

Hedging Volatility Shocks to the Canadian Investment Opportunity Set

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A volatility shock model is used to assess whether Canadian investors are interested in an asset's ability to hedge volatility shocks (the asset's coskewness) and shocks to the market mean return (the asset's beta). This study indicates that the risk premiums for beta and coskewness vary over time and are sensitive to market conditions in the United States (coskewness) and Canada (beta and coskewness). The risk premium for coskewness shows a seasonal effect in January and September, while the risk premium for beta is sensitive to the month of September. The performance of the volatility shock model using U.S. data is similar to the outcomes found using Canadian data. The Canadian investor, however, is willing to pay more for an asset's potential protection capabilities.

Introduction

Investors that are interested in protecting themselves from shocks to their investment opportunity set will be interested in an asset's ability to hedge against unexpected shocks to the mean of the market return and to the volatility of the market. The ability of an asset to hedge shocks to the market mean is the asset's systematic risk, as measured by its beta. The ability of an asset to hedge shocks to the market's variance is the asset's systematic skewness or coskewness with the market. A volatility shock model that incorporates these two factors is applied to weekly Canadian data from January 19, 1977 to December 20, 1994 and compared to the results for the United States.

The results obtained indicate that the risk premiums for beta and coskewness vary over time and are sensitive to market conditions in the U.S. (coskewness) and in Canada (beta and coskewness). The risk premium for coskewness shows a seasonal effect in January and September, while the risk premium for beta is sensitive to the month of September. The performance of the volatility shock model

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in the U.S. over the same period is similar, although Canadian investors appear to be willing to pay more to hedge shocks to their investment opportunity set.

The Volatility Shock Model

The volatility shock model, VSM, assumes that investors are concerned with unanticipated shocks to their investment opportunity set and that these shocks can be represented by unexpected movements in the market return and in the variance of the market. The model can be derived both on the basis of arbitrage pricing theory and as a three-moment capital asset pricing model, CAPM, where investors are interested in the mean, variance, and skewness of their wealth.¹ An outline of these derivations is contained in Appendix 1. VSM consists of an equilibrium asset pricing equation, equation (1), and a return-generating process, equation (2).

$$(1) \quad E[r_{it}|I_{t-1}] = \lambda_0(I_{t-1}) + \lambda_1(I_{t-1})\beta_{it,t-1} + \lambda_2(I_{t-1})\gamma_{it,t-1}$$

$$(2) \quad r_{it} = \alpha_{it,t-1} + \beta_{it,t-1}(R_{mt} - \mu_m)_t + \gamma_{it,t-1}[(R_{mt} - \mu_m)_t^2 - h_{mt}] + \varepsilon_{it}$$

where:

- r_{it} = The log of the return on firm i in period t ;
- I_{t-1} = Information available at time $t-1$;
- λ_0 = Intercept coefficient;
- λ_1 and λ_2 = The risk premiums for beta and gamma (normalized coskewness), respectively;
- $\lambda_j(I_{t-1})$ = Indicates that the risk premium depends on information available up to period $t-1$ for $j = 0$ to 2 ;
- $\alpha_{it,t-1}$ = Expected return on the i th asset, conditional on the information available at time $t-1$;
- $\beta_{it,t-1}$ = Asset's ability to hedge against unexpected movements in the market return, conditional on the information available at time $t-1$;
- $\gamma_{it,t-1}$ = The asset's ability to hedge against unexpected movements in the market variance, conditional on the information available at time $t-1$;
- R_{mt} = Log of the return on the market portfolio in period t with mean μ_m and a GARCH (1, 1) variance process, h_{mt} ;
- $[(R_{mt} - \mu_m)^2 - h_{mt}]$ = Volatility shock to the market return because $h_{mt} = E[(R_{mt} - \mu_m)^2]$ is the predicted variance and $(R_{mt} - \mu_m)^2$ is the current observation on the market variance; and
- ε_{it} = The random error term.

¹ The three moment CAPM was originally derived by Rubinstein (1973). Kraus and Litzenberger (1976) were the first to estimate this model for the U.S. using a time invariant coskewness normalized by the skewness of the market. This normalization is cumbersome when the market is symmetric and has led to misinterpretations of the appropriate sign for the price of coskewness. Friend and Westerfield (1980), Barone-Adesi (1985), and Lim (1989) also have estimated versions of the three moment CAPM for monthly U.S. equity data.

In VSM an asset's ability to hedge shocks to the market mean and variance is given by its beta and gamma, respectively. Beta, the systematic risk of the asset, is the covariance of the asset's return with the return on the market portfolio normalized by the variance of the market return. Gamma is a measure of the asset's (normalized) coskewness. It is the covariance of the asset's return with the volatility shock to the market, normalized by the variance of the market volatility shock. The normalization is used in order to avoid scaling problems and thus the terms *gamma* and *coskewness* can be used interchangeably.

Systematic risk is a familiar concept that is thoroughly described in the literature, while conditional coskewness is newer and has received less attention. The systematic skewness, or more simply the coskewness, between the market portfolio and an asset with return r_i and mean return μ_i , is defined as: $E[(r_i - \mu_i)(R_m - \mu_m)^2]$. An asset that is positively coskewed will be able to offset an unusually low market return or enhance an unusually high return on the market. Equivalently coskewness can be expressed as $E[(r_i - \mu_i)((R_m - \mu_m)^2 - h_m)]$. In this form, it is clear that a positively coskewed asset hedges against unexpected movements in the market's variance. This asset's return will be increasing when there is an unanticipated increase in the volatility of the market.

Data

The performance of VSM in Canada is investigated using weekly data for the Toronto Stock Exchange, TSE. The logarithm of daily data is taken from the Canadian Financial Markets Research Centre, CFMRC, database and converted to weekly data. A week is defined as Tuesday close to Tuesday close to avoid problems with holiday Mondays. Ninety-four firms had continuously listed stocks between January 8, 1975 to December 20, 1994 with a limited number of consecutive missing observations.^{2, 3} Individual firms are studied because forming portfolios undoes skewness.⁴ The equally weighted index portfolio is used as the proxy for the market portfolio, and the variance of the market portfolio is found to follow a generalized autoregressive conditional heteroscedastic process of order (1, 1), GARCH (1, 1).⁵ These data are used in a two step estimation procedure. First, the

² In any given week, a minimum of three daily observations is required.

³ See Fowler, Rorke, and Jog (1980) for a discussion of thin trading on the TSE. These authors conclude that a technique to avoid the bias induced by thin trading is not yet available. Calvet and Lefoll (1988) also use continuously listed stocks to circumvent thin trading problems with the potential side effect of a survivorship bias.

⁴ It has been suggested that investors are not fully diversified because forming portfolios undoes skewness. Also, Jobson and Korkie (1985) find that the rejection of the CAPM, in Canada and the U.S., depends critically on the portfolio formation procedure.

firm's ability to hedge shocks to the market mean and variance must be estimated. Once beta and gamma are accurately measured, the willingness of investors to pay for these attributes is assessed using equation (1).

Estimation of the Return-Generating Process

Before the importance of an asset's hedging capabilities can be analyzed, the betas and gammas must be estimated. Beta and gamma are parameters to be estimated from the linear return-generating mechanism as given in equation (2). *There are many reasons why these parameters may be time varying. Beta, the firm's systematic risk may change over time as the underlying characteristics of the firm evolve due to, for example, variations in the degree of financial leverage, changes in management strategy or direction, technological advances, and taste shifts, (Jagannathan and Wang (1996)). Similarly, the systematic skewness of a firm, as measured by its gamma, may be adapting to evolving firm fundamentals. In addition, Maddala (1977) and Raj and Ullah (1981), suggest that parameters may be time dependent if, for example, there is an omitted variable, or the true variable is difficult to measure and thus a proxy is used instead or if the functional form is incorrect. Each of these three scenarios may be relevant for VSM, as there is a possibility that more than two factors are required to explain returns, the reference (market) portfolio is hard to measure, and a linear relationship has been assumed. To determine whether beta and gamma are varying over time and in order to help choose an appropriate estimation procedure, the Hansen test (1992) for parameter stability is applied to equation (2) and the data described above. The Hansen test rejects the null hypothesis of a jointly stable beta and gamma (coskewness) in equation (2) for 60 percent of the firms.⁵ This suggests an estimation procedure, such as the Kalman filter that explicitly allows the extraction of time dependent parameters. This method is described below. Previous studies of the two-moment CAPM for Canada also have found beta to be unstable over time. See, for example, Robinson (1993b), Morin (1980), and Ellert (1979). Although Calvet and Lefolli (1988) and Morin (1980) add the skewness of an asset to their models in an attempt to capture return asymmetry in Canadian markets, this study is the first to examine coskewness in a Canadian context.*

⁵ The model is chosen on the basis of Lagrange multiplier tests, examination of autocorrelation and partial autocorrelation functions and the results of Ljung-Box tests. A GARCH(1,1) process is commonly found for financial data. See Bollerslev, Chou, and Kroner (1992).

⁶ All testing is done at the 5 percent level of significance unless otherwise stated.

The Kalman Filter

The Kalman filter procedure can accurately characterize the time dependency of beta and gamma. It is a popular approach for engineers to use when extracting information from noisy data, and it is used by economists in models that have time-varying parameters. This method works by analyzing new information and assessing the portion of that information that is signal (and therefore useful) and the portion that is noise (and thus should be ignored). The signal, as well as the size of the prediction error, is used to update the current estimate of the parameters (beta and gamma). This updating equation is presented in equation (3).

$$(3) \quad \theta_{it} = \theta_{it-1} + \frac{\text{Cov}[(\theta_{it}, r_{it}) | I_{t-1}]}{\text{Var}[r_{it} | I_{t-1}]} (r_{it} - r_{it-1})$$

where the vector θ_{it} is $[\beta_{it}, \gamma_{it}]'$ for firms $i = 1$ to 94 in each period t . θ_{it-1} is the optimal estimate of θ_{it} , based on data up to period $t-1$. The prediction error, $(r_{it} - r_{it-1})$, is the difference between the actual return this period and its optimal estimate, r_{it-1} . Equation (2) yields the optimal estimate of the return, conditional on available information. The relevance of the current observation is measured by

$$\frac{\text{Cov}[(\theta_{it}, r_{it}) | I_{t-1}]}{\text{Var}[r_{it} | I_{t-1}]},$$

where Cov and Var are the covariance and variance operators, respectively.

To start the Kalman filter initial estimates of beta and gamma are required. For this study the first two years of data are reserved, January 2, 1975 to January 18, 1977, and an OLS regression on equation (2) is used to calculate the base values of beta and gamma.⁷ These estimates are updated for each week between January 19, 1977 and December 20, 1994 using equation (3). Technical details for the Kalman filter are available in Hamilton (1994) and Harvey (1989).

Expected Risk Premiums for Beta and Gamma

Once the beta and gamma are accurately measured they are used in the cross-sectional tests, equation (1), to determine if investors are willing to pay for the coskewness and beta properties of assets. The risk premium on coskewness, λ_2 , is expected to be negative because an asset that can provide protection against a volatility shock should require a lower return than otherwise to attract investors. The price of beta, λ_1 , should be positive. The higher the systematic risk of the firm, the more susceptible it is to unexpected movements in the market and thus the higher

⁷ Various ways of obtaining the initial values are considered. In addition to the one adopted, these include estimating beta and gamma separately, estimating beta while holding gamma constant, detrending the data and allowing for structural breaks in the data. The resulting Kalman filter estimates are similar.

the return required in order to induce investors to hold the asset. The results of these equilibrium pricing tests are reported after a discussion of the diagnostic tests of the model.

Results

To determine if investors are willing to pay for a firm's ability to hedge unanticipated movements in the market return (firm beta) and unexpected shifts in the market variance (firm coskewness), the Kalman filter estimates of beta and gamma are used in equation (1). The asset pricing equation is estimated weekly from January 19, 1977 to December 20, 1994. There are 94 firms in each of the 935 weekly pricing regressions. Before analyzing the pricing results, several misspecification tests are conducted on the residuals from these regressions. These diagnostics include tests for heteroscedasticity, tests for autocorrelation of first and higher orders, and the Jarque Bera test for normality.

Diagnostics

If VSM is able to explain asset pricing, there should be no systematic patterns remaining in the residuals of equation (1). Nonrandom sequences in the residuals could indicate an omitted risk factor. To determine if the residuals are white noise, tests for autocorrelation, heteroscedasticity, and normality are performed. The White test for misspecification of the first and second moments, White (1980), is rejected in 93 percent of the periods. The Durbin-Watson test of the null hypothesis of no autocorrelation is rejected in 4 percent and inconclusive in 6 percent of the sample. Because the data are weekly, higher order tests of autocorrelation are also considered. The Lagrange multiplier, LM, test for no autocorrelation against the alternative of autocorrelation of order p , AR (p), for $p = 4$ to 20 in steps of 4, is rejected for 10 percent ($p = 4$), 15 percent ($p = 8$ and $p = 12$), 16 percent ($p = 16$), and 13 percent ($p = 20$) of the periods. The LM test of the null of no autoregressive conditional heteroscedasticity, ARCH, against the alternative hypothesis of an ARCH process of order p , ARCH(p), is rejected for 10 percent ($p = 4$), 16 percent ($p = 8$), 19 percent ($p = 12$), and 21 percent ($p = 16$ and $p = 20$) of the periods. The Jarque Bera test of the null of normality is rejected for 94 percent of the sample. Thus, the diagnostic tests on the residuals generally support VSM, although there is some evidence of serial correlation and heteroscedasticity. The residuals are not normally distributed, consistent with other studies of TSE data. (See Robinson, 1993a.)

Averaged Risk Premiums

The misspecification tests support VSM so a detailed analysis of the individual regression coefficients is appropriate. On average, the signs of the risk premiums are as expected, but only the average risk premium on beta is priced significantly differ-

Table 1—Averaged Equilibrium Pricing Results, January 19, 1977 to December 20, 1994 (935 observations)

	Mean	Standard Deviation	Minimum	Maximum	t test Mean = 0 ^{1,2}
λ_0	-.0030	.0466	-.8139	.2296	-1.98 *
λ_1	.0044	.0670	-.4234	1.1148	2.019 *
λ_2	-.0002	.0152	-.1425	.1910	-.3397
R^2	.0901	.1049	0	.988	26.25 **

These are the summary statistics for the results from the equilibrium pricing equation, equation (1):

$$r_{it} = \lambda_0 + \lambda_1 \beta_{it}^{KF} + \lambda_2 \gamma_{it}^{KF} + \varepsilon_{it}$$

where the superscript KF refers to the Kalman filter estimate of either beta or gamma and the risk premiums are averaged across 935 weekly observations. There are 94 Canadian firms per cross section

Notes:

1. t test that population mean is zero. $t = \text{mean}(\lambda_i) / \text{standard deviation}(\lambda_i) / \sqrt{935}$

* Significant at 5 percent

** Significant at 1 percent

ent from zero. Table 1 presents summary statistics of the pricing results averaged over the 935 weeks used for testing the pricing of beta and gamma. The goodness of fit measure, R^2 , varies widely, suggesting that the model's applicability changes over time. Correspondingly, across the entire sample investors are willing to pay for the beta and coskewness properties of their assets in 37 percent and 32 percent of the periods, respectively. Further, in 54 percent of the priced beta periods, λ_1 is positive, as anticipated. Of the priced coskewness periods, 51 percent of the λ_2 are negative, the anticipated sign. Because beta and gamma are not priced in every period, it is important to investigate why and to determine when they are priced. The impact of multicollinearity on these pricing results must be analyzed before economic explanations for the behavior of the risk premiums is considered.

Collinearity Between Beta and Gamma

The pricing results presented in the previous section indicate that investor interest in beta and gamma also changes over time. A possible explanation for the changing signs and significance of the risk premiums in the tests of the proposed multivariate equilibrium asset pricing relationship is collinearity between the estimates of beta and gamma. The changing signs and significance of the risk premiums is common to most studies of univariate or multivariate models of asset pricing, whether for American or Canadian data. See for example, Robinson (1993a), Morin (1980), and Ellert (1979) for Canadian studies and Racine (1996), Lim (1989), Friend and Westerfield (1980), and Kraus and Litzenberger (1976) for American studies. If the variables are collinear the individual parameters in equation (1) are estimated with less accuracy, but the overall explanatory power of the model is unaffected. To determine if multicollinearity is a problem in the current study, the correlations between the Kalman filter estimates of beta and gamma for each time period are calculated. The average correlation is -.21 while an absolute value of .9 or

Table 2—Information Variables

Variable	Acronym	Source	Original Frequency ¹
Value-Weighted TSE Market Return	VW _{TSE}	CFMRC	daily
Level of Risk Aversion ²	RA	CANSIM ³	monthly ²
Canadian Leading Indicator	CLI	CANSIM	monthly
TSE Stock Dividend Yield	DIV	CANSIM	monthly
Value-Weighted U.S. Market Return	VW _{US}	CRSP ⁴	daily
Equally Weighted U.S. Market Return	EW _{US}	CRSP	daily
Volatility Shock to VW _{US}	VS _{VW,US}	calculated from CRSP data	weekly
Volatility Shock to EW _{US}	VS _{EW,US}	calculated from CRSP data	weekly
Foreign Exchange Rate (Canada/U.S.)	FX	CANSIM	monthly

Notes:

1. All daily returns are converted to weekly returns using Tuesday close to Tuesday close as the definition of a week. The volatility shock variables are calculated from the weekly returns. There are 935 weekly observations for each weekly series and 215 monthly observations for each monthly series, unless otherwise noted

2. The level of risk aversion is measured as the difference between the long-term Canadian Corporate bond rate and the long-term Canadian government bond rate. A weekly rate of RA is implied from the monthly rate. It has 894 weekly and 206 monthly observations because the corporate bond series is not available until November 1977

3. CANSIM = the Canadian Socio-Economic Information System of Statistics Canada

4. CRSP = Center for Research in Security Pricing

greater is used as an indicator of problems due to multicollinearity (Kennedy, 1992). In addition, the Kalman filter estimation is repeated separately for beta and gamma. The asset pricing tests are replicated individually for the new estimates of beta and gamma. The results are similar to those from the joint estimation and pricing tests. For example, when beta is the only factor sensitivity in the model, it is priced in 35 percent of the cross sections and has the correct sign in 52 percent of the priced periods. If gamma is the only risk measure, it is priced 35 percent of the time and negatively priced in 53 percent of those periods. Consequently, the changing signs and significance of the risk premiums cannot be attributed to collinearity between the estimates of beta and gamma; instead, we turn to an investigation of the relationships between the prices and variables that represent economic information available to the investor.

Information Variables

The possibility that multicollinearity is responsible for the observed time-varying risk premiums has been dispelled, and we can seek economic explanations of the observed behavior. The risk premiums represent the price that investors are willing to pay for protection against beta risk and exposure to coskewness risk. This section investigates the possibility that the willingness to pay depends on the economic conditions that prevail when the pricing decisions are made. To characterize the periods of priced beta and coskewness, various lagged macroeconomic and stock market variables are used to proxy information that investors have when pricing decisions are made. The risk premiums are regressed on each

information variable to detect significant co-movements. Ferson and Harvey (1991) use this method to explain the time variation in the risk premiums of their multibeta model for monthly U.S. data. There is no generally accepted theory to dictate which variables should be considered, nor to predict the anticipated sign on the chosen variables. The current choices largely reflect predetermined variables used in other studies and the availability of suitable Canadian series. The Canadian instruments that are considered are the value-weighted return on the TSE market index, the difference between the returns on long-term government bonds and corporate bonds, the stock dividend yield on the TSE, and a Canadian leading indicator variable. The acronyms adopted for the variables, the data source, and the frequency of each variable are given in Table 2.

In the absence of clear theoretical guidance, it is useful to consider the variables that have been successfully incorporated in previous studies of market returns. For example, Conrad and Kaul (1989) and Fama and French (1988a) have found that expected future returns in the U.S. follow an autoregressive process. If this is also true of weekly Canadian data, the lagged TSE market index should help explain the time-varying risk premiums. Several American studies, including Ferson and Harvey (1991) and Fama and French (1988b), have successfully connected dividend yields and returns or risk premiums. Schmitz (1996) shows that the TSE dividend yield and the return spread between Canadian government and corporate bonds are current measures of the health of the Canadian economy and are positively related to market risk premiums. Smith (1993a) and Koutoulas and Kryzanowski (1996) use the difference in the long-term Canadian corporate bond rates and long-term Canadian government bond rates as a factor in an arbitrage pricing theory, APT, framework. In the current study, consistent with Schmitz (1996), the lagged difference in the long-term Canadian corporate and government bond rates is used as an indicator of the level of risk aversion. As the level of risk aversion increases, the difference between corporate bonds and government bonds should increase. The assets that are considered more sensitive to shifts in the degree of risk aversion should require a higher mean return. As in Koutoulas and Kryzanowski (1996), a Canadian leading indicator is used to forecast real domestic activity.

The pricing of Canadian assets may be responsive to conditions in the U.S. due to the close ties between the countries. If the U.S. market has experienced unusually high returns or volatility shocks, these may cause Canadians to believe a similar experience is likely in their domestic economy. The influence of the U.S. economy is measured by regressing a given Canadian risk premium on the equally weighted or the value-weighted U.S. market portfolio, the volatility shock to the U.S. market,

using both market proxies,⁸ and the Canada/U.S. foreign exchange rate. Koutoulas and Kryzanowski (1996) also use the Canada/U.S. foreign exchange rate as a macrofactor. Table 2 gives the source and frequency of these variables.

These comparisons generally use a weekly risk premium and a weekly information variable. Several of the Canadian variables, however, are monthly. Where applicable, a weekly implied rate of return is calculated from a monthly rate of return for an information variable; otherwise, the risk premiums are compounded to get a monthly risk premium to compare to the monthly macrovariables. Descriptive statistics for the information variables are listed in Table 3 (weekly series) and Table 4 (monthly series). Table 4 also gives summary statistics for the monthly risk premiums. The analysis for the weekly premiums and information variables is presented followed by the outcomes for the monthly variables.

Table 3—Summary Statistics for Information Variables, Weekly Data,¹ January 19, 1977 to December 20, 1994 (935 observations, unless otherwise noted)

Variable	Mean	Standard Deviation
Value-Weighted U.S. Market Return VW_{US}	.0026	.0210
Equally Weighted U.S. Market Return EW_{US}	.0040	.0207
Volatility Shock to VW_{US} ($VS_{VW,US}$)	-.0001	.0021
Volatility Shock to EW_{US} ($VS_{EW,US}$)	-.0001	.0025
Value-Weighted TSE Market Return VW_{TSE}	.0030	.0209
Level of Risk Aversion $RA^{2,3}$	.0002	.0001

Notes:

1. Weekly data converted from daily CRSP data for the U.S. series and converted from daily CFMRC data for Canada

2. Weekly implied rates from monthly CANSIM data

3. RA is the difference between the long-term Canadian Corporate bond rate and the long-term Canadian government bond rate. It has 894 observations because the corporate bond series was not available until November 1977

The weekly risk premium for Canadian coskewness is correlated with last week's U.S. volatility shock and with the lagged equally weighted U.S. market return. As Table 5 shows, a volatility shock in the U.S. market last week has a negative correlation with the risk premium for coskewness in the current week. A volatility shock in the U.S. market is perceived by investors as an indication that volatility shocks are likely to occur in the Canadian market. Consequently, investors become more willing to pay extra for an asset with desirable coskewness properties and thus the return on such an asset declines. Notice that the equally weighted U.S. market portfolio (but not the value-weighted market return) tracks λ_2 . The equally weighted portfolio, relative to the value-weighted index, puts more weight on small

⁸ The market variances for both the equally weighted and value-weighted indices follow a GARCH(1,1) process, and these variances are used to calculate the volatility shocks, as defined earlier. See footnote 5 for a description of how the GARCH processes are identified.

Table 4—Summary Statistics for the Monthly Risk Premiums and Information Variables nuary 1977 to December 1994 (215 observations, unless otherwise noted)

Variable ¹	Mean	Standard Deviation
$\bar{\lambda}_0$	-0.0112	0.1030
$\bar{\lambda}_1$	0.0206	0.1621
$\bar{\lambda}_2$	-0.0004	0.0435
DIVIDEND YIELD DIV	0.0359	0.0079
Canadian Leading Indicator CLI	122.21	24.5333
Foreign Exchange: Canada/U.S. FX	1.2332	0.0885

1. $\bar{\lambda}_0$, $\bar{\lambda}_1$, $\bar{\lambda}_2$ are the monthly compounded value of the weekly lambdas. There are either four or five weeks in a month. The remaining variables are measured on a monthly frequency as supplied by CANSIM

firms and thus is more typical of the stocks on TSE. This index may be capturing a seasonal effect, however, because it becomes insignificant when a January dummy variable is added to the regression while the January dummy variable is highly significant. This is the only instance where the addition of a January dummy variable, which is usually significant, substantially changes the magnitude or significance of an information variable.

The weekly risk premium on beta, λ_1 , has a significant and positive relationship with the weekly change in the level of risk aversion lagged two weeks;⁹ see Table 5. If current business conditions are unhealthy, the level of risk aversion increases; investors require a higher return on assets with greater systematic risk. On the other hand, neither the lagged value-weighted TSE market portfolio nor the equally and value-weighted American market proxies has a relationship with λ_1 , regardless of the lag length. The lack of a significant relationship between the market proxy and the risk premiums is similar to the U.S. evidence presented in Ferson and Harvey (1991). This is also consistent with studies on monthly and daily Canadian data that do not find significant autoregressive behaviour in equity returns. See Foerster (1993).

The mean of the cross section of returns that is left unexplained by both beta and gamma is captured by λ_0 . This should be the risk free rate, if one exists. Although Canadian studies by Smith (1993b), Abeysekera and Mahajan (1987), and Hughes (1984) find a general lack of support for their APT models, the intercept in each model is approximately equal to the riskless rate. Smith uses the overnight loan

⁹ The regressions of the weekly risk premiums on the weekly lagged information variables are performed for lag lengths ranging from one to five weeks. The significant lags are indicated in Table 5.

Table 5—Relationship Between the Risk Premiums and the Information Variables¹

$$\lambda_j = a_0 + a_1 * (\text{information variable})_{t-1} \text{ for } j = 1, 2$$

January 19, 1977 to December 20, 1994 (935 observations unless otherwise noted)

Variable ²	λ_1	λ_2
VW _{US}	.11 (.10)	.03 (.02)
EW _{US}	.14 (.11)	.05 (.02) *
VS _{VW,US}	-.28 (1.05)	-.78 (.24) **
VS _{EW,US}	-.35 (.88)	-.51 (.20) *
VW _{TSE}	.09 (.11)	.03 (.024)
RA ³	29.25 (42.61) ⁴	-6.81 (9.74)

The table gives the regression coefficient on the information variable lagged one period and the associated standard error of the regression coefficient is given in parentheses

* Significant at 5 percent

** Significant at 1 percent

The information variables were lagged one to five weeks. Significance at later lags is marked, see note 4
Notes:

1. All data are weekly. The information variables for the U.S. are converted from daily CRSP data. The value-weighted TSE market return is converted from daily CFMRC data. RA is a weekly implied rate from monthly CANSIM data

2. The acronyms for the variables are defined in Table 2

3. RA is the difference between the long-term Canadian Corporate bond rate and the long-term Canadian government bond rate. It has 894 observations because the corporate bond series was not available until November 1977

4. Significant if information variable is lagged two, three, four, or five weeks

rate of the Bank of Canada while Hughes and Abeysekera and Mahajan use the three month treasury bill rate as a measure of the risk free rate.¹⁰ In this study, a t-test reveals that λ_0 is insignificantly different from the risk free rate in 76 percent of the periods considered. This outcome holds for both proxies of the risk free rate.

The results of time series regressions of the monthly risk premiums on the monthly information variables are presented in Table 6.¹¹ The monthly risk premium on beta, $\bar{\lambda}_1$, shows a significant positive correlation with the TSE stock dividend yield variable. Similarly, Ferson and Harvey (1991) find that dividend yield is useful in tracking the risk premium on their U.S. market variable. Schmitz (1996) finds that the TSE dividend yield is the best predictor of the Canadian market risk premium. The addition of a monthly dummy variable for January, although significant, does not change the significance of the relationship between the dividend yield and the monthly risk premium on beta. No other monthly information variable has a significant relationship with either $\bar{\lambda}_1$ or $\bar{\lambda}_2$.

¹⁰ The overnight loan rate of the Bank of Canada is available from the CFMRC and a weekly rate for the three-month Treasury bill is implied from the monthly CANSIM data. Korkie (1990) discusses the problems of using the three-month Treasury bill rate, as reported by CFMRC, as a riskless rate.

¹¹ The monthly estimates of the risk premiums are the compounded weekly estimates. The number of weeks in any given month is either 4 or 5.

Table 6—Relationships Between the Risk Premiums and the Monthly Information Variables¹ $\bar{\lambda} = a_0 + a_1 \cdot \text{(information variable)}_{t-1}$ for $j = 1, 2$ Jan. 1977 to Dec. 1994 (215 observations)

Variable ²	$\bar{\lambda}_1$	$\bar{\lambda}_2$
Canadian Leading Indicator ³ CLI	-.00 (.00)	-.00 (.00)
Stock Dividend Yield DIV	3.80 (1.39) *	-.06 (.38)
Foreign Exchange FX	.04 (.12)	.03 (.03)

The table gives the regression coefficient on the monthly information variable lagged one period and the associated standard error of the regression coefficient is given in parentheses

* Significant at 5 percent

** Significant at 1 percent

Notes:

1. Variables are measured as monthly by CANSIM. The risk premiums, the lambdas, are converted to a monthly frequency from weekly data by compounding over the number of weeks (four or five) in the given month. $\bar{\lambda}_1$ and $\bar{\lambda}_2$ denote the monthly risk premium for beta and gamma respectively

2. The acronyms for the variables are defined in Table 2

3. The Canadian leading indicator is tested contemporaneously as well as lagged by one month

January Effect

The results of the previous section suggest that there may be seasonality in the pricing of beta and gamma. A size-related January effect has been found in Canadian data by, for example, Foerster and Porter (1993) and Tinic, Barone-Adesi, and West (1987). Similarly, Tinic and West (1984) have suggested that the two-moment CAPM only holds in January for U.S. data. To investigate seasonality, VSM pricing outcomes are analyzed by month, and the results are presented in Table 7 for coskewness and in Table 8 for beta. The analysis reveals that the proportion of January weeks with priced coskewness is larger, 42 percent, than in any other month. This proportion is only statistically significantly different than (greater than) those for July (23 percent) and April (26 percent).¹² Interestingly, 79 percent of the coskewness pricing that occurs in January involves a positive coskewness premium, whereas a negative premium is anticipated. This is statistically greater than the percent of positively priced coskewness weeks in every other month except for June, July, and October, (Table 7, column (5)). On the other hand, September has the highest frequency of negatively (correctly) priced coskewness, 73 percent, followed by April (65 percent).

The percent of the January weeks that involve a priced beta is not statistically different from any other month. In addition, of the periods with a priced premium for beta, a significant positive risk premium is no more likely in January than in any of the other months, except for April and September, see Table 8 column (4).

¹² The null hypothesis in each case is that the proportion in January is equal to the proportion in month(k), where month(k) ranges from February to December. The normal approximation to the binomial distribution is appropriate for each test.

Table 8—Seasonality in the Risk Premium for Beta, Pricing of λ_1 by Month (January 19, 1977 to December 20, 1994)

Month	Number of Weeks in Month (j) Over the Sample Period N_j	λ_{1sig}	$\lambda_{1(sig\ and\ pos)}$ ¹	$\lambda_{1(sig\ and\ neg)}$
		N_j Fraction of Weeks for Which λ_1 is Significant	λ_{1sig}	λ_{1sig}
January	79	0.39	0.68	0.32
February	72	0.43	0.48	0.52
March	81	0.37	0.57	0.43
April	76	0.38	0.38	0.62
May	81	0.35	0.64	0.36
June	77	0.40	0.65	0.35
July	79	0.32	0.60	0.40
August	80	0.45	0.47	0.53
September	79	0.35	0.21	0.79
October	77	0.32	0.52	0.48
November	78	0.33	0.65	0.35
December	76	0.38	0.62	0.38

Notes:

1. This column, column (4), and column (5), are the breakdowns of column (3). Thus, column (4) gives the fraction of the significantly priced beta weeks that involve a significantly positive price on beta while column (5) gives the fraction of the significantly priced beta weeks that involve a significantly negative price on beta

the 830 weeks of the overlap period, both models reject the pricing of coskewness but not of beta. The riskless rate is lower and the market prices of beta and gamma risk are higher in Canada than in the U.S., suggesting that Canadian investors demand more reward for systematic risk and that they are willing to pay more for protection from volatility shocks. Morin (1980) also finds a lower riskless rate and a higher beta premium in Canada relative to the U.S.

Averaging the risk premiums across weeks is misleading because the pricing of beta and coskewness are time dependent. The U.S. VSM model has a priced beta in 54 percent of the periods; of those periods, 56 percent have a positive risk premium on beta. Investors are willing to pay for coskewness in 33 percent of the periods and the risk premium is negative in 55 percent of these periods. For the Canadian VSM, 38 percent of the periods have a priced beta and 53 percent of those prices are positive. Thirty-two percent of the periods price coskewness, and 51 percent of those prices are negative. Thus, there is stronger evidence for the pricing of beta in the U.S. but the evidence for gamma is the same across the two countries.

Conclusions

The volatility shock model is estimated for weekly Canadian data and compared to the outcomes obtained using American data. In Canada, investors' willingness to pay for an asset's ability to hedge against unexpected movements in the market

Table 8—Seasonality in the Risk Premium for Beta, Pricing of λ_1 by Month (January 19, 1977 to December 20, 1994)

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Notes:

1. This column, column (4), and column (5), are the breakdowns of column (3). Thus, column (4) gives the fraction of the significantly priced beta weeks that involve a significantly positive price on beta while column (5) gives the fraction of the significantly priced beta weeks that involve a significantly negative price on beta

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Conclusions

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Table 9—Equilibrium Pricing Results for the U.S. and Canada, January 19, 1977 to December 20, 1992 (830 Observations)

	Mean	Standard Deviation	Minimum	Maximum	t test Mean = 0 ¹
Panel A: Canada					
λ_0	-.0035	.0491	-.8319	.2296	-2.041*
λ_1	.0048	.0697	-.4234	1.1148	1.970*
λ_2	-.0002	.0157	-.1425	.1910	-.294
R ²	.0931	.1079	0	.9880	24.86**
Panel B: U.S.					
λ_0	.0008	.0153	-.1047	.0738	1.55
λ_1	.0031	.0307	-.2597	.1956	2.92 *
λ_2	-.00003	.0042	-.0412	.0655	-.198
R ²	.0777	.0977	0	.9964	22.91**

The following regression is run:

$$r_{it} = \lambda_0 + \lambda_1 \beta_{it}^{KF} + \lambda_2 \gamma_{it}^{KF} + \varepsilon_{it}$$

where the superscript KF refers to the Kalman filter estimate of either beta or gamma. This asset pricing equation is applied to the Canadian weekly and the American weekly cross sections of beta and gamma. There are 94 Canadian firms per cross section and 191 American firms per cross section

Notes:

1. t test that population mean is zero. $t = \text{mean}(\lambda_i) / \text{standard deviation}(\lambda_i) / \sqrt{830}$

* Significant at 5 percent

** Significant at 1 percent

return and in the volatility of the market is time varying and responsive to market conditions in both Canada and the United States. Canadian investors become interested in the coskewness properties of assets after a volatility shock in the U.S. while the premium for beta risk increases in response to an increase in the level of risk aversion or the level of dividend yields in the Canadian economy.

The Canadian risk premiums for coskewness are sensitive to the months of January and September; however, there is virtually no January effect in the pricing of beta. September is historically a poor month for Canadian equities, and investors seek positively coskewed assets in order to offset the unusually low returns or perhaps to enhance a potential future gain. Thus, coskewness is priced when investors anticipate a need for protection, either due to past U.S. or Canadian market experience.

The American risk premiums on coskewness and beta are also time varying. Finally, although there is stronger support for the pricing of beta in the U.S. than in Canada, the averaged risk premiums are consistent with a more conservative Canadian investor.

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Appendix 1—Derivations of a Volatility Shock Model from a Two Factor Arbitrage Pricing Theory Model and From a Three Moment Capital Asset Pricing Model

APT Model for Volatility Shocks

According to arbitrage pricing theory (APT) there are certain underlying forces in the economy that are unavoidable and affect all firms. In equilibrium a firm's sensitivity to these forces is important in determining how it is priced. The asset's sensitivity to these factors, also called the asset's factor loadings, are unknown and must be estimated before their pricing can be tested. If one of the factors is unexpected change in the market variance, then the firm's sensitivity to that factor is its coskewness with the market portfolio.

There are two key equations in an APT model: the equilibrium asset pricing equation and the hypothesized return-generating mechanism for firm returns. The equilibrium model relates an asset's expected returns to the asset's sensitivity to the common economic factors that systematically affect all firms:

$$(a1) \quad E(r) = \lambda_0 + \lambda_1 a_{11} + \lambda_2 a_{12}.$$

where:

a_{ij} = Firm i 's sensitivity to factor j for $j = 1, 2$;

λ_j = The premium for bearing the risk associated with factor j .

The premium on an asset with no exposure to factor risk is given by λ_0 . It is the risk free rate, if one exists. The firm factor sensitivities are unknown and are estimated from the return-generating mechanism. The hypothesized return-generating mechanism is linear:

$$(a2) \quad r_{it} = E(r) + a_{i1} R_1 + a_{i2} R_2 + e_{it}$$

Equation (a2) represents the return-generating process for asset i in period t , r_{it} . The return on factor j , R_j , is defined as the unexpected changes in the common factor j , $(R_j - E(R_j))$, for $j = 1, 2$. a_{ij} is, as above, the i th firm's sensitivity to, or factor loading on R_j , and e_{it} is the idiosyncratic risk.

An arbitrage argument is used to arrive at the equilibrium pricing equation from the linear generating mechanism. The existence of an arbitrage portfolio (that is, a portfolio that uses no wealth, involves no risk, and yet earns a nonzero reward) is inconsistent with equilibrium. All investors would want to own such an arbitrage portfolio and thus its price would be bid up until its arbitrage return was dissipated. To demonstrate this, an arbitrage portfolio is formed using equation (a2). Let the weights or the proportion of wealth devoted to each asset in the arbitrage portfolio be given by x_i . The no arbitrage argument implies that equation (a2) becomes:

$$\sum_i x_i r_{it} = \sum_i x_i E(r_{it}) + \sum_i x_i a_{i1} R_1 + \sum_i x_i a_{i2} R_2 + \sum_i x_i e_{it} = 0$$

By the definition of the arbitrage portfolio there is no risk, $\sum x_i a_{i1} = \sum x_i a_{i2} = \sum x_i e_{it} = 0$ and equilibrium requires $\sum x_i E(r_{it}) = 0$. The same weights are orthogonal

to the expected returns and to each of the factor loadings. Consequently, expected returns and the factor loadings must be linearly dependent. That is, the no-arbitrage argument means that the expected returns can be expressed as a linear combination of the factor loadings, as given in equation (a1). Thus, in equilibrium, the APT maintains that an asset's expected return will be a linear combination of the risk free rate plus premiums for the degree of factor risks characterizing the specific asset.

Most studies of the APT have identified the factor risks from factor or principle components analysis and hence lack an economic interpretation. Recently attempts have been made to choose factors on the basis of an economic rationale. The APT model presented here conjectures that the prevailing forces behind asset returns can be captured by a market factor and by a volatility risk factor. The market factor is the inherent uncertainty underlying movements in the economy and, as such, is unavoidable. Volatility risk is the risk associated with unanticipated shocks to the market variance. Hence, the return-generating process, equation (a2), and the equilibrium pricing model, equation (a1), specialize to equations (a3) and (a4), respectively.

$$(a3) \quad r_{it} = E(r_{it}) + \beta_{im}(R_m - \mu_m)_t + \gamma_{im}[(R_m - \mu_m)^2 - h_m]_t + \varepsilon_{it}$$

$$(a4) \quad E(r_{it}) = \lambda_0 + \lambda_1 \beta_i + \lambda_2 \gamma_i$$

$\beta = a_{i1}$ is firm i 's sensitivity to market risk, $(R_m - M_m)$. A volatility shock occurs when the squared deviation of the market return is different from the expected squared deviation, $[(R_m - M_m)^2 - h_m]$. $\gamma = a_{i2}$ is a measure of firm i 's responsiveness to volatility shocks. γ is the (normalized) coskewness of an asset. λ_1 and λ_2 are the premiums for market and volatility risk, respectively.

Coskewness and the Three Moment CAPM

A model of asset pricing where the expected return reflects an asset's systematic risk as well as its coskewness properties also can be derived from a three moment capital asset pricing model. In this model, the risk averse investors have utility functions, ϕ , that depend on the first three moments of their wealth. Utility increases with mean, μ_w , and skewness of wealth, K^3_w , and decreases with variance of wealth σ^2_w . Investors maximize utility $\phi(\mu_w, \sigma^2_w, K^3_w)$ with respect to mean, variance, and skewness of wealth by choosing the proportion of wealth to allocate to each asset and subject to the initial wealth constraint that the sum of the investments in the assets must equal the initial wealth. The equilibrium for these utility-maximizing investors is an asset pricing model that requires higher returns to compensate for higher systematic risk and lower returns in exchange for assets with desirable (positive) coskewness properties. The resulting pricing equation is the same as the equilibrium APT volatility shock model given in (a4).

Under the APT volatility shock model or the three moment CAPM, an asset's returns are priced according to equation (a4). In the three moment CAPM an investor's preference for a positively skewed investment position translates into a

search for assets with a positive coskewness with the market. In the APT context the motivation behind seeking positively coskewed assets is to provide protection against volatility shocks. Regardless of the model, coskewness of an asset is theoretically attractive to an investor. In either model, investors try to reduce exposure to downside risk by searching for assets that contribute to a positively skewed portfolio.