The Effects of Inaccurate Parameter Estimates in Cost Variance Investigation Decisions

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This study uses the Dittman-Prakash Markovian model to investigate how the cost variance investigation decision is affected by inaccuracies in the following four parameters: standard deviation of the cost variance distributions; state transition probability; investigation cost; and correction cost. Among the many patterns of effects, errors in investigation and correction costs are usually not damaging. In contrast, transition probability should be estimated more accurately, as inaccuracy in this parameter can lead to serious cost consequences.

An effective approach of defining different CVID situations using normalization also is presented. This approach should be useful in future CVID evaluation studies.

INTRODUCTION

Formulating a cost variance investigation decision (CVID) as a cost minimization problem for a multiperiod two state process (Kaplan, 1969, 1975, 1982) is accepted as theoretically sound; well-defined solution procedures are available (Kaplan, 1969; Dittman and Prakash, 1978, 1979). This model, however, uses several parameters that are often difficult and expensive to estimate accurately. The purpose of this study is to investigate how inaccuracies in these parameters affect the quality of the CVID made with Dittman-Prakash's procedure. Our results are useful for estimating the appropriate expenses warranted for obtaining each cost parameter and in evaluating the robustness and superiority of the two state cost minimization CVID models relative to the earlier and simpler alternatives.

Many managerial accounting researchers have been concerned about the mathematical and/or computational complexity of the more sophisticated models; therefore, some researchers considered the use of simpler models. (See, e.g., Magee's (1976) study in the case of CVID models.) Given today's proliferation of computers and software, however, the computation cost of even the most complex managerial accounting models (e.g., Kaplan's (1969) CVID model) is almost always trivial compared to the cost of determining the values of the model's parameters. Rejecting a managerial accounting model may be justified by the high cost of parameter estimation or the sensitivity of the solutions' quality to parameter errors, but seldom by mathematical/computational considerations.

BRIEF REVIEW OF THE CVID PROBLEM

Reviews of CVID models are found in Kaplan (1975, 1982), Magee (1976), and Greenberg (1986). We consider only the following formulation:

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- At the end of each period the cost generation process can be either in control or out of control;
- One decides whether an investigation (at a cost) is warranted on the basis of a reported cost (or cost variance) x for the period;
- If an investigation reveals that the process is out of control, it is corrected (at a cost) so that the next period begins in control;
- One has to determine a policy R to be used in all periods as follows: investigate if $x > R$, otherwise do not investigate.

Among procedures for solving this formulation, Kaplan's (1969) dynamic programming solution gives the multiperiod theoretical optimum, but Dittman-Prakash's procedure (1978, 1979) is accepted more widely because it is much simpler and yet gives solutions close to the optima.

SUMMARY OF DITTMAN-PRAKASH'S APPROACH

Dittman and Prakash state their model in terms of cost distributions; the following modification uses the cost-variance distributions, which matches our needs better.

Dittman and Prakash show that C'(R), the expected system cost (including operating, investigation, and correcting costs) per period for any policy R, is:

$$(1)\ C'(R) = M_i + C(R)$$

where:

$$(2)\ C(R) = \delta - [gI.F_i(r) - I - (1\text{-}g)K + g\delta] \cdot \frac{1\text{-}F_o(r)}{1\text{-}gF_o(r)}\ .$$

The parameters are:

M_i = Mean operating cost/period when the process is in control;
g = Transition probability, the probability that the cost process remains in control at the end of a period given that it was in control at the beginning of the period;
$F_i(.)$ = Distribution function of the in control cost-variance distribution, with mean = 0;
$F_o(.)$ = Distribution function of the out-of-control cost-variance distribution, with mean = δ;
I = Investigation cost; and
K = Cost to correct an out-of-control process.

The objective is to find R^* that minimizes C'(R) in equation (1), which is equivalent to finding R^* that minimizes C(R) in equation. (2) because M_i is fixed and does not vary with R for a given process. This paper considers only C(R).

AN ILLUSTRATIVE STATEMENT OF OUR PROBLEM

INACCURACY IN ESTIMATING THE STANDARD DEVIATION OF THE VARIANCE DISTRIBUTIONS

Assume that the in control and out-of-control cost variance distributions are both normally distributed and have the same standard deviation (σ) and that the parameters of the CVID problem are estimated to be:

(3) $\delta = 20, \sigma = \sigma_i = \sigma_o = 25, g = 0.95, I = 100, K = 30$

Using a Fortran procedure of golden section search (Wagner, 1969) with these parameters shows that C(R) in equation (2) is minimized at R = 49.59; i.e., the policy R = 49.59 will be implemented. Assume now that all other parameters are correct, but σ has been estimated incorrectly; the correct value is $\sigma_c = 12.5$ (the subscript c denotes correct). In this study the magnitude of parameter error will be measured by the correction factor (CF) needed to correct the wrong estimate, i.e.,

(4) Correct Parameter Value = CF $\times$ Estimated Parameter Value or

$$CF = \frac{\text{Correct Parameter Value}}{\text{Estimated Parameter Value}} .$$

Hence, CF = 12.5/25 = 0.5 here. Minimizing C(R) with the parameters in equation (3) but $\sigma = 12.5$ shows that if the correct σ had been known, the correct policy $R_c^* = 28.8$ would have been implemented with an attainable cost of $C(R_c^*) = 10.07$. Substituting the actual R = 49.59 into equation (2) with the correct parameters, however, gives $C(R_w) = 18.084$ (the subscript w denotes "wrong"). The percentage cost increment (PCI) of the wrong decision caused by an error of CF = 0.5 in estimating σ is:

$$(5)\ PCI = 100 \cdot [C(R_w) - C(R_c^*)]/C(R_c^*)$$

$$= \frac{100*(18.08-10.074)}{10.074} = 79.5 \text{ percent, a substantial figure.}$$

Consider another situation where the parameters are estimated as:

(6) $\delta = 20, \sigma = \sigma_i = \sigma_o = 25, g = 0.85, I = 4, K = 12.$

R = 6.676 minimizes C(R) with these parameters. Assume again that the correct σ_c is 12.5, but all other parameters have been estimated correctly. Minimizing C(R) with the corrected parameters gives $C(R_c^*$ = 6.658, but substituting R = 6.676 into C(R) with the correct parameters gives $C(R_w) = 6.741$. The wrong decision produced by the same substantial error in σ(CF = 0.5) now leads to a negligible PCI of 100*(6.741 - 6.658)/6.658 = 1.2 percent.

One cannot generalize whether it is important to determine σ accurately. One should allocate more resources to estimate σ accurately in the situation defined in equation (3), but not in the situation defined in equation (6).

INACCURACY IN ESTIMATING THE TRANSITION PROBABILITY

Consider the situation with estimated parameters:

(7) $\delta = 10$, $\sigma = 2.5$, $g = 0.95$, $I = 10$, $K = 30$,

where $R = 6.88$ minimizes $C(R)$. Assume now that only g is incorrect, and CF = 0.6; i.e., $g_c = 0.6*0.95 = 0.57$. The minimum cost achievable under the correct parameters is $C(R_c^*) = 10$, but substituting $R = 6.88$ into equation (2) with the correct parameters gives $C(R_w) = 20.958$. The PCI is 100*(20.958-10)/10 or 109.6 percent.

Repeating the analysis for the situation with parameters:

(8) $\delta = 10$, $\sigma = 2.5$, $g = 0.95$, $g_c = 0.57$ (i.e., CF = 0.6), $I = 10$, $K = 3$

gives $R = 6.793$ using the wrong $g = 0.95$; $C(R_c^*) = 9.905$ with g_c, and $C(R_w) = 9.913$ with $R = 6.793$ and the correct parameters. The PCI is now a trivial 100*(9.913-9.905)/9.905 = 0.08 percent; note that the g's error magnitudes are identical in both situations.

Similar analyses on the effects of errors in the other parameters such as I, K, and δ point to the same conclusion: errors in these parameters lead to substantial cost increments in some situations but only negligible cost increments in other situations. The natural question at this point is whether there is a simpler way to estimate the effect of a suspected error in a parameter.

DEFINING THE PARAMETERS AND THEIR RANGES

To investigate the preceding question, tighter definitions of the CVID problem and its parameters are necessary.

SPECIFYING THE PARAMETERS TO BE CONSIDERED

Estimating the actual functions of $F_i(.)$ and $F_o(.)$ is rarely practical. Following most earlier studies, we assume $F_i(.)$ and $F_o(.)$ are Gaussian with equal variances, which reduces the problem to one of estimating δ and σ ($= \sigma_i = \sigma_o$). To reduce the number of free parameters (to keep the problem manageable) further, we now normalize the problem by setting $\delta = 1$ and expressing σ, I, and K in terms of δ. To illustrate, if a problem 1 with parameters $\{\delta = 1, \sigma = 0.5, g, I = 1.5, K = 2\}$ has optimal solutions R_1^* and $C_1(R_1^*)$, the optimal solutions for problem 2 with $\{\delta = 20, \sigma = 10, g, I = 30, K = 40\}$ are simply $R_2^* = 20R_1^*$ and $C_2(R_2^*) = 20C_1(R_1^*)$. Therefore, for the same CF, both problems have the same PCI, and problem 1 can be considered as a normalized problem 2.

With $\delta = 1$, there are four free parameters: σ, g, I, and K for the cost process. We have to study how the PCIs are affected by CFs of various magnitudes in each of these 4 parameters under different situations defined by different combinations of these four parameters.

Finley (1990) recently goes to the extreme of advocating that the estimation of g and δ be avoided completely by using the minimax criterion. He also assumes that errors in σ, I, and K are negligible.

RANGES OF THE PARAMETERS

Table 1 summarizes the parameters and their ranges considered in this study. With $\delta = 1$, σ now corresponds to Dittman and Prakash's coefficient of intrusion, and our range of σ corresponds to Dittman and Prakash's range of coefficient of intrusion. We fixed our g-range by noting that g should be high and at least somewhat greater than 0.5 for realistic systems. In contrast, earlier numerical CVID studies (e.g., Magee, 1976; Dittman and Prakash, 1978, 1979; Greenberg, 1986) consider g as low as 0.5 but no higher than 0.9. The problematic situations turn out to be those with high g's.

The possible ranges for I and K are necessarily wide for the normalized problem of realistic situations, and we have pegged K to I because they should have the same order of magnitude. Magee (1975), Dittman and Prakash (1978, 1979), and Greenberg (1986) consider only situations with K = 0. The realism of assuming K = 0 is questionable; moreover, the problem's behavior is sensitive to the value of K.

Table 1—Summary of Ranges and Values of Parameters Considered

Parameter	Range in Regressions	Values Considered in Graphs for Errors in σ, I, and K
CF	0.4 to 2.0	0.4 to 2.0, in steps of 0.2; for errors in g: .65, .75, .85, .95, 1., 1.05, 1.1, 1.2, 1.3
σ	0.25 to 1.25	0.25, 0.50, 0.75, 1.00, 1.25
g	0.75 to 0.95	0.75, 0.85, 0.90, 0.95
I	0.10 to 10.0	0.1, 0.2, 1.0, 5.0, 10.0
K	0.3KI to 3KI	0.3I, I, 3I

It easily can be verified that the situations considered in Magee (1976) correspond to normalized situations with $\delta = 1$ or 0.4, I = 0.2 to 1.2, and K = 0 (see his Tables 1 and 2). A narrower range of situations are considered in Dittman and Prakash (1978, 1979) and Greenberg (1986). The advantage of using our normalizing approach to range situations should be apparent. If a numerical study considers too narrow a range of situations, the conclusions may be invalid for many overlooked realistic situations.

APPROACHES FOR SUMMARIZING THE EFFECTS OF PARAMETER INACCURACIES

Our initial approach of constructing tables of PCI for various combinations of parameter values proves to be inappropriate for two reasons: because there are five parameters, the number of different parameter-combinations that must be considered is large; and the relationships among PCI and the five parameters are complex and cannot be revealed properly in tabular forms.

Reported below are results for each of the four system parameters (σ, g, I, and K) from two alternative approaches: finding a regression function for PCI and presenting the variation of PCI graphically. Although still not completely adequate, they are more effective than the tabular approach.

COST EFFECT OF ERROR IN σ

SEARCHING FOR A REGRESSION FORMULA TO PREDICT PCI

A random problem can be generated by generating a random value for each parameter CF, σ_w, g, I, and K in the ranges given in Table 1. Given CF, the value of the σ_c is determined, and PCI is computed with these parameters. For example, a random problem might be:

(9) $CF = 0.9493, I = 1.7264, g = 0.7673, \sigma_w = 1.6349, K = 0.7806 \cdot I$
giving PCI = 0.0669 percent.

Dittman and Prakash show that only situations that satisfy the criterion:

(10) $gI > [I + (1 - g)K - g\delta]$

are of interest, and we generate 5,000 random problems that satisfy this criterion. The resultant 5,000 PCIs have the following statistical characteristics:

(11) mean = 5.77, s.d. = 9.03, skewness = 3.33, range: 0.0 to 96.6
quantiles: 50% (median): 2.24, 75%: 7.32, 90%: 15.9, 95%: 22.7.

The PCI distribution is highly skewed. In most situations the PCI is small; i.e., the effect of the error in σ is not serious. There are situations where the PCI is high, however, and it is important to identify such situations.

With the 5,000 sets of values like those shown in equation (9), we use stepwise regression to search for a formula that can predict PCI as a function of σ_w, g, I, K, and CF (either σ_w or σ_c can be used here; we choose σ_w because it is implicitly assumed that the analyst does not know σ_c, while CF is merely a hunch of the possible error magnitude). Although many different functional forms were tried, we are unable to find a sufficiently reliable formula for predicting PCI (i.e., one with a R^2 of > 0.9). We then turn our attention to finding a reasonably parsimonious formula to depict the effects of the various parameters on PCI. One of the better performing formulas is:

(12) $PCI = 50.3 + 18.4g^6(\sqrt[4]{I}) - 0.27(CF)^7 - 73.2\sqrt{CF} + 10.9(CF)^3 + 4.95\sigma_w - 0.80K,$
$R^2 = 0.5003.$

The stepwise improvement in R^2 is summarized in Table 2. Equation (12) and Table 2 indicate that the relationship between PCI and the parameters is complex; that PCI is sensitive to g, as the first term to enter the stepwise regression involves the sixth power of g; that I is also an important factor; and that either σ_w and K have relatively little effect on PCI or the effect of σ_w and/or K on PCI is complex and cannot be captured by simple functional forms because terms involving σ_w and K enter late in the stepwise regression procedure and have small partial-R^2. (Large numbers of functional forms involving σ_w and K were tried in the stepwise regression procedure).

Although a large sample size (n = 5,000) is used to obtain the preceding results, we nevertheless repeat the experiment with two additional samples of 5,000 random problems; they lead to the same results. Therefore, a crude rule for handling errors in σ may be that in situations where both g and I are high, errors in estimating σ can lead to substantial PCI.

Table 2-Summary of Forward Stepwise Procedure

Step	Variable Entered	Partial R^2	Model R^2
1	$g^6(\sqrt[4]{I})$	0.1657	0.1657
2	$(CF)^7$	0.0955	0.2612
3	$\sqrt{CF}$	0.0806	0.3417
4	$(CF)^3$	0.1154	0.4571
5	σ_W	0.0248	0.4819
6	K	0.0185	0.5004

GRAPHS DEPICTING THE VARIATION OF PCI WITH CF

To facilitate the use of the preceding rule quantitatively, we plot numerous curves depicting the variation of PCI with CF for different combinations of parameters (σ_w, g, I, K), and consider many different ways of grouping them; values considered for each parameter are given in Table 1. This is accomplished by importing the mainframe-computed PCIs and their associated parameters to a Macintosh microcomputer and using Cricket software to plot the hundreds of required graphs. The subset of curves presented in Figures 1 to 4 to depict the general pattern is selected and grouped to best illustrate the following characteristics:

- The total set of curves confirms the suggestion from the preceding regression analyses on K; i.e., the effect of K on PCI is small, and higher K tends to cause lower PCI. Therefore we present only curves with K = 0.3I.
- Our curves also confirm that I has a strong and simple positive effect on PCI. Figure 1 presents all cases when I = 0.1; at this I-level, PCI is never large, no matter what values other parameters may have. Figures 2 to 4 present cases for larger Is. The effect of I is obvious from the different vertical scales in the figures.
- Our curves reveal an effect of σ_w that is missed by the regression analyses. This effect is highlighted by grouping curves for the same σ_w value (σ_w = 0.25, 0.50 and 1.25 in Figures 2, 3, and 4, respectively). In addition to confirming that higher σ_w tends to give higher PCI, the curves also reveal a more interesting effect: the curves' left is much lower than their right when σ_w is low (Figure 2), but the left becomes higher than the right when σ_w is larger Figure 4). This means that when σ_w is low, overestimating σ (CF < 1) causes little damage although underestimating σ leads to a significant

Figure 1—Error in SD (I = 0.1 and K = 0.3I in all cases)

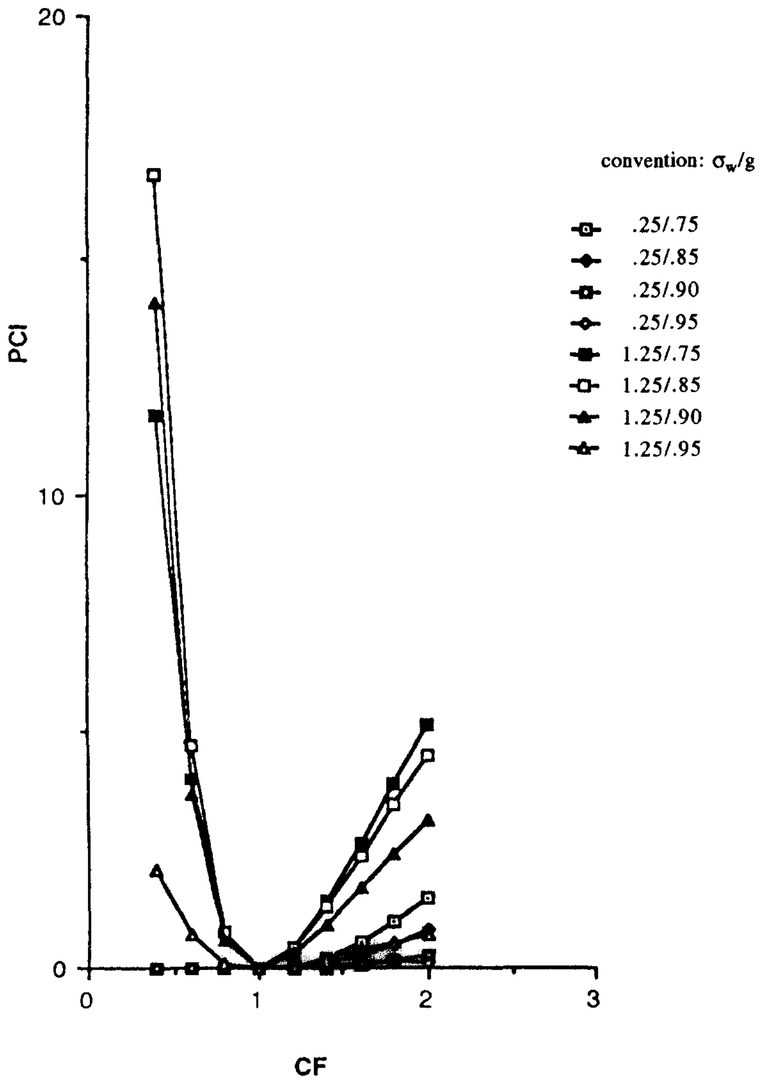


Figure 2—Error in SD (σ_w = 0.25 and K = 0.3I in all cases)

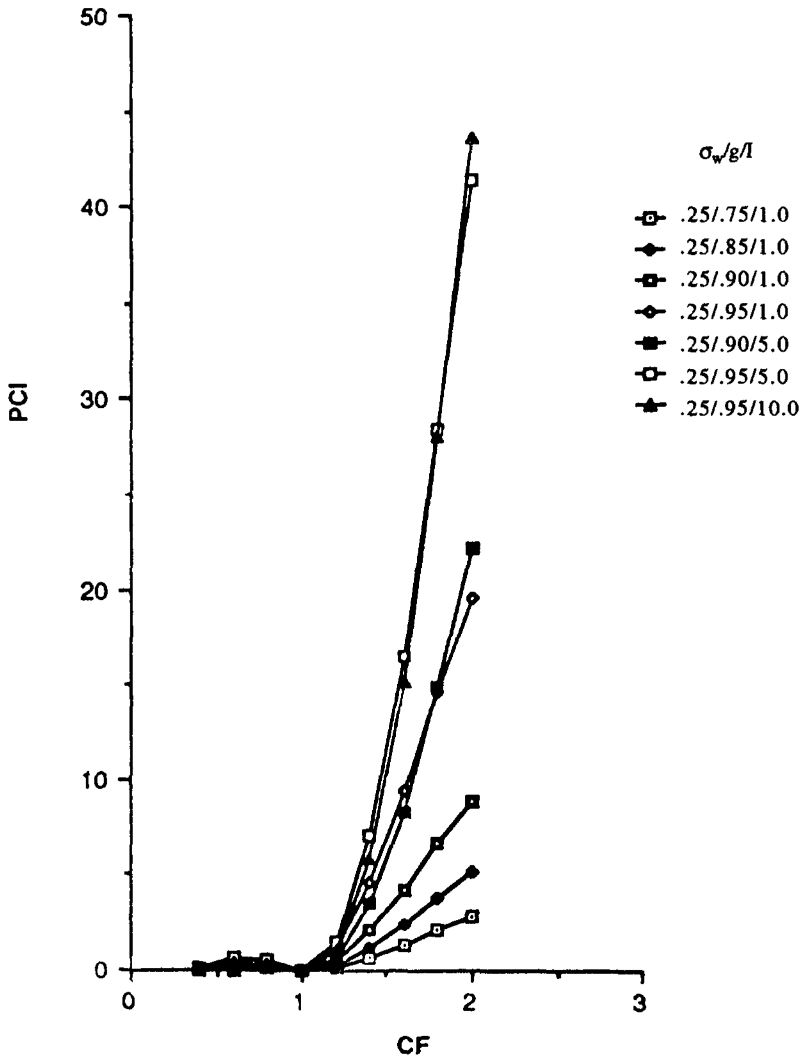


Figure 3—Error in SD (σ_w = 0.50 and K = 0.3I in all cases)

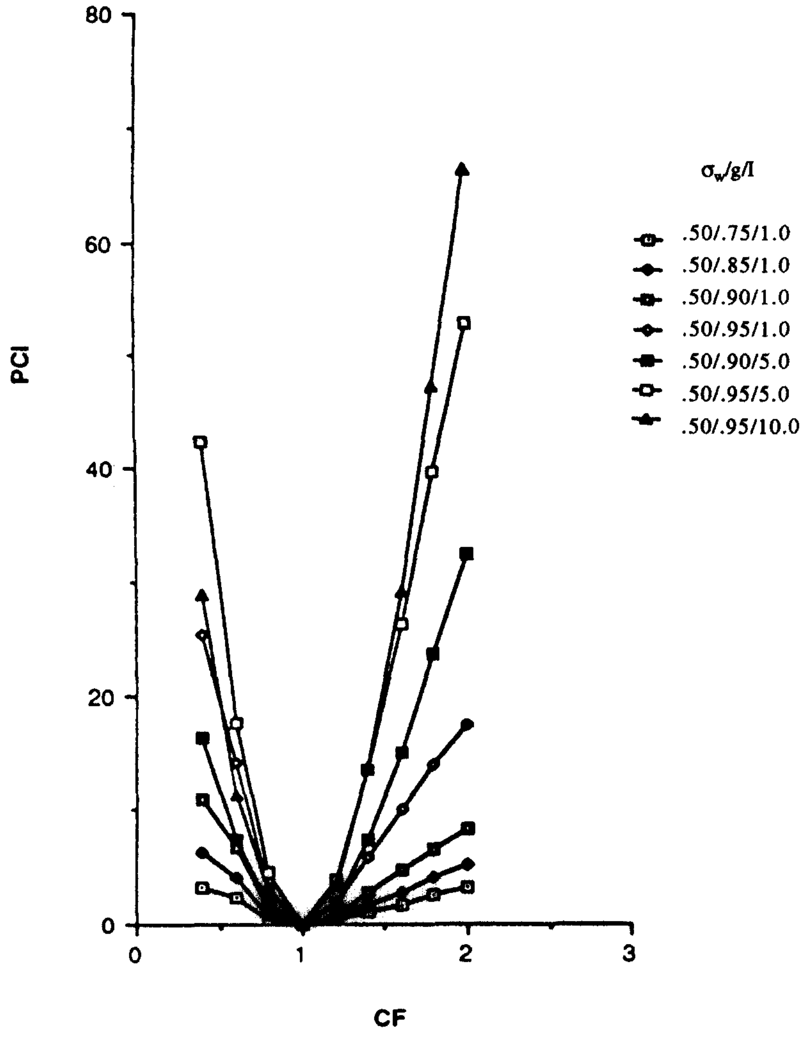
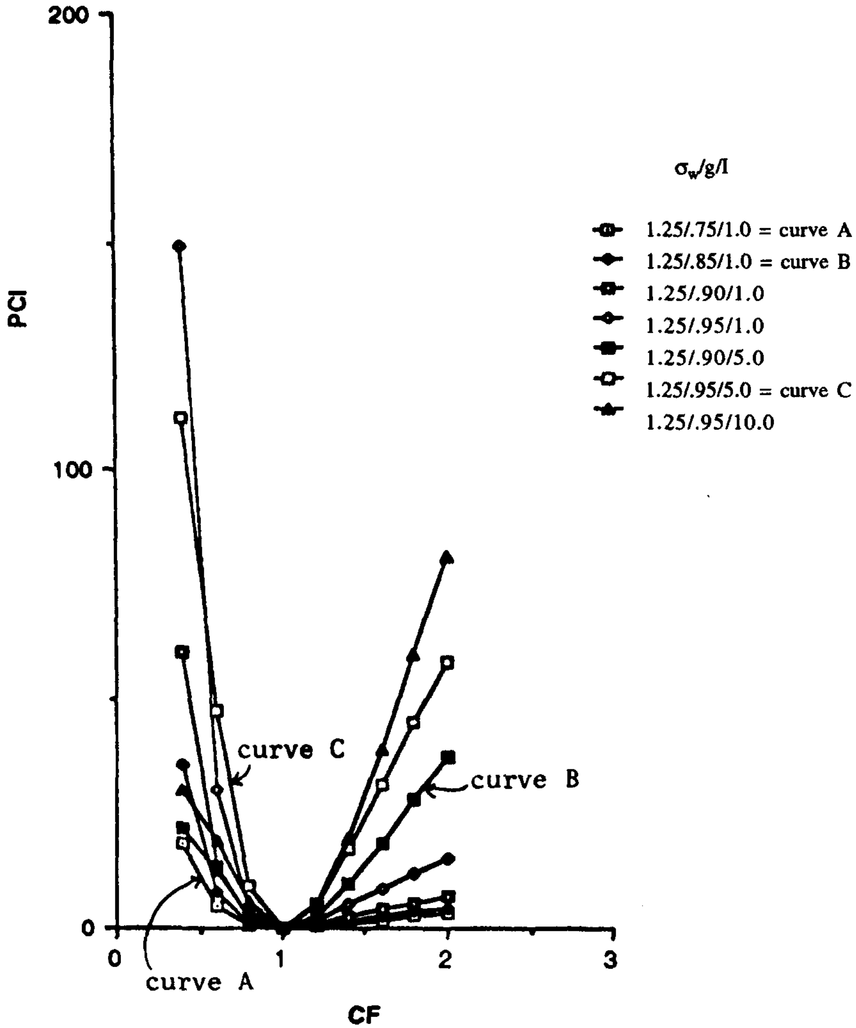


Figure 4—Error in SD (σ_w = 1.25 and K = 0.3l in all cases)



PCI. In situations with the largest PCIs depicted in Figure 4, however, overestimating σ (i.e., CF < 1) is more damaging than underestimating it. With hindsight, this complex phenomenon explains the negative coefficients for the CF terms in equation (12) and σ_w's low partial-R^2 and late entry in the stepwise regression procedure.

AN ILLUSTRATION APPLYING THE PRECEDING RESULTS

Assume now that we face a situation with estimated parameters $\delta = 100$, $\sigma_w = 120$, g = 0.8, I = 200, and K = 100. The normalized problem (with $\delta = 1$) has parameters σ_w = 1.2, g = 0.8, I = 2.0, and K = 0.5I. We know that the value of K is not important—noting that curves in Figures 1 to 4 are identified by the convention σ_w/g/I, we need a curve for 1.2/.80/2.0. Although this curve is not presented, it can be roughly interpolated with the curves A, B, and C in Figure 4 (i.e., curves for σ_w/g/I of 1.25/.75/1.0, 1.25/.85/1.0, and 1.25/.95/5.0). Curves for 1.25/.85/5.0 and 1.25/.75/5.0 are not presented because these situations violate equation (10).

COST EFFECT OF ERROR IN K

SEARCHING FOR A REGRESSION FORMULA TO PREDICT PCI

The procedure in the preceding section is repeated for the effects of error in K. After random values for parameters CF, σ, g, I, and K_w are generated for each of the 5,000 random problems, the value of the K_c is determined with CF, and PCI is computed. The 5,000 PCIs have the following characteristics:

(13) mean = 1.53, s.d. = 4.38, skewness = 4.47, range: 0.0 to 49.8;
quantiles: 50% (median): 0.06, 75%: 0.49, 90%: 4.3, 95%: 10.1

In contrast to the figures in equation (11) for errors in σ, the figures in equation (13) show that errors in K lead to considerably lower PCIs. That is, in general, it is more important to estimate σ than K accurately.

We are unable to obtain a predictive formula for PCI with sufficiently high R^2. One of the better parsimonious predictive formulas for PCI is:

(14) $PCI = -0.318 + 0.1956(CF)^5 + 1.078K_w - 8.11g^{12}$, $R^2 = 0.344$
(independent variables listed in the order of entry in the forward stepwise regression procedure, K_w = estimated (wrong) K value).

Associated information similar to Table 2 on the marginal contributions of various parameters to R^2 is also obtained but not presented for the current and future cases for the sake of brevity. These results suggest that:

- σ and I probably have little effect on PCI;
- PCI increases with larger K_w but decreases rapidly with increasing g.

The second characteristic tends to limit PCI's magnitude, because situations with large K_w and/or small g will violate the condition stated in equation (10). Note that a similar upper bounding mechanism does not exist for the effect of σ-errors considered in the preceding section.

These suggested characteristics are confirmed by the many PCI-versus-CF curves plotted for combinations of parameters specified in Table 1. Because PCI is insensitive to I and σ, we present only curves for $I = 1$ and $\sigma = 0.75$ in Figure 5. The eight curves are for $g = .75/.85/.95$ and $K_w = .3/1/3$ (the case of $g = .75$ and $K_w = 3$ does not satisfy the criterion stated in equation (10). The dashed line in Figure 5 for ($g = .85$, $K_w = 3$) covers points where equation (10) is violated by K_c ($= CF \times K_w$), even though K_w does not violate equation (10) (the condition for inclusion in Figure 5). Note that corresponding dashed lines do not arise in the preceding section on σ-errors because equation (10) does not contain σ, but later sections on I- and g-errors will have corresponding dashed lines because equation (10) contains I and g as well as K. Figure 5 shows that the only situations where PCIs can become large, and hence K needs to be estimated accurately, are ($g = .85$, $K = 3$) and ($g = .75$, $K = 1$). Surprisingly, for most other situations PCI remains small even when the error in K is substantial ($CF = 0.4$ or 2.0). Note that in Figure 5 (and also in some later figures) the lower curves for several situations are so close to each other that they become hidden.

Given any situation, one can normalize the parameters and use the preceding information and Figure 5 to estimate the effect of a K-error on PCI.

COST EFFECT OF ERROR IN I

The procedure in the preceding two sections is repeated for the effects of error in I. The 5,000 PCIs now have the following characteristics:

(15) mean = 1.96, s.d. = 3.34, skewness = 3.93, range: 0.0 to 45.8
quantiles: 50% (median): 0.79, 75%: 2.3, 90%: 4.7, 95%: 8.5.

They indicate that errors in I, like errors in K, generally lead to less serious PCI than errors in σ. One of the better parsimonious formulas for PCI is:

(16) $PCI = 12.3 - 0.13(CF)^6 + 11.5I_w - 11.6gI_w - 17.5\sqrt{(CF)} + 3.1(CF)^3$, $R^2 = .47$

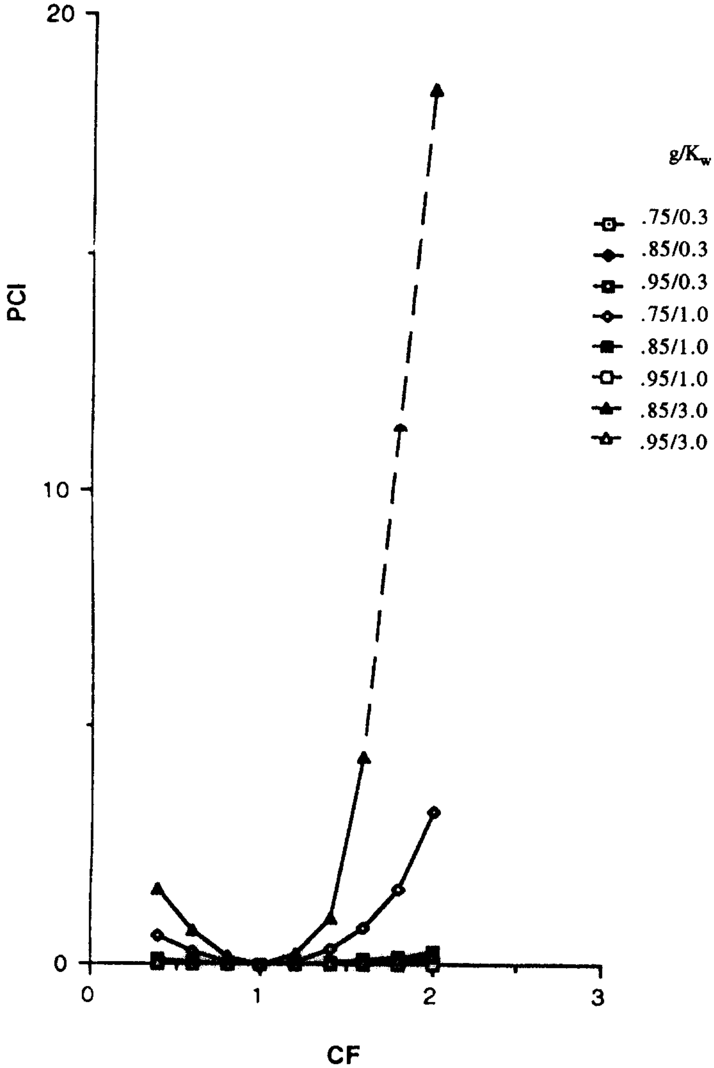
(independent variables listed in the order of entry in the forward stepwise regression procedure)

Equation (16) and associated regression results suggest that:

- The parameters K and σ probably have little effect on PCI;
- PCI increases with larger I_w and lower g.

Again, through condition (10), the second characteristic tends to limit PCI's magnitude. The many PCI-versus-CF curves we plotted confirm that K's effect is small and PCI increases with larger I. The curves reveal that g and σ have complicated effects on PCI, however, which explain why σ's significant effect was not properly revealed by our regression analyses.

Figure 5—Error in K (I = 1 and σ = 0.75 in all cases)



Because K's effect is small and PCI tends to decrease with large K, only curves with $K = 0.3I_w$ will be presented. Figure 6 depicts the main effects of the parameters: while PCI tends to increase somewhat with larger I_w and g, the most prominent feature is that the curves are divided into two distinct groups. The group with $\sigma = 0.25$ is much lower than the group with $\sigma = 1.25$. The positive effect of σ on PCI is so dominant here that one wonders why it was not revealed by the preceding regression analyses. An examination of situations with $I > 2$ depicted in Figure 7 (where $\sigma = 0.25$) and Figure 8 (where $\sigma = 1.25$) may provide an explanation. The dashed lines in these figures (similar to Figure 5) cover points that violate condition (10) (which include most higher PCI-points). For example, in the situation $\{\sigma = .25, g = .75, I_w = 2\}$ (curve A in Figure 7), condition (10) is violated when CF > 1.4 (i.e., when I > 1.4 Iw). Note how the curve shoots up when CF goes beyond 1.4; in other words, PCI increases sharply when the real situation violates condition (10), which only can happen (given the ranges of parameters considered here) when $I_w > 1$ and g is sufficiently small. In contrast, the curves B for $\sigma = .25$, $g = .85$, $I_w = 2$ and $\sigma = .25$, $g = .95$, $I_w = 2$ remain low throughout the CF range, because with g = .85 and .95, condition (10) is not violated even when CF = 2. When I_w increases to 4, however, curve C for $\sigma = .25$, $g = .85$, $I_w = 4$ exhibits a sharp rise, while the companion curve B for $\sigma = .25$, $g = .95$, $I_w = 4$ still remains low throughout. With g = .95, the sharp rise can be observed only when I_w is as large as 10 (curve D in Figure 7). A major reason for this phenomenon is that with a higher g value (e.g., g = 0.95), a higher I_w value is needed (say, $I_w \geq 10$) to violate condition (10), and condition 10's violation is in turn the main cause of the high PCIs.

Compare now Figure 7's curve A for $\sigma = .25$, $g = .75$, $I_w = 2$ with Figure 8's curve A $\sigma = 1.25$, $g = .75$, $I_w = 2$. The latter curve has lower PCIs, even though σ is higher. Several other pairs of curves also exhibit the same phenomenon; i.e., the situation with the higher σ (other parameters being the same) has lower PCIs. As these situations all involve PCIs of large magnitudes, their effect overwhelms the positive effect of σ on PCI. Therefore, our regression analyses fail to identify a significant effect of σ on PCI.

Although the effect of inaccuracies in I on PCI is complicated, it is consolatory that most large PCIs involve situations where the correct parameters violate condition (10), and one might argue that such situations are uncommon. Figure 6 also shows that when I_w is sufficiently small and g is sufficiently large, PCI is never very large and also follows a simple pattern. The PCIs only become large and complicated when I_w is large ($I_w > 1$) and g is sufficiently small. For example, Figure 8 shows that when g = 0.95, PCI never exceeds 10 percent (for any CF) until $I_w > 8$. At this point, it is necessary to point out that the earlier figures for errors in σ and K also include situations where condition (10) is violated, but these situations do not produce as sharp breaks in Figures 1 to 6 as they do in Figure 7.

COST EFFECT OF ERROR IN g

The same procedure used in the preceding three sections is used here with only one modification. In the preceding sections we consider CF in the range of 0.4 to 2.0. Besides being reasonable, there is little common sense justification for using this or some other ranges. In contrast, g must be less than 1; for a reasonable process, the lower bound of g should be somewhat larger than 0.5. Therefore, CF should be such that $0.5+ < g_w(CF) <$

Figure 6—Error in I (K = 0.3I in all cases)

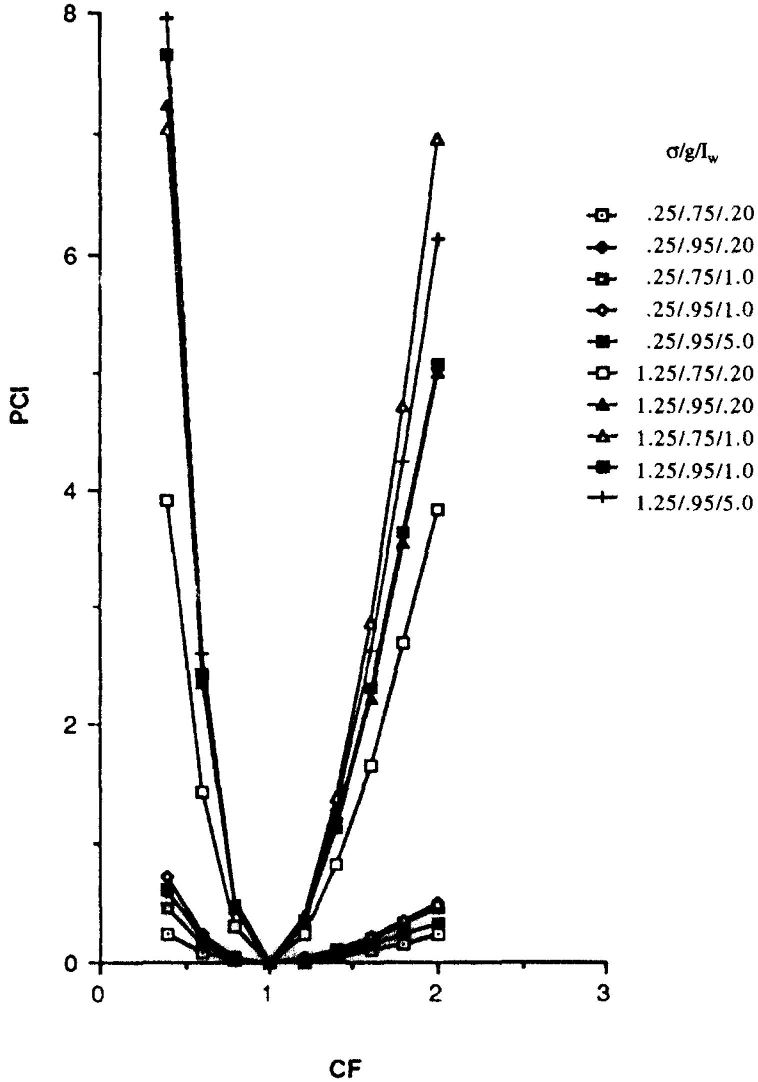


Figure 7—Error in I (Large I, σ = .25, and K = 0.3I in all cases)

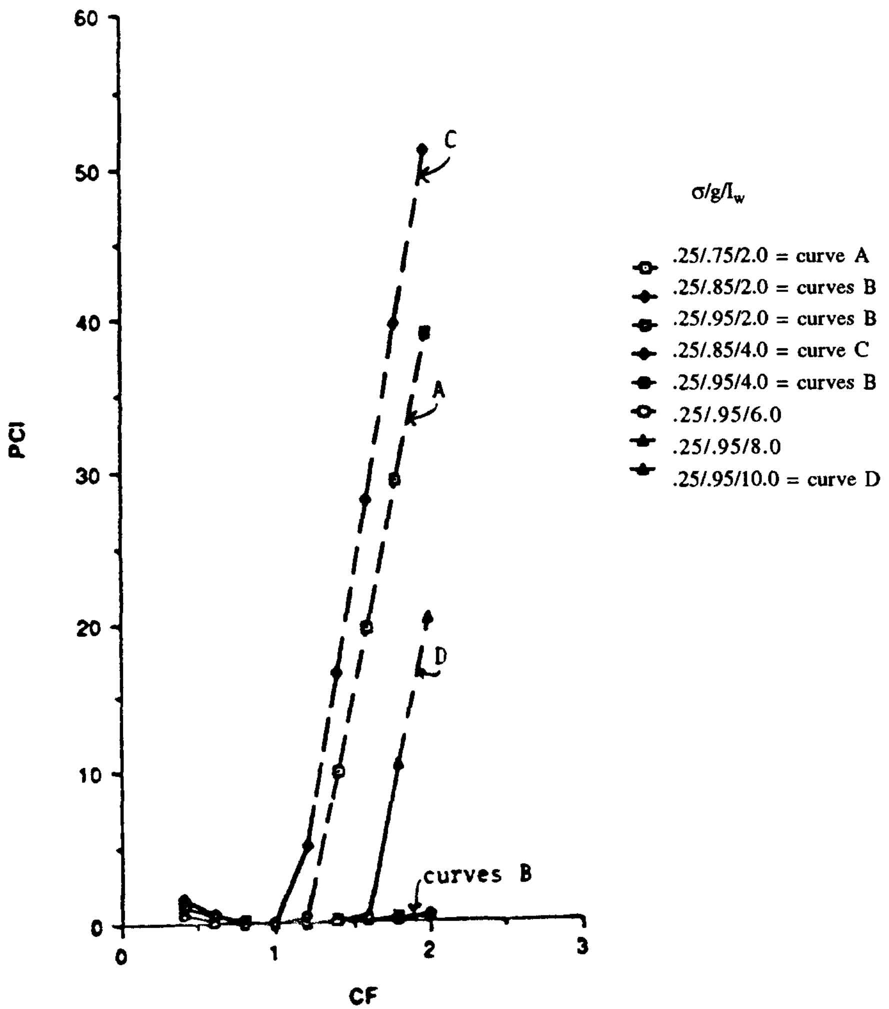
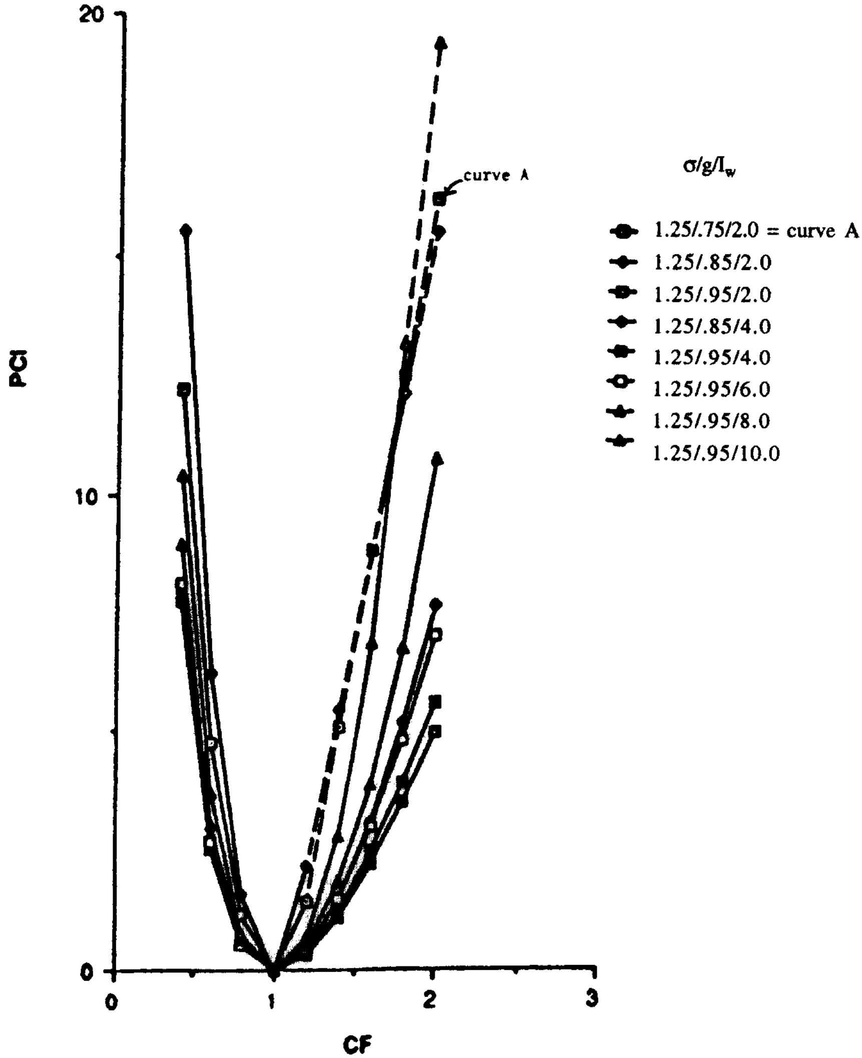


Figure 8—Error in I (Large I, σ = 1.25, and K = 0.3I in all cases)



1. Because $0.75 < g_w < 0.95$, the 5,000 random problems in this section are generated with CF in the range of 0.65 to $(0.999/g_w)$. The characteristics of the resultant 5,000 PCIs are:

(17) mean = 23.1, s.d. = 36.7, skewness = 2.64, range: 0.0 to 371.4
quantiles: 50% (median): 3.98, 75%: 35.1, 95%: 69.0, 95%: 95.2

Clearly, errors in g are more serious than errors in I, K, or even σ. One of the better parsimonious regression formulas for PCI is:

$$\text{(18) PCI} = 476.8 + 19.3\sqrt{K} - 474.2\sqrt{CF} + 34.3(CF)^6 - 74.1(\sqrt[4]{\sigma}) + 5.69(g_w{}^5)I,$$
$$R^2 = 0.55,\ g_w = \text{estimated (wrong) g value}$$

The many PCI-versus-CF curves confirm that PCI increases with g_w, I, and K. Figure 9 presents curves for situations with g = 0.75, where CF's upper limit ≈ (1/0.75) or 1.3. Similarly, Figures 10 and 11 are for situations with g = 0.85 and 0.95, where CF's upper limits are 1.1 and 1.05, respectively. In Figure 9, the four curves with I = 1 rise higher than the four curves with I = 0.1. Among the four curves with I = 1, the two curves with K = I are higher than the corresponding curves with K = 0.3I. The same effect is seen in Figures 10 and 11. Note the large PCIs in Figure 11 when I = 10. Note that the upper K value considered in Figures 10 and 11 is 3I, but in Figure 9 the upper K value is I because at g = 0.75 (the g value in Figure 9) condition (10) will be violated when K = 3I.

It is obvious from Figures 9 to 11 that errors in g are more serious than errors in σ, I, and K; moreover, the effects of g-errors are also more erratic (i.e., the PCI-versus-CF curves in Figures 9 to 10 do not form well-behaved envelopes as they do in Figures 1 to 6). Furthermore, Figures 10 and 11 show that when g_w and I are sufficiently high, PCI can be substantial even when CF differs only slightly from 1.0. In contrast, curves in the Figures 1 to 9 show that even in situations when the PCI curves rise to great heights, they do so only when CF deviates from 1.0 by 0.1 or more.

SUMMARY OF MAIN EFFECTS OF PARAMETER ERRORS

This study investigates the effects of incorrect parameter estimates on the CVID. The effects of making an incorrect decision were measured by dividing the incremental costs resulting from using a wrong parameter estimate by the decision costs resulting from using the correct parameter estimate (equation (5)).

The study uses the following normalization procedure. There are five parameters:

- δ (mean of the out-of-control cost variance distribution);
- σ (the standard deviation of both the in-control (σ_i) and out-of-control (σ_o) cost variance distributions ($\sigma_i = \sigma_o$));
- g (the state transition probability);
- I (investigation cost); and
- K (cost to correct an out-of-control process).

The CVID problem is normalized by setting δ = 1 and then expressing σ, I, and K in terms of δ.

Figure 9—Error in g (g_w = 0.75 in all cases)

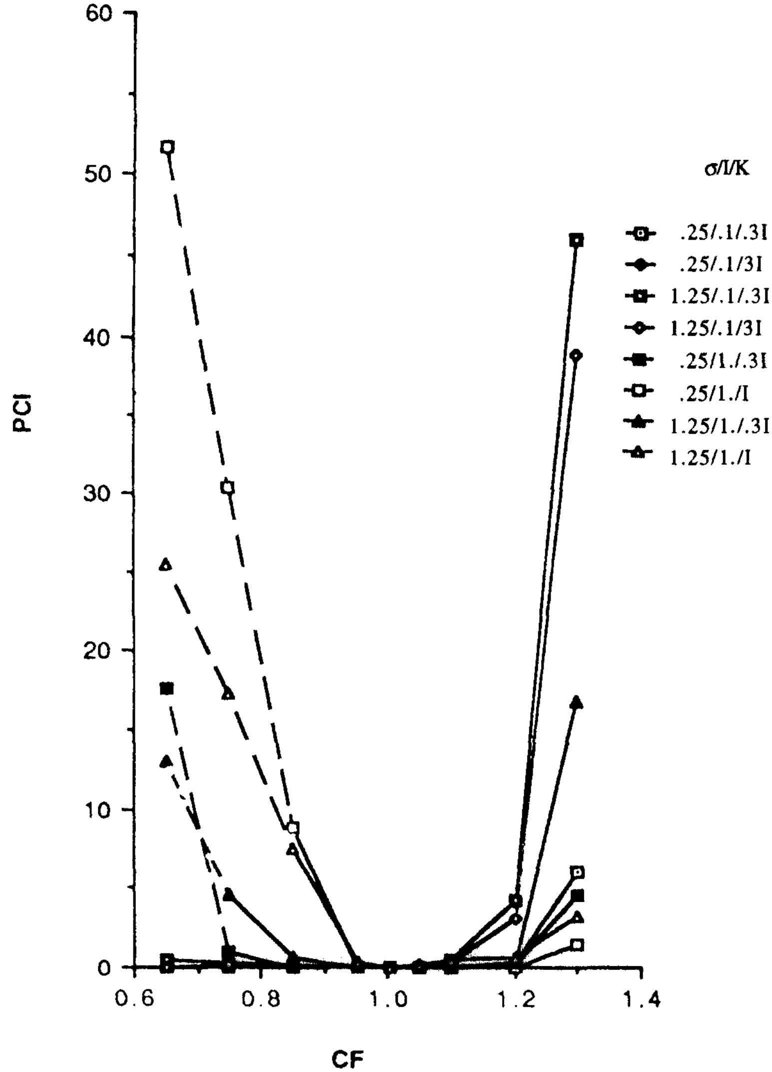


Figure 10—Error in g (g_w = 0.85 in all cases)

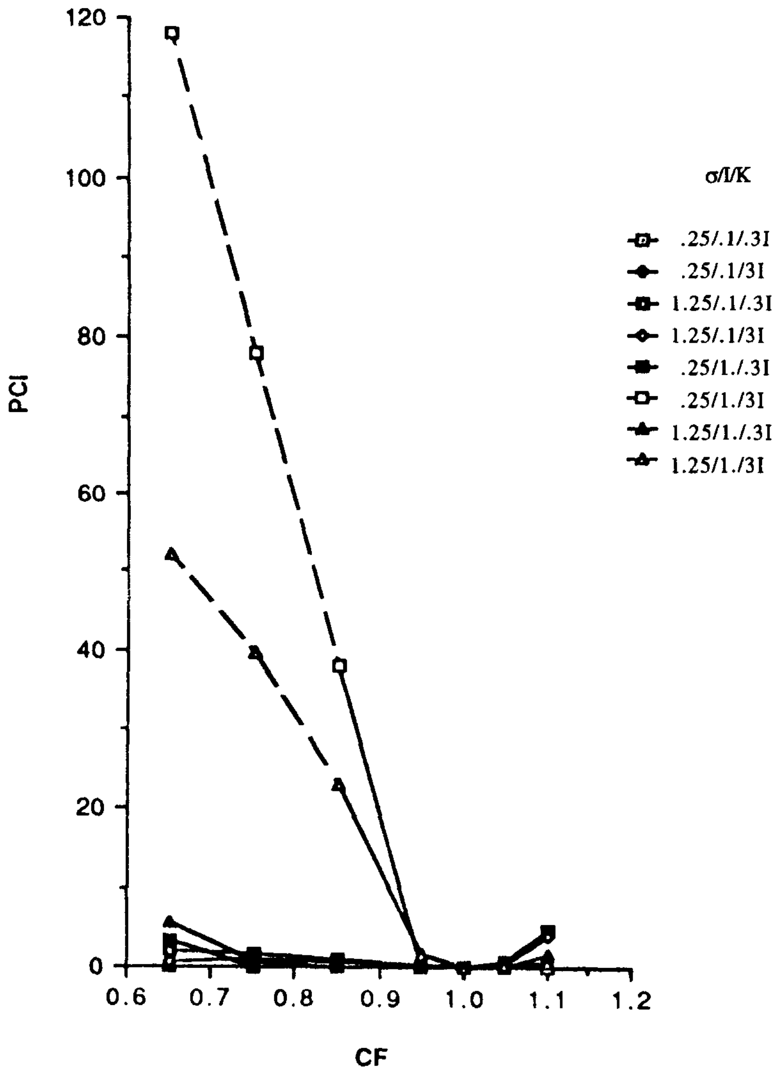
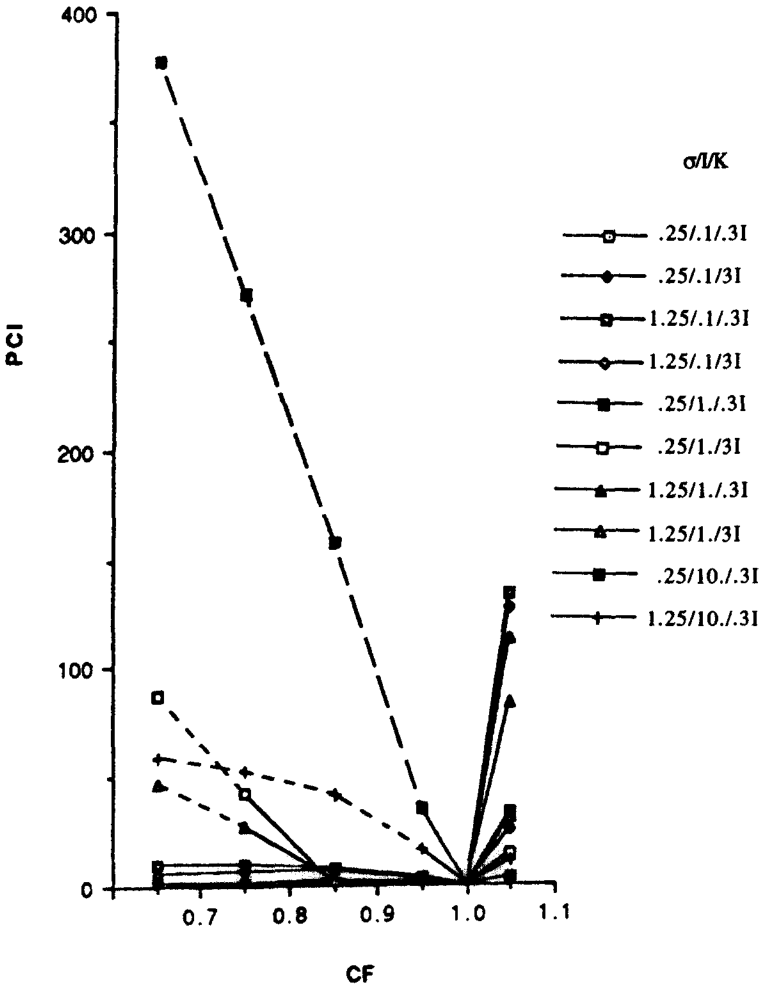


Figure 11—Error in q (q_w = 0.95 in all cases)



For example, assume we have estimated the following parameters: $\delta = 100$, $\sigma_w = 125$, $g = 0.85$, $I = 100$, and $K = 30$. The normalized problem (with $\delta = 1$) has parameters $\sigma_w = 1.25$, $g = 0.85$, $I = 1.0$ and $K = 0.3I$. Four of the parameters have been estimated correctly, with the only error being in the estimation of the standard deviation (σ). Equation (2) is used to determine the optimal R value for both (the correct and wrong) sets of parameters. Using equation (1), the expected system cost per period is determined for both the correct parameters ($C(R_c)$) and wrong parameters ($C(R_w)$). The seriousness of making a wrong parameter estimate is measured in terms of the percentage cost increment (PCI, equation (5)).

In this example, we assume a wrong estimate of σ. Figures 1 to 4 depict the PCIs resulting from a wrong estimate of σ with various combinations of the other parameters ($K = .3I$ for all figures because the results suggest K is unimportant). We need a curve for 1.25/.85/1.0 (the parameters σ_w/g/I, respectively). This is curve B in Figure 4. This curve indicates if the correct standard deviation (σ_c) is 250 (thereby resulting in CF = 2), the PCI would be approximately 40 percent. In other words, because we make a wrong estimate of σ, the expected system cost per period is approximately 40 percent more than if the correct estimate of σ had been made.

Estimates of the PCIs resulting from wrong assumptions of other parameters can be determined in a similar manner using the other figures. For those parameter combinations for which an exact curve is not given, interpolation and Figures 1 to 12 can be used to estimate the PCIs.

The results of this study support the following conclusions regarding parameter estimates. First, error in K is the least important; in most situations PCI is less than 10 percent even when CF is 0.4 or 2.0. It causes moderately large PCI (i.e., > 10 percent) only when K is substantially underestimated (i.e., CF > 1.5) in situations where K is large and g is small.

Second, error in I is also unimportant. Except for situations that violate condition (10), PCI is moderately large only when I > 2 and I is substantially overestimated. Third, error in σ is more serious. Substantial PCI (> 40 percent) can arise if I > 1, g > 0.9 and σ_w's error is more than 50 percent. Fourth, error in g is the most serious. PCI can be extremely large (> 100 percent) for even a small error in g_w if g, I, and K are all large.

As illustrated in the section on σ's errors, the PCI for any situation can be estimated by normalizing the situation's parameters and interpolating with Figures 1 to 11.

RELATIONSHIP WITH EARLIER STUDIES

This study extends the CVID literature in many respects. First, the earlier works of Magee (1976), Dittman and Prakash (1978, 1979), and Greenberg (1986) consider only a limited range of parameter estimates. Our normalizing approach is more general and facilitates consideration of a wider range of parameter estimates.

Second, this study considers the CVID problem from a different perspective than earlier studies. Magee (1976) and Greenberg (1986) consider the comparative merits of different CVID procedures, and both show that the more sophisticated procedures (such as Dittman and Prakash's) are significantly better than simpler CVID procedures in some situations, but not in others. There is no simple way, however, to distinguish one type of situation from the other. Our study suggests a different perspective to the problem. For example, consider the rule to investigate all cost observations that exceed the standard by at least 2σ, which is the best-performing simpler procedure in Magee's study. This rule uses information on σ, but not on g, I, and K. Therefore, its performance could be as good as using Dittman and Prakash's procedure but with unreliable estimates on g, I, and K. Results in this study indicate that errors in I and K are usually not serious, but errors in g can be large when g, I, and K are all large. Thus, Dittman and Prakash's (1978, 1979) procedure is more likely to perform much better than the simpler alternative in situations where g, I, and K are all large. Note, however, that such a situation does not guarantee that Dittman and Prakash's procedure will perform much better. It is always possible that the simpler procedure produces, coincidentally, a solution close to Dittman and Prakash's. It is common knowledge that for many decision problems naive or even illogical decision procedures often produce solutions similar to those obtained from sophisticated and logical approaches. It is unfortunate if this can be used as an argument against using or developing the more logical approaches.

Third, we measure CVID costs differently than prior studies. When comparing the costs of different CVIDs, one can use C'(R) defined in equation (1), C(R) defined in equation (2), or C(R) - δ. If C'(R) is used to compute the percentage cost difference in using two different R-decisions, the percentage will be smaller because of the typically overwhelming fixed component ($M_i + \delta$). If C(R) - δ is used, the percentage difference for the same two R-decisions will appear much larger. Magee (1976) and Greenberg (1986) use C'(R), Dittman and Prakash (1978, 1979) use C(R) - δ. We use C(R), a compromise.

SUMMARY AND CONCLUSION

We show initially that the quality of a CVID is sensitive to the accuracy of the fundamental parameters σ, I, K, and g in some situations, but may be insensitive to these accuracies in other situations. Although the relationship between this sensitivity and the parameters σ, I, K, and g that characterize a situation is extremely complicated, we obtain several simple rules-of-thumb and graphs for identifying situations where parameter errors lead to serious cost consequences and for estimating the appropriate level of effort for determining each parameter.

Our results also suggest a new approach and perspective for future evaluations of alternative CVID procedures.

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