional variance equation for this series (log $h_{11,t}$). Of particular note is the estimated value of ξ_1 in equation (6a). Taking a value of 1.091 and significant at the 5 percent level, this estimated parameter lends strong support to the contended positive relationship between inflation uncertainty and the level of inflation, that is, that inflation volatility rises in response to positive inflation surprises and falls in response to negative surprises. This finding suggests that reductions in inflation uncertainty can be practicably attained, at least in part, by reducing the level of inflation.

In addition to the parameter estimates of the conditional variance equations, Table 2 highlights the absence of EGARCH-M terms, that is, the absence of $h_{11,t}^{.5}$ and $h_{22,t}^{.5}$ in the conditional mean equations for inflation and output growth, respectively. Consistent with the findings of previous research, such an absence suggests that neither the growth of per capita real GDP nor inflation in the U.S. is influenced by its own volatility. As for the estimate of the parameter testing the validity of Friedman's hypothesis, γ_{22} in equation (4b), the data indicate that output growth does respond significantly and negatively to increases in the volatility of inflation: The parameter estimate is –0.766 with significance at the 1 percent level. Given the battery of likelihood ratio tests conducted to generate Table 2, this evidence can be construed as robust.

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7 The maximum lag length considered in the execution of the likelihood ratio tests is 12.

8 Estimation is performed in this paper using the *Estima* RATS software. Large numbers of likelihood ratio tests were performed to generate Table 2 as the precise specification of the EGARCH-M model; the authors will provide results from these tests upon request. Jansen (1989) estimates a similar model, specifically a bivariate ARCH-M model, to examine the empirical relationship between inflation uncertainty and output growth. Jansen (1989) fails, however, to find evidence supporting the relationship. We attribute his findings to two features of his analysis:

- The use of the GNP deflator data to measure inflation, a series with well-known dubious ARCH properties; and
- The overparameterization of the estimated model.

Table 2—Estimated EGARCH-M Model of U.S. Per Capita Real GDP Growth and Inflation

$$\Delta p_t = \underset{(3.161)}{0.161} + \underset{(7.130)}{0.451}\Delta p_{t-1} + \underset{(0.342)}{0.028}\Delta p_{t-2} + \underset{(5.565)}{0.360}\Delta p_{t-3} + \varepsilon_{1,t},$$

$$\Delta q_t = \underset{(3.805)}{0.484} + \underset{(2.686)}{0.224}\Delta q_{t-1} - \underset{(-4.109)}{0.463}\Delta p_{t-2} - \underset{(-2.701)}{0.766}h_{11,t}^{.5} + \varepsilon_{2,t},$$

$$\log h_{11,t} = \underset{(-10.019)}{-2.050} + \underset{(2.250)}{0.455}\{|v_{1,t-1}| - E|v_{v1,t-1}| + \underset{(2.198)}{1.091}v_{1,t-1}\} + \underset{(1.722)}{0.248}\{|v_{1,t-2}| - E|v_{v1,t-2}| + \underset{(2.198)}{1.091}v_{1,t-2}\},$$

$$\log h_{22,t} = \underset{(-2.490)}{-278},$$

$$h_{12,t} = \underset{(1.735)}{0.143}\sqrt{h_{11,t}\ h_{22,t}}$$

LLF = 29.021

p_t and q_t are the logarithms of the quarterly U.S. Consumer Price Index and per-capita real gross domestic product, respectively. LLF denotes the value of the log likelihood function. The sample period evaluated is 1959:Q4–1996:Q4. The asymptotic t-values are in parentheses

Following Clark (1997), we test the sensitivity of our results to different episodes in the sample. Such tests are typically performed with a procedure similar to the Chow (1960) test. Due to the sample size requirements of ARCH estimation, we conduct an alternative test employing dummy variables. The dummy variables we construct are twofold: DUM1, a dummy variable equal to 1 for observations between 1959:Q1-1971:Q4, and DUM2, a dummy variable equal to 1 for observations between 1972:Q1–1983:Q1, the volatile oil-price years. The base case is the sample period 1983:2–1996:Q4. Our choice of dummies is based on our desire to isolate and thus assess the relative importance of oil-price shocks, via their impact on inflation volatility, to the estimated output-growth/inflation-volatility relationship. Interacting these dummy variables with $h_{11,t}^{.5}$ in the conditional mean equation for output growth, we re-estimate the EGARCH-M model displayed in Table 2. The results are presented in Table 3. Given the lack of evidence indicating significant structural breaks in U.S. growth, we do not include corresponding intercept dummies in this estimation.[9] As this estimation shows, the inclusion of the multiplicative dummies has little impact on the results of Table 2. All estimates are nearly the same as before with similar statistical significance. Moreover, the estimated slope coefficients on both multiplicative dummies are individually statistically insignificant. Not surprisingly, however, real GDP growth does

[9] See Ben-David and Papell (1998) for a discussion on structural breaks in real GDP.

Table 3—Estimated EGARCH-M Model of U.S. Per Capita Real GDP Growth and Inflation with Time Dummies

$$\Delta p_t = \underset{(3.068)}{0.159} + \underset{(7.012)}{0.452}\Delta p_{t-1} + \underset{(0.412)}{0.034}\Delta p_{t-2} + \underset{(5.312)}{0.356}\Delta p_{t-3} + \varepsilon_{1,t},$$

$$\Delta q_t = \underset{(2.984)}{0.475} + \underset{(2.697)}{0.226}\Delta q_{t-1} - \underset{(-4.400)}{0.595}\Delta p_{t-2} - \underset{(-1.609)}{0.628}h_{11,t}^{.5} + \underset{(0.318)}{0.180}\,DUM1\bullet h_{11,t}^{.5} - \underset{(-0.346)}{0.180}DUM2\bullet h_{11,t}^{.5} + \varepsilon_{2,t},$$

$$\log h_{11,t} = \underset{(-10.022)}{-2.048} + \underset{(2.300)}{0.460}\{|v_{1,t-1}| - E|v_{v1,t-1}| + \underset{(2.168)}{1.108}v_{1,t-1}\} + \underset{(1.759)}{0.253}\{|v_{1,t-2}| - E|v_{v1,t-2}| + \underset{(2.168)}{1.108}v_{1,t-2}\},$$

$$\log h_{22,t} = \underset{(-2.514)}{-277},$$

$$h_{12,t} = \underset{(1.771)}{0.152}\sqrt{h_{11,t}\ h_{22,t}}\ ,$$

LLF = 29.026

p_t and q_t are the logarithms of the quarterly U.S. Consumer Price Index and per-capita real gross domestic product, respectively. DUM1 is a dummy variable equal to 1 for observations between and including the years 1959:Q4–1971:Q4. DUM2 is a dummy variable equal to 1 for observations between and including the years 1972:Q1–1983:Q1. LLF denotes the value of the log likelihood function. The sample period evaluated is 1959:Q4–1996:Q4. The asymptotic t-values are in parentheses. Likelihood ratio test that the estimated slope coefficients on the multiplicative dummy variable are jointly equal to 0: 0.01 $\sim \chi^2$ (2) (marginal significance 0.99501)

respond more negatively to inflation volatility during the volatile oil-price years. In addition to the individual statistical insignificance of the multiplicative dummies, a likelihood ratio test that the estimated slope coefficients of these two variables are jointly equal to zero cannot be rejected. In other words, we cannot reject the null that the output-growth/inflation-volatility relationship is stable over the entire sample. Hence, the results of this test with those presented in Table 2 lend strong support to Friedman's hypothesis.

Concluding Remarks

In this study we construct a bivariate EGARCH-M model of inflation and output growth to assess the hypothesized negative relationship between inflation volatility and economic growth. The following conclusions emerge from our investigation. First, inflation volatility, as measured by the conditional variance of inflation, robustly and significantly negatively affects output growth. Second, inflation volatility is significantly and positively related to the level of inflation. Combined, these conclusions support the widely held belief that inflation uncertainty adversely affects economic growth. Given that the empirical analysis in this paper has focused only on a single country, and a relatively low inflation country at that, it would be of interest to extend our inquiry to an examination of multiple countries employing panel analysis. Although it is true that such longitudinal studies exist, the fundamentally dynamic nature of the problem has never been exploited.

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