Valuing Convertible Bonds Under the Assumption of Stochastic Interest Rates: An Empirical Investigation

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Empirical examinations of models studying the pricing of default-free U.S. Treasury bonds in general and Treasury bonds with embedded options in particular often have appeared in the literature; however, the empirical evaluation of models dealing with the pricing of their corporate counterparts has not received great attention. This paper empirically investigates a corporate convertible bond valuation model developed within a contingent claims framework under the assumption of stochastic interest rates.

Evidence in this paper suggests that the model significantly overprices bonds that are deep out-of-the-money (i.e., the conversion value of the issue is lower than its straight debt value) and slightly underprices at- or in-the-money convertible bonds. Whether the biases observed in this model are unique to the financial instrument examined or a result of the methodology employed for pricing corporate liabilities is not known. An interesting direction for future work would be to apply the model to corporate debt issues other than convertible bonds and look for similar biases.

INTRODUCTION

Numerous models for pricing default-free fixed income securities have appeared, but the contingent claims approach to pricing corporate debt has not received a great deal of attention. This paper empirically tests the contingent claims approach to the valuation of corporate convertible bonds under the assumption of stochastic interest rates.

A contingent claims approach to the valuation of convertible bonds first appeared in the late 1970s. Ingersoll (May 1977, September 1977) and Brennan and Schwartz (1977) introduce similar valuation models where the convertible bond price depends on one underlying variable: the firm value. This underlying variable is assumed to follow a diffusion-type stochastic process, and the price of the convertible bond is obtained by solving the resulting partial differential equation under appropriate terminal and boundary conditions. These conditions are determined by the bond's cash flow at maturity, the firm's optimal call policy, and the bondholders' optimal conversion strategy. Brennan and Schwartz (1980) extend their previous work and allow for uncertainty inherent in interest rates by introducing the short-term risk-free interest rate as an additional stochastic variable. As a result, the model captures the stochastic nature of interest rates and the impact of the correlation that likely exists between firm value and the risk-free interest rate.

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An empirical test of a simpler model that uses the value of the firm as the only underlying variable is provided by King (1986). King argues that when market prices are compared with model valuations, the means are not significantly different. Furthermore, he argues that 90 percent of model predictions fall within 10 percent of market values. The model generally generates prices that are slightly higher than actual market prices. A partition of the sample into out-of-the-money bonds (i.e., the bond's conversion value is lower than its straight debt value) and in-the-money bonds (i.e., conversion value exceeds the straight debt value) indicates that the former are slightly overpriced by the model while the latter are slightly underpriced. There is evidence in the paper, however, that the magnitude of the pricing differences can vary significantly even within the sample partitions considered. Further analysis is necessary to assess the accuracy of the model.

This paper takes a closer look at the application of a contingent claims valuation model for pricing corporate convertible bonds and extends the results suggested by King's (1986) study. It empirically tests a model that considers the stochastic nature of interest rates. Prices are generated for 30 of the most commonly traded convertible bonds for a number of term structures observed during the fourth quarter of 1989 and the first three quarters of 1990. These model-generated prices are compared to actual prices observed in the market.

THE MODEL

The model is developed under the usual set of assumptions of capital structure irrelevancy and perfect capital markets, i.e., no taxes, no transaction costs, and access to information for all investors. It further is assumed that if the bond is callable, bondholders must immediately surrender their bonds upon the firm's call notice. It is similar to a model developed by Brennan and Schwartz (1980); the only difference is the interest rate process assumed.[1] The short-term interest rate, r, follows the mean-reverting process suggested by Cox, Ingersoll, and Ross (1985):

$$(1)\ dr = g(l - r)dt + \sigma_1 \sqrt{r}dz_1\ ,$$

where:

g, l, σ_1	=	Positive constants; and
dz_1	=	A standard Wiener process with mean zero and variance dt.

The process retains the benefit of mean reversion while it precludes negative interest rates. The existence of a closed form solution for the pricing of a pure discount default-free bond makes empirical estimation of the parameters associated with the process easier.

The value of the firm, V, is assumed to follow the process:

[1] Brennan and Schwartz (1980) assume an interest rate process of the form

$$dr = g(l - r)dt + \sigma_1 r dz_1$$

$$(2) \; dV = (\mu V - c)dt + \sigma_2 V dz_2$$

where:

μ = The drift rate per unit time;
c = The total dollar payments by the firm to all investors per unit time;
σ_2 = The instantaneous standard deviation of the return on the firm per unit time; and
dz_2 = The standard Wiener process.

Given these processes and assuming that the market price of interest rate risk is of the form $\lambda r^{1/2}$, the value of a convertible bond F(V, r, t) satisfies the partial differential equation

$$(3) \; \frac{\partial F}{\partial t} + (rV - c)\frac{\partial F}{\partial V} + [g(1 - r) - \lambda\sigma_1 r]\frac{\partial F}{\partial r} + \frac{1}{2}\sigma_1^2 r\frac{\partial^2 F}{\partial r^2}$$

$$+ \frac{1}{2}\sigma_2^2 V^2\frac{\partial^2 F}{\partial V^2} + \rho\sigma_1\sigma_2\sqrt{r}V\frac{\partial^2 F}{\partial r\partial V} - rF + cpn = 0$$

where:

cpn = The payout of F per unit time; and
ρ = The instantaneous correlation between the processes that the interest rate and the value of the firm follow.

The bond price is derived by solving equation (3) subject to appropriate boundary and terminal conditions. The three dimensional lattice suggested by Hull and White (1990) is used to solve equation (3). The use of the risk-free rate in the valuation model requires the introduction of conditions that capture the risky nature of the convertible bond being valued, i.e., the likelihood of default during the life of the bond and the fact that at maturity the bond may not pay what originally was promised. The following condition determines the bond's cash flow at maturity T:

$$F(V, r, T) = \begin{bmatrix} \dfrac{V}{n_1} & \text{if } V \leq n_1 FV \\ FV & \text{if } V\dfrac{q}{n_1 q + n_2} \leq FV < \dfrac{V}{n_1} \\ V\dfrac{q}{n_1 q + n_2} & \text{if } FV < \dfrac{q}{n_1 q + n_2} \end{bmatrix}$$

where:

n_1	=	The number of convertible bonds outstanding;
n_2	=	The number of common shares outstanding;
FV	=	The face value of the convertible bonds at maturity; and
q	=	The number of common shares into which each bond can be converted.

In other words, at maturity bondholders take over the firm if the firm value is less than the aggregate value of the convertible bond issue; they receive an amount equal to the bond's face value if the value of the firm is greater than the aggregate value of the bonds and the conversion value of each bond is less than the bond's face value. Alternatively, they receive the conversion value of the bond if this value exceeds the face value. The above condition assumes the existence of only the convertible issue and common equity in the firm's capital structure. Furthermore, default can occur any time during the life of the bond if the firm does not have enough proceeds to make the required coupon payment. It is assumed that this happens when firm value reaches a minimum equal to the aggregate coupon payment of the issue.

Certain pricing bounds also are imposed during valuation. For example, as Ingersoll (September 1977) and Brennan and Schwartz (1977) demonstrate, a convertible bond never can trade at a price lower than its conversion value. If the bond is callable, the optimal time for the firm to call the issue during the calling period is when the conversion value of the issue equals the prevailing effective call price. This defines a maximum firm value that must be considered in the lattice. During the calling period the bond also may be called if declining interest rates cause the price of the bond to rise to its effective call price. As a result, the value of the convertible bond cannot be greater than its effective call price.

EMPIRICAL SIMPLIFICATION OF THE MODEL AND SAMPLE SELECTION

Moving from a theoretical model to an associated empirical one introduces nontrivial variable specification problems. King (1986) addresses some of these problems and introduces certain simplifying assumptions. The theoretical model presented earlier assumes relatively simple capital structures with the existence of only common stock and convertible debt. In reality, the existence of senior debt, preferred equity, and multiple classes of common equity complicates the problem. Brennan and Schwartz (1980) allow for the existence of senior debt in the firm's capital structure. Senior debt reduces the value of the firm that is available to convertible bondholders and, therefore, affects all terminal and boundary conditions necessary for the valuation of the convertible issue. Although theoretically attractive, valuing senior debt prior to the valuation of the convertible issue can be intractable in practice. In addition to computational complexity, nonsynchronous trading and the availability of data on nonpublicly traded issues are serious empirical problems. Thus, for any senior debt in existence, a simplifying assumption is made similar to that made by King (1986). $V - D_S$ instead of V is assumed to follow the process in equation (2), where D_S represents the market value of senior debt. Empirically, $V - D_S$ can be specified as the sum of the market values of equity and the convertible bond issue. It also replaces V in all terminal and boundary conditions. This assumption is equivalent to viewing any senior debt as riskless. As King (1986) notes, this simplification may bias the results only for relatively low firm values because the risk of senior debt is most important in this case. It also is assumed that if V - D_S reaches a minimum value equal to the aggregate coupon payment of the convertible bond issue, default occurs. Furthermore, it is assumed that default and bankruptcy occur at the same time and that at this point the convertible bondholders equally share the value of the firm.

There are two restrictions imposed in the sample selection process. First the issuing firm must have a relatively simple capital structure in the sense that it should have no preferred equity or multiple classes of common equity outstanding. This restriction is made to avoid the problem of junior priority issues. As in the case of senior debt, junior issues require additional boundary conditions and, while theoretically attractive, a model dealing with them would be extremely complex empirically. Consequently, the capital structure of the firms considered consists only of senior debt, convertible bonds, and common stock. Unlike the secondary market of Treasury securities (one of the most liquid markets in the world), the secondary market for corporate bonds is not as liquid; only a fraction of corporate bonds outstanding trade regularly at an organized exchange. Therefore, the second restriction limits our sample to convertible bonds that were traded frequently at the NYSE and AMEX during the period considered in this study, i.e., from the fourth quarter of 1989 to the third quarter of 1990. These restrictions result in the selection of 30 corporate convertible bonds (from well over 100 convertible bonds that are traded at the previously mentioned exchanges). Information pertaining to these 30 convertible bonds is presented in Table 1. Weekly closing convertible bond prices for the fourth quarter of 1989 and the first three quarters of 1990 are collected from *Barron's* and adjusted for accrued interest. The terms of each convertible issue are collected from *Moody's Industrial Manual*. Weekly closing common stock prices as well as dividend information are collected from the NYSE and the AMEX daily records from October 1989 to September 1990.

The sample is examined for violations of the optimal call and conversion strategies. No issue is found to violate the optimal conversion strategy. At no time was any bond traded at a price lower than its conversion value throughout the study period. Four of the issues, however, were traded at conversion prices higher than the bonds' effective call prices at various times during the fourth quarter of 1989 and the first three quarters of 1990. This groups includes the bonds issued by Mark IV Industries Incorporated, Petrie Stores Corporation, Trinity Industries Incorporated, and Western Digital Corporation. Asquith and Mullins (1991) suggest that this is possible (despite the predictions of finance theory) because some firms pay less after tax interest than they would pay for dividends if the bonds were converted. When the call policy is violated, an upper boundary condition on the bond value based on an optimal call policy becomes irrelevant. In such case, the appropriate upper boundary condition is unclear. In this empirical investigation (and for the bonds that violated the optimal call policy), the maximum conversion value of the bond observed during the period of a year prior to the valuation date is used as the upper boundary condition. Although such an assumption has no theoretical justification, it is based on the belief that because the firm has not called the bonds for such a high firm value previously, it would not call them for at least the same firm value in the future.

Table 1—Description of Convertible Bond Issues

Company	Coupon	Maturity	Conversion Ratio	Call Price (10/06/89)
Ashland Oil	6.75	Jul 1, 2014	1.948	Not Callable
Bolt Beranek	6.00	Apr 1, 2012	3.333	104.80
BSN Corp.	7.75	Apr 15, 2001	6.410	104.43
Businessland Inc.	5.50	Mar 1, 2007	4.878	103.85
Cray Research	6.125	Feb 1, 2011	1.190	103.68
Dana Corp.	5.875	Jun 15, 2006	1.983	100.00
Genrad Inc.	7.25	May 1, 2011	6.957	105.08
Guilford Mills	6.00	Sep 15, 2012	2.260	104.50
Health- Chem	10.375	Apr 15, 1999	5	102.08
Hudson General	7.00	Jul 15, 2011	3.053	104.20
Int'l Rectifier	9.00	Jun 15, 2010	6.107	105.40
Jamesway Corp.	8.00	May 15, 2005	7.246	104.80
J.P. Industries	7.25	Apr 15, 2013	4.638	105.80
Legget & Platt	6.50	Apr 1, 2006	2.667	104.55
Lynch Corp.	8.00	Jul 15, 2006	3.226	105.60
Mark IV Ind.	7.00	Jun 15, 2011	4.742	104.90
Nasta Int'l	12.00	Jun 1, 2002	7.063	106.00
National Ed.	6.50	May 14, 2011	4.000	104.55
Noble Affiliates	7.25	Jun 1, 2012	5.096	Not Callable
Pep Boys	6.00	Nov 1, 2011	5.568	104.80
Petrie Co., Inc.	8.00	Dec 15, 2010	4.520	104.80
Pittston Co.	9.20	Jul 1, 2004	2.000	104.60
Trinity Ind.	6.75	Apr 1, 2012	3.065	105.40
UNC Inc.	7.50	Mar 31, 2006	6.494	105.25
Vestron Inc.	9.00	Mar 1, 2011	7.692	106.30
Wendy's Int'l	7.25	Dec 1, 2010	5.734	105.08
Western Digital	9.00	Jun 1, 2014	6.920	Not Callable
Willcox & Gibbs	7.00	Aug 1, 2014	4.115	Not Callable
WMS Industries	12.75	Nov 1, 1996	4.958	102.75
Zenith Electronics	6.25	Apr 1, 2011	3.200	103.75

PARAMETER ESTIMATION

PARAMETERS ASSOCIATED WITH THE FIRM VALUE PROCESS

The model parameters associated with firm value are firm value volatility σ_2, the firm's expected dividend payments, and the correlation coefficient between changes in firm value and changes in the short-term interest rate, ρ. The estimate of the standard deviation of returns on common equity is used as a proxy for the standard deviation of returns on the entire firm value. Again, empirical tractability dictates the use of this estimate. Market prices for all equity and liability claims (observed concurrently to avoid nonsynchronous trading problems) would be required otherwise. The volatility of common equity returns represents an upper bound on the volatility of returns on the entire firm value.[2] The direction of the bias introduced in the pricing of a convertible bond is not evident. With an upward biased volatility estimate, the model would tend to underestimate the straight debt value of the issue and overestimate its option value.[3] Annualized estimates of the standard deviation of weekly equity returns are obtained using information for the 52 weeks prior to the valuation date. A trade-off must be made when an appropriate time

[2] See King (1986).

[3] Simulation results indicate that the effect on the straight debt portion dominates for almost the entire range of firm values considered.

frame for estimation of the parameter under consideration is chosen. More data generally lead to more accurate results. Volatility changes over time and stale data, however, may not accurately represent a current volatility measure. We believe that a 52 week period prior to the valuation date represents a good compromise.[4]

Incorporating dividends into the model also requires assumptions about the firm's future dividend policy. In this study it is assumed that the current dividend is the best expectation concerning future dividends. Given the presence of bond indentures that usually limit the amount that can be paid to common shareholders, an assumption of a constant dividend policy probably has a small effect on the valuation results.

Estimates of the correlation, ρ, between changes in equity returns and changes in the short-term rate of interest, r, also are obtained using the 52 weeks prior to the valuation date. These estimates are used as a proxy for the instantaneous correlation between firm value changes and changes in the short-term interest rate. The yield on Treasury bills with maturity closest to one month is used as a proxy for the value of the short-term interest rate.

PARAMETERS ASSOCIATED WITH THE INTEREST RATE PROCESS

The introduction of a second stochastic variable, the short-term interest rate, further complicates empirical application of the model. Four additional parameters g, l, σ_1, and l must be estimated. The usual procedure has been to estimate g, l, σ_1 based on a long time series of previous values of proxies for the short-term interest rate and then make an explicit assumption about the value of l, a utility-based parameter that cannot be observed directly.[5] An assumption, for example, that l = 0 is in line with the expectations hypothesis. Such a procedure, however, implies a bond price curve that does not necessarily fit the observed bond price curve.

In order to avoid estimation that is not consistent with an observed term structure, a methodology similar to the one used by Brown and Dybvig (1986) is employed next. The partial differential equation (3) can be written in the form

$$(4)\ \frac{\partial F}{\partial t} + (rV - c)\frac{\partial F}{\partial V} + (\Phi_1 - \Phi_2 r)\frac{\partial F}{\partial r} + \frac{1}{2}\sigma_1^2 r\frac{\partial^2 F}{\partial r^2}$$

$$+ \frac{1}{2}\sigma_2^2 V^2\frac{\partial^2 F}{\partial V^2} + \rho\sigma_1\sigma_2\sqrt{r}V\frac{\partial^2 F}{\partial r\partial V} - rF + cpn = 0$$

where:

Φ_1 = gl; and
Φ_2 = $g + \lambda\sigma_1$.

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[4] Because most of the companies considered do not have options on their stocks currently trading, it is not possible to use option theory to calculate implied volatilities.

[5] See Brennan and Schwartz (1980) and Ogden (1987).

Therefore, three instead of four parameters associated with the particular interest rate process must be estimated. Φ_1, Φ_2, and σ_1 can be estimated on the basis of data on the prices of default-free pure discount U.S. Treasury bonds trading at a particular point in time. The CIR default-free pure discount bond pricing formula is of the form

$$(5)\ P(r, T) = A(t, T) \cdot e^{-B(t,T) \cdot r}$$

with

$$B(t, T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + \Phi_2)(e^{\gamma(T-t)} - 1) + 2\gamma}$$

and

$$A(t, T) = \left(\frac{2\gamma e^{(\gamma + \Phi_2)(T-t)/2}}{(\gamma + \Phi_2)(e^{\gamma(T-t)} - 1) + 2\gamma}\right)^{2\Phi_1/\sigma_1^2}$$

where:

$$\gamma \quad = \quad \sqrt{\Phi_2^2 + 2\sigma_1^2}.$$

Of the U.S. Treasury issues trading on a particular date, only Treasury bills are pure discount issues priced by equation (5). Treasury bills, however, have relatively short maturities and do not span a long maturity space. One way to resolve this problem is to treat other coupon-bearing issues as portfolios of discount bonds, one for each coupon payment and one for the final payment of each Treasury issue. Brown and Dybvig (1986) use a similar approach to estimate a slightly different set of parameters emanating from the CIR process. Results from such an approach, unfortunately, may be distorted due to the differential tax treatment of coupon payments versus capital gains.

Alternatively, and in order to avoid the differential tax treatment of coupon payments versus capital gains, observed prices of U.S. Treasury zero-coupon strips can be used. It is possible to obtain maximum likelihood estimates of the parameters Φ_1, Φ_2, σ_1, using nonlinear least squares procedures applied to data on the prices of Treasury zero-coupon strips of different maturities trading at a given point in time. The methodology allows us to take full advantage of information contained in an observed term structure. Furthermore, an explicit estimation of, or an assumption regarding, the value of λ is not necessary.

The methodology is applied on weekly closing prices of U.S. Treasury zero-coupon strips for the fourth quarter of 1989 and the first three quarters of 1990 as reported in *Barron's*. For each date equation (5) is fitted to a cross-section of approximately 120 zero-coupon strip prices, spanning a maturity space of up to 30 years. Parameter estimates for term structures observed at the end of each of the 52 weeks during the last quarter of 1989 and the first three quarters of 1990 are available upon request. In general, results for Φ_1

suggest low speeds of adjustment for the aforementioned period, whereas values of Φ_2 suggest negative values of λ, something consistent with theoretical expectations.[6]

RESULTS AND ANALYSIS

This study examines the validity of the model for a cross-section of bonds not only at a specific point in time, but also across time. We hope to better understand how changing conditions and parameter values affect convertible bond prices. Thus, the model is used to generate prices for the 30 convertible bonds for certain dates during the third quarter of 1989 and the first three quarters of 1990. We try to select common valuation dates for all convertible bonds during the period. The first Friday of each month initially is selected as a valuation date. One problem that arises is nonsynchronous trading. The price of the convertible bond and the price of the corresponding common stock must be observed concurrently. For almost all issues the convertible bonds are thinly traded relative to the corresponding common stock. Because weekly closing prices are used for valuation and comparison purposes, a requirement for a valuation date is that both the convertible bond and the corresponding common stock are traded on this date. If this does not happen on the first Friday of the month, a subsequent Friday on which both issues are traded is chosen. Twelve valuation dates ideally would be found for each convertible, one for each month. Due to thin trading, however, this is not possible for some issues.

A comparison of model-generated prices to observed market prices for each convertible bond is presented in Table 2. The minimum, maximum, and average ratios of model-generated prices to market prices are presented. The minimum, maximum, and average dollar differences between model-generated prices and actual market prices also are presented. A wide range of results is observed. For 15 of the 30 convertible issues the mean model-generated price is within 10 percent of the mean observed market price. For four more convertibles the mean model-generated price is within 15 percent of the mean market price. For the rest of the convertible issues the mean model-generated price is more than 115 percent of the mean market price. A maximum mean model to market price ratio of 143.55 percent is observed, that for BSN Corp. In terms of dollar pricing errors, the average dollar error is within \$5 for 11 of the 30 convertible issues. For nine more convertible bonds the average mean dollar error is between \$5 and \$10, and for seven more the average mean dollar error is between \$10 and \$15. The maximum mean dollar error observed is \$26.64, that for BSN Corp. Dollar pricing errors are based on a \$100 face value for all bonds. Furthermore, for 25 of the 30 convertible issues results suggest that the model on average overprices relative to actual market prices. Evidence in Table 2 also indicates that the accuracy of the model when pricing a particular issue varies across time.

This diversity of results suggests that further analysis is required. In an effort to discover systematic biases in the model, the sample is partitioned on the basis of the bond's conversion value. The relative measure used is the ratio of the bond's conversion value to the value of an otherwise equivalent nonconvertible bond, i.e., the straight debt value of

[6] See Hull (1989), p. 159.

Table 2—Comparison of Actual to Stochastic Interest Rate Model-Generated Prices

Company	n	Model-Generated Price as % of Actual Min	Max	Mean	Model-Generated Price Minus Actual Price Min	Max	Mean
Ashland Oil	8	96.48	103.78	99.78	-3.50	3.28	-0.29
Bolt Ber.	12	116.02	141.50	127.54	7.19	19.65	13.57
BSN Corp.	10	136.83	153.07	143.55	23.37	30.24	26.64
Bus'land Inc.	12	103.21	137.94	115.02	2.22	20.27	9.06
Cray Res.	12	102.09	123.86	110.99	1.76	17.08	8.18
Dana Corp.	12	105.05	114.87	110.00	4.2	10.71	7.72
Genrad Inc.	12	118.49	183.03	139.19	12.96	30.83	19.97
Guilford M.	11	102.73	118.87	110.25	2.12	12.8	6.76
Health-Chem	10	108.20	136.08	118.37	5.28	18.07	10.52
Hudson Gen.	12	108.38	123.90	117.01	6.08	15.77	11.30
Int'l Rect.	12	98.40	133.36	113.91	-1.40	17.72	8.18
Jamesway	12	109.42	126.77	120.56	8.54	18.95	14.57
J.P. Ind.	11	103.19	115.20	108.03	2.97	12.02	6.71
Legg. & Platt	12	97.86	103.99	99.97	-2.22	3.77	-0.07
Lynch Corp.	9	100.02	113.62	105.26	0.02	11.04	4.46
Mark IV	12	95.81	105.46	100.73	-4.47	5.07	0.58
Nasta Int'l	11	94.04	134.24	111.67	-3.25	18.09	8.14
Nat'l Ed'n	12	108.96	133.55	123.85	3.67	15.78	10.48
Noble Aff.	12	94.80	103.45	98.13	-5.45	3.35	-1.98
Pep Boys	12	90.95	105.76	97.19	-8.79	4.78	-2.77
Petrie Co.	12	95.90	103.77	100.46	-5.00	3.90	0.41
Pittston Co.	12	103.60	111.05	106.77	3.38	10.00	6.28
Trinity	12	91.78	96.54	94.47	-8.24	-3.90	-5.52
UNC Inc.	12	102.49	130.08	118.33	1.48	19.07	10.96
Vestron Inc.	12	113.79	176.48	149.44	3.07	15.87	10.58
Wendy's	12	93.83	115.73	103.33	-4.69	11.80	2.42
Western Dig.	12	92.33	118.18	104.29	-8.46	15.52	3.44
Willcox & Gibbs	12	94.37	123.27	108.30	-5.34	13.58	5.31
WMS Ind.	12	96.52	107.75	101.76	-2.96	7.23	1.63
Zenith	11	120.98	139.15	129.12	10.78	24.10	17.31

n represents the number of valuation dates for each bond. Pricing differences are based on an assumed $100.00 face value for all bonds

the issue.[7] A ratio of less than one indicates that the bond is out-of-the-money. A ratio greater than one suggests the bond is in-the-money. Results are reported in Table 3. The results suggest that the model significantly overprices when dealing with a deep out-of-the-money bond. When the conversion value is less than 60 percent of the issue's straight debt value, the model on average overprices more than 20 percent. If the conversion value is between 60 and 80 percent of the straight debt value, the model on average overprices 7.54 percent. For conversion values more than 80 percent of the straight debt value, the model on average slightly underprices relative to observed market data. For the overall sample, the model on average overprices 12.90 percent and the average error is $7.10.

Does the poor performance of the model when the conversion value of the issue is small spring from the relative riskiness of the bond and not from the fact that the bond is out-of-the-money? The ratio of the conversion value to the issue's straight debt value also can serve as a measure of the riskiness of the convertible bond. A low conversion value

[7] The value of such a bond is calculated using the same model that is used to calculate the value of the convertible bond, with the conversion ratio set equal to zero.

Table 3—Valuation for Various Conversion Value Ranges

		Model-Generated Price as % of Actual			Model-Generated Price Minus Actual Price		
CV/SB	n	Min	Max	Mean	Min	Max	Mean
(0, .20]	15	108.20	176.48	126.39	3.07	18.07	9.72 (7.22)
(.20, .40]	59	93.83	183.03	123.54	-4.69	30.83	10.65 (11.12)
(.40, .60]	101	95.40	154.08	120.81	-3.60	30.24	12.70 (17.15)
(.60, .80]	87	94.37	126.55	107.54	-5.34	15.01	5.72 (12.58)
(.80, 1]	62	90.95	112.30	99.19	-8.79	9.64	-0.94 (-1.84)
(1, ∞)	21	91.78	103.57	97.96	-8.24	3.90	-2.21 (-2.89)
Total	345	90.95	183.03	112.90	-8.79	30.83	7.10 (16.55)

CV/SB denotes the ratio of conversion value to straight debt value. n denotes the number of observations in each category. Number in parentheses is the value of the t-statistic. Pricing differences are based on an assumed $100.00 face value for all bonds

generally suggests low overall firm value and a higher probability of default. It is possible that the model underestimates this probability of default and, as a result, produces prices that are greater than the ones observed in the market. To examine this hypothesis, the sample is further partitioned into classes of risk. As a measure of riskiness the ratio of the aggregate book value of debt (which is the amount bondholders try to recover in the case of bankruptcy) to the firm's market value is chosen. The higher the value of this ratio, the higher the risk of the issue. Results are summarized in Table 4. Even within each risk class, the model still overprices deep out-of-the-money bonds. There is also evidence that the degree of overpricing is higher for classes of higher risk. This indicates that for relative low firm values the market requires a higher yield premium than what the model suggests.

VARIABLE SPECIFICATION ANALYSIS AND EXAMINATION OF THE BIASES INTRODUCED

Given the evidence of suboptimal call policy that issuing firms follow in general,[8] it is expected that a model that assumes an optimal call policy in order to determine an upper bound on the value of the convertible bond will introduce a certain bias in the results. Because by the time a firm calls the issue the conversion value of the bond usually exceeds its effective call price (and assuming investors anticipate this delay), a model that assumes an optimal call policy should generate price consistently lower than those observed in the marketplace. This is evident in the results produced in this empirical study where bonds with relative high conversion value are, in general, slightly underpriced by the model. There is no explanation, however, for the poor performance of the model when dealing with bonds with relatively low conversion value. This section examines whether

[8] See Ingersoll (May 1977).

Table 4—Valuation for Various Conversion Value Ranges and Risk Classes

	Book Value of Debt/Firm Value					
CV/SB	(0, .10]	(.10, .20]	(.20, .30]	(.30, .50]	(.50, _)	Total
				8.44	10.04	9.72
(0, .20]	-	-	-	139.66	123.08	126.39
				(3)	(12)	(15)
	4.95	14.03	9.44	13.61	13.02	10.65
(.20, .40]	105.84	154.64	133.54	130.12	126.99	123.54
	(17)	(3)	(4)	(15)	(20)	(59)
	6.92	3.90	13.81	15.89	13.31	12.70
(.40, .60]	109.05	105.82	120.97	126.93	123.85	120.81
	(11)	(10)	(25)	(35)	(20)	(101)
	3.33	4.99	6.16	6.33	10.94	5.72
(.60, .80]	104.17	106.44	107.80	108.34	116.17	107.54
	(17)	(27)	(19)	(16)	(8)	(87)
	-0.74	-3.49	1.70	2.31		-0.94
(.80, 1]	99.47	96.53	101.86	102.49	-	99.19
	(22)	(22)	(11)	(7)		(60)
	-2.06	-2.24	-2.19			-2.21
(1, ∞)	98.31	97.88	97.94	-	-	97.96
	(4)	(16)	(1)			(21)
	2.71	1.32	8.61	11.88	12.24	
Total	103.54	103.66	113.75	121.90	123.72	
	(71)	(78)	(60)	(76)	(60)	

CV/SB denotes the ratio of conversion value to straight debt value. The first number denotes the $ difference between the model-generated prices and the market prices. They are based on an assumed $100.00 face value for all bonds. The second number expresses the model-generated price as percentage of the market price. The number in parentheses denotes number of observations in each category

the simplifying assumptions made during the empirical application of the model can explain some of the biases observed.

RISKLESS SENIOR DEBT

One of the assumptions for the empirical application of the model is that any senior debt in the issuing firm's capital structure is riskless. This simplification may bias the results for relatively low firm values, as the risk of any senior debt would be most important in this case. It is possible, therefore, that the overpricing of the model for relatively low firm and conversion values could be attributed, to a certain extent, to the simplifying assumption of senior riskless debt. It does not explain, however, the overpricing by the model for relatively high firm values when the bond's conversion value is small.

To analyze the impact of riskless senior debt assumption, the empirical analysis is repeated for six convertible issues where the issuing firm does not have any senior debt traded in the market.[9] The six firms are Bolt Beranek & Newman Inc., Genrad Inc., Health-Chem Corp., National Education Corp., Petrie Stores Corp., and Vestron Inc. Results indicate the presence of the same biases found in the entire sample; therefore, it can be argued that the assumption of riskless senior debt does not explain the biases observed in the model-generated prices.

[9] The only form of senior debt outstanding that some of those six firms have is a line of credit whose size is small when compared to the principal amount of the convertible issue.

VOLATILITY OF EQUITY RETURNS

The volatility of equity returns is used as a proxy for the volatility of firm value returns. The volatility of common equity returns represents an upper bound on the volatility of returns on the entire firm value. Consequently, such a volatility measure represents an upwardly biased estimate of the true volatility measure. The use of an upwardly biased volatility estimate is expected to result in convertible bond prices where the straight debt value of the issue is underestimated and its option value overestimated. The effect on the straight debt value is expected to dominate, especially for relatively low firm values. Thus, the use of the biased volatility estimate cannot explain the overpricing by the model when dealing with deep out-of-the-money bonds and low firm values.

DIVIDEND POLICY

It is assumed that the current dividend level paid to common shareholders represents the best estimation of future dividends. In the case where the firm has yet to pay a dividend, the no dividend policy is expected to continue in the future. Given the presence of bond indentures that usually limit the amount that can be paid to common shareholders, an assumption of a constant dividend policy probably has a small effect on the valuation results. To verify this argument, an alternative dividend policy is examined. Only 11 of the 30 firms in the entire sample currently pay a dividend. Prices for a particular date for these 11 firms are generated by the model assuming not a constant dividend, but a constant dividend yield. This dividend yield is defined as percentage of firm value. Results are similar to those obtained under the assumption of constant dividend. The assumption concerning the firm's dividend policy cannot explain the biases observed in the prices generated by the model.

CONTINUOUS COUPON PAYMENTS

A continuous coupon payment assumption is made in the model. This assumption is a simplification of the real situation where coupons are paid at discrete points in time. Such an assumption is expected to bias results in the following manner: model-generated prices are expected to be higher than the corresponding market prices immediately after a coupon has been paid. As time progresses toward the next coupon payment, however, the degree of the bias decreases and eventually reverses in sign. Just before a coupon is paid, the model-generated price should be lower than the market price. Over the course of half a year, assuming semi-annual coupon payments, the average bias is expected to equal zero. Therefore, the assumption of a continuously paid coupon should introduce no bias on average and should affect only the volatility of the pricing error.

DEFAULT CONDITIONS

It is assumed that default occurs when the value of the firm reaches a minimum value equal to the aggregate coupon payment of the issue. Bankruptcy also is assumed to occur at the same time. Valuation stops at this point, and the convertible bondholders receive the remaining value of the firm. In reality, however, default may, but need not, trigger bankruptcy. To the extent that other hidden costs exist during bankruptcy (such as legal costs, delayed distribution of residual value, and other costs), the model should overvalue the issue at low firm values. Nevertheless, it is believed that the magnitude of these costs is not high enough to explain the serious overpricing of the model at relatively low firm values.

A POSSIBLE EXPLANATION OF THE OBSERVED BIASES

The empirical investigation presented in this paper is a joint test of the validity of the model, the parameter estimation procedure, and the assumption of market efficiency. An examination of the simplifying assumptions in the model, and the resulting variable specification errors, yields no explanation for the observed biases in prices generated by the model. A possible explanation could lie in the nature of the particular financial instrument considered and the market players. Discussions with traders and interested parties indicate that convertible bonds are traded by equity traders. It is possible that such traders attach more value to the equity than to the straight debt value of the bond. In practice, the value of the convertible may be estimated by adding the bond's conversion value to the present value of the difference between the coupon and dividend payments over the next five years. Such an approach should result in a price that is lower relative to the issue's real value when the bond is deep out-of-the-money. Furthermore, it should result in a market price higher than the issue's real value when the conversion value of the bond is relatively high. Although such an explanation is consistent with the results observed in this study, it implies some degree of market inefficiency. One must be careful in adopting such an argument. An interesting piece of future work would be to apply the same model to the pricing of nonconvertible corporate bonds and test for the presence of similar biases.

COMPARISON WITH THE CONSTANT INTEREST RATE MODEL

Results obtained with the model that assumes stochastic interest rates are compared to results obtained with the simpler model that assumes constant interest rates. Using simulated results, Brennan and Schwartz (1980) argue that pricing differences between the more complicated valuation model that assumes stochastic interest rates and the simpler one that assumes a constant interest rate are likely to be slight; therefore, the benefits of using the more complicated model are negligible. The empirical results in this paper suggest that pricing differences between the two models are not large. On average, prices derived with the simpler model exceed the prices derived by the more complicated one by a little less than 1 percent. The 90 percent range of pricing differences is between 4.1 percent and -1.83 percent. These differences are within the limits suggested by Brennan and Schwartz (1980). For about 10 percent of all cases, however, pricing differences of 6 percent or more are observed with a maximum percentage difference of 12.5 percent. Furthermore, mean square error analysis suggests that the more complicated model provides marginally better results with respect to actual prices than the less complicated constant interest rate model. The root mean square error (RMSE) for the stochastic interest rate model is 10.66 versus 11.18 for the simpler model.

SUMMARY AND DIRECTION OF FUTURE WORK

This research empirically investigates the pricing of corporate convertible bonds within a contingent claims valuation framework under the assumption of stochastic interest rates. Evidence suggests that the model significantly overprices bonds that are deep out-of-the-money and slightly underprices bonds that are at- or in-the-money. There is also evidence that the model overprices issues of higher risk. When the book value of debt over the market firm value ratio is used as a proxy of risk, model-generated prices are significantly greater than actual market prices for higher values of this ratio.

Whether the biases produced by the application of the model are unique to the financial instrument examined or a result of the general methodology employed is not known. An interesting direction of future work would be to apply the model to pricing corporate liability issues other than convertible bonds. It will be worthwhile to investigate the presence of biases similar to the ones observed in this paper. This will provide a better insight into the validity of the contingent claims approach as a corporate liabilities pricing tool.

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