

The Equivalence of the Cascading Scenario and the Backward-Bending Demand Curve Theory of the 1987 Stock Market Crash

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Two of several theories of the 1987 stock market crash are the cascading scenario and Leland's backward-bending demand curve theory. This paper shows that previous descriptions of the cascading scenario are incomplete and that the logical completion of the scenario leads to Leland's backward-bending demand curve. Therefore, whether the cascading scenario led to the crash depends primarily on whether the demand curve for stock or stock equivalents was backward bending.

Explanations of the 1987 crash initially blamed the market's institutional structure and practices such as program trading, futures, and portfolio insurance. These explanations include the cascading scenario,¹ the Brady Commission report, and the SEC Report.² Academic researchers also have considered speculative bubbles, changes in expectations, and increased uncertainty perceptions as possible alternative explanations of the crash. (For example, see Fama, 1989; Roll, 1989; and Leland and Rubinstein, 1988). New evidence now contradicts some assertions of these academic researchers.³ As a result, academics have revived their interest in the institutional explanations of the crash.⁴

If institutional theories of the crash receive more attention, we need to understand these theories fully. This paper discusses two such theories: the cascading scenario, which blames the crash on the interaction between portfolio insurance and index arbitrage,⁵ and Leland's (1987) backward-bending demand curve theory. (See also Eagle, 1988; Ferguson, 1989; and Gennotte and Leland, 1990.)

This paper shows that previous descriptions of the cascading scenario are incomplete and that the logical completion of the scenario leads to Leland's backward-bending demand

¹ John Phelan and the SEC discussed the cascading scenario during the year preceding the 1987 crash. See "The Market Meltdown Made Phelan a Prophet," *Wall Street Journal* (October 23, 1987), by George Melloan.

² For excerpts of the major reports on the crash, see Kamphuis, *et al.* (1989).

³ For example, both Roll (1989) and Fama (1989) argue that there is no evidence of overreaction during the 1989 crash. Seyhun (1990—this citation is missing from the references, as are several others in the footnotes. Please check and correct), however, concludes that an overreaction did occur based on insider buying during or shortly after the crash.

⁴ Academics have researched how institutional factors affected stock prices in the crash. Blume, *et al.* (1989) provide evidence that index arbitrage may have contributed significantly to NYSE stocks in the S&P 500 index dropping seven percentage points more than did the NYSE stocks not in the index during the crash. Also, Greenwald and Stein (1991) present a model where circuit breakers do improve the economic efficiency of the markets.

⁵ The media in 1987 blamed portfolio insurance and program trading because they did not distinguish index arbitrage from program trading. From the context of their reports, however, it is clear that the media meant index arbitrage rather than all program trading.

curve. Therefore, the cascading scenario and Leland's backward-bending demand curve theory are, in essence, the same theory.

A DESCRIPTION OF THE CASCADING SCENARIO

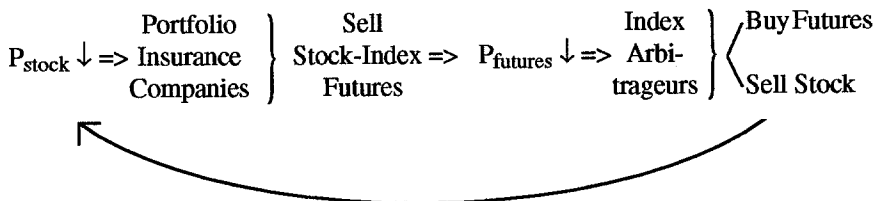
The cascading scenario describes a sequence of events involving portfolio insurance and index arbitrage that leads to a snowball effect on stock prices. In 1987 the product sold as *portfolio insurance* was a dynamic investment strategy based on the Black-Scholes option pricing formula. This strategy attempted to replicate a put option in stocks. The portfolio insurance companies bought stock index futures (or stock) on behalf of their insureds when the price of stock increased and sold stock index futures when the price of stock decreased. (For more details on portfolio insurance, see Rubinstein, 1985.)

The form of index arbitrage included in the cascading scenario is arbitrage between the stock index futures market and the stock market. As stock index futures are almost perfect substitutes for the stocks⁶ themselves, arbitrageurs would buy at the lower price and sell at the higher price to take advantage of the price disparity.⁷ The index arbitrage would bring the prices of the two markets into alignment.

Previous descriptions of the cascading scenario start with stock prices falling for some reason (such as bad economic news). The drop in stock prices triggers portfolio insurance companies to sell stock index futures which in turn triggers index arbitrage. Stock prices fall further in response to the index arbitrage, triggering more selling of stock index futures by the portfolio insurance companies. This process then repeats itself *ad infinitum*. Figure 1 presents a causal schematic of this process.

Several questions about the cascading scenario remain unanswered by the causal schematic in Figure 1 which represents the essence of previous descriptions of the sce-

Figure 1—Causal Schematic of the Cascading Scenario



⁶ The stocks would be the stocks in the particular index in the portions of that index.

⁷ To be comparable to the stock price, the futures price must be adjusted for the time value of money, dividends, and tax effects. See Cornell and French (1983), Chang and Loo (1987), Gastineau and Madansky (1983), and Goodman and Hanweck (1986).

nario: "When will this process end?" "Won't this process always occur when the price drops?" "Does this process always lead to a stock market crash?"

Table 1 presents two scenarios that are consistent with the process depicted in Figure 1. Scenario I is consistent with a stock market crash, whereas Scenario II is not. In both scenarios the price of stock first decreases \$1 for some reason such as bad economic news. This triggers a round of portfolio insurance selling and index arbitrage that leads to the price of stocks falling farther.

Table 1—Two Scenarios Consistent With Figure 1

Needs subheads	Scenario I		Scenario II	
	Price of Stock	Drop in Price	Price of Stock	Drop in Price
	101	⇒ 1	101	⇒ 1
	100	⇒ 2	100	⇒ 0.1
	98	⇒ 4	99.9	⇒ 0.01
	94	⇒ 8	99.89	⇒ 0.001
	86	⇒ 16	99.889	⇒ 0.0001
	70	⇒ 32	99.8889	⇒ 0.00001
	38		99.88889	
	.	.	.	.
	.	.	.	.
	CRASH		99.88	1.11

In Scenario I the drop in the price of stock doubles with every round. Scenario I depicts a snowball effect that leads to a stock market crash. Just as a snowball becomes bigger and gains more momentum as it rolls down a steep hill, the price falls more rapidly the more it drops.

In Scenario II every round of portfolio insurance selling and index arbitrage results in an additional price drop that is only one tenth of the previous price drop. Even if this process continues *ad infinitum*, the total price drop is only \$1.11. This price drop is only slightly more than the initial \$1 price drop.

Scenario II does not represent a stock market crash, yet it is consistent with the causal schematic in Figure 1. Hence, previous descriptions of the cascading scenario have been incomplete. They do not indicate when the price drops will snowball and when they will taper.

The appendix derives a condition that determines when the price drops will snowball and when they will taper. This condition can be understood more clearly, however, in the context of a consolidated futures-stock market.

A CONSOLIDATED FUTURES-STOCK MARKET

As long as index arbitrage keeps the prices of stock index futures and stock consistent, there is little reason to distinguish between the two markets. This is a major theme of the Brady Commission Report (1988). One way of consolidating the two markets is to define an investor's net demand for stocks or stock equivalents as the investor's desired net

exposure to stocks. To determine this net demand, we sum the investor's long stock and futures positions and subtract the investor's short stock and futures positions.

Our concept of demand for this consolidated market is a demand to hold rather than a demand to buy. For example, suppose a particular investor owns 15,000 shares but wants to hold 20,000 shares. The investor demands to hold 20,000 shares. Alternatively, we could say the investor demands to buy 5,000 shares. Both concepts of demand are legitimate. For the consolidated market the demand for holding stocks is more convenient than the demand for buying stocks.

The supply of stocks that can be held is the number of shares that exist, whether or not they are brought to market. In the short run the supply curve for stock is fixed because firms cannot react immediately to issue new stock.

Neither futures nor selling stock short affect the fixed nature of supply. Demand includes the buying of stock index futures. Also, by definition, selling stock index futures and selling stock short indicate a negative demand for stocks.

With supply fixed in the short run, we now turn our attention to demand. The market demand curve must reflect the reaction of portfolio insurance companies to changes in price.⁸ Under normal conditions the demand curve would be downward sloping, as in Figure 2. To determine whether price drops snowball or taper, consider a shift of the demand curve to the left.

Immediately after the shift, the market price⁹ equals the old equilibrium price, resulting in a surplus. The surplus pushes the price downward. As the price falls, the surplus decreases. Because the size of the surplus determines the amount of pressure on the price to fall, the smaller surplus leads to less pressure on the price to fall. Therefore, the price drops taper, as shown in the table accompanying Figure 2.

For any downward-sloping demand curve with a fixed supply, the price drops taper rather than snowball because the surplus decreases as the price falls. Therefore, our search for a snowball effect must lead us to consider upward-sloping demand curves.

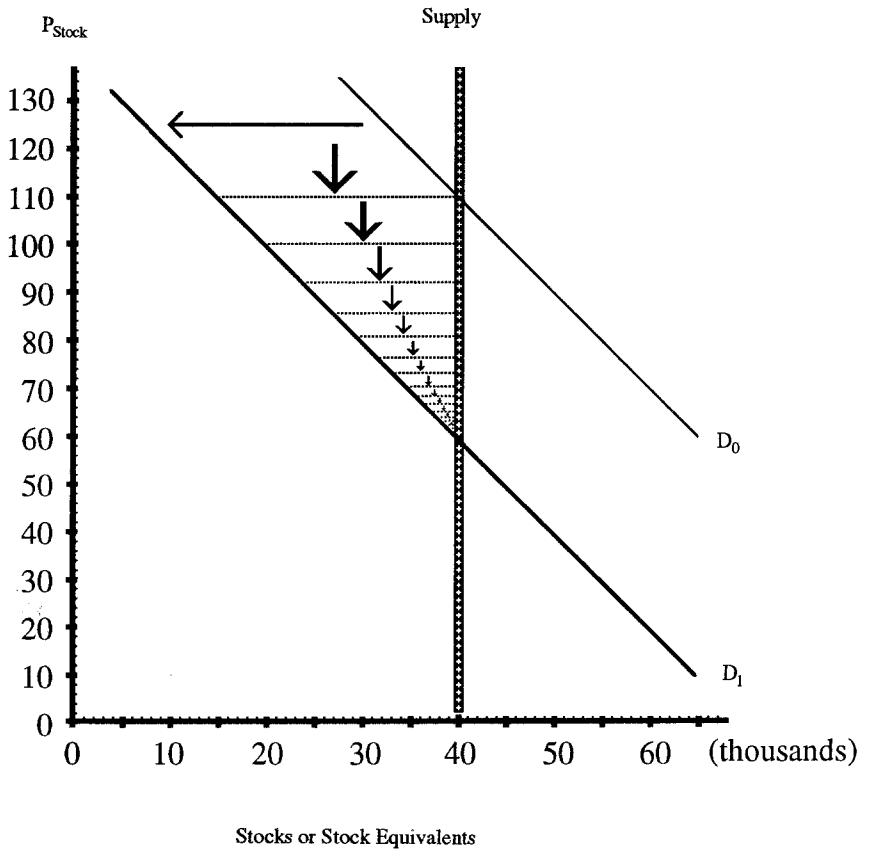
A leftward shift in an upward-sloping demand curve does lead to a snowballing effect, as demonstrated in Figure 3. When demand is D_0 , the equilibrium price is $P_0 = 90$. When the demand curve shifts to the left, a surplus results at the price of 90, which pushes the price down. As the price falls, however, the surplus increases which increases the pressure on the price to fall. As a result, the price falls at a faster and faster rate.

Figures 2 and 3 have helped answer the question about when the interaction of portfolio insurance and index arbitrage leads to a snowballing sequence of price drops and when it does not. When the demand for stocks or stock equivalents is downward sloping, the price drops taper; when demand is upward sloping, the price drops snowball.

⁸ Selling by portfolio insurance companies in response to a price drop should not be confused with a shift in the demand or supply curves because it is the price drop that precipitates selling. Basic economic principles point out that the demand curve is the relationship between the price of the good and the quantity of the good that is demanded when everything else stays the same. If the price of the good decreases, the quantity demanded of the good will increase. The increase in quantity demanded is not a shift of the demand curve, however, but a movement along the demand curve.

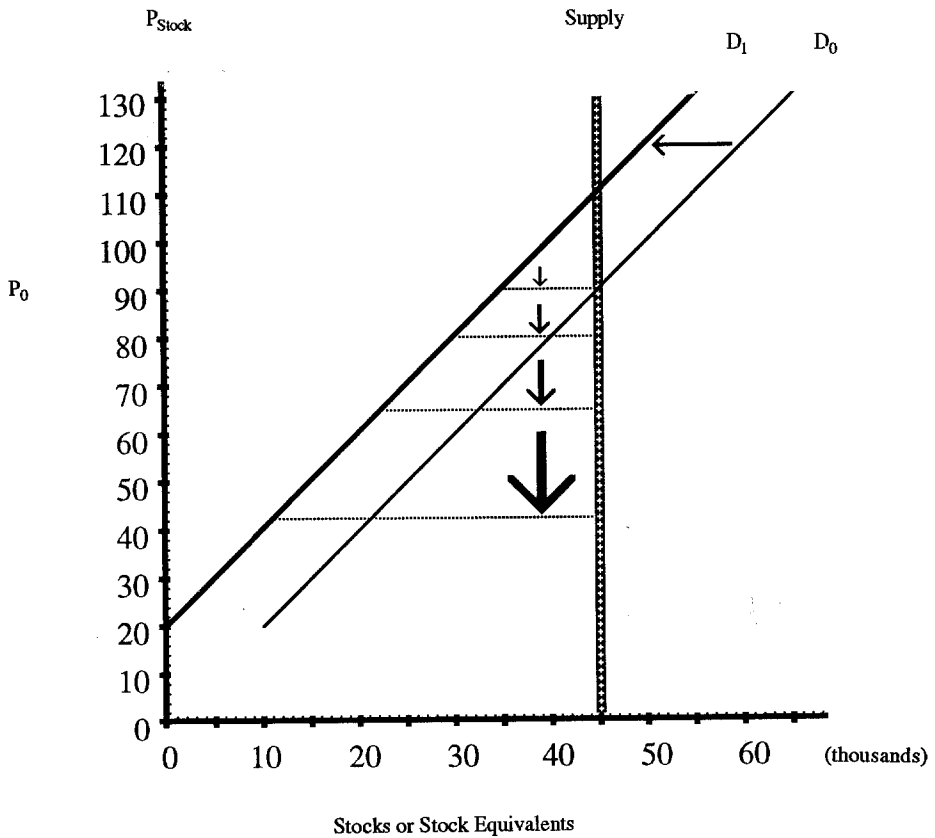
⁹ The term *market price* means the current price in the market, not necessarily the equilibrium price.

Figure 2—Downward-Sloping Demand Curve: No Snowball Effect



Numerical Values of Surpluses and Implied Price Changes in Figure 2 Assuming Each Price Drop = Surplus/2500

Stock Price	Surplus	Price Change
110.00	25,000 ⇒	10.00
100.00	20,000 ⇒	8.00
92.00	16,000 ⇒	6.40
85.60	12,800 ⇒	5.12
80.48	10,240 ⇒	4.10
76.38	8,192 ⇒	3.28
73.11	6,554 ⇒	2.62
70.49	5,243 ⇒	2.10
68.39	4,194 ⇒	1.68
66.71	3,355 ⇒	1.34
65.37	2,684 ⇒	1.07
64.29	2,147 ⇒	0.86
63.44	1,718 ⇒	0.69
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮

Figure 3—Upward-Sloping Demand Curve, Snowball Effect**Numerical Values of Surpluses and Implied Price Changes in Figure 3 Assuming Each Price Drop = Surplus/2500**

Price	Q^S	Q^D	Surplus	Price Change
90.0	45	35.00	10,000 $\Rightarrow$	10.00
80.0	45	30.00	15,000 $\Rightarrow$	15.00
70.0	45	22.50	22,500 $\Rightarrow$	22.50
42.5	45	11.25	33,750 $\Rightarrow$	33.75
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

We have not yet completed the cascading scenario. The initial equilibrium in Figure 3 is unstable from both above and below. Because the stock market is volatile, the market almost certainly would push us away from this equilibrium. We would almost never be at an unstable equilibrium, such as in Figure 3. Therefore, we need to look beyond this equilibrium to learn where the market would be.

Because of limited liability to stockholders, the price of stock cannot become negative. Instead, as the market price nears zero, the market demand curve must become downward sloping (Figure 4). Also, at some price high enough, the quantity demanded of stock must decrease. The market in Figure 4 has three equilibria: A is stable, B is unstable, and C is stable. There is almost no chance the market will be at the unstable equilibrium B; instead, it must be at either equilibrium A or equilibrium C.

Figure 4—The Logical Extensions to an Upward-Sloping Demand Curve

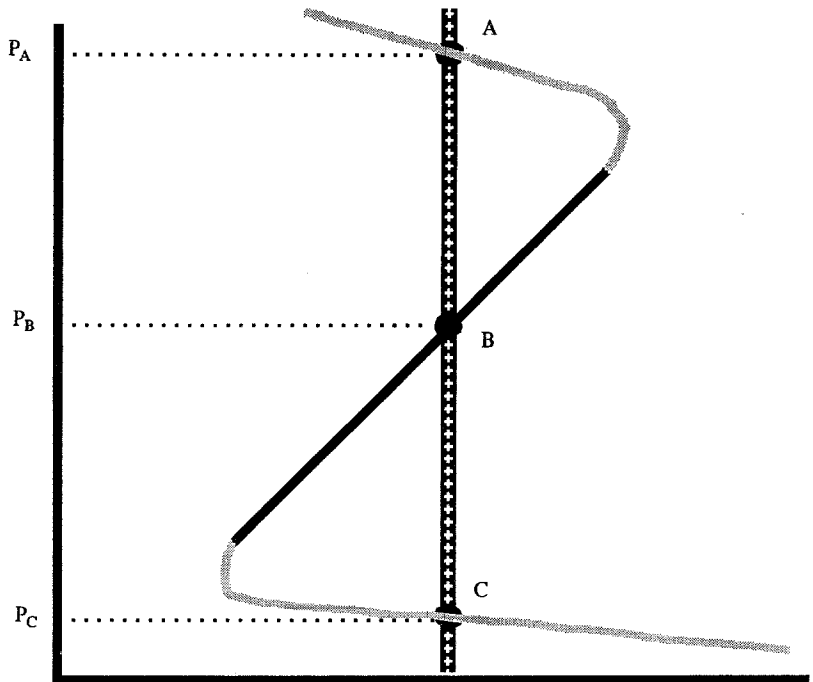
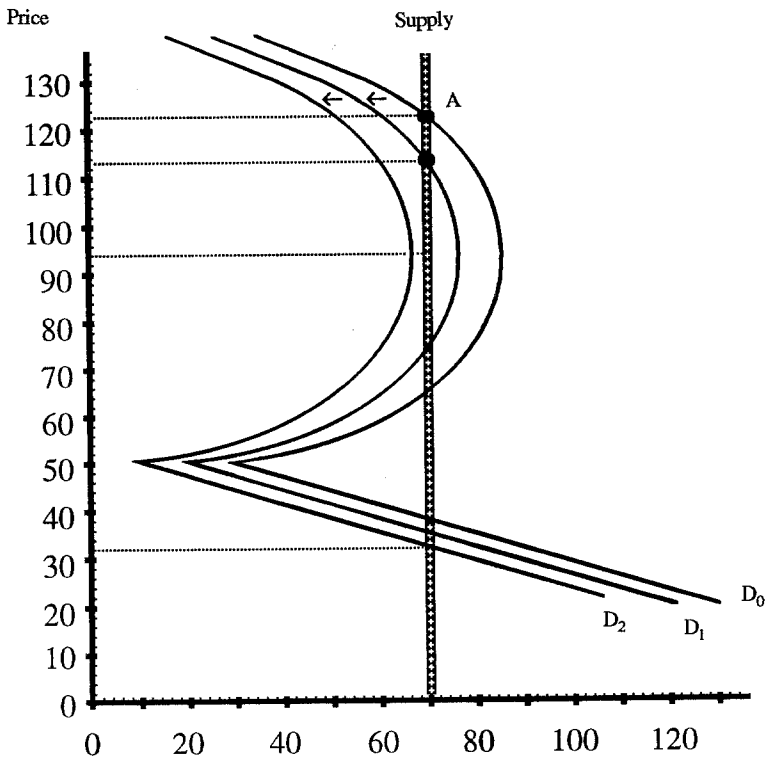


Figure 4 is similar to Leland's backward-bending demand curve. He argues that if the economy were at equilibrium A and the demand curve were to shift continuously to the left (as in Figure 5), the equilibrium price initially would decrease continuously. Once the demand curve became tangent to the supply curve and shifted slightly to the left, the equilibrium price would drop discontinuously from 94 to 33.

Immediately after the demand curve shifts to D_2 , there is a small surplus if the market price is 94 (Figure 6). This surplus nudges the price downward. As the price falls, the surplus becomes larger and increases the pressure on the market price to fall. This increase in the pressure on the market price to fall continues until the price reaches 50. As the price drops below 50, the surplus declines, which in turn decreases the pressure on the market price to fall.

Figure 5—Leland's Crash Scenario of a Leftward-Shifting Backward-Bending Demand Curve

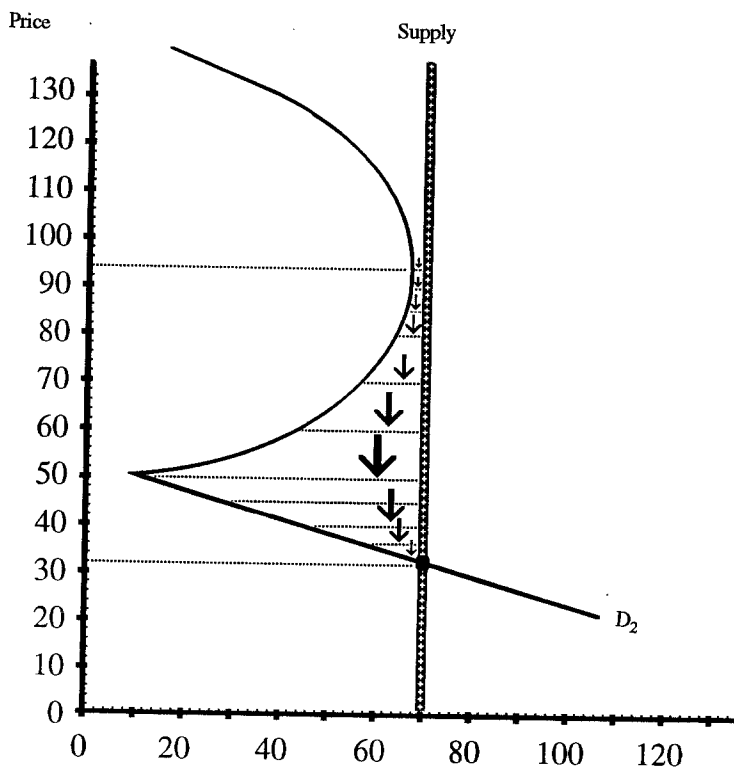


The increasing price drops between 94 and 50 are similar to the price drops in Scenario I of Table 1. The preceding discussion shows that in terms of a consolidated futures-stock market, a cascading scenario involving snowballing price drops must involve a backward-bending demand curve similar to that in Leland (1987).

THE SOURCE OF THE BACKWARD-BENDING DEMAND CURVE

As discussed by Leland (1987), those who use portfolio insurance implicitly are buying when the price of stock rises and selling when the price falls. As a result, they demand to hold more stock at higher prices than they do at lower prices. In other words, they have upward-sloping demand curves for holding stock or stock equivalents.

If all investors used portfolio insurance, the market demand curve would be upward sloping. When there is a combination of portfolio-insured investors and other investors, the market demand curve may or may not be upward sloping, depending on whether the portfolio-insured investors dominate other investors.

Figure 6—Price Dynamics During A Crash Involving a Backward-Bending Demand Curve

Whether portfolio insurance dominates the market depends not on the relative number of portfolio-insured investors, but on the relative size of their buying or selling activities. If a price drop triggers more selling by portfolio-insured investors than it triggers net buying by other investors,¹⁰ the market demand curve will be upward-sloping in that price range.¹¹

In summary, if a price drop induces more selling by portfolio-insured investors than it induces net buying by the other investors, then the price drops will snowball. On the

¹⁰ It is possible that investors other than those using portfolio insurance will sell stocks as a result of a price drop. For example, investors with stop-loss orders and margin calls may sell as the price drops. Also, panicking investors will sell when the price drops. Thus, it is important that we compare the portfolio insurance selling to the net buying by other investors.

¹¹ The demand curve will be upward sloping if, and only if, the price increase causes an increase in the quantity demanded in the whole market. If the price increase causes the portfolio-insured investors to increase their quantity demanded more than it causes other investors to decrease their quantity demanded, the overall quantity demanded will increase for the market.

other hand, if a price drop induces less portfolio insurance selling than it induces buying by other investors, then the price drops will not snowball.

REFLECTIONS

Recognizing that the cascading scenario depends on the existence of a backward-bending demand curve can help focus the debate concerning this theory. In particular, consider Santoni's (1988) criticism of the cascading scenario: The cascading scenario suggests that past changes in cash prices should help explain changes in futures prices; yet Santoni's empirical research indicates otherwise. The portfolio insurance companies were trying various methods to reduce the implicit costs of their service. One method they used was to delay sales of futures. On the Friday before Black Monday, portfolio insurance companies had delayed about \$8 billion of sales. (See Brady, 1988.) This delay of portfolio insurance selling would have masked the statistical relationship between cash prices and futures prices.

Brennan and Schwartz (1988) also criticize the cascading scenario. They develop a model using a demand elasticity based on a typical portfolio consumption model and portfolio insurance controlling 5 percent of the stock. Their model does not result in a crash.

During the 1987 crash only 3 percent of publicly owned stock changed hands (Cohen, 1988), which is difficult to reconcile with typical portfolio consumption models. When investors infer future price changes from current price changes (as in Grossman and Stiglitz, 1980), the actual demand elasticity is much less than is predicted by the typical portfolio consumption models. Gennotte and Leland (1990) present a model consisting of three groups of investors:

- Uninformed investors who have no informational advantages in the market;
- Price-informed investors who receive noisy but unbiased signals about future prices; and
- Hedging-informed investors¹² who receive signals about the portfolio-insurance hedging in the market.

Following Brennan and Schwartz's work, they find that a backward-bending demand curve could result when they use reasonable parameters in their model.

In actuality, the portfolio insurance companies sold much less than indicated by their formulas because of their delayed trading strategies. Despite a carryover of \$8 billion going into Black Monday (and that amount was doubled or tripled by the price drops during the crash), the portfolio insurance companies only sold a total of \$6 billion of futures and stocks on Black Monday. Nevertheless, many trading companies anticipated heavy selling by the portfolio insurance companies and sold early to try to profit from the resulting price changes. (See Brady, 1988.)

Miller (1988) criticizes the cascading scenario because much of the portfolio insurance selling of futures was absorbed on the futures market rather than transmitted to the stock market via index arbitrage. A probable reason for this absorption was the lack of

¹² Gennotte and Leland call the hedging-informed investors *supply-informed investors*. Because those using portfolio insurance normally are net demanders of stocks, I believe the term *supply-informed* is a misnomer.

index arbitrage during the afternoon on Black Monday. Therefore, any portfolio insurance selling of futures in the afternoon had to be absorbed in the futures market. This feat was accomplished often by radically lowering the futures price.

Without index arbitrage, there are other ways for portfolio insurance selling to affect the stock market. In particular, some portfolio insurance companies did start selling stocks directly instead of selling stock index futures. Thus, to determine the validity of the cascading scenario, one needs to consider the behavior of the consolidated futures-spot market.

Ferguson (1989) criticizes Leland's backward-bending demand curve theory because it does not comply with other empirical observations. In particular, Ferguson notes that large price increases occur much less frequently than do large price decreases. Yet according to Leland's backward-bending demand curve theory, large price increases should be as likely as large price decreases.

Ferguson's criticism is due to a simplification in Leland's model. Leland's demand by portfolio-insured investors is invariant to past prices. In reality, the demand by portfolio investors does depend on past prices. As a result, the market demand will be less likely to be backward bending after a crash than before.¹³

One factor that would have increased the likelihood of a backward-bending demand curve is the lack of short-term liquidity in the market. In essence, the lack of short-term liquidity means that price drops lead to small increases in buying activity. In such a situation, portfolio insurance selling would have dominated these small increases in buying activity resulting from price drops.

The purpose of this paper has not been to present the cascading scenario as the theory of the crash, but to clear some confusion by proving that the cascading scenario is equivalent to Leland's backward-bending demand curve theory. Future arguments concerning the cascading scenario should focus on the issue of whether the demand curve for stock or stock equivalents is backward bending or not, i.e., whether price drops induced more portfolio-insurance selling than price drops induced buying by other investors in the market.

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¹³ The change in an investor's portfolio value is what triggers portfolio insurance buying or selling. Portfolio insurance is supposed to insure that an investor's portfolio value does not fall below a certain minimum level. Therefore, if the price of stock falls sufficiently to cause the investor's portfolio value to equal the minimum allowable level, the investor would have to exit the stock market completely. If the price then increased, the investor's portfolio value would not be affected because the investor had no exposure to the stock market. Therefore, the investor would be unable to increase his or her exposure to the stock market. Hence, the demand for stocks by portfolio-insured investors will be less sensitive to stock price increases after a crash than before a crash.

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Appendix—Derivation of the Mathematical Condition Determining Whether Price Drops Snowball or Taper

Let $Q_S^D(P_S)$ and $Q_S^S(P_S)$ be the market demand and supply for stock excluding index arbitrage as functions of the price of stock. Let $Q_F^D(P_F^*)$ and $Q_F^S(P_F^*, P_{-1})$ be the market demand and supply functions for stock index futures excluding index arbitrage. The P_{-1} represents the stock price of the previous moment. This argument's effect on the futures supply function represents the portfolio insurance companies' reactions to changes in the price of stock.

Define A to be a variable representing the amount of index arbitrage. If index arbitrageurs buy futures and sell stocks, A is the stock equivalent of the futures being bought, which also equals the stocks being sold. A negative A represents futures being sold while simultaneously buying stocks.

Equilibrium in the futures market requires that:

$$(A1) \quad Q_F^D(P_F^*) + A = Q_F^S(P_F^*, P_{-1})$$

Equilibrium in the stock market requires that

$$(A2) \quad Q_S^D(P_S) = Q_S^S(P_S) + A$$

In equilibrium, index arbitrage will eliminate any differences between the adjusted futures price and the stock price:

$$(A3) \quad P_F^* = P_S = P$$

Equations (A1) and (A2) imply that:

$$Q_S^D(P_S) - Q_S^S(P_S) = -(Q_F^D(P_F^*) - Q_F^S(P_F^*, P_{-1}))$$

Totally differentiating the above gives:

$$\left(\frac{\partial Q_S^D}{\partial P_S} - \frac{\partial Q_S^S}{\partial P_S} \right) dP_S = - \left[\left(\frac{\partial Q_F^D}{\partial P_F^*} - \frac{\partial Q_F^S}{\partial P_F^*} \right) dP_F^* - \frac{\partial Q_F^S}{\partial P_{-1}} dP_{-1} \right]$$

By equation (A3), $dP_F^* = dP_S = dP$. Solving the above equation for dP/dP_{-1} gives the following:

Portfolio Insurance Selling
Caused by a \$1 Drop
in the Price of Stock

$$\frac{dP}{dP_{-1}} = \frac{\frac{\partial Q_F^S}{\partial P_{-1}}}{\left(\frac{\partial Q_S^D}{\partial P_S} - \frac{\partial Q_S^S}{\partial P_S} \right) + \left(\frac{\partial Q_F^D}{\partial P_F} - \frac{\partial Q_F^S}{\partial P_F} \right)}$$

Increase in Excess
Demand in the
Stock Market
When the
Stock Price
Falls \$1

Increase in Excess
Demand in the
Futures Market
When the
Futures Price
Falls \$1

If $dP/dP_{-1} > 1$ then the price drops will snowball. If $dP/dP_{-1} < 1$, then the price drops will taper.

The above ratio is the portfolio insurance selling as a result of a \$1 price drop divided by the net buying of all other investors as a result of a \$1 price drop. Hence, when a price drop causes more portfolio insurance selling than buying by other investors, the price drops will snowball; when a price drop induces less portfolio insurance selling than buying by other investors, the price drops taper.