Nuisance OLS Correlations in Market Model Parameter Shift Studies

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Much financial research allows for the possibility that the market model parameters alpha and beta undergo shifts. This paper examines correlations between parameter shifts that can arise as an artifact of ordinary least squares (OLS) regression. The presence of such correlations in some cases can lead to misinterpretation of the results of empirical analyses. Formulas are given for the correlation between the estimated shifts in alpha and beta and the correlation between estimated shifts in abnormal returns and beta. These results have implications for empirical studies involving OLS estimation of alphas, betas, and abnormal returns over disjoint subsets of market data. Examples are given with reference to papers by Fabozzi and Francis (1977), Hays and Upton (1986), and Copeland and Lee (1991).

INTRODUCTION

There now exists a large body of literature addressing the hypothesis that, at any point in time, securities prices fully reflect all available relevant information. An important tool in this research is the *single index market model*:

(1) $R_t = \alpha + \beta m_t + e_t$

which describes the return, R_t, on a security or portfolio at time t as a simple linear function of the return, m_t, on the market portfolio, plus random error ε_t with variance $\sigma^2[\varepsilon_t]$. The slope coefficient β (henceforth, beta) has been of particular interest because of its use as a measure of the systematic component of the riskiness of a security. (We have suppressed the usual i index for the security to avoid cumbersome notation later—α and β are different for each security. For the same reason, we use m_t for the market return instead of the more conventional Rm_t.)

In the simplest applications of the market model, an ordinary least squares (OLS) fit of equation (1) to historical security and market returns provides estimates of alpha and beta and permits estimation of abnormal returns (differences between actual and expected returns) attributable to events in the financial life of the firm or to general market conditions. Implicit in these applications is the assumption that the parameters α, β, and $\sigma^2[\varepsilon_t]$ are constant over the study period. Numerous studies, such as those of Hamada (1972), Blume (1975), Kon and Lau (1979), Sunder (1980), Fisher and Kamin (1985), DeJong and Collins (1975), and Ghosh (1992), however, document evidence that some or all of these parameters are not constant over time. These and other researchers have offered more general models than equation (1) that account for instability in the parameters, but none of these approaches has gained general acceptance to date.

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Variations on the simple linear regression approach still predominate in applications. One common variation assumes that alpha and beta are stable over subsets of time periods but may shift between subsets. For example, in typical event studies the time subsets are pre- and post- some significant financial event. In studies of seasonal anomalies (such as turn-of-the-year effects) the subsets are those including or excluding the season under consideration. In any of these variations, separate OLS regressions can be fitted to the different stable subsets, and the shifts in estimated parameters between subsets can be used to test hypotheses.

Such studies, particularly the event studies, are so common that we will not attempt a complete survey here. (A cursory survey of *The Journal of Financial and Quantitative Analysis*, *The Journal of Finance*, *The Journal of Financial Economics*, and *Financial Management* for 1985 through 1992 uncovered thirteen event studies, five seasonality studies, and three general OLS-based beta-shift studies.

Any OLS-based study that examines shifts in the parameters of the market model or shifts in abnormal returns between two sets of time periods should make allowance for correlations that arise as artifacts of the OLS procedure. These correlations can be characterized as nuisance correlations because they convey no information about market fundamentals, can obscure real information, and may lead to misinterpretation of empirical results if allowance is not made for them. Our objectives in this paper are to: 1) alert empirical researchers to the existence of nuisance OLS shift correlations; 2) explain the source of the correlations and show how to calculate them; and 3) illustrate the potential for misinterpretation of empirical results with reference to three previous studies.

OLS SHIFT CORRELATIONS

CORRELATION BETWEEN ESTIMATED SHIFTS IN ALPHA AND BETA

We begin by developing an outline of a typical OLS-based experiment that allows for shifts in market model parameters. Consider a sequence of returns $\{R_t: t = 1, 2, \dots\}$ for a security and a corresponding sequence of returns $\{m_t: t = 1, 2, \dots\}$ on the market portfolio. Suppose that the returns on the security are governed by a market line with intercept and slope α_1 and β_1 for a set T_1 containing n_1 time periods, and that parameters α_2 and β_2 prevail for a second, distinct set T_2 of n_2 periods. In some cases (for example, in a typical event study on monthly data), these sets of time periods will be both internally sequential and contiguous; that is, $T_1 = \{1, 2, \dots, n_1\}$ and $T_2 = \{(n_1 + 1) = 1, 2, \dots, (n_1 + n_2)\}$; however, this is by no means necessary. For example, in a study by Fabozzi and Francis (1977), the two sets of time periods are distinguished by whether the market is bull or bear; thus, T_1 and T_2 are interleaved. In other cases, T_1 and T_2 will be internally sequential but not contiguous. This typifies event studies using daily data in which a number of days before and after the event are excluded from the estimation process to avoid local effects. This is true in a Copeland and Lee (1991) study discussed below. For convenience we will use terms such as *preshift* and *postshift* somewhat loosely to refer to time sets T_1 and T_2, respectively.

For the pre- and postshift models we can write:

$$R_{1t} = \alpha_1 + \beta_1 m_{1t} + \varepsilon_{1t} \text{ for } t \in T_1\text{, and}$$

$R_{2t} = \alpha_2 + \beta_2 m_{2t} + \varepsilon_{2t}$ for $t \in T_2$;

where:

ε_{1t} and ε_{2t} = Random errors in the pre- and postshift regimes, respectively.

In typical applications it is assumed that ε_{1t} and ε_{2t} have stationary variances $\sigma^2[\varepsilon_1]$ and $\sigma^2[\varepsilon_2]$, respectively.; it often further is assumed that $\sigma^2[\varepsilon_1] = \sigma^2[\varepsilon_2]$. This assumption of constancy of variances is not empirically justified. For example, papers by Christie (1982), Fisher and Kamin (1985), and Ghosh (1992) discuss various aspects of nonconstant variance and provide references to earlier empirical studies demonstrating heteroscedasticity. This assumption, however, underlies the use of OLS regression to fit the market model. Also, the OLS shift correlation derived below is robust to modest departures from constancy of the variances.

Let a_1, b_1, a_2, and b_2 denote the OLS estimators for α_1, β_1, α_2, and β_2, respectively. Then expected returns are estimated from

(2) $\hat{R}_{1t} = a_1 + b_1 m_{1t}$ for $t \in T_1$,

(3) $\hat{R}_{2t} = a_2 + b_2 m_{2t}$ for $t \in T_2$.

Define the parameter shifts $\delta\alpha = \alpha_2 - \alpha_1$ and $\delta\beta = \beta_2 - \beta_1$ and their unbiased estimators $\delta\alpha = a_2 - a_1$ and $\delta b = b_2 - b_1$. We first examine the correlation $\rho(\delta a, \delta b)$ between the estimated shifts in alpha and beta.

It is well-known that the slope and intercept estimates for any model fitted by OLS may be correlated, and simple expressions are available for calculating the size of this correlation. Appendix A shows that, as a consequence, the shifts in slope and intercept between two regressions also will be correlated. Furthermore, it is shown that if the estimates (a_1, b_1) are statistically independent of (a_2, b_2) (henceforth, the *independence assumption*), and if $\sigma^2[\varepsilon_1] = \sigma^2[\varepsilon_2]$, then the OLS alpha/beta shift correlation can be calculated from:

$$(4)\ \rho[\delta\alpha, \delta\beta] = \frac{-\left(\dfrac{\bar{m}_1}{SSm_1} + \dfrac{\bar{m}_2}{SSm_2}\right)}{\sqrt{\left(\dfrac{\Sigma m_{1t}^2}{n_1 SSm_1} + \dfrac{\Sigma m_{2t}^2}{n_2 SSm_2}\right)\left(\dfrac{1}{SSm_1} + \dfrac{1}{SSm_2}\right)}}$$

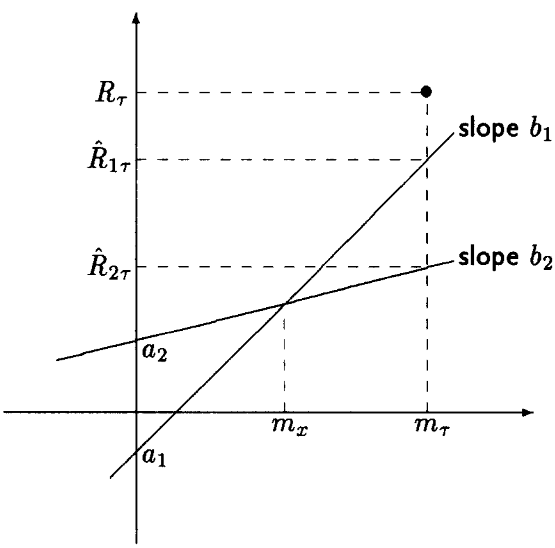
where, for k = 1 or 2, $\bar{m}_k$ denotes the sample mean market return over T_k and SSm_k is the sum of squares:

$$\Sigma_{t \in T_k} (m_{kt} - \bar{m}_k)^2.$$

This expression for $\rho[\delta\alpha, \delta\beta]$ is relatively insensitive to departures from the equality of variances assumption; also, equation (4) can be rewritten in a less compact form in terms of the sample means and variances of the market returns. Thus, if the assumptions are satisfied, the OLS shift correlation for a single security will depend only on the market returns. In particular, $\rho[\delta\alpha, \delta\beta]$ will not depend on the size of the true shifts, $\delta\alpha$ and $\delta\beta$, or on the actual returns of the security, R_t. We emphasize that this correlation will arise whenever the market model is fitted to two separate subsets of returns data; it is independent of the rule used to select T_1 and T_2 (subject to the independence assumption) and will be present even if these sets are picked arbitrarily.

OLS shift correlations can be visualized as consequences of some simple geometry that is illustrated in Figure 1. If the estimated pre- and postshift betas are not equal, the pre- and postshift estimated market lines will intersect at some market return $m_x = -\delta a/\delta b$. This return is positive in Figure 1, but easily could be negative. If m_x is positive, then (as depicted) the shift in the estimated intercept, $(a_2 - a_1)$, will have the opposite sign to the shift in the estimated betas, $(b_2 - b_1)$. If m_x is negative, the two shifts will have the same sign. Thus, the strength of the correlation between these shifts depends in large part on the behavior of the random variable m_x. Note for future reference that the shift in expected returns, $(\hat{R}_{2\tau} - \hat{R}_{1\tau})$, predicted for some time point τ exhibits similar behavior depending on the position of the market return m_τ relative to m_x. The quantities a_1, b_1, a_2, b_2, and m_x are all random variables on the sample space of security and market returns. If it were possible to observe multiple realizations of returns on the security over the same set of returns on the market, the value of m_x easily might be negative for some realizations and positive for others. Equation (4) takes into account both the geometry and the random behavior and provides an estimate of the resulting OLS shift correlation.

Figure 1—Shifted Market Model



AN APPROXIMATION FORMULA FOR THE OLS ALPHA/BETA SHIFT CORRELATION

Equation (4) can be simplified under certain circumstances, and the simplified formula permits some insight into the behavior of the OLS shift correlation. In particular, it is possible to identify conditions that lead to extreme values for $\rho[\delta\alpha, \delta\beta]$.

For a large time subset T_k (large n_k), the absolute mean market return $|\bar{m}_k|$ generally will be small. This is particularly true if the returns have been adjusted for a risk-free rate. If n_k is relatively small, however, extreme values of $|\bar{m}_k|$ are much more likely. Also, SSm_k is nondecreasing in n_k. Thus, if one time subset is much smaller than the other time subset, the market statistics from the small regime can dominate in equation (4). Seasonality studies are particularly vulnerable to this situation since one time set often contains a single month from each year under consideration.

Under these circumstances equation (4) simplifies to the following approximation (subscripts have been suppressed to denote either the pre- or postshift time regime):

$$(5)\ \rho[\delta\alpha, \delta\beta] \approx \frac{\pm 1}{\sqrt{1 + \left(\frac{n}{n-1}\right) v^2}}$$

where:

v = $S[m]/\bar{m}$ is the sample coefficient of variation over the dominant subset; and

$S[m]$ = The sample standard deviation of the market returns obtained from $S^2[m] = SSM/(n-1)$;

The correlation is negative if $\bar{m}$ is positive and vice versa. The same approximation is obtained if it is assumed that $n_1 \approx n_2$, $\bar{m}_1 \approx \bar{m}_2$, and $SSM_1 \approx SSm_2$. An extreme OLS shift correlation will result if the coefficient of variation of the market returns happens to be unusually small over the dominant subset. This is not likely in a large time subset but can arise easily in a short one.

THE CORRELATION BETWEEN ESTIMATED SHIFTS IN ABNORMAL RETURNS AND BETA

We now turn to the existence of an OLS correlation between shifts in beta and shifts in abnormal returns in the market model. This is a more significant phenomenon because it implies a spurious systematic relationship between the beta shift and the shift in abnormal returns calculated from pre- and postshift models.

Let τ denote the time for which abnormal returns are calculated, and let $\hat{R}_{1\tau}$ and $\hat{R}_{2\tau}$ denote expected returns predicted at τ by the pre- and postshift models (2) and (3), respectively. The market return at time τ is m_τ. Corresponding abnormal returns at τ are given by $(R_\tau - \hat{R}_{1\tau})$ and $(R_\tau - \hat{R}_{2\tau})$, respectively. Then the shift in abnormal returns between the two models, $\delta\hat{A}_\tau$ is $(\hat{R}_{1\tau} - \hat{R}_{2\tau})$, the negative of the corresponding shift in expected returns at τ. We are interested in the correlation $\rho[\hat{A}_\tau, \delta b]$. In Appendix B we show that:

$$(6)\ \rho[\hat{A}_\tau, \delta b] = \frac{\left(\frac{\bar{m}_1}{SSm_1} + \frac{\bar{m}_2}{SSm_2}\right) - m_\tau\left(\frac{1}{SSm_1} + \frac{1}{SSm_2}\right)}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{(m_\tau - \bar{m}_1)^2}{SSm_1} + \frac{(m_\tau - \bar{m}_2)^2}{SSm_2}\right)\left(\frac{1}{SSm_1} + \frac{1}{SSm_2}\right)}}.$$

As with equation (4), this expression is only exact under the independence assumption and assumption of equality of variances; however, it exhibits a similar robustness with respect to departures from the equality of variances assumption. A key difference between this correlation and $\rho[\delta a, \delta b]$ is the presence of the variable m_τ. (As should be expected, $\rho[\hat{A}_\tau, \delta b]$ reduces to $-\rho[\delta b, \delta b]$ if $m_\tau = 0$.) The correlation between shifts in abnormal returns and shifts in beta is highly sensitive to the value of the market return that happens to prevail at the time for which abnormal returns are calculated. As a consequence, extreme values of $\rho[\hat{A}_\tau, \delta b]$ are much more likely to occur.

TYPICAL VALUES OF OLS SHIFT CORRELATIONS

To investigate the possible magnitude of the OLS shift correlations, we apply equations (4) and (6) to every possible shift point from month 2 to month 167 of the 1968 to 1981, S&P 500 market return series obtained from the University of Chicago Center for Research in Security Prices (CRSP) data tapes. (These data are selected as part of a replication of the Hays and Upton (1986) study.) For a shift in month t, the time set T_1 comprises months $\{2, 3, \ldots, (t-1)\}$, and T_2 comprises months $\{t, (t+1), \ldots, 167\}$. Returns are adjusted for the risk free rate by dividing by 12 the annualized rate of return on 90 day Treasury bills available in each month. This calculation provides an estimate of the OLS shift correlations for any securities that experienced single shifts over the 168 month series, excluding the first and last months. The results are summarized in Table 1.

Table 1—Empirical Shift Correlations 1968-1981

	$-\rho[\delta b, \delta b]$	$\rho[\hat{A}_\tau, \delta b]$
Mean	0.11	–0.03
Standard Deviation	0.14	0.59
Minimum	–0.24	–0.99
Maximum	0.99	0.94

The two extreme alpha/beta shift correlations of –0.24 and 0.99 both occur within the first or last few months of the series. This is consistent with the possible extreme behavior of this correlation over short time regimes. Most of the calculated correlations are much closer to the mean value, 0.11. (The minimum and maximum shift correlations over the middle 132 months of the data series are 0.03 and 0.26, respectively.)

In contrast, the correlations between shifts in abnormal returns and shifts in beta are dispersed more widely. (Note the larger standard deviation.) High positive and negative correlations are observed throughout the data series. This is consistent with the sensitivity of this correlation to the particular value of the market return at the shift point—large positive or negative market returns at the shift point tend to produce high positive or negative correlations. Over half of the correlations have absolute values of 0.60 or greater.

IMPLICATIONS OF SHIFT CORRELATIONS FOR EMPIRICAL STUDIES

In this section we discuss the implications of OLS correlations between shifts in alpha and beta and between shifts in abnormal returns and beta. We will summarize these implications below and illustrate them in more detail by referring to three prior studies selected from the literature of the 1970s, 1980s, and 1990s—Fabozzi and Francis (1977), Hays and Upton (1986), and Copeland and Lee (1991).

Tests of Significant Shifts: Some studies have examined the shifts in alpha and beta over distinct sets of time periods and have used standard hypothesis testing methods to test separately for significant shifts in alpha and beta. The difficulty with this approach is that when estimates are correlated, the frequencies with which significant shifts in alpha and beta occur will not be independent. For example, in two-tailed tests at the 5 percent significance level, if estimated alpha and beta shifts are independent, the probability that a particular alpha shift is significant (given that the corresponding beta shift is significant) is 0.05. With a weak correlation of 0.15 between the alpha and beta shifts, however, the same conditional probability rises to 0.104. In the extreme case that there is a very high correlation between shifts, a shift in beta almost always will be accompanied by a shift in the corresponding alpha. Studies in which oneof the time sets T_1 and T_2 is small (where high shift correlations are possible) are particularly vulnerable to this effect. Seasonality studies fall into this category when the time span considered is less than 12 years. Even a weak correlation can have a significant systematic effect when the same shift time is studied across many firms. This is typical of studies investigating the impact of some external event (e.g., an oil crisis) across all firms in an industry sector.

Pre– and Postshift Abnormal Returns: The difference between preshift and postshift abnormal returns may be highly correlated (positively or negatively) with the estimated shift in beta. Refer again to Figure 1. Depending on the location of m_τ with respect to m_x, use of the postshift model instead of the preshift model could give the appearance of revealing abnormal returns that are obscured by the preshift model or of eliminating abnormal returns that are falsely implied by the preshift model. In either case, the apparent systematic changes in abnormal returns may be attributable largely to the location of m_τ. Any study that compares pre– and postshift abnormal returns or relationships between shifts in abnormal returns and shifts in beta should take this into account, particularly if the shift point is the same across multiple firms.

FABOZZI AND FRANCIS (1977)

We choose for our first example a somewhat dated but well-known study to illustrate the case in which the time sets T_1 and T_2 are not contiguous. Fabozzi and Francis (1977) conduct empirical tests of the hypothesis that market model parameters shift between bull and bear markets. They apply dummy variable regressions to 700 stock return series over a 72 month time period (January 1966 to December 1971) using the following model (the notation has been changed for consistency with our own):

$$R_t = (\alpha + \delta\alpha d_t) + (\beta + \delta\beta d_t)m_t + \varepsilon_t,$$

where d_t is a dummy variable taking the value unity in bull markets and zero otherwise. Tests of significance for $\delta\alpha$ and $\delta\beta$ for each security are tests for significant shifts in the alpha and beta parameters during bull markets. To allow for variations in opinion of what constitutes a bear or bull market, Fabozzi and Francis use three different definitions: bull and bear (BB), up and down (UD), and substantial up and down (SUD). (For more complete definitions, see Fabozzi and Francis.)

It is important to note that in this study: 1) the time period sets T_1 and T_2 are the same across all 700 securities within each of the bull/bear market definitions; and 2) T_1 and T_2 do not consist of contiguous time regimes, particularly with the UD and SUD definitions. The significance of 1) is that every security in the sample is subject to the same alpha/beta shift correlation; the significance of 2) is that the abnormal return/beta correlation is not relevant (there is no specific shift point or market return separating T_1 and T_2.)

Fabozzi and Francis conduct partial F-tests of the hypothesis: $\delta a = \delta b = 0$ for each of the bull/bear definitions and find that regardless of the bull/bear definition used, the proportion of rejections is consistent with the significance level of the tests. They conclude that there is no evidence of parameter changes between bull and bear markets. This portion of their study seems entirely reasonable within the context of their data. [Other studies using different approaches find evidence for differential behavior between bull and bear markets. See, for example, Kim and Zumwalt (1979).]

Fabozzi and Francis also conduct separate, two-tailed t-tests for the significance of the shifts in alphas and betas. Their objective is "To determine if either the alpha or the beta coefficients were less stable between bull and bear market conditions ..." (Fabozzi and Francis, p. 1096). It is this portion of their study that we wish to reinterpret in light of our findings on the shift correlation. Over the total sample of 700 securities for the BB bull/bear definition, Fabozzi and Francis find 27 (3.9 percent) significant shifts in alpha and 46 (6.5 percent) significant shifts in beta at the 5 percent level of significance. For UD they find 34 (4.8 percent) significant alpha shifts and 35 (5.0 percent) significant beta shifts, and for SUD, 32 (4.6 percent) and 33 (4.7 percent), respectively. Fabozzi and Francis comment that these percentages of significant shifts are generally consistent with the 5 percent level of the test, but that "... there were fewer T-statistics ... which were significantly different from zero at both the 1 percent and 5 percent levels of significance than sampling theory suggests would normally occur" (Fabozzi and Francis, p. 1097). They point out in a footnote, however, that only the 6.5 percent significant shifts for the BB market betas are significantly different from 5 percent when tested with a simple binomial test. It is evidently the consistency of the lower than expected percentages rather than their size that prompted their comment.

The difficulty with their treatment of these results is that the alpha and beta shifts are correlated. From our own calculations, the OLS alpha/beta shift correlations corresponding to the time periods considered by Fabozzi and Francis are 0.13 (BB), 0.21 (UD), and 0.16 (SUD). Fortunately, these are rather weak correlations; however, they are sufficient to increase the likelihood of simultaneous significance in some of the alpha and beta shifts. This has two implications:

- Separate tests on the proportions of occurrences of significant shifts in alpha and beta are not warranted because the level of the tests is distorted by the shift correlations.
- It is possible that some of the remarkable consistency in results for alpha/beta shifts, particularly in the UD and SUD cases, is attributable to the OLS shift correlation. Fabozzi and Francis do not report the number of times that alpha and beta shifts are simultaneously significant, so it is difficult to check this. The key point is that it is not possible to reach a conclusion regarding differences in the behavior of alphas and betas without allowing for the presence of the shift correlation.

HAYS AND UPTON (1986)

Hays and Upton (1986) review some of the prior work on market model parameter stability. They point out that the shifting-regimes model is adequate for most purposes. In this model the time series of returns for any security can be divided into a number of contiguous time regimes. The parameters of the market model are assumed to remain constant within any time regime but change from one regime to another.

Working from the assumption of a shifting-regimes model, Hays and Upton report on their application of the method of recursive residuals of Brown, Durbin, and Evans (1975) to test for shifts in betas for a sample of 73 securities. [Howe and Upton (1992) report that the recursive residuals method performs poorly relative to other shift detection methods; however, the principal results do not depend on the particular shift detection method used.] Readers interested in the details of the recursive residuals method are referred to the discussion in Brown, Durbin, and Evans or the summary in Hays and Upton.

Hays and Upton report finding no significant shifts in 12 of the sampled firms and a total of 116 significant shifts in the remaining 61. Hays and Upton report that when a shift in beta is detected there is an overwhelming likelihood that the estimate of alpha will shift in the opposite direction. Specifically, they fit the following simple linear model of the estimated shifts in alpha against the estimated shifts in beta to the 116 significant shifts:

(7) $E[\delta a] = \eta_0 + \eta_1 \delta b.$

They find a highly significant fit with F-value = 46,323.6 and adjusted R^2 of 0.997. This R^2 corresponds to an observed shift correlation of −0.99.

Hays and Upton speculate that this negative correlation arises because of a fundamental tendency in the market to compensate for shifts in beta:

> This result suggests that the change in the regime is in the nature of a rotation of the characteristic line rather than an upward or downward shift. This, in turn, implies that the previous studies that were restricted to changes in the beta parameter may have been misleading. The negative relationship found here means that changes in beta will be somewhat offset by changes in alpha; consid-

eration of only the beta would overstate the effect of the change." (Hays and Upton, 1986, p. 317).

This interpretation of the shift-correlation is significant because it reflects on the validity of a large number of empirical studies, particularly event studies, done before and after the Hays and Upton study.

As we have seen, some correlation between alpha and beta shifts is to be expected as an artifact of the OLS procedure; however, a correlation as extreme as –0.99 across a large sample of securities is highly unlikely unless, as suggested by Hays and Upton, there is some fundamental property of the market operating above and beyond the OLS correlation. We repeat the Hays and Upton experiment in the hopes of obtaining some explanation for their extreme shift correlation.

Table 2 summarizes the results of a replication of the Hays and Upton experiment on a random sample of 74 securities with complete monthly returns in the time span 1968 to 1981 (the time span studied by Hays and Upton). Following Hays and Upton, we correct all returns for the risk-free rate obtained from prevailing monthly returns on Treasury bills.

Table 2—Means and Standard Deviations of Shifts in Parameters

	H&U	This Study		H&U	This Study
mean δa	.0660	.0032	mean δb	–.0670	–.1065
$\sigma[\delta a]$	.5840	.0224	$\sigma[\delta b]$	.5860	.7211
mean $\lvert\delta a\rvert$	.4490	.0176	mean $\lvert\delta b\rvert$	.4520	.5405
$\sigma[\lvert\delta a\rvert]$	.3770	.0141	$\sigma[\lvert\delta b\rvert]$	.3760	.4859
max $\lvert\delta a\rvert$	2.081	.0757	max $\lvert\delta b\rvert$	2.081	3.1347

The results are comparable between the two studies with respect to the shifts in beta, but there are significant discrepancies in all of the results relating to the shifts in alpha. In particular, Hays and Upton report that the maximum absolute change in alpha and beta are the same (2.081) while our results are 0.0757 for alpha and 3.1347 for beta. Also, the shift correlation, calculated in the same manner as Hays and Upton using equation (7), is an insignificant 0.056—far from the -0.99 obtained by Hays and Upton. Equation (4) can be used to estimate the OLS shift correlation that should arise as an artifact of the OLS procedure. Accordingly, equation (4) is applied to each of the 91 shift points obtained in our study. The mean value of the resulting set of OLS shift correlations is 0.087, which accords well with our empirical shift correlation of 0.056. (The appropriateness of averaging across estimated shift correlations is verified in a separate study. Details are available from the authors on request.)

The maximum and minimum OLS shift correlations obtained in the application of equation (4) are 0.65 and –0.23, respectively. The maximum value is obtained for a case in which the postshift regime is only 12 months long and contains some unusually severe negative market returns. For the minimum value, the preshift regime is 17 months and contains some unusually high positive market returns. In both cases the absolute mean market return is inflated, and the coefficient of variation is correspondingly low.

These observations are consistent with circumstances leading to high positive or negative shift correlations; however, neither extreme is near –0.99.

It is not possible to speculate on the discrepancy between the Hays and Upton study and the present one without access to the original data in the Hays and Upton study. It is clear, however, that the slight shift correlation that we observe can be accounted for as an artifact rather than as a reflection of any underlying market fundamentals.

COPELAND AND LEE (1991)

Our final example is a typical event study based on daily returns data in which the events are firm-specific; hence, the time period sets T_1 and T_2 are different for each firm under consideration. The sets are sequential in time internally (in contrast to the Fabozzi and Francis study), but they are not contiguous because Copeland and Lee exclude 15 days before and after the events from their samples for estimating alpha and beta. Copeland and Lee investigate decisions to recapitalize firms through exchange offers or stock swaps (henceforth, events). They review a number of possible explanations for such decisions by firms and argue that empirical evidence supports one of these (the class of signaling hypotheses). Their empirical analysis considers systematic behaviors of a number of accounting measures as well as the changes in the riskiness of the firms as represented by shifts in betas before and after events. Their sample consists of 90 leverage-increasing events and 127 leverage-decreasing events scattered throughout the 22 year period from mid-1962 to mid-1984. Calculations of betas are based on daily returns for 250 days before and 250 days after a period of 30 days centered on the initial announcement time of the event. They use both standard OLS betas and betas corrected for nonsynchronous trading effects by the method of Scholes and Williams (1977).

The general conclusions of the Copeland and Lee study are not questioned here; however, we will comment on two elements of the study that we believe are related to shift correlations. For the first, we provide in Table 3 a portion of the results reported in Copeland and Lee's Exhibit 3 (p. 39).

Table 3—Changes in Beta and Intercept Around the Announcement Date (Copeland and Lee, Exhibit 3)

Type of Transaction		Changes in Alpha	Significant Changes @ 5%	Changes in Beta	Significant Changes @ 5%
Leverage-Increasing	Positive	29	8	46	2
	Negative	61	23	44	1
	Total	90	31	90	3
Leverage-Decreasing	Positive	75	38	49	2
	Negative	52	16	78	6
	Total	127	54	127	8

The effect of alpha/beta shift correlation is most apparent with the leverage-decreasing events. The distribution of shifts in beta between positive and negative shifts is almost the exact reverse of that for alpha (75 positive and 52 negative for alpha versus 49 positive and 78 negative for beta). We suspect this pattern can be attributed to a negative alpha/beta shift correlation in most of the securities tested. Copeland and Lee find that the proportion of positive changes in beta is significantly greater than 0.5 at the 5 percent significance level. They also find that the proportion of positive changes in alpha is significantly less than 0.5; however, in light of the possibility of negative OLS correlation between alpha and beta shifts, the latter test must be regarded as inconclusive. It is possible that the proportion of positive shifts in alpha is attributable to the shifts in beta and not to any underlying systematic behavior of the intercept.

The same behavior is not exhibited by the shifts around leverage-increasing events: while the proportion of negative shifts in alpha is significantly greater than 0.5, the same is not true of the shifts in beta. It is conceivable that the shift correlation is near zero for all of the 90 leverage-increasing events, but this seems unlikely and would constitute a phenomenon worth exploring. Without access to the Copeland and Lee data, we are unable to comment further.

A second comment regarding the Copeland and Lee study relates to the correlation between shifts in abnormal returns and shifts in beta. Copeland and Lee report (p. 40) that average abnormal returns are obtained "... from a market model regression estimated during the pre-event benchmark period." They comment (footnote 11, p. 40) that "The average abnormal return estimates were not significantly different when estimated ... using a post-event benchmark period." The difficulty here is that the shift in abnormal returns is likely to be correlated with the shift in beta, and such correlations can be much stronger than the alpha/beta shift correlations.

Consider as an example the leverage-increasing events that are found by Copeland and Lee to be associated with negative shifts in beta. Copeland and Lee find that there is an average two day abnormal return of 5.4 percent associated with such events, and evidently they find a similar result when the postevent model is used to estimate abnormal returns. For a particular firm exhibiting a positive abnormal return for both pre- and postevent models, the actual returns on the event day(s) must be higher than the predictions provided by both models. This is the situation depicted in Figure 1. With a negative shift in beta, two situations are possible depending on whether the market return at the time of the shift, m_τ, is greater than m_x. If $m_\tau > m_x$, the postevent model will show a greater abnormal return than the pre–event model. The opposite will be true if $m_\tau < m_x$. Similar regularity of behavior will result for all of the other possible situations; for example, in the case that the beta shift is negative and there is a positive abnormal return with the preshift model and a negative abnormal return with the postshift model. The relative values of abnormal returns calculated with pre- and postshift models can be influenced strongly by the relative position of m_τ and m_x, which are determined largely by happenstance because the event time τ is firm-specific. The abnormal return/beta correlation given by equation (6) can be taken as an index of the severity of this tendency.

Across a suitably large number of firms, the number of times $m_\tau > m_x$ approximately will equal the number of times $m_\tau < m_x$. This being the case, the systematic under- or overestimations of the postevent model associated with the abnormal return/beta shift correlation will tend to balance. Thus, it is not surprising that Copeland and Lee find little difference between average abnormal returns computed with the pre- and postevent models. A decision to use the pre- or postevent model should be based on a more detailed analysis than a simple comparison of averages.

DEALING WITH SHIFT CORRELATIONS

As is evident from the examples, there is no simple prescription for dealing with shift correlations because they can arise in many different contexts and because their importance and treatment depends on how parameter shifts are being interpreted. Nonetheless, we can offer the following suggestions:

- The first and most important action is to calculate the theoretical shift correlation whenever a study involves estimation of differences between OLS estimates across different data sets. The shift correlation may be so small that it can safely be ignored, but it is certainly necessary to check this.
- If tests of hypothesis are being carried out on more than one parameter shift, and a shift correlation is present, then the tests should be done using simultaneous methods such as those discussed in Johnson and Wichern (1992, p. 188).
- Suppose it is important to test whether a particular empirical shift correlation, r, is significantly different from the nuisance correlation, ρ, predicted by equations (4) or (6). Then it is appropriate to construct a test of the hypothesis in which the null hypothesis value of the shift correlation is ρ, and r is tested using Fisher's z–transformation. (The Fisher transformation to approximate normality is given by

  $$z_r = \frac{1}{2}(\ln(1 + r) - \ln(1 - r)),$$

  and the standard error of the transformed sample value is $1/\sqrt{n-3}$. See, for example, Snedecor (1967) for further details.)
- This test finds no significant difference between the sample correlation 0.056 and theoretical value 0.087 found in our replication of the Hays and Upton study.

CONCLUSION

In this paper we demonstrate the existence of nuisance correlations between shifts in alphas, betas, or abnormal returns that are estimated by OLS regression of the market model over distinct sets of time periods or time regimes. These shift correlations arise from the OLS procedure itself and have no bearing on market fundamentals. We provide formulas for the shift correlation between alpha and beta and between abnormal returns and beta. These formulas are valid under a mild independence assumption and are robust to departures from the assumption that unsystematic variances are equal between the two sets of time periods.

We describe circumstances in which nuisance correlations are likely to be the most severe: 1) when one or both time regimes are small; 2) when the coefficient of variation of market returns is particularly high over one time regime; 3) when the market return at the time of the event or shift is distant from the market return at the point of intersection of the pre- and postshift models; and 4) when shifts in abnormal returns are calculated for a single time point across multiple securities. (In the final case, the difficulty is that the same shift correlation, which is possibly high, will apply to all of the securities under consideration.)

We illustrate difficulties created by the nuisance correlations with reference to three prior studies and conclude with a brief discussion of remedial measures.

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APPENDIX A—THE OLS CORRELATION BETWEEN SHIFTS IN BETA AND SHIFTS IN ALPHA

Let $\bar{R}_1$ and $\bar{m}_1$ denote the mean security and market returns over time set T_1 and $\bar{R}_2$ and $\bar{m}_2$ those over T_2. For k = 1 or 2, the OLS estimates of beta and alpha are given by:

$$b_k = \frac{SSrm_k}{SSm_k} \quad \text{and } a_k = \bar{R}_k - b_k\bar{m}_k;$$

where:

$$SSm_k = \Sigma_{t\in T_k} (m_{kt} - \bar{m}_k)^2 \text{ and}$$

$$SSrm_k = \Sigma_{t\in T_k} (R_{kt} - \bar{R}_k)(m_{kt} - \bar{m}_k).$$

It is well-known (cf. Draper and Smith, 1983, p. 83) that if the error terms ε_k follow independent normal distributions with constant variances $\sigma^2[\varepsilon_k]$, the estimators (a_k, b_k) will have a joint bivariate normal distribution with covariance matrix:

$$(8)\ \frac{\sigma^2[\varepsilon_k]}{SSM_k}\begin{pmatrix} \frac{\Sigma_t m_{kt}^2}{n_k} & -\bar{m}_k \\ -\bar{m}_k & 1 \end{pmatrix} \text{ for } k = 1, 2.$$

The shift estimates $(\delta a, \delta b) = (a_2 - a_1, b_2 - b_1)$ also will be bivariate normal with a covariance that will depend, in part, on any dependencies that exist between the estimates across the time sets T_1 and T_2; that is, among the pairs $(a_2 - a_1)$, $(a_1 - b_2)$, $(b_1 - a_2)$, and $(b_1 - b_2)$. Such dependencies conceivably can arise if the returns in T_2 are statistically dependent on those in T_1. Such dependence is highly unlikely if the shift point is an arbitrary point not dependent on the data such as a point determined by some exogenous event like the 1970s oil crisis. Under these circumstances it will be satisfied if the security and market return series are well-approximated as the first differences of a random walk process. In the event that shift points are estimated from the individual security returns data [as in the Hays and Upton (1986) study], however, they will be random variables on the same sample space as the returns. It is conceivable that, conditional upon their relative placement before or after the estimated shift point, returns will not be independent. Nonetheless, the experimental conditions of the Hays and Upton study make such a dependency extremely unlikely. An assumption of independence is even more likely to be satisified in each of the the other two studies that we examine in this paper.

In the following analysis, then, we stipulate the independence assumption that slope and intercept estimates across the time sets T_1 and T_2 are statistically independent.

Under the independence assumption, we have

$\mathrm{Cov}\ (a_1, a_2) = \mathrm{Cov}\ (a_1, b_2) = \mathrm{Cov}\ (b_1, a_2) = \mathrm{Cov}\ (b_1, b_2) = 0$; furthermore,

$\sigma^2[\delta a] = \sigma^2[a_1] + \sigma^2[a_2]$, $\sigma^2[\delta b] = \sigma^2[b_1] + \sigma^2[b_2]$, and

$\mathrm{Cov}\ [\delta a, \delta b] = \mathrm{Cov}\ [a_1, b_1] + \mathrm{Cov}\ [a_2, b_2]$.

By definition, $\rho[\delta a, \delta b] = \mathrm{Cov}\ [\delta a, \delta b]/\sigma[\delta a]\sigma[\delta b]$. Then, using equation (8) and simplifying, we have:

$$(9)\ \rho[\delta a, \delta b] = \frac{-\left(\frac{\sigma^2[\varepsilon_1]\bar{m}_1}{SSm_1} + \frac{\sigma^2[\varepsilon_2]\bar{m}_2}{SSm_2}\right)}{\sigma[\varepsilon_1]\sigma[\varepsilon_2]\sqrt{\left(\frac{\Sigma m_{1t}^2}{n_1 SSm_1} + \frac{\Sigma m_{2t}^2}{n_2 SSm_2}\right)\left(\frac{1}{SSm_1} + \frac{1}{SSm_2}\right)}}.$$

Under the assumption that $\sigma^2[\varepsilon_1] = \sigma^2[\varepsilon_2]$, we obtain:

$$(10)\ \rho[\delta a, \delta b] = \frac{-\left(\frac{\bar{m}_1}{SSm_1} + \frac{\bar{m}_2}{SSm_2}\right)}{\sqrt{\left(\frac{\Sigma m_{1t}^2}{n_1 SSm_1} + \frac{\Sigma m_{2t}^2}{n_2 SSm_2}\right)\left(\frac{1}{SSm_1} + \frac{1}{SSm_2}\right)}}.$$

The assumption of equality of variances is not as limiting as it might seem—the correlation given by equation (9) is insensitive to inequality between $\sigma^2[\varepsilon_1]$ and $\sigma^2[\varepsilon_2]$. For example, in the case that

$$\frac{\bar{m}_1}{SSm_1} = \frac{\bar{m}_2}{SSm_2},$$

the variation in the value of $\rho[\delta a, \delta b]$ as $\sigma^2[\varepsilon_1]$ ranges between

$$\frac{\sigma^2[\varepsilon_2]}{2} \text{ and } 2\sigma^2[\varepsilon_2]$$

is no more than 20 percent of its maximum value. If the maximum correlation over this range of variance values is 0.10 (a typical value), the minimum value will be 0.08. Similar insensitivity is exhibited in the case that

$$\frac{\bar{m}_1}{SSm_1} \approx \frac{\bar{m}_2}{SSm_2}\ .$$

APPENDIX B—THE CORRELATION BETWEEN SHIFTS IN BETA AND SHIFTS IN ABNORMAL RETURNS

Let τ denote the time for which abnormal returns are to be calculated using either the pre- or postshift model. The market return at time τ is m_τ. The shift in abnormal returns, $\delta\hat{A}_\tau$, is simply the negative of the shift in predicted expected returns from equations (2) and (3); that is,

$$(11)\ \delta\hat{A}_\tau = -\delta a - m_\tau \delta b.$$

Multiplying both sides of equation (11) by δb and taking the expectation yields

$$E[\delta\hat{A}_\tau \delta b] = -E[\delta a \delta b] - m_\tau E[(\delta b)^2].$$

Then

$$\begin{aligned}
\text{Cov}\,[\delta\hat{A}_\tau, \delta b] &= E[\delta\hat{A}_\tau \delta b] - E[\delta\hat{A}_\tau]E[\delta b] \\
&= E[\delta a \delta b] - m_\tau E[(\delta b)^2] + (E[\delta a] + m_\tau E[\delta b])E[\delta b) \\
&= -(E[\delta a \delta b] - E[\delta a]E[\delta b]) - m_\tau(E[(\delta b)^2] - (E[\delta b])^2)).
\end{aligned}$$

Thus

$$\text{Cov}\,[\delta\hat{A}_\tau, \delta b] = -\text{Cov}\,[\delta a, \delta b] - m_\tau \sigma^2[\delta b].$$

Dividing both sides of this expression by $\sigma[\delta\hat{A}_\tau]\sigma[\delta b]$ and rearranging terms yields:

$$(12)\ \rho[\delta\hat{A}_\tau, \delta b] = \frac{-1}{\sigma[\delta\hat{A}_\tau]}(\sigma[\delta a]\rho[\delta a, \delta b] + m_\tau \sigma[\delta b]).$$

Note that, as should be expected, this correlation is equivalent to $-\rho[\delta a, \delta b]$ in the case that $m_\tau = 0$.

By using equation (10) and performing substitutions and simplifications similar to those in Appendix A, equation (12) can be written in the computational form:

$$(13)\ \rho(\delta\hat{A}_\tau, \delta b) = \frac{\left(\frac{\bar{m}_1}{SSm_1} + \frac{\bar{m}_2}{SSm_2}\right) - m_\tau\left(\frac{1}{SSm_1} + \frac{1}{SSm_2}\right)}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{(m_\tau - \bar{m}_1)^2}{SSm_1} + \frac{(m_\tau - \bar{m}_2)^2}{SSm_2}\right)\left(\frac{1}{SSm_1} + \frac{1}{SSm_2}\right)}}$$