Stock Return Volatility and Time-Varying Betas in the Toronto Stock Exchange

Athanasios Episcopos*
Clarkson University

This paper investigates the time series properties of Canadian stock returns using daily data from the TSE300 index and several sub-index portfolios. The sub-index portfolios include stocks from the major Canadian industries such as financial services, gold and silver mines, communications and media, paper and forest products, consumer products, and others. Stock returns are modeled as autoregressive processes with time-varying parameters and errors following an EGARCH process. There is evidence that the first order autocorrelations are negatively linked to volatility and that sub-index betas are time varying.

INTRODUCTION

Estimating the volatility of stock returns is essential for measuring the systematic risk of a portfolio. Recent time series studies use the ARCH methodology which provides a useful way of jointly modeling the first two moments of the returns on an asset. The conditional variance exhibits a memory that can last several weeks, and the duration of such persistence depends on the asset under study. For a discussion of volatility and applications of the ARCH methodology, see Bera and Higgins (1993) and Bollerslev, Chou, and Kroner (1992).

Schwert and Seguin (1990) estimate a single factor model yielding time-varying betas. They discuss the implications of heteroskedasticity for testing capital asset pricing models and find that risk-adjusted returns are related to firm size. Accounting for heteroskedasticity increases the evidence that risk-adjusted returns are related to firm size. Koutmos, Lee, and Theodossiou (1994) apply the Schwert-Seguin (1990) technique to a world portfolio of several country indexes including Canadian stocks. They find that market betas with respect to a world index are also time dependent and that markets with high volatility persistence exhibit higher systematic risk during periods of high world market volatility.

ARCH modeling is useful in the study of the relationship of volatility and autocorrelation of asset returns. An empirical regularity in financial market indexes has been that first order autocorrelations are higher during periods of low volatility and lower during periods of high volatility. This phenomenon can be attributed to fads and feedback traders (e.g., Shiller, 1984) and to nonsynchronous trading (e.g., Lo and MacKinlay, 1989; Atchison, Butler, and Simonds, 1987), although the issue is not settled in the literature. (See, for instance, Conrad and Kaul, 1988). In addition, the autocorrelation-volatility relationship appears in assets other than stocks [for instance, in exchange rates (e.g., Koutmos, 1994)]. Sentana and Wadhwani (1992) and LeBaron (1992) use GARCH models in

* The author would like to thank Andreas Pericli, Panayiotis Theodossiou, Jon Vilasuso, and two anonymous referees for helpful comments and suggestions. Any errors are my responsibility. Financial support from the Embassy of Canada under the 1993-94 Faculty Research Grant Program is gratefully acknowledged.

0747-5535/96/1400-0028 $02.50

which they impose a specific functional form on the AR(1) coefficient in terms of the conditional variance. Both papers find a negative relationship between volatility and autocorrelation.

The present study focuses on the Toronto Stock Exchange (TSE), a major market in Canada. Since 1954 the value of shares traded each year in the TSE has been as large as 5 percent of the respective value of shares traded in the NYSE. The TSE is important for American and Canadian investors because of the increased volume of international trade between the U.S. and Canada. About two-thirds of Canada's imports come from the U.S., and about two-thirds of its exports are delivered to the United States. The North American Free Trade Agreement has strengthened trading ties between the two countries, and one would expect increased investment activity in the TSE and derivatives markets, as investors attempt to hedge the risks of international trade.

Current empirical work on the Toronto Stock Exchange using the latest time series techniques is lacking. Fowler, Rorke, and Jog (1978) find evidence of heteroskedasticity and relate it to the frequency of trading. Koutmos (1992) investigates the issue of asymmetric volatility using an EGARCH-M model on Canadian and other stocks. The present study adds new evidence and updates the available empirical literature on the TSE. This study uses about four years of recent data and employs the latest techniques from the ARCH methodology to estimate time-varying volatilities and betas. The influence of the variance on the autocorrelation parameter is modeled in an approach similar to that of Sentana and Wadhwani (1992), and the statistical behavior of the major sub-index portfolios of the TSE300 is examined. These portfolios include the most important Canadian industries such as mining, oil, manufacturing, retailing, and financial services.

The major findings of this paper are that the returns of most stock index portfolios in the TSE follow distributions that deviate from the normal, that variances are time dependent, that autocorrelation of returns is inversely related to volatility, and that there is evidence that portfolio betas are time varying.

DATA DESCRIPTION AND PRELIMINARY FINDINGS

The data include daily closing prices in \$CA of the TSE300 composite index and its sub-indexes[1] from 7/30/1990 to 6/30/1994. Table 1 contains a description of the indexes and the stocks they contain. Continuously compounded percentage changes are calculated using the formula

$$r_t = 100 \times \log(y_t/y_{t-1})$$

where:

y_t = Price of each index on day t.

Table 2 shows preliminary statistics for the series of returns. The sample skewness estimate is statistically zero for the series gold and silver, consumer products, communica-

[1] The source of data is Infomart, Southam Electronic Publishing, Don Mills, ON M3B 2X7, Canada. The TSE300 does not include dividends.

Table 1—TSE300 Sub-Index Portfolios

Communications & Media (Broadcasting, Cable & Entertainment, Publishing & Printing)
Conglomerates
Consumer Products (Food Processing, Tobacco, Distilleries, Breweries, Household Goods, Autos & Parts, Packaging, Biotechnology, Pharmaceuticals)
Financial Services (Banks, Trusts, Savings & Loans, Investment Companies & Funds, Insurance, Financial Management)
Gold & Silver (Gold & Silver Mines, Precious Metal Funds)
Industrial Products (Steel, Fabricating & Engineering, Transportation Equipment, Technology, Cement/Concrete, Chemicals, Business Services)
Merchandising (Wholesale Distributors, Retail Stores, Hospitality)
Metals & Minerals (Integrated Mines, Metal Mines, Non-Base Metal Mining)
Oil & Gas (Integrated Oils, Oil & Gas Producers)
Paper & Forest Products
Pipelines
Real Estate & Construction
Transportation
Utilities (Telephone, Gas, Electrical)

tions and media, financial services, and conglomerates. The rest of the series are skewed either to the right or to the left. The excess kurtosis measure, however, is highly significant for all series. Therefore, returns are leptokurtic and do not follow the normal distribution. The Bera-Jarque test statistic[2] rejects normality for all series. Ljung Box (LB) statistics for 12 lags on the series indicate the presence of serial correlation with the exception of pipelines and conglomerates. Also, LB statistics on the squared series show high autocorrelation—which implies nonlinear dependency—with the exception of industrial products, pipelines, and conglomerates. The last two columns show the augmented Dickey-Fuller (ADF) test statistics[3] for unit root in the price and return series. Returns are stationary, while prices are not.

To gain further insight on the above issues, the model $r_t = kr_{t-1} + u_t$, where u_t is an error term, is estimated by the Newey-West (1987) method which corrects for heteroskedasticity and autocorrelation. (For space reasons, detailed results are not reported). The lag 1 for this autoregressive model is determined using the Schwarz criterion.[4] The autocorrelation coefficient is statistically significant, and LB tests show little or no serial correlation in the residuals. The squared residuals, on the other hand, are highly correlated except for pipelines, industrial products, and conglomerates. Examination of LB statistics at shorter and longer lags shows that these three sub-indexes do not exhibit any nonlinear dependencies. The residual series are basically white noise and, therefore, the three sub-indexes are dropped from the sample.

[2] The Bera-Jarque statistic is given by $(n - k)(S^2/6 - K^2/24)$, where n is the number of observations, k is zero for an ordinary series and equal to the number of regressors when working with the residuals of an equation, S is skewness, and K is excess kurtosis. See Bera and Jarque (1980).

[3] See Dickey and Fuller (1981) on the procedure of this test. Critical values at 1 percent, 5 percent, and 10 percent are -3.9702, -3.4159, and -3.1300, respectively.

[4] The Schwarz criterion (SC) is given by n log(RSS) + k(log(n)), where n is number of observations and k is number of regressors (Schwarz, 1978). The method rests on finding the model with the minimum SC. SC penalizes adding regressors, and it leads to a parsimonious model. The SC selected AR(2) for the cases of real estate & construction and merchandising. The AR(1) is retained here for uniformity and because the results of later estimation do not differ much from those reported, at least qualitatively.

Table 2—Preliminary Statistics for Return Series 7/30/90—5/3/94

Index	m	s	S	K	BJ	LB(12)	LB2(12)	ADFr	ADFy
TSE300	0.01	0.56	-0.39	2.14	212.4	109.8	47.15	-23.1	-1.97
Metals & Minerals	0.01	1.06	0.26	0.93	46.96	58.14	47.44	-24.94	-2.40
Gold & Silver	0.042	1.75	-0.14	2.64	287.57	19.24	38.96	-28.26	-2.06
Oil & Gas	0.01	0.72	-0.53	3.12	444.25	233.90	15.84	-20.08	-1.34
Paper & Forest Products	0.01	0.80	0.47	2.32	256.87	137.89	45.73	-22.46	-1.83
Consumer Products	0.03	0.71	-0.14	1.54	99.73	77.31	30.87	-25.77	-1.47
Industrial Products	0.02	0.85	-0.35	1.94	173.28	59.6	9.62	-24.70	-2.04
Real Estate & Construction	-0.16	1.20	0.25	2.86	344.66	170.38	65.68	-23.33	-1.51
Transportation	-0.09	1.67	-0.81	6.88	2047.08	21.49	22.83	-27.85	-2.84
Pipelines	-0.01	0.75	-0.99	9.99	4247.46	14.42	6.26	-32.94	-1.59
Utilities	0.02	0.58	-0.39	2.48	277.59	2.8	27.54	-26.55	-1.84
Communications & Media	0.03	0.76	-0.07	1.13	53.13	25.29	37.02	-28.35	-2.27
Merchandising	-0.01	0.61	-0.62	3.76	640.31	68.92	25.48	-26.46	-0.90
Financial Services	0.01	0.56	-0.39	2.14	212.38	109.8	47.15	-26.56	-1.98
Conglomerates	0.00	1.00	-0.02	0.74	22.34	20.04	13.34	-28.25	-1.82

m, s, S, and K = Sample mean, standard deviation, skewness, and excess kurtosis, respectively
BJ = Bera-Jarque statistic for normality distributed as a χ^2
LB(12) and $LB^2(12)$ = Ljung-Box statistics following a χ^2
ADF_r, ADF_y = Augmented Dickey-Fuller statistics for the return and price series, respectively; critical value at 5 percent is -3.4159
Sample size = 982

The conclusion of the preliminary analysis is that the distribution of returns is more leptokurtic than the normal, that the returns exhibit autocorrelation, and that there is evidence that the conditional variance may be time dependent.

AN EGARCH(1-1) - LINEAR AR MODEL

High autocorrelation in stock returns tends to be accompanied by low market volatility. Papers such as Sentana and Wadhwani (1992) and LeBaron (1992) successfully address the joint issues of volatility-autocorrelation and heteroskedasticity in returns. To investigate these issues, a modified version of the EGARCH model of Nelson (1991) is used. The model is written as follows.

(1) $e_t = r_t - (a + bh_t)r_{t-1}$

(2) $\log h_t = a_0 + a_1 N_t + g_{t-1} + a_2 \log h_{t-1}$

(3) $g_t = a_3 x_t + a_4(|x_t| - E(|x_t|))$

(4) $x_t = e_t/(h_t)^{1/2}$

where:

h_t = The conditional variance;
e_t = The residuals;
x_t = Standardized residuals; and
N_t = Number of nontrading days preceding each observation.

The errors e_t are assumed to follow the generalized error distribution (GED) with scaling v. The density of GED is given by

$$f(e_t) = v(\lambda 2^{(1+1/v)}\Gamma(1/v)h_t^{1/2})^{-1} \exp(-(1/2)|e_t \lambda^{-1} h_t^{1/2}|^v)$$

where:

$$\lambda = 2^{(-2/v)v)}\Gamma(1/v)\Gamma(3/v))^{1/2},$$

and $\Gamma(\cdot)$ is the gamma function. For $v = 2$, the GED becomes the normal distribution. For $v < 2$, the density is more peaked (leptokurtic) and has fatter tails than the normal. The expected value of $|x_t|$ is

$$(7)\ E(|x_t|) = \lambda 2^{(1/v)}\Gamma(2/v)\Gamma(1/v).$$

The log-likelihood function is given by

$$(8)\ L = \Sigma\ [\log(v/\lambda) - (1/2)|e_t \lambda^{-1} h_t^{-1/2}|^v - (1+1/v)\log(2) - \log(\Gamma(1/v)) - (1/2)\log(h_t)]$$

where sum index t runs through the observations.

In summary, there are five main features of the model. First, the conditional variance cannot become negative as it does in the GARCH model of Bollerslev (1986) or the ARCH model of Engle (1982). Second, the conditional variance is asymmetric, allowing for the *leverage effect* [that is, the negative relation between current volatility and past returns (Black, 1976)]. Third, the effect of nontrading on volatility is captured by N. As noted in Nelson (1991), we expect coefficient a_1 to be positive but much less than one. Fourth, the autocorrelation-volatility relationship is captured by the function $(a + bh_t)$ as in Sentana and Wadhwani (1992), and we expect b to be negative. Fifth, this model uses a GED instead of a normal distribution because GED better approximates the leptokurtosis feature of the returns observed in the previous section and because the normal distribution is a special case of the GED.

The Bernt-Hall-Hall-Hausman (1974) iterative procedure is used to maximize the log-likelihood function L. Table 3 presents estimation results from this EGARCH model. In the second row the restricted model of TSE300 with $b = 0$ is included for comparison. Adding the variance improves the model slightly, based on the value of the likelihood function. Coefficient b has a negative sign in all but paper & forest products series.[5] Coefficient a_2 shows a significant influence from past variances on today's variance. Coefficient a_1 is positive and much less than one, as expected, which corroborates the findings of Fama (1965) and French and Roll (1986). Coefficient a_4 is positive and statistically significant for all series. (For consumer products it is significant at 10 percent.)

[5] Second and higher order polynomials are tried to find further nonlinearity. None give significant results. This result is different from LeBaron (1992) where variance enters exponentially. His model is a GARCH(1,1) with a normal distribution without nontrading day effects; this could be the reason for the difference.

Table 3—EGARCH Parameter Estimates

$$e_t = r_t - (a + bh_t)\, r_{t-1}$$
$$\log h_t = a_0 + a_1 N_t + g_{t-1} + a_2 \log h_{t-1}$$
$$g_t = a_3 x_t + a_4(|x_t| - E(|x_t|)),\ x_t = e_t/(h_t)^{1/2}$$

Index	a	b	a_0	a_1	a_2	a_3	a_4	v
TSE300	0.4676	-0.6217	-0.2295	0.2263	0.9047	-0.0441	0.1417	1.3693
	(5.901)	(-2.5844)	(-3.9894)	(3.9293)	(25.1854)	(-1.7306	(3.0615)	(16.6651)
TSE300R	0.2785		-0.3333	0.2823	0.8456	-0.0567	0.1588	1.3927
	9.2004		(-4.5722)	(4.8604)	(16.3948)	(-1.7833)	(2.8623)	(16.958)
Metals & Minerals	0.3528	-0.1343	-0.063	0.1415	0.965	-0.0078	0.105	1.5472
	(3.706)	(-1.6295)	(-2.5635)	(2.6586)	(53.5132)	(-0.4610)	(3.3284)	(15.531)
Gold & Silver	0.2627	-0.055	0.0537	0.1983	0.8639	0.0029	0.1761	1.3573
	(2.8747)	(-2.1637)	(1.0419)	(3.3423)	(19.6948)	(0.1044)	(3.7936)	(16.5842)
Oil & Gas	0.6226	-0.4035	-0.2575	0.1919	0.8292	-0.0323	0.2606	1.3633
	(10.5873)	(-3.6624)	(-4.1897)	(3.1354)	(15.59)	(-1.1571)	(5.0728)	(18.922)
Paper, Forest Products	0.2789	0.0084	-0.2926	0.239	0.7011	0.0292	0.2478	1.4886
	(3.1046)	(0.0697)	(-3.7737)	(3.7656)	(6.257)	(0.8186)	(3.7834)	(15.4589)
Consumer Products	0.4198	-0.4714	-0.0661	0.1359	0.9922	-0.0204	0.0273	1.4948
	(4.6255)	(-2.5233)	(-2.9066)	(3.1434)	(146.329)	(-1.9492)	(1.6818)	(15.2675)
Real Estate, Construction	0.3693	-0.0683	-0.028	0.1107	0.9073	0.0238	0.2509	1.3751
	(5.2948)	(-1.8524)	(-0.8242)	(1.5578)	(25.514)	(0.9359)	(4.5503)	(16.4872)
Transportation	0.0961	0.0015	0.0863	0.0869	0.8631	-0.0121	0.2458	1.3868
	(1.1668)	(0.0657)	(1.3753)	(1.1096)	(16.2045)	(-0.3472)	(3.9814)	(19.6442)
Utilities	0.2757	-0.3294	-0.1174	0.1959	0.9762	-0.0009	0.0849	1.3686
	(2.9715)	(-1.2627)	(-3.3230)	(3.4967)	(60.4271)	(-0.0550)	(2.7069)	(17.0628)
Communications, Media	0.1994	-0.1818	-0.1395	0.1205	0.8569	-0.0338	0.1909	1.4335
	(1.717)	(-1.1000)	(-2.8965)	(1.9558)	(13.1626)	(-1.0367)	(2.9556)	(15.2326)
Merchandising	0.1478	-0.0322	-0.0847	0.1314	0.9764	-0.01	0.0664	1.3742
	(1.6492)	(-0.1440)	(-2.4937)	(2.6657)	(50.7339)	(-0.6289)	(2.1052)	(18.6556)
Financial Services	0.3353	-0.1878	-0.0807	0.1602	0.9718	-0.0459	0.0978	1.5148
	(4.2798)	(-2.0651)	(-3.1626)	(3.0442)	(64.6724)	(-2.3396)	(2.844)	16.136)

h_t = The conditional variance

e and x_t = The residuals and standardized residuals, respectively. x_t are assumed to follow GED with scaling v

N_t = Number of nontrading days between observations t-1 and t

t-statistics are in parentheses

This means that past volatility shocks affect today's variance. Coefficient a_3 is negative but generally weak in statistical significance. This means that for all series (except for the market portfolio, consumer products, and financial services) the asymmetry effect of unexpected increases or decreases in the index on volatility is not strong.[6] This corroborates earlier work on Canadian stocks by Koutmos (1992). Overall, the results indicate that volatility tends to persist over time. With respect to predictability of stock returns, coefficient a is statistically significant. This implies some inefficiency in the TSE market, although accounting for transaction costs may eliminate riskless profit opportunities. Koutmos, Lee, and Theodossiou (1994) find similar results with respect to predictability of the TSE300 using weekly data. Including a regression constant in equation (1) for the

[6] The effect of variance on the conditional mean also is explored. The coefficients have the correct positive sign, but p-values are between 0.5 and 0.9 for most of the indexes. Therefore, it is not included in the model. The weak statistical significance of that coefficient agrees with the results of Nelson (1991), French, Schwert, and Stambaugh (1987), and Koutmos (1992).

Table 4—Goodness-of-Fit Tests; Models From Table 3

Index	LB(12)	LB2(12)	S	K	D	Log L
TSE300	14.68	2.81	-0.36	1.42	0.02	-718.85
TSE300R	11.62	3.14	-0.33	1.47	0.03	-724.03
Metals & Minerals	5.81	15.18	0.23	0.91	0.02	-1396.31
Gold & Silver	10.02	15.11	0.15	1.68	0.02	-1887.19
Oil & Gas	8.90	11.36	-0.19	2.31	0.03	-890.81
Paper & Forest Products	15.58	9.73	0.26	0.99	0.03	-1078.19
Consumer Products	27.02	10.78	-0.07	0.90	0.04	-998.45
Real Estate & Construction	37.52	13.20	0.04	1.48	0.06	-1474.98
Transportation	6.25	5.28	-0.50	4.17	0.03	-1833.29
Utilities	9.06	6.40	-0.29	1.83	0.05	-790.84
Communications & Media	13.27	5.80	-0.17	1.13	0.03	-1084.08
Merchandising	27.31	7.16	-0.44	2.52	0.02	-845.31
Financial Services	7.46	5.78	-0.02	1.09	0.02	-1228.33

TSE300R shows estimates from fitting the TSE300 index by using the restricted model (b = 0)
t-statistics in parentheses
LB(12) and $LB^2(12)$ = Ljung-Box statistics for the residuals and squared residuals, respectively, from the restricted model of Table 7
S and K = Skewness and excess kurtosis of standardized residuals
D = Kolmogorov-Smirnov statistic which tests whether the standardized residuals come from a GED with the estimated scaling ν; critical value at 5 percent is 0.0434
Log L = Maximum value of the log-likelihood function
Sample size = *980*

mean does not perceptibly alter the estimation results. Finally, parameter n is less than two for all the sub-index portfolios. This result corroborates the earlier finding about leptokurtosis of the returns.

Table 4 presents additional tests. The models fare well in terms of accounting for heteroskedasticity as evidenced by the LB statistics for the squared standardized residuals. Excess kurtosis of the standardized residuals is positive, indicating that the estimated kurtosis is higher than in the normal, as expected. Because the scaling parameter is free in GED, one way to check the adequacy of the models is to test whether the standardized residuals come from a GED distribution with scaling equal to the maximum likelihood estimate of ν. The Kolmogorov-Smirnov test statistic,[7] D, is less than the critical value of 0.0434 at 5 percent for most series (less than the critical value of 0.0520 at 10 percent for utilities). Therefore, the models fit the data adequately in this respect.

SUB-INDEX PORTFOLIO BETAS

One way of calculating a portfolio beta is by estimating the model

$$(9)\ r_t = c_0 + \beta_0 r_{mt} + \varepsilon_{0t}$$

where:

[7] The Kolmogorov Smirnov test is constructed as the maximum absolute difference (over all observations) between the empirical distribution and the postulated distribution one wants to test. A convenient feature of this statistic is that its own distribution is independent of the hypothesized distribution and it depends only on the number of observations, n. For n = 980 the critical values at 1 percent, 5 percent, and 10 percent significance are 0.0520, 0.0434, and 0.0391, respectively.

r_t = Returns of each portfolio;
r_{mt} = The return on the TSE300 index;
c_0 = The intercept;
ε_{0t} = The error term; and
β_0 = The usual beta.

The last column of Table 5 shows estimated beta coefficients for each portfolio. For simplicity, the method used is Newey-West (1987). Following Koutmos, Lee, and Theodossiou (1994) and Schwert and Seguin (1990), one can model the returns of each sub-index portfolio via equation (10)

(10) $r_t = c + b_1 r_{mt} + b_2 r_{mt} + \eta_t$

where:

c = The intercept term;
b_1 and b_2 = Slope coefficients;
R_{mt} = r_{mt}/h_{mt};
h_{mt} = The conditional variance of the market portfolio; and
η_t = The error term.

The time-varying beta for each sub-index is given by

(11) $\beta_t = b_1 + b_2/h_{mt}$.

Derivations of these formulas can be found in Koutmos, Lee, and Theodossiou (1994).

Table 5 includes estimates of this model by the Newey-West (1987) method. Coefficient b_2 is somewhat weak in terms of statistical significance. The sign of b_2 determines the effect of market volatility on each portfolio. If b_2 is negative (positive), increases in the variance of the TSE300 index lead to an increase (decrease) in the sub-index beta, other things held constant. In addition, for portfolios with average betas less than one b_2 is always negative, while for those with average betas greater than one, b_2 tends to be positive. This supports the hypothesis that market volatility affects safer and riskier stocks differently. Thus, the systematic risks of these two classes of stocks would tend to move in opposite ways in periods of high volatility. If firm size and beta are related, as much research has shown (e.g., Curley, Hexter, and Choi, 1982), then the result is at least consistent with Schwert and Seguin (1990) where the spread between the systematic risk of small and large stocks in the U.S. is higher during periods of high market volatility.[8]

Table 6 shows the percent difference between conventional betas and time-varying betas as estimated above. The percentile figures show that the difference can be considerable, although there is no clear pattern relating percent differences in betas and type of sub-index portfolios.

[8] Testing directly the hypothesis that volatility affects small and large stocks differently in Canada is beyond the scope of this paper.

Table 5—Estimates of Time-Varying Betas

$$r_t = c + b_1 r_{mt} + b_2 R_{mt} + \eta_t$$

Index	c	b_1	b_2	R^2	SEE	β_0
Metals & Minerals**	-0.0104	1.2911	-0.0003	0.4671	0.7718	1.2903
	(-0.4235)	(8.7046)	(-0.0079)	(27.1889)		
Gold & Silver**	0.0209	0.7986	0.1104	0.1469	1.6196	1.1934
	(0.4045)	(2.1734)	(1.1957)	(9.4322)		
Oil & Gas*	0.0005	0.7739	-0.0348	0.2581	0.6193	0.6518
	(0.0255)	(5.4682)	(-1.0034)	(14.5410)		
Paper & Forest Products*	0.0006	0.9471	-0.0318	0.3419	0.6491	0.8837
	(0.0306)	(8.5933)	(-1.0879)	(20.3591)		
Consumer Products*	0.0186	1.0798	-0.0564	0.4814	0.5140	0.8783
	(1.1347)	(10.4853)	(-2.1070)	(26.4143)		
Real Estate & Construction*	-0.1641	0.8344	-0.0314	0.1145	1.1283	0.7231
	(-4.5685)	(4.6266)	(-0.6246)	(10.6912)		
Transportation**	-0.1082	1.2755	0.0477	0.2354	1.4684	1.4441
	(-2.2957)	(5.4410)	(0.7506)	(16.6375)		
Utilities*	0.0150	0.7018	-0.0307	0.3309	0.4744	0.5928
	(0.9855)	(6.2283)	(-1.0661)	(16.5151)		
Communications & Media*	0.0206	1.0051	-0.0575	0.3527	0.6102	0.7989
	(1.0526)	(10.8316)	(-2.3688)	(21.7235)		
Merchandising*	-0.0173	0.8776	-0.0674	0.3521	0.4896	0.6381
	(-1.1126)	(7.2385)	(-2.2800)	(17.5592)		
Financial Services**	-0.0047	1.1161	0.0344	0.5941	0.5737	1.2380
	(-0.2549)	(9.1290)	(1.0907)	(31.3426)		

Models are estimated by the Newey-West (1987) method
rt = Daily returns of each index portfolio
r_{mt} = Return on the TSE300
c = The intercept
η_t = The error term
R_{mt} = r_{mt}/h_{mt}, where h_{mt} is its variance as estimated with EGARCH
The time-varying betas are given by $b_t = b_1 + b_2/h_{mt}$
The last column shows the standard beta, β_0. It is the slope in the regression $r_t = c_0 + \beta_0 r_{mt} + \varepsilon_{0t}$
t-statistics are in parentheses
* Index has an average beta less than one
** Index has an average beta greater than one

CONCLUDING COMMENTS

This paper presents some new evidence about the time series behavior of stock returns in the Toronto Stock Exchange. It establishes that nonlinearities in the variance exist for most of the TSE300 sub-index portfolios and finds that first order autocorrelations are negatively liked to the conditional variance of returns. The estimated EGARCH models document the phenomenon of volatility persistence in each sub-index. Time-varying betas are computed as functions of the market volatility. The evidence indicates that the spread between betas of safe and risky sub-index portfolios may increase during periods of increased aggregate volatility in the Canadian market. We also show that the discrepancy between time-varying and constant betas can be significant, although there is no clear relation between such discrepancy and portfolio type.

There are at least two issues that need to be mentioned about the estimation of the models in this paper. Thinness of individual stocks leads to autocorrelation in an index, and it can affect the significance and consistency of the parameter estimates (Fowler, Rorke, and Jog, 1979), although the EGARCH models presented account for heteroske

Table 6—Fractiles of Discrepancy Between Time-Varying Betas and Constant Betas

Index	1 Percent	10 Percent	Median	90 Percent	99 Percent
Metals & Minerals	-0.0443	-0.0013	0.0320	0.0707	0.0978
Gold & Silver	-28.5995	-15.4312	-3.1831	10.6223	26.5621
Oil & Gas	-10.5127	-4.9223	2.3125	12.2239	20.2851
Paper & Forest Products	-7.9236	-3.7580	1.4378	8.2180	13.4646
Consumer Products	-12.7008	-6.2295	2.3615	14.5396	24.8123
Real Estate & Construction	-0.8302	-0.4061	0.0784	0.6467	1.0435
Transportation	-8.8847	4.1818	1.7123	9.5315	15.6838
Utilities	-12.7228	-4.9875	1.9989	11.5148	19.2087
Communications & Media	-14.0782	-6.9908	2.5905	16.5203	28.6001
Merchandising	-19.1696	-9.6754	4.1793	26.7310	49.0731
Financial Services	-7.9970	-5.2865	1.0737	2.8691	6.6224

The difference between time-varying betas and constant betas, $100\times(\beta_0 - \beta_t)/\beta_t$, is shown in terms of percentiles

dasticity. Volatility spillovers also can be a problem. Perhaps due to the geographic and economic proximity of Canada to the United States, volatility shocks in the S&P500 index tend to pass to the TSE300 index. Testing for thinness using firm level data and comparing the U.S. and Canada markets are subjects of ongoing research.

REFERENCES

1. Atchison, M., K. Butler, and R. Simonds, "Nonsynchronous Security Trading and Market Index Autocorrelation," *Journal of Finance*, 42 (March 1987), pp. 111-118.

2. Bera, Anil, and Matthew Higgins, "ARCH Models: Properties, Estimation and Testing," *Journal of Economic Surveys*, 7 (1993), pp. 305-366.

3. Bera, Anil, and C.M. Jarque, "Efficient Tests for Normality, Heteroskedasticity, and Serial Independence of Regression Residuals," *Economics Letters*, 6 (1980), pp. 255-259.

4. Bernt, E., B. Hall, R. Hall, and J. Hausman, "Estimation and Inference in Nonlinear Structural Models," *Annals of Economic and Social Measurement*, 4 (October 1974), pp. 653-666.

5. Black, Fischer, "Studies of Stock Market Volatility Changes," *1976 Proceedings of the American Statistical Association, Business and Economic Statistics Section* (1976), pp. 177-181.

6. Bollerslev, Tim, "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 31 (April 1986), pp. 307-328.

7. Bollerslev, Tim, Ray Chou, and Kenneth Kroner, "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence," *Journal of Econometrics*, 52 (April/May 1992), pp. 5-59.

8. Curley, A., L. Hexter, and D. Choi, "The Cost of Capital and the Market Power of Firms: A Comment," *Review of Economics and Statistics* (August 1982), pp. 513-523.

9. Dickey, David, and Wayne Fuller, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica*, 49 (July 1981), pp. 1057-1072.

10. Engle, Robert, "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica*, 50 (July 1982), pp. 987-1007.

11. Fama, Eugene, "The Behavior of Stock Market Prices," *Journal of Business*, 38 (January 1965), pp. 34-105.

12. Fowler, D., H. Rorke, and V. M. Jog, "Heteroskedasticity, R^2 and Thin Trading on the Toronto Stock Exchange," *Journal of Finance*, XXXIV (December 1978), pp. 1201-1210.

13. French, Kenneth, and Richard Roll, "Stock Return Variances: The Arrival of Information and the Reaction of Traders," *Journal of Financial Economics*, 17 (September 1986), pp. 5-26.

14. French, Kenneth, William Schwert, and Robert Stambaugh, "Expected Stock Returns and Volatility," *Journal of Financial Economics*, 19 (September 1987), pp. 3-29.

15. Koutmos, Gregory, "Asymmetric Volatility and Risk Return Tradeoff in International Stock Markets," *Journal of Multinational Financial Management*, 2 (1992), pp. 27-43.

16. Koutmos, Gregory, "Time Dependent Autocorrelation in EMS Exchange Rates," *Journal of International Financial Markets, Institutions & Money*, 3 (1994), pp. 65-84.

17. Koutmos, Gregory, Unro Lee, and Panayiotis Theodossiou, "Time-Varying Betas and Volatility Persistence in International Stock Markets," *Journal of Economics and Business*, 46 (May 1994), pp. 101-112.

18. LeBaron, Blake, "Some Relations Between Volatility and Serial Correlations in Stock Market Returns," *Journal of Business*, 65 (April 1992), pp. 199-219.

19. Lo, Andrew, and Craig MacKinlay, "An Econometric Analysis of Nonsynchronous Trading," *Journal of Econometrics*, 45 (July/August 1990), pp. 181-211.

20. Nelson, Daniel, "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, 59 (March 1991), pp. 347-370.

21. Newey, Whitney, and Kenneth West, "A Simple Positive-Definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55 (May 1987), pp. 703-708.

22. Sentana, Enrique, and Sushil Wadhwani, "Feedback Traders and Stock Return Autocorrelations: Evidence from a Century of Daily Data," *Economic Journal*, 102 (March 1992), pp. 415-425.

23. Schwarz, Gideon, "Estimating the Dimension of a Model," *Annals of Statistics*, 6 (March 1978), pp. 461-464.

24. Schwert, William, and Paul Seguin, "Heteroskedasticity in Stock Returns," *Journal of Finance*, 45 September 1990), pp. 1129-1155.

25. Shiller, Robert J., "Stock Prices and Social Dynamics" *Brookings Papers on Economic Activity*, 2 (1984), pp. 457-498.