Misestimation of Systematic Risk and Magnitude of the Size Effect: Evidence Using Direct and Reverse Regressions

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This study examines a lower boundary estimate of the well-known company size effect using reverse regression. Estimates of abnormal returns in prior studies use direct regression that provides upper bound estimates of the small firm abnormal return. The size effect is diminished materially with the reverse regression. The effect is still material and significant, however, even with reverse regression. Results are consistent with expectations and run counter to prior reverse regression results reported by Booth and Smith (1985).

INTRODUCTION

The issue of measurement errors in financial economics has received much attention in recent years. Many researchers caution about evidence of large abnormal returns on portfolios based on firm-specific variables such as firm size, stock price, price to earnings ratio, and dividend yield (e.g., Banz, 1981; Reinganum, 1981; Keim, 1983) may be due to measurement errors (e.g., Roll, 1977, 1980, 1981a,b; Dimson, 1979; Fowler and Rorke, 1983).

Roll (1981a), along with others, indicates that tests of the CAPM require *ex ante* returns on all assets as well as the market portfolio. Due to the inability of meeting these requirements, proxy measures are used. The use of proxy variables results in two sources of errors. First, observed returns are used, which leads to measurement errors in both asset and portfolio returns. Second, broad based market portfolios are used as proxies for the market portfolio. As a result, the market portfolio is characterized by a second source of error. Roll further points out that the beta estimates, in general, tend to be biased downward when using these surrogate measures. Consequently, the estimates of abnormal returns using these betas are biased upward.

In addressing the issue of the effect of measurement errors on the size effect, Booth and Smith (1985) use the direct and reverse regression approach to derive bounds on the coefficients of systematic risks (betas) and abnormal returns (alphas). They show that estimates of systematic risk of size-based portfolios, using the traditional ordinary least squares (OLS) regression employed in prior studies, are downward biased. Surprisingly, after adjusting for this bias using the risk estimates from reverse regressions, the abnormal returns are even larger. This finding is counterintuitive. An increase in risk estimates

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should cause a corresponding decrease in abnormal returns because both the average market risk premium and the portfolio returns are positive over their sample period.

In this study, we employ the direct and reverse regression methodology developed in Booth and Smith (1985) and show that estimates of abnormal returns derived using the traditional OLS regression procedure are overestimated. These findings hold over the current 63 year study period, 1926-1988, as well as over the 19 year period, 1963-1981, analyzed by Booth and Smith. Results from 20 size-based portfolios show that the magnitude of the size effect is diminished considerably when systematic risk estimates from reverse regressions are used for risk adjustment. We also find that the choice of benchmark portfolio impacts the magnitude of the size effect significantly. Size effect estimated with OLS regression is much larger when a value-weighted market portfolio is employed instead of an equally weighted market portfolio.

DATA AND METHODOLOGY

The data set comprises all NYSE and AMEX common stocks with monthly returns and year-end market values available on the Center for Research in Security Prices (CRSP) monthly master tape. At the beginning of each year from 1926 to 1988, all stocks in the sample are divided equally into 20 size-based portfolios, MV1 (smallest) to MV20 (largest), according to the market value of equity at the end of the prior year. Monthly returns on portfolios are obtained for each month in the year under consideration by averaging returns across all stocks in each portfolio.

Frisch (1934) and Kalman (1982) address the issue of errors in variables in OLS estimation of regression coefficients. These studies show that if there are errors in observed variables, the slope coefficient of regression, or beta for a market model, will lie within a range where the lower bound is the ordinary least squares (direct regression) coefficient and the upper bound is the reverse regression coefficient.

To examine the impact of measurement error on beta, we estimate systematic risks (betas) and abnormal returns (alphas) for the 20 size-based portfolios using the following direct and reverse regressions (see, e.g., Booth and Smith (1985)):

Direct regression:

(1) $R_t - R_{ft} = \alpha_D + \beta_D(R_{mt} - R_{ft}) + u_t$

Reverse regression:

(2) $R_{mt} - R_{ft} = \alpha + \beta(R_t - R_{ft}) + v_t$

where:

R_t, R_{mt}, and R_{ft} = The return in month t on a particular size portfolio, return on market portfolio (CRSP equally weighted or value-weighted portfolio), and risk free rate (U.S. Treasury bills).

α_D and β_D in the direct regression (1) are the traditional estimates of abnormal return and systematic risk employed in prior studies. The corresponding reverse regression estimates α_R and β_R are obtained by rearranging the reverse regression model (2) as:

(3) $R_t - R_{ft} = -\alpha/\beta + (1/\beta)(R_{mt} - R_{ft}) - v_t/\beta$

or

(4) $R_t - R_{ft} = \alpha_R + \beta_R(R_{mt} - R_{ft}) + w_t$

where:

$a_R = -\alpha/\beta$;
$\beta_R = 1/\beta$; and
$w_t = -v_t/\beta$.

Therefore, β_D and β_R are respectively the lower and upper bounds of the beta, i.e., $\beta_D \leq \beta \leq \beta_R$. Note that if the means of $(R_t - R_{ft})$ and $(R_{mt} - R_{ft})$ are positive then α_D and α_R are the upper and lower bounds, respectively, of the alpha (estimate of abnormal return). Further, the closer the means $(R_t - R_{ft})$ and $(R_{mt} - R_{ft})$ are to zero, the smaller will be the difference between α_D and α_R. Empirically, the average return on portfolios of smaller firms is greater than the average return on large firm portfolios. Therefore, the alpha bounds should be wider for small firm portfolios than for large firm portfolios. This leads to the likelihood that the abnormal return estimate, α_D, for a small firm portfolio is likely to be more biased than those for a large firm portfolio.

RESULTS

Descriptive statistics on the 20 size-based portfolios, MV1-MV20, over the period 1926 to 1988 are presented in Table 1. The average market value of equity ranges from \$2.36 million for portfolio MV1 to \$2,537.58 million for portfolio MV20. As expected, the smallest portfolio has both the highest average return and standard deviation, and the largest portfolio has the lowest average return and standard deviation. An inverse relationship between firm size and returns is evident. Consistent with Keim (1983), most of the difference in returns between the smallest and the largest portfolios occurs in January. Outside of January, except for portfolios MV1 and MV2, there is no noticeable difference in average returns across portfolios. Similar statistics for returns on different market portfolios and the risk free rate are reported in the lower panel of Table 1.

Table 2 reports estimates of systematic risk (betas) and average abnormal returns (alphas) for selected portfolios MV1 (smallest), MV5, MV10, MV15, and MV20 (largest) with direct (OLS) and reverse regressions. The sample period is 1926 to 1988. The CRSP equally weighted index is used to proxy the market portfolio. Results are reported for all months as well as for January and non-January months. Separation of January from other months is required because the size effect documented in prior studies is predominantly a January phenomenon. The results contain evidence that estimates of systematic risk for both the smallest and largest size portfolios are lower with direct (OLS) regression than

Table 1—Summary Statistics of Portfolios, 1926-1988

		Average Returns and Standard Deviation of Returns					
		All Months (756)		Januarys (63)		Other Months (693)	
Portfolio[a]	Average Market Value[b]	Average Return	Standard Deviation	Average Return	Standard Deviation	Average Return	Standard Deviation
MV1	2.36	2.81%	13.50%	17.17%	17.00%	1.50%	12.35%
MV2	4.63	2.03	11.96	12.69	13.59	1.06	11.32
MV3	6.84	1.59	10.92	10.08	10.91	0.82	10.60
MV4	9.44	1.61	10.23	9.01	10.65	0.93	9.93
MV5	12.45	1.33	9.68	7.59	9.93	0.76	9.46
MV6	16.01	1.29	8.98	7.41	8.90	0.73	8.78
MV7	20.25	1.40	9.14	6.33	8.47	0.96	9.08
MV8	25.22	1.26	8.42	5.89	8.33	0.84	8.31
MV9	31.29	1.21	8.20	5.34	8.06	0.84	8.11
MV10	39.30	1.19	8.12	5.29	7.67	0.82	8.06
MV11	49.67	1.22	7.65	4.80	7.26	0.89	7.61
MV12	63.04	1.20	7.98	4.41	7.79	0.91	7.94
MV13	81.28	1.19	7.32	3.85	6.67	0.95	7.33
MV14	106.37	1.15	7.39	3.36	6.68	0.95	7.43
MV15	141.67	1.05	7.03	3.30	6.39	0.85	7.06
MV16	194.43	0.98	6.79	2.66	5.86	0.83	6.85
MV17	275.15	1.09	6.81	2.56	5.72	0.96	6.89
MV18	413.11	1.07	6.44	2.61	5.51	0.93	6.50
MV19	666.77	0.90	5.97	1.87	5.07	0.81	6.04
MV20	2537.58	0.86	5.60	1.14	4.82	0.83	5.67
Market:							
CRSP E-W portfolio		1.33	7.96	5.95	7.75	0.91	7.85
CRSP V-W portfolio		0.93	5.71	1.87	4.90	0.85	5.77
E-W portfolio of all sampled stocks		1.32	8.00	5.87	7.75	0.91	7.85
Risk Free Rate (U.S. T-Bills)		0.29	0.27	0.29	0.25	0.29	0.27

[a]MV1 is the smallest market value portfolio, and MV20 is the largest market value portfolio. The 20 market value portfolios, MV1-MV20, are revised at the beginning of each year over the 1926-1988 period

[b]Average market value is in millions of dollars

with reverse regression. This pattern, which is consistent with Booth and Smith's (1985) results, also holds for all intermediate portfolios over all months as well as in January and non-January months. Average abnormal returns are reduced considerably when risk is adjusted using betas obtained from the reverse regression. Panel A contains results for all months including Januarys. Using direct regression, the average abnormal return is 0.99 percent per month on the smallest MV1 portfolio and -0.08 percent on the largest MV20 portfolio. With reverse regression these are reduced to an insignificant 0.47 percent and a significant -0.26 percent, respectively. Consequently, the magnitude of the size effect represented by MV1 to MV20 is reduced from 1.07 percent per month (or about 12.84 percent per year) to 0.73 percent (or about 8.76 percent per year). This reduction occurs both in January and non-January months (Panels B and C, respectively). The January size effect of 6.00 (2.36 percent + 3.64 percent) percent is still significantly large and positive, however, although materially smaller than the 8.15 percent found with direct regression.

Table 2—Estimates of Systematic Risks and Abnormal Returns on Market Value Portfolios With Direct and Reverse Regressions Using the CRSP Equally Weighted Market Portfolio, 1926-1988

The two regression models are:

Direct regression: (1) $R_t - R_{ft} = \alpha_D + \beta_D(R_{mt} - R_{ft}) + u_t$

Reverse regression: (2) $R_{mt} - R_{ft} = \alpha + \beta(R_t - R_{ft}) + v_t$

The reverse regression estimates of α_R and β_R are derived as $\alpha_R = -\alpha/\beta$ and $\beta_R = 1/\beta$

	Systematic Risk		Average Abnormal Return	
Portfolio	β_D	β_R	α_D	α_R
Panel A: All Months (756)				
MV1	1.46***	1.96***	0.0099***	0.0047
MV5	1.18***	1.26***	-0.0018**	-0.0026***
MV10	1.00***	1.04***	-0.0013**	-0.0018***
MV15	0.85***	0.92***	-0.0012*	-0.0019**
MV20	0.62***	0.80***	-0.0008	-0.0026***
Panel B: Januarys (63)				
MV1	1.88***	2.56***	0.0622***	0.0236
MV5	1.25***	1.32***	0.0024	0.0015
MV10	0.95***	1.03***	-0.0038	-0.0084**
MV15	0.79***	0.86***	-0.0145***	-0.0187***
MV20	0.49***	0.79***	-0.0193***	-0.0364***
Panel C: Non-January Months (693)				
MV1	1.37***	1.80***	0.0036	0.0009
MV5	1.16***	1.25***	-0.0025***	-0.0029***
MV10	1.01***	1.05***	-0.0009	-0.0012*
MV15	0.87***	0.93***	0.0002	-0.0002
MV20	0.65***	0.80***	0.0014	0.0004

* Significant at the 10 percent level based on a two tailed t-test
** Significant at the 5 percent level based on a two tailed t-test
*** Significant at the 1 percent level based on a two tailed t-test

The results with the CRSP value-weighted market portfolio are in Table 3. The January size effect of 8.5 percent (7.77 percent + 0.73 percent from Table 3, Panel B) with direct regression is much larger when the CRSP value-weighted market portfolio is used than with the CRSP equally weighted market portfolio used in Table 2. The proportional reduction in the January size effect in Table 3, however, is considerably greater with the reverse regression approach.

We conclude from both Tables 1 and 2 that the magnitude of size effect is impacted significantly by the procedure used for risk estimation where the size effect is smaller with reverse regression than with direct regression. Additionally, market portfolio choice has a significant impact on the size effect. The equally weighted portfolio provides the smallest size effect while the value-weighted portfolio demonstrates the largest proportional decrease in the size effect when using reverse versus direct regression. With both portfolio proxies, however, the size effect is smaller with reverse over direct regression.

Table 3—Estimates of Systematic Risks and Abnormal Returns on Market Value Portfolios With Direct and Reverse Regressions Using the CRSP Value-Weighted Market Portfolio, 1926-1988

The two regression models are:

Direct regression: (1) $R_t - R_{ft} = \alpha_D + \beta_D(R_{mt} - R_{ft}) + u_t$

Reverse regression: (2) $R_{mt} - R_{ft} = \alpha + \beta(R_t - R_{ft}) + v_t$

The reverse regression estimates of α_R and β_R are derived as $\alpha_R = -\alpha/\beta$ and $\beta_R = 1/\beta$

	Systematic Risk		Average Abnormal Return	
Portfolio	β_D	β_R	α_D	α_R
Panel A: All Months (756)				
MV1	1.58***	3.52***	0.0150***	0.0024
MV5	1.42***	2.02***	0.0013	-0.0025
MV10	1.31***	1.54***	0.0006	-0.0009
MV15	1.18***	1.28***	0.0000	-0.0006
MV20	0.97***	0.99***	-0.0006*	-0.0007**
Panel B: Januarys (63)				
MV1	2.09***	5.75***	0.1357***	0.0777*
MV5	1.62***	2.54***	0.0475***	0.0329***
MV10	1.33***	1.84***	0.0289***	0.0209***
MV15	1.22***	1.39***	0.0108***	0.0081**
MV20	0.97***	1.00***	-0.0068***	-0.0073***
Panel C: Non-January Months (693)				
MV1	1.52***	3.02***	0.0036	-0.0047
MV5	1.39***	1.92***	-0.0030	-0.0060***
MV10	1.30***	1.50***	-0.0020*	-0.0031**
MV15	1.18***	1.27***	-0.0010	-0.0015**
MV20	0.97***	0.99***	-0.0000	-0.0001

* Significant at the 10 percent level based on a two tailed t-test
** Significant at the 5 percent level based on a two tailed t-test
*** Significant at the 1 percent level based on a two tailed t-test

Our results conflict with the evidence reported in Booth and Smith (1985). They find abnormal returns on all portfolios increase with reverse regression. Their results are surprising given that they point out that

> ... the direct (and reverse) estimate of α may be either an upper or lower bound depending on the signs of the two data means [i.e., means of $R_t - R_{ft}$ and $R_{mt} - R_{ft}$]. If both are positive (as would typically be expected *ex ante* for capital market research), then a_D will be an upper bound and a_R will be a lower bound. However, if either (or both) is negative, the opposite will be true. (p. 506)

Table 1 shows that average returns on all (20) size-based portfolios and the (three) market portfolios are greater than the average risk free rate over the whole 756 month sample period 1926 to 1988. This also holds for both January and non-January months and (in Table 5) over the 228 month subperiod 1963 to 1981 that corresponds with the Booth and Smith (1985) study. Thus, the means of $R_t - R_{ft}$ and $R_{mt} - R_{ft}$ are positive over both

Table 4—Estimates of Systematic Risks and Abnormal Returns on the Smallest and Largest Market Value Portfolios With Direct and Reverse Regressions Using Different Market Portfolios, 1963-1981

The two regression models are:

Direct regression: (1) $R_t - R_{ft} = \alpha_D + \beta_D(R_{mt} - R_{ft}) + u_t$

Reverse regression: (2) $R_{mt} - R_{ft} = \alpha + \beta(R_t - R_{ft}) + v_t$

The reverse regression estimates of α_R and β_R are derived as $\alpha_R = -\alpha/\beta$ and $\beta_R = 1/\beta$

Portfolio	Systematic Risk β_D	Systematic Risk β_R	Average Abnormal Return α_D	Average Abnormal Return α_R
Panel A: All Months (228)				
Using CRSP Equally Weighted Market Portfolio:				
MV1	1.45***	1.84***	0.0099***	0.0068*
MV20	0.54***	0.86***	-0.0032*	-0.0058**
Using CRSP Value-Weighted Market Portfolio:				
MV1	1.43***	3.96***	0.0178***	0.0117***
MV20	0.98***	1.01***	-0.0013**	-0.0014**
Using Equally Weighted Portfolio of All Sampled Stocks:				
MV1	1.44***	1.86***	0.0104***	0.0073***
MV20	0.54***	0.86***	-0.0031*	-0.0055**
Panel B: Januarys (19)				
Using CRSP Equally Weighted Market Portfolio:				
MV1	1.54***	1.69***	0.0864***	0.0748***
MV20	0.50***	0.76***	-0.0321***	-0.0513***
Using CRSP Value-Weighted Market Portfolio:				
MV1	1.96***	3.34***	0.1699***	0.1479***
MV20	0.96***	1.00***	-0.0099***	-0.0104***
Using Equally Weighted Portfolio of All Sampled Stocks:				
MV1	1.55***	1.72***	0.0881***	0.0762***
MV20	0.51***	0.76***	-0.0318***	-0.0502***
Panel C: Non-January Months (209)				
Using CRSP Equally Weighted Market Portfolio:				
MV1	1.25***	1.63***	0.0027	0.0020
MV20	0.61***	0.90***	-0.0005	-0.0010*
Using CRSP Value-Weighted Market Portfolio:				
MV1	1.22***	3.08***	0.0034	0.0013
MV20	0.98***	1.02***	-0.0006	-0.0006
Using Equally Weighted Portfolio of All Sampled Stocks:				
MV1	1.25***	1.64***	0.0029	0.0023
MV20	0.61***	0.89***	-0.0004	-0.0008

* Significant at the 10 percent level based on a two tailed t-test
** Significant at the 5 percent level based on a two tailed t-test
*** Significant at the 1 percent level based on a two tailed t-test

Table 5—Average Monthly Market Returns and Risk Free Rate 1963-1981

Market	All Months (228)	Januarys (19)	Other Months (209)
CRSP E-W Portfolio	1.29%	7.97%	0.69%
CRSP V-W Portfolio	0.75	2.09	0.63
Risk-Free Rate U.S. T-bills	0.51	0.50	0.51

sample periods, 1926 to 1988 and 1963 to 1981. This supports the claim that α_D and α_R, respectively, are the upper and lower bounds of the estimated level of abnormal return.[1]

SUMMARY

This paper presents evidence that the magnitude of the size effect documented in prior studies using direct (OLS) regression are probably overstated. We show that betas when estimated with direct regression are considerably lower than those estimated with reverse regression. As a result, estimates of abnormal returns using direct regression beta (as reported in previous studies) essentially represent the upper boundary of the abnormal return. Further, this phenomenon holds irrespective of the market portfolio used for risk adjustment. Finally, the size effect is found only in January, and is more pronounced using the value-weighted market portfolio. In all cases, this size effect is diminished with reverse regression betas. However, material and significant January size effects are still generally found with the lower boundary estimate of abnormal returns generated with reverse regressions.

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[1] We also follow an alternative approach that examines the feasibility of evidence reported in Booth and Smith (1985). Subtracting our regression equation (1) from equation (4), taking expectation of both sides, and simplifying yields an expression for the mean of $R_{mt} - R_{ft}$ as $(\alpha_D - \alpha_R)/(\beta_R - \beta_D)$. Using their reported estimates, $\alpha_D = 0.023$, $\alpha_R = 0.042$, $\beta_D = 1.383$, and $\beta_R = 2.005$, for the smallest size portfolio of Booth and Smith (1985) from the results in their Table 1 (p. 510), we find that mean $R_{mt} - R_{ft}$ would have to be $(0.023 - 0.041)(2.005 - 1.383) = -0.02894$ or -2.894 percent per month to validate their results. Such a large negative average monthly market premium over their 228-month test period does not match historical evidence. Over this period, the average $R_{mt} = 1.29$ percent using the CRSP equally-weighted index and average $R_{ft} = 0.51$ percent based on U.S. Treasury bill rates. This yields a positive average market premium of 0.78 percent per month.

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