# The Detection of Nonstationarity in the Market Model

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*A Bayesian shifting regimes model is used to reexamine the problem of detecting nonstationarity in the market model parameters. In contrast to previous studies, the model provides a more general technique for locating a multiple number of unknown regime shift points. A close examination of the nature of the shifts is made by investigating the extent of the stationarity violations; the temporal distribution of the shift points; the size, direction, and significance of the parameter shifts; and the relationship between the number of shift points and firm size. We also indicate the presence of so-called transition betas in a large percentage of the firms, suggesting that changes in beta sometimes may follow a gradual period of transition rather than an abrupt or distinct change from one value to another.*

## INTRODUCTION

Financial theorists and practitioners for many years have used the well-known market model for the development and use of financial theories. As a return-generating equation, the market model has found extensive application in portfolio theory, the capital asset pricing model (CAPM), and event study methodology. Because of its importance, considerable investigation has centered on issues related to the estimation and behavior of the model, especially the systematic risk parameter, beta. In particular, a series of studies by Blume (1971, 1975) and Levy (1971) concludes that the systematic risk measure of individual securities is not stationary, but has a tendency to change over time. Since then there has been additional evidence to support the conclusion that the market model parameters of individual securities are nonstationary.

This nonstationarity of the market model parameters is important when one considers the informational as well as the methodological implications for applications of the model. In the first case, assuming that the market model parameters define the return-generating process for an individual firm, then it stands to reason that a change in the parameters may indicate a change in the fundamental factors influencing a firm's stock price. Research has shown that certain fundamental factors may influence a firm's systematic risk. For example, Hamada (1972) and Lev (1974) show that a firm's systematic risk is related to its degree of financial and operating leverage. Since a theoretical basis exists

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for the contention that fundamental firm characteristics may influence systematic risk, it naturally follows that changes, or announcements of changes, in these factors may engender changes in the market model parameters. Hence, an empirically identified parameter change may be associated with a change in a fundamental firm characteristic. In other words, given that a parameter change has been observed, there should be a fundamental reason for observing the change. As Bey (1983b, p. 286) suggests, "the ability simply to identify nonstationarity in individual securities in a given period may serve as a viable starting point for identifying the economic factors causing the structural change in the generating process and developing a superior model to describe the stochastic process generating security returns."

In the second case, the important role played by beta and its normative implications in capital market theory highlight the importance of the impact of nonstationarity on the accurate and valid estimation of the systematic risk parameter (Sunder, 1980; Lee and Chen, 1982; Howe and Upton, 1992). In addition, because nonstationarity can bias the true measure of abnormal returns, the detection of stationarity violations calls into question previous tests of market efficiency and event studies that rely on the assumption of a stationary market model (Brown, Lockwood, and Lummer, 1982; McDonald and Nichols, 1984).

The purpose of this study is to reexamine the problem of detecting nonstationarity in the market model parameters using a more general detection scheme and to make a closer examination of the nature and extent of the problem in the daily stock returns of individual firms. In the first objective, a Bayesian shifting regimes (BSR) model presented by Mehta and Beranek (1982) is used to detect nonstationarity in the market model parameters because it provides a general technique for detecting multiple regime shift points. In many studies (e.g., Hsu, 1982; Bey, 1983; McDonald and Nichols, 1984; and Brown, Lockwood, and Lummer, 1985) the technique used or experimental design constrains the number or locations of the shift points, making it difficult to ascertain the extent of the violations. The Bayesian shifting regimes model incorporates the entire time series of returns to determine the number and location of the regimes. While the Bayesian shifting regimes technique has no inherent limitations on the number of shift points, for computational reasons we limit the maximum number of shift points to three.[1] Implementing the model is a computationally intensive procedure, and Mehta and Beranek only demonstrate the technique by analyzing a time series of quarterly returns for just one company—the AT&T Corporation from 1950 to 1974. While their analysis detects a single shift point in the 34th quarter (1958), a wider application of the model has not been made. This study will provide such an application by considering a random sample of 150 NYSE and AMEX firms.

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[1] Several attempts to search for more than three shift points with just one of the firms in the sample were all beyond the feasible capabilities of even a CRAY supercomputer. Therefore, a completely accurate claim is that, within the limits of our computational resources, we place no prior restrictions on the number of parameter shift points. The important point is that we do not artificially constrain the number or locations of the shift points as some techniques do.

To determine the robustness of our results to this limitation, we examine the stability of the shift point locations when we estimate the model for one, two, and three shift points. We find that in almost 40 percent of the firms the change point locations are exactly the same in each case. In almost 70 percent, the locations in each case are all within ± ten days.

Our second objective is to examine the nature and character of the nonstationarity problem in the daily returns of individual firms. Because many studies demonstrate only the existence of nonstationarity of the market model parameters, insight into the nature of the problem will allow researchers to better understand the implications of nonstationarity to their application. In particular, we investigate the extent of the stationarity violations; the temporal distribution of the shift points; the size, direction, and significance of the parameter shifts; and the relationship between the number of shift points and firm size.

While our results indicate stationarity violations in about 71 percent of the firms, the remaining 29 percent of the firms show no evidence of structural changes in the underlying market model. Testing the hypothesis that larger firms generate fewer shift points, we find only slight evidence of a relationship between the number of shift points and firm size. A close examination of the results also seems to indicate the presence of so-called transition betas in about 43 percent of the firms with more than one shift point. These transition betas are characterized by extreme or abnormal values occurring in short regression regimes. Although not conclusive, it seems to suggest that changes in beta are not necessarily abrupt and distinct, but sometimes may go through a period of transition as they shift from one value to another.

## METHODOLOGY

The objective of this study is to reexamine the problem of detecting structural changes in the market model using daily returns of individual securities. Stochastic parameter models, including the random coefficients model (RCM), have been used in a number of studies (Fabozzi and Francis, 1978; Sunder, 1980; Lockwood and Kadiyala, 1988) that generally conclude that beta behaves as a random coefficient. In contrast to stochastic parameter models that are unable to pinpoint the location of the nonstationarity, many studies have applied switching regression or shifting regimes models to the study of nonstationarity in individual securities. In shifting regimes models, nonstationarity of the parameters is assumed to occur at discrete locations, or shift points, in the time series of observations. These discrete points of nonstationarity form the distinct regression regimes associated with the shifting parameter values. Kon and Jen (1978), Kon and Lau (1979), Hsu (1982), Mehta and Beranek (1982), Bey (1983), Brown, Lockwood, and Lummer (1985), and Hays and Upton (1986) have employed switching regression or shifting regimes methodologies. While many are based upon the Brown, Durbin, and Evans (1975) and Quandt (1958, 1960) methodologies, Hsu (1982) and Mehta and Beranek (1982) provide Bayesian shifting regimes models, and Miller and Gressis (1980) utilize a partition regression model developed by Guthery (1974).

In this study, no prior expectations about the number and/or locations of the shift points are held. Instead, the more general Bayesian shifting regimes technique is used to isolate the time points corresponding to significant parameter changes. This section briefly explains the Bayesian shifting regimes model used in this study.

The model being estimated is the well-known market model:[2]

[2] Note that our assumption of the market model as the applicable return-generating process does not necessarily imply or preclude the existence of the CAPM.

(1) $R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$

where:

$R_{it}$ = The return on security i for time period t;
$R_{mt}$ = The return on the market for time period t;
$\alpha_i$ and $\beta_i$ = The regression parameters;
$t$ = 1, 2, ... , T; and
$\varepsilon_{it} \sim N(0,\sigma_i^2)$.

If the shift points are designated as $\tau_1, \ldots, \tau_p$ with $0 = \tau_0 < \tau_1 < \ldots < \tau_p < \tau_{p+1} = T$, then there are p shift points that form r = 1, 2, ... , p + 1 regression regimes. Generalizing the market model to allow for the different regimes yields (individual firm subscripts suppressed):

(2) $R_t = \alpha_t + \beta_t R_{mt} + \varepsilon_t$

where:

$\beta_t = \beta_r$;
$\alpha_t = \alpha_r$; and
$\varepsilon_t \sim N(0,\sigma_r^2)$

when t is in the interval $[\tau_{r-1} + 1, \tau_{r-1} + 2, \ldots, \tau_r]$. In essence, the model can be viewed as a (p + 1) equation regression system with each regression equation separated by a shift point. Therefore, the model can be written as:

$$
(3) \quad \begin{aligned}
y_1 &= X_1\zeta_1 + \varepsilon_1 \\
y_2 &= X_2\zeta_2 + \varepsilon_2 \\
&\vdots \\
y_{p+1} &= X_{p+1}\zeta_{p+1} + \varepsilon_{p+1}
\end{aligned}
$$

where:

$\zeta_r = (\alpha_r, \beta_r)$.

Each regression regime contains $n_r$ observations where, in terms of the change points, $n_1 = \tau_1$, $n_2 = \tau_2 - \tau_1$, ... , and $n_{p+1} = T - \tau_p$. For the entire model, the parameter vector is $\phi = (\gamma, \delta)$ where $\gamma = (\zeta_1, \ldots, \zeta_{p+1}, \sigma_1, \ldots, \sigma_{p+1})$ and $\delta = (\tau_1, \ldots, \tau_p, p)$, all conditional on the number of change points, p.

Because Bayesian techniques are used to estimate the model, for the model as a whole, Bayes' theorem can be expressed as:

(4) $h(\gamma, \delta|Z) = \dfrac{L(\gamma, \delta|Z)\, g(\gamma, \delta)}{f(Z)}$

where:

| | | |
|---|---|---|
| $h(\gamma, \delta\|Z)$ | = | The posterior probability density function (pdf) for the parameter vector $\phi$; |
| $g(\gamma, \delta)$ | = | The prior pdf; |
| $L(\gamma, \delta\|Z)$ | = | The likelihood function conditional on the data $Z=(y, X)$; and |
| $f(Z)$ | = | The marginal pdf of the data. |

Because our objective is to determine the number and locations of the shift points, the regression parameters are nuisance parameters. In order to make inferences on $\delta$, the nuisance parameters are integrated out to obtain the marginal posterior pdf for $\delta$:

$$(5)\ h(\delta|Z) = \frac{\left[\int L(\gamma, \delta|Z)\, g(\gamma|\delta)\, d\gamma\right] g(\delta)}{f(Z)}\ .$$

The expression in brackets is the so-called integrated likelihood function, ILF($\delta$, Z), $g(\gamma|\delta)$ is the conditional prior pdf for the regression parameters, and $g(\delta)$ is the prior pdf for the shift point parameters. Assuming independent, noninformative priors for $\gamma$, conditional on $\delta$, the ILF is given by:

$$(6)\ L^*(\tau, p, Z) = \prod_{r=1}^{p+1} \frac{1}{2}\pi^{-\frac{\upsilon_r}{2}}\Gamma\left(\frac{\upsilon_r}{2}\right)|X_r'X_r|^{-\frac{1}{2}}\left[(y_r - X_r b_r)'(y_r - X_r b_r)\right]^{-\frac{\upsilon_r}{2}}$$

where:

| | | |
|---|---|---|
| $\upsilon_r$ | = | $n_r - k$; |
| $b_r$ | = | $(X_r'X_r)^{-1}X_r'y_r$; and |
| $\Gamma(\bullet)$ | = | The gamma function. |

The marginal posterior pdf for the shift point parameters then becomes:

$$(7)\ h(\delta|Z) = L^*(\tau, p, Z)\,\frac{g(\delta)}{f(Z)}\ .$$

Assuming a step loss function, the estimator of $\tau$ corresponds to the mode of the posterior pdf so that inference on the shift point locations can be based on maximization of the posterior pdf over all possible combinations of shift points. If we assume independent, noninformative priors for $\tau$, then the vector of shift points, $\hat{\tau}$, that maximizes the ILF also maximizes the marginal posterior pdf, $h(\delta|Z)$ and therefore corresponds to the most likely set of shift points (conditional on the number of shift points, p).

Because maximizing the posterior pdf (or ILF) yields shift point estimates for a given value of p, it remains to determine the appropriate value of p for the model. Choosing the value of p that maximizes the (already maximized) posterior pdf always will lead to choosing the largest value of p. Mehta and Beranek propose using the Schwarz (1978) adjustment information criterion to determine the dimension of the model. The Schwarz criterion is appropriate as it selects the *a posteriori* most probable model and is a consistent estimator of the dimension of the model. The criterion calls for choosing the value of p that maximizes the expression:

(8) $Ln(L_p) - 1/2K_pLn(T)$

where:

$L_p$ = The likelihood function evaluated at $(\hat{\gamma}, \hat{\tau})$ conditional on p; and
$K_p$ = A constant that indicates the number of continuous parameters being estimated $( = 3(p + 1))$.

In this manner, the Schwarz criterion is used to select the number (and hence locations) of shift points.

## RESULTS

A random sample of 150 NYSE and AMEX firms is chosen for the study. For each of the firms a one year time series of daily returns is taken from the Center for Research in Security Prices (CRSP) data tapes for the period from December 1, 1985 to January 31, 1987. The two 20+ trading day periods outside of 1986 are used to ensure that the initial and final regression regimes are estimated with at least 20 return observations, but shifts in the underlying market model parameters are identified only for the days in 1986. For intermediate regimes, the shift points are allowed no fewer than ten trading days apart.

For each of the firms in the sample, the Bayesian shifting regimes methodology is used to search for up to three shift points within the time series of returns. The methodology calls for maximizing the ILF portion of the posterior pdf, $h(\delta|Z)$, across all combinations of shift points for each value of p and then applying the Schwarz criterion to determine the dimension of the model. An example of the ILF for the case of one shift point is shown in Figure 1. As the location of the shift point is varied across each of the 253 trading days in the time series, the ILF reaches a maximum at $\hat{\tau} = 175$, implying that the most likely shift point is September 10. A portion of the three-dimensional ILF for the case of two shift points is shown in Figure 2. The ILF is calculated for each possible pair of shift points, $(\tau_1, \tau_2)$. The graph is symmetrical because, structurally, the shift point pair $(\tau_1, \tau_2)$ is the same as the pair $(\tau_2, \tau_1)$. In this example the ILF reaches a maximum at $\hat{\tau}_1 = 175$ and $\hat{\tau}_2 = 234$ (September 10 and December 3). Applying the Schwarz criterion for each value of p from zero to three indicates a single parameter shift point for this firm.

**Figure 1—Integrated Likelihood Function for One Shift Point**

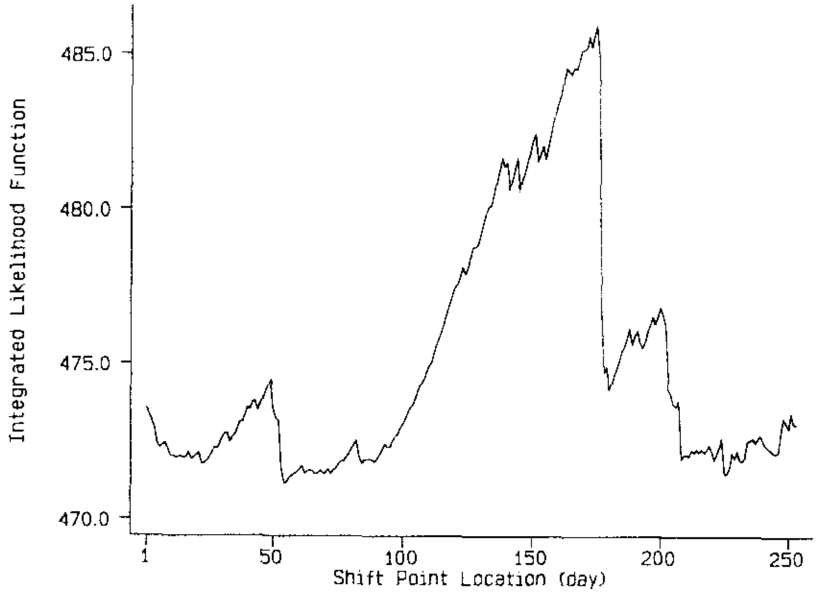



**Figure 2—Integrated Likelihood Function for Two Shift Points**

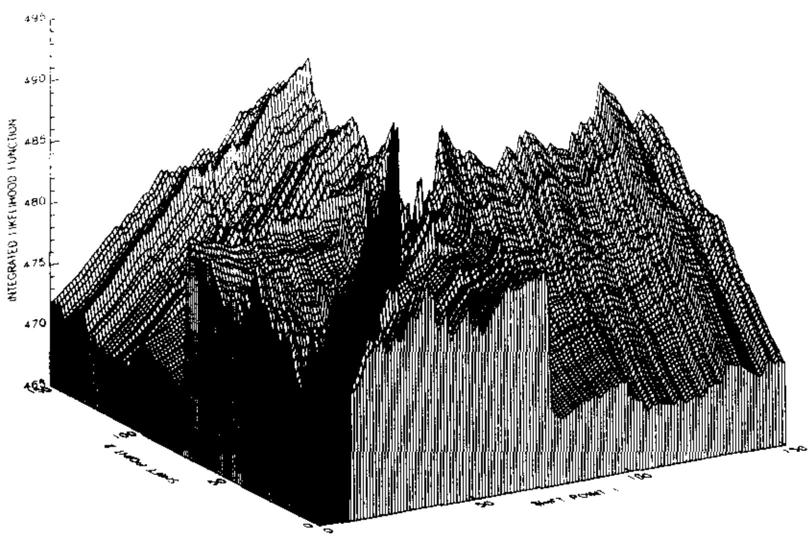


**Table 1—Distribution of the Number of Shift Points Detected**

| Number of Shift Points Detected | Number of Firms |
|---|---|
| 0 | 43 (28.7%) |
| 1 | 19 (12.7%) |
| 2 | 47 (31.3%) |
| 3 | 41 (27.3%) |
| TOTAL | 150 |

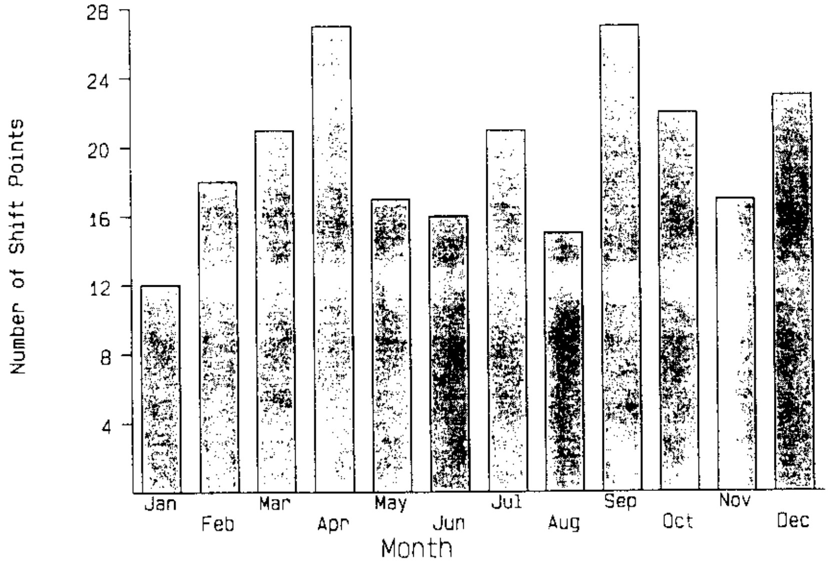
*SHIFT POINTS*

Some 236 shift points are detected for the 150 firms in the sample. Table 1 presents the distribution of the number of shift points detected in each firm. Stationarity violations are detected in 71 percent of the firms in the sample, with the maximum number of shift points detected in about 27 percent of the firms. While this indicates a problem extensive enough to warrant consideration in applications of the market model, it remains that 29 percent of the firms exhibit relatively stable return series with no stationarity violations. The extent of nonstationarity in individual securities varies considerably from study to study depending on the methodology and return period used. Hence, direct comparisons are impossible. Previous studies have reported nonstationarity in 15 percent to 74 percent of the sample.

Figure 3 illustrates the temporal distribution of the 236 shift points by month of the year. The months of April, September, and December have the largest numbers of shift points. Thompson, Olsen, and Dietrich (1987) find the highest number of news announcements occurring in April (due mainly to earnings announcements), which may explain the large number of shift points in April. A similar argument would indicate large numbers in July, October, and January, but not September. A possible explanation for the large number in September may reflect the market's reaction to the Tax Reform Act of 1986 which was passed in September. The large number of shift points in December may be attributed to the well-known January effect, especially because six of the shift points occur on the last day of the year. Overall, the shift points tend to occur randomly throughout the year. Chi-square tests using both monthly and weekly distributions of the shift points do not reject the null hypothesis that the shifts are uniformly distributed across the respective time periods. Hence, there is no statistical evidence that stationarity violations are more likely to occur in any particular part of the year.

Because firms with zero or even one shift point exhibit relatively stable return series, we seek to determine if the number of shift points is related to firm size. In particular, is it the case that smaller (larger) firms generate more (fewer) shift points than larger (smaller) firms? As Roll (1977) suggests, "larger firms generally have many divisions and often operate in more than a single industry and market." As such, "they superficially resemble diversified portfolios of smaller firms ...". Hence, we would expect larger firms to exhibit fewer structural changes with respect to the underlying market model. Table 2 presents several distributions of the number of shift points detected according to firm size. Firm size is measured by the market value of equity, calculated as the number of shares outstanding multiplied by the price per share on the first trading day of 1986. In Panel A,

**Figure 3—Frequency of Shift Point Occurrences (by Month)**




the sample is ordered according to firm size and then divided into ten equal groups, from the smallest ten percent of the firms to the largest ten percent. The chi-square value for a test of the hypothesis that the number of shift points detected is independent of firm size is given at the bottom of the table. The p-value of 0.114 indicates only a slight, and statistically insignificant, relationship between firm size and the number of shift points.

In Panel B the distinction between small and large firms is defined more narrowly. In this case, only the ten, 20, and 30 smallest and largest firms are compared. In each case the number of shift points is skewed toward a larger number of shifts for small firms, but the null hypothesis of independence is rejected only when comparing the 20 smallest and largest firms. The same test using the 25 smallest and largest firms also rejects, but other intermediate categories do not. Therefore, even though small firms seem to generate more shift points, overall the sample provides only slight evidence of a relationship between the number of shift points and firm size.

### *PARAMETER CHANGES*

Because the Bayesian shifting regimes technique detects structural changes in the entire market model, not just the beta coefficient, we can examine the changes in each of the parameters. For the shift points detected,[3] the overall average change in beta is 0.066

[3] Excluding the parameter shifts associated with a possible transition period as explained in the next section.

## Table 2—Number of Shift Points Detected and Firm Size

Panel A: Full Sample Ordered and Grouped From Smallest to Largest Firms

| Number of Shift Points Detected | Smallest 10% of Firms | | | | | | | | | Largest 10% of Firms |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 2 | 2 | 5 | 8 | 6 | 2 | 5 | 5 | 2 | 6 |
| 1 | 1 | 3 | 1 | 2 | 1 | 0 | 2 | 1 | 5 | 3 |
| 2 | 3 | 5 | 7 | 3 | 4 | 8 | 6 | 5 | 4 | 2 |
| 3 | 9 | 5 | 2 | 2 | 4 | 5 | 2 | 4 | 4 | 4 |
| TOTAL | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 |
| Avg Size (millions) | $15.00 | | $70.43 | | $184.03 | | $672.60 | | $4,278.90 | |

Ordered Sample (Grouped According to Firm Size)

$\chi^2 = 36.06$ $p = 0.114$

Panel B: Smallest and Largest Firms in the Sample

| Number of Shift Points Detected | Smallest Ten Firms | Largest Ten Firms | Smallest 20 Firms | Largest 20 Firms | Smallest 30 Firms | Largest 30 Firms |
|---|---|---|---|---|---|---|
| 0 | 2 | 5 | 2 | 7 | 4 | 8 |
| 1 | 1 | 2 | 1 | 7 | 4 | 8 |
| 2 | 1 | 1 | 6 | 2 | 8 | 6 |
| 3 | 6 | 2 | 11 | 4 | 14 | 8 |
| TOTAL | 10 | 10 | 20 | 20 | 30 | 30 |
| Average Size (millions) | $5.50 | $8,562.40 | $9.60 | $5,663.65 | $15.00 | $4,278.90 |
| | $\chi^2 = 3.62$ $p = 0.306$ | | $\chi^2 = 12.54$ $p = 0.006$ | | $\chi^2 = 4.59$ $p = 0.205$ | |

and the average change in alpha is -0.000219. In each case, the numbers of positive and negative changes are almost the same. The average absolute value of the change in beta is 0.494 ($\sigma = 0.430$) and the average magnitude of the change in alpha is 0.00548 ($\sigma = 0.00621$).

Previous research has suggested significant relationships between changes in alpha and beta and changes in beta and the size of beta. In the first case, Hays and Upton (1986) find a highly significant, negative relationship between the changes in alpha and beta. This relationship suggests that the shift points are associated with rotations of the characteristic regression line rather than shifts one way or the other. Our results indicate neither a shift nor a rotation. A regression of the change in alpha against the change in beta produces the following model:

$$(9)\ \Delta\alpha = \underset{(-0.407)}{-0.000247} + \underset{(0.470)}{0.000437}\ \Delta\beta$$

where the coefficient t-statistics are shown below the estimates. With an R-squared less than 1 percent, we find no relationship between the changes in alpha and beta.

In the second case, Blume (1971, 1975) finds that, over time, estimated betas have a tendency to regress toward the grand mean or one. To determine if the changes in beta exhibit a similar tendency, we regress the change in beta against the previous regime beta value. In this case, our results agree with Hays and Upton (1986) in offering additional support for Blume's conclusions regarding the regression tendencies of beta. The following model results:

$$(10)\ \Delta\beta = \underset{(9.018)}{0.497} - \underset{(-10.68)}{0.751}\ \beta_{-1}\ .$$

The overall regression is significant with an F-statistic for the model equal to 114.12 and an R-squared equal to 0.379. The negative coefficient on $\beta_{-1}$ indicates that shift points associated with increases in beta tend to follow lower betas and those associated with decreases tend to follow higher betas.

The Bayesian shifting regimes model locates the most likely set of shift points. Bayesian highest posterior density (HPD) intervals consider the probabilities that the regime parameters are different and are used to determine in which of the parameters a statistically significant change occurs. Highest posterior density intervals are constructed for the posterior distributions of $\beta_j - \beta_i$, $\alpha_j - \alpha_i$, and $\sigma_i^2/\sigma_j^2$ (for regimes i and j) to determine the probabilities that $\beta_j = \beta_i$, $\alpha_j = \alpha_i$, and $\sigma_i^2 = \sigma_j^2$ across regimes. The posterior distributions of $\beta_j - \beta_i$ and $\alpha_j - \alpha_i$ can be closely approximated by t-distributions, and the posterior distribution of $\sigma_i^2/\sigma_j^2$ is an F-distribution. (See Zellner, 1971; Box and Tiao, 1973; and Patil, 1964.) In the case of beta, the null hypothesis, $H_0$: $\beta_j - \beta_i = 0$, is rejected if zero is outside of a $(1 - \alpha)$ highest posterior density interval, implying that the probability $\beta_j = \beta_i$ is less than $\alpha$. For example, a 50 percent highest posterior density interval would indicate if there are better than even posterior odds that the market model parameter is different across the two regimes. Both 50 percent and 90 percent highest posterior density intervals are reported in Table 3.

Using a 90 percent (50 percent) highest posterior density interval, 75 (162) of the shift points exhibit a significant change in beta, 55 (168) a significant change in alpha, and all but five (two) of the shift points are associated with a significant change in the

**Table 3—Bayesian Tests of Significance for Changes in the Market Model Parameters (236 Shift Points)**

| | 90% Highest Posterior Density Interval | | 50% Highest Posterior Density Interval | |
|---|---|---|---|---|
| Parameter | Number of Significant Changes | Positive Changes | Number of Significant Changes | Positive Changes |
| $\beta$ | 75 | 44 (58.7%) | 162 | 83 (51.2%) |
| $\alpha$ | 55 | 24 (43.6%) | 168 | 82 (48.8%) |
| $\sigma^2$ | 231 | 123 (53.2%) | 234 | 126 (53.8%) |
| $\beta$ and $\alpha$ | 20 | - | 111 | - |
| $\beta$ without $\alpha$ | 55 | 32 (58.2%) | 51 | 29 (56.9%) |
| $\alpha$ without $\beta$ | 35 | 16 (45.7%) | 57 | 31 (54.4%) |

variance of the residuals. For each of the parameters, about half of the changes are positive and half negative. Of the 236 shift points, 20 (111) are associated with significant changes in both of the parameters, while 90 (108) experience a significant change in only one of the parameters. The remaining 126 (17) of the shift points did not elicit a significant change in either the beta or alpha parameter.

Naturally, there are fewer total significant parameter changes with a 90 percent highest posterior density interval. When only one of the parameters significantly changes, however, the difference is fairly small, 90 versus 108. At the 90 percent level there are actually a greater number of significant changes in beta when not accompanied by a change in alpha, 55 versus 51. Hence, the main difference in the total number of significant parameter changes comes from the different number of simultaneous changes in both parameters, 20 versus 111. The difference can be explained by considering the joint probability of a significant change in both alpha and beta. The probability of rejecting the joint hypothesis, $H_0$: $\beta_j - \beta_i = 0$ and $\alpha_j - \alpha_i = 0$, is represented by the joint probability of a significant change in alpha and beta. With a 90 percent highest posterior density interval this joint probability is 0.01 (assuming independence), compared to 0.25 for a 50 percent highest posterior density interval. This large difference in the two joint probabilities explains the big difference in the number of simultaneous parameter changes at the 50 percent and 90 percent levels.

### *TRANSITION BETAS*

By definition, the Bayesian shifting regimes model assumes a distinct and abrupt shift in the market model parameter(s) at the identified shift point(s). A close examination of the changes in the beta parameter, however, reveals that a transition period seems to be present in the beta shifts of a substantial number of the firms. This characteristic of the shift points is common to 46 of the firms in the sample and is typified by regimes of short duration and extreme or abnormal beta values that are out of line with the adjacent beta values. As an example, consider the case of one of the firms in the sample, Federal Express (FDX). Two shift points are detected for FDX, the first on September 12 and the second just 11 trading days later on September 29. The value of the FDX beta in the regime preceding September 12 is 1.20, and the value in the regime following September 29 is 1.02, both of which represent reasonable betas for FDX.[4] In the 11 day regime between September 12 and September 29, however, the estimated value of beta is -1.94, about 275 percent lower than the average of the two adjacent beta values. It seems reasonable that FDX's beta value is not -1.94 in the period from September 12 to September 29, but is going through a short transition period as it adjusts to its new value. In other words, instead of some informational event causing an immediate impact on beta around September 12, there is an adjustment period occurring prior to the new value of beta. A similar effect is noted by Miller and Gressis (1980) who also suggest the possibility of a transition period between regression regimes.

In the case of FDX it is possible to trace the transition period to September 17 when the company announced a decline in its quarterly earnings. Independent of its struggling

[4] At the time, Value Line was reporting a beta of 1.30 for FDX.

Zapmail service, the company actually had increased earnings. In light of the fact that its Zapmail service had continued to lose money, the announcement increased uncertainty about FDX's future earnings and intensified speculation that it would discontinue the unprofitable Zapmail service. Hence, it seems reasonable that the increased uncertainty concerning FDX's future earnings and response to a failing product would prompt a period of adjustment while the market revalued FDX stock. On September 29, the same time the transition period ended, the uncertainty was resolved when the company announced that it was dropping its Zapmail service.

The beta changes of each firm in the sample with two or more shift points are examined in order to identify possible transition periods and transition betas. Beta values are defined as extreme in relation to the beta values in the adjacent regimes if they are more than 100 percent greater than or less than the average of the adjacent regime betas. According to the prescription, transition betas are indicated in 46 of the firms. The average duration of the transition periods is 14.1 days, with 80 percent of them less than 20 days and 91 percent less than 25 days. Recalling that the minimum regime size is ten days, it seems reasonable that these effects are only transitory.

Thirty-nine (85 percent) of the transition betas are either negative or greater than 2.0. On average they are 815 percent greater than or less than the average of the adjacent betas. Twenty-four of the firms have negative betas in the transition period, with an average value of -3.08. The remaining 22 firms have positive beta values in the transition period with an average value of 3.33. These values compare to average pre- and post-transition betas of 0.53 and 0.60. In 30 of the 46 firms there is a significant (50 percent highest posterior density) change between the pre- and post-transition betas, with an average increase of 0.63 (n = 17) and an average decrease of -0.62 (n = 13). In the remaining 16 firms the post-transition beta is not significantly different from the pre-transition beta, with the average magnitude of the change equal to 0.13.

## SUMMARY AND CONCLUSION

This study reexamines the issue of market model stationarity and provides insight into the nature and extent of the problem. Using daily returns and a more general shifting regimes model presented by Mehta and Beranek (1982), stationarity violations are detected in 71 percent of the firms in the sample, with the violations occurring uniformly throughout the year. Some 236 parameter shift points are detected in the 150 firms. At better than even posterior odds (50 percent highest posterior density interval), all but 17 can be associated with a significant change in at least one of the market model parameters.

Although small firms appear to generate a higher number of shift points, we find only slight statistical evidence to indicate an inverse relationship between firm size and number of shift points. We also find no evidence of a relationship between the changes in alpha and beta, but we do find further evidence of the tendency for increases in beta to follow lower betas and decreases to follow higher betas.

Of particular interest is the indication in many of the firms that changes in beta may not occur instantaneously, but may go through a transition period from one value to another. These transition periods are characterized by short regimes containing extreme or abnormal beta values. The possibility of observing gradual changes in beta may explain why there are far fewer firms in the sample with only one shift point. If changes in beta are often gradual rather than abrupt, we would expect to have fewer detections of just one shift point. While the evidence presented here is not conclusive, in the future techniques to model a gradual shift in beta [such as those in Bacon and Watts (1971) and Tsurumi (1980)] should be used to investigate this phenomenon further.

## REFERENCES

1. Bacon, D.W., and D.G. Watts, "Estimating the Transition Between Two Intersecting Straight Lines," *Biometrika*, 58 (1971), pp. 525-534.

2. Bey, R.P., "Market Model Stationarity of Individual Public Utilities," *Journal of Financial and Quantitative Analysis*, 18, no. 1 (March 1983a), pp. 67-85.

3. Bey, R.P., "The Market Model as an Appropriate Description of the Stochastic Process Generating Security Returns," *Journal of Financial Research*, 6 (1983b), pp. 275-288.

4. Blume, M.E., "On the Assessment of Risk," *Journal of Finance*, 26, no. 1 (March 1971), pp. 1-10.

5. Blume, M.E., "Betas and Their Regression Tendencies," *Journal of Finance*, 30, no. 3 (June 1975), pp. 785-795.

6. Box, G.E.P., and G.C. Tiao, *Bayesian Inference in Statistical Analysis* (Addison-Wesley, 1973).

7. Brown, Keith C., Larry J. Lockwood, and Scott L. Lummer, "An Examination of Event Dependency and Structural Change in Security Pricing Models," *Journal of Financial and Quantitative Analysis*, 20, no. 3 (September 1985), pp. 315-334.

8. Brown, R., J. Durbin, and J. Evans, "Techniques for Testing the Constancy of Regression Relationships Over Time," *Journal of the Royal Statistical Society*, 37, no. 2 (1975), pp. 149-163.

9. Fabozzi, F.J., and J.C. Francis, "Beta as a Random Coefficient," *Journal of Financial and Quantitative Analysis*, 13 (1978), pp. 101-116.

10. Guthery, Scott, "Partition Regression," *Journal of the American Statistical Association*, 69, no. 348 (December 1974), pp. 945-947.

11. Hamada, R.S., "The Effects of the Firms Capital Structure on the Systematic Risk of Common Stocks," *Journal of Finance*, 27 (1972), pp. 435-52.

12. Hays, P.A., and D.E. Upton, "A Shifting Regimes Approach to the Stationarity of the Market Model Parameters of Individual Securities," *Journal of Financial and Quantitative Analysis*, 21, no. 3 (September 1986), pp. 307-21.

13. Howe, T.S., and D.E. Upton, "Detection of Beta Shifts," *Quarterly Journal of Business and Economics*, 31, no. 3 (Summer 1992), pp. 20-37.

14. Hsu, D.A., "A Bayesian Robust Detection of Shift in the Risk Structure of Stock Market Returns," *Journal of the American Statistical Association*, 77, no. 377 (March 1982), pp. 29-39.

15. Kon, S.J., and F.C. Jen, "Estimation of Time-Varying Systematic Risk and Performance for Mutual Fund Portfolios: An Application of Switching Regression," *Journal of Finance*, 33, no. 2 (May 1978), pp. 457-475.

16. Kon, S.J., and W.P. Lau, "Specification Tests for Portfolio Regression Parameter Stationarity and the Implications for Empirical Research," *Journal of Finance*, 34, no. 2 (May 1979), pp. 451-465.

17. Lee, C.F., and C.R. Chen, "Beta Stability and Tendency: An Application of a Variable Mean Response Regression Model," *Journal of Economics and Business*, 34, no. 3 (1982), pp. 201-206.

18. Lev, B., "On the Association Between Operating Leverage and Risk," *Journal of Financial and Quantitative Analysis*, 9 (1974), pp. 627-641.

19. Levy, Robert A., "On the Short-Term Stationarity of Beta Coefficients," *Financial Analysts Journal*, 27, no. 6 (November-December 1971), pp. 55-62.

20. Lockwood, Larry J., and K.R. Kadiyala, "Risk Measurement for Event-Dependent Security Returns," *Journal of Business and Economic Statistics*, 6 (1988), pp. 43-49.

21. McDonald, B., and W.D. Nichols, "Nonstationarity of Beta and Tests of Market Efficiency," *Journal of Financial Research*, 7 (1984), pp. 315-322.

22. Mehta, C.R., and W. Beranek, "Tracking Asset Volatility by Means of a Bayesian Switching Regression," *Journal of Financial and Quantitative Analysis*, 17, no. 2 (June 1982), pp. 241-63.

23. Miller, Tom W., and Nicholas Gressis, "Nonstationarity and Evaluation of Mutual Fund Performance," *Journal of Financial and Quantitative Analysis*, 15, no. 3 (September 1980), pp. 639-654.

24. Patil, V.H., "The Behrens-Fisher Problem and Its Bayesian Solution," *Journal of the Indian Statistical Association*, 2 (1964), pp. 21-31.

25. Quandt, R.E., "The Estimation of the Parameters of a Linear Regression System Obeying Two Separate Regimes," *Journal of the American Statistical Association*, 53, no. 284 (December 1958), pp. 873-880.

26. Quandt, R.E., "Tests of the Hypothesis That a Linear Regression System Obeys Two Separate Regimes," *Journal of the American Statistical Association*, 55, no. 290 (June 1960), pp. 324-330.

27. Roll, R., "$R^2$," *Journal of Finance*, 43, no. 2 (July 1988), pp. 541-566.

28. Schwarz, G., "Estimating the Dimension of a Model," *Annals of Statistics*, 6, no. 2 (March 1978), pp. 461-464.

29. Sunder, S., "Stationarity of Market Risk: Random Coefficients Tests for Individual Stocks," *Journal of Finance*, 35, no. 4 (September 1980), pp. 883-896.

30. Thompson, R.B., C. Olsen, and J.R. Dietrich, "Attributes of News About Firms: An Analysis of Firm-Specific News Reported in the Wall Street Journal Index," *Journal of Accounting Research*, 25, no. 2 (Autumn 1987), pp. 245-274.

31. Tsurumi, H., "A Bayesian Estimation of Structural Shifts by Gradual Switching Regressions with an Application to the U.S. Gasoline Market," in Arnold Zellner (ed.), *Bayesian Analysis in Econometrics and Statistics*, (1980) pp. 213-240.

32. Zellner, Arnold, *An Introduction to Bayesian Inference in Econometrics* (Wiley, 1971).