Estimation of Food Demand Elasticities Using Hicksian Composite Commodity Assumptions

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*This paper demonstrates, step by step, how income and price elasticities of demand as defined in a Hicksian composite commodity model can be estimated from data on expenditures and physical quantities. The econometric model is a modified version of one used by Deaton for a study of food demand in the Ivory Coast. This paper extends Deaton's model to incorporate the assumption of Hicksian composite commodities, and reestimates the model using data from the **1985 U.S. Consumer Expenditure Survey.***

INTRODUCTION

Cross-section surveys of household food expenditure often have been used for the estimation of income elasticities of demand for food items; recently the potential for using them for the estimation of price elasticities has received renewed attention (Timmer and Alderman, Deaton, Cox and Wohlgenant, Nelson). When a survey contains information on both household expenditure on an item and the physical quantity purchased, one seems to have both a measure of quantity and the ability to derive a measure of price by dividing expenditure by physical quantity. These measures are not, however, in general directly useful in demand equation estimation. The price paid may reflect endogenous choice among items of differing characteristics or quality levels, instead of being exogenous to the household. For example, a household may purchase six pounds of oranges for \$3, at an implicit price of \$.50 per pound. If, however, the household also bought (or could have bought) less juicy oranges for \$.30 per pound, neither \$.50 per pound nor \$.30 per pound can be taken as the price of oranges. The complications such quality variation creates for the measurement of price (on the right side of the demand equation) have been discussed by Theil, Houthakker, Deaton, and Cox and Wohlgenant.

Nelson (1991) shows that quality variation (or, more generally, commodity heterogeneity) also complicates the left side of the demand equation: the simple sum of physical quantities is not the appropriate measure of quantity demanded except in special cases. It is possible, for example, to obtain different estimates of the income and price elasticities when quantity is measured in different physical dimensions. Consider a case where richer households buy fewer but bigger oranges than poorer households: the income elasticity by count would be negative, even if the income elasticity by weight is zero or positive. Nelson suggests that a measure of quantity based on Hicksian composite commodity assumptions provides a conceptually more adequate measure of quantity. The Hicksian quantity measure is a price-level-deflated expenditure and hence is independent of the choice of physical quantity metric. This paper goes beyond the theoretical explanation in

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Nelson (1991) to explain, step by step, how estimation using the Hicksian assumptions can be implemented using an extension of an empirical method developed by Deaton (1986). The implementation is illustrated with an application to the study of demand for eleven food commodities in the United States. While the final results of this study briefly have been reported elsewhere (Nelson, 1990), only this article contains a description of the method sufficiently detailed to allow the interested researcher to replicate the study, a detailed description of the data, a presentation of the results of the intermediate steps, and a detailed discussion of the results.

THE HICKSIAN COMPOSITE COMMODITY APPROACH[1]

Suppose that there are R individual, homogeneous, elementary goods, whose quantities we denote by x_i ($i = 1, \ldots, R$). Suppose further that these goods can be grouped such that within each group, prices of all goods vary proportionally. Let P_G denote the factor of proportionality common to all elementary goods in group G. Then by Hicks' composite commodity theorem, a measure of the demand for the composite commodity G can be defined as

$$(1)\ Q_G \equiv \sum_{i \in G} p_i^* x_i$$

where:

$p_i^* \equiv$ The base price of good x_i

The demand function for such a composite commodity is

$$(2)\ Q_G(P,Y)$$

where:

$P \equiv$ The vector of P_Gs.

Such composites can be treated, in every respect, as if they were elementary goods.

Such an approach can incorporate measures of physical quantity, measures of the "quality" of an aggregate good, and the concept of unit value that have been discussed elsewhere in the demand literature. The (unweighted) physical quantity of an aggregate good can be noted as

$$q_G \equiv \sum_{i \in G} x_i.$$

[1] This section draws on Nelson (1991).

Following influential work by Theil and Cramer, one also may define a quantity-weighted sum of elementary base prices as the measure of average "quality" within a group:

$$v_G \equiv \sum_{i \in G} \left(\frac{x_i}{q_G}\right) p_i^*$$

That is, the larger the proportions bought of higher priced goods, the higher is the measure of quality. Figure 1 illustrates iso-quality lines for the case of a composite consisting of a low priced (low quality) elementary good x_1 and a high priced (high quality) elementary good x_2. Iso-quality lines are simply rays from the origin, with points on lines closer to the x_2 (high priced) axis representing bundles of higher quality. Last, assuming one can observe expenditures E_G and physical quantity, one can calculate unit values

$$V_G \equiv \frac{E_G}{q_G}.$$

It is easy to derive, by manipulating the various definitions, the following expressions for composite quantity, expenditure, and unit value:

(3) $Q_G = v_G q_G$

(4) $E_G = P_G v_G q_G$

(5) $V_G = P_G v_G$

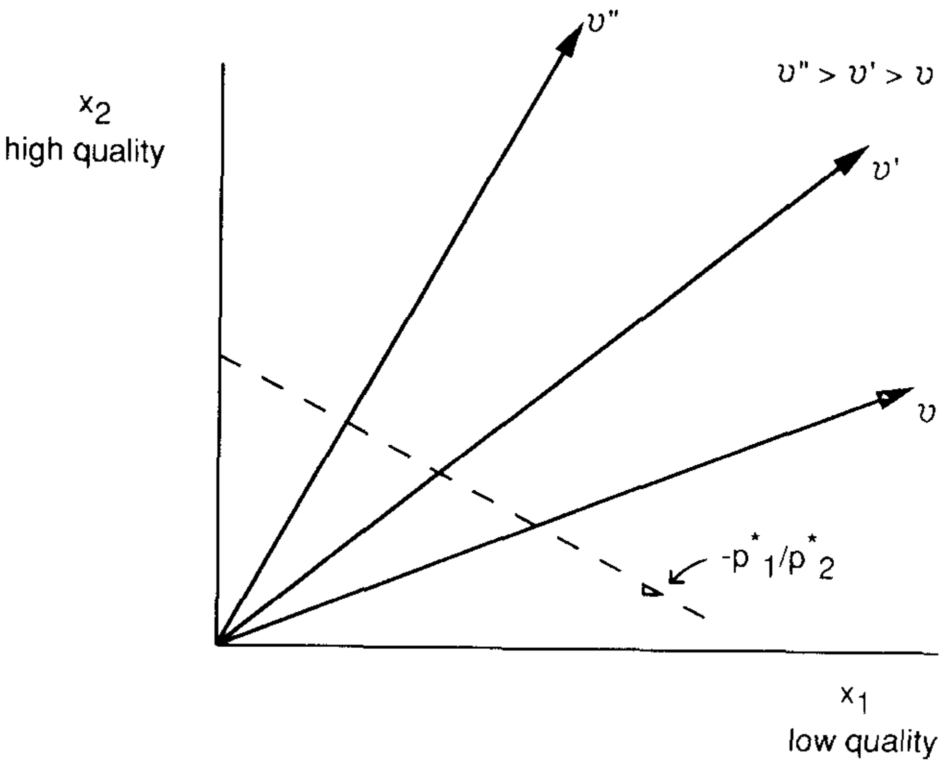
All of the income and price elasticities of interest are derived from these equations by taking natural logarithms and differentiating with respect to ln Y or ln P_G. Of particular interest is the price derivative of the logarithm of the first equation: the price elasticity of the Hicksian composite commodity is equal to the sum of the price elasticity of quality and the price elasticity of physical quantity.

IMPLEMENTATION OF HICKSIAN COMPOSITE COMMODITY ESTIMATION

Techniques for estimating the price elasticity of *physical quantity* from household survey data, adjusting for the effect of quality choice on unit values, have been suggested by Cox and Wohlgenant and Deaton (1986, 1987, 1988). Cox and Wohlgenant assume that quality choice depends only on household characteristics such as family size, income, race, etc. They assume that any variation in household-level unit values from regional/quarterly unit values, after controlling for the effects of these variables, represents real price variation.

Deaton's (1986, 1987, 1988) approach to estimating physical quantity elasticities is similar in relating quality choice to household characteristics, but takes into account two

Figure 1—Iso-Quality Lines for the Case of One High Priced and One Low Priced Good



additional factors.[2] First, he considers that quality choice also may be dependent on commodity price. Second, he notes that the derivation of unit values by division of expenditure by physical quantity creates a spurious negative correlation between physical quantity and unit value if the variables are measured with error. This creates bias in the estimates of elasticities of physical quantity with respect to price, which may act to countervail the usual attenuation bias and produce estimates that are more negative than the true price elasticities. His method of correcting for measurement error makes use of the clustered structure of most household surveys (that is, the fact that households surveyed often are clumped based on geographical proximity within primary sampling units). He assumes that real price variation exists only across clusters, while within any geographic area variations in unit value reflect either quality choice or measurement error.

[2] The notation in this article differs somewhat from that used by Deaton. Deaton's (1988) Q is equivalent to this article's q, both indicating the physical quantity demanded. His price level λ is equivalent to this article's P. His q is equivalent to this article's x (taking x to be measured directly in the relevant units so that k^0 in Deaton's expression $k^0 \cdot q$ is a vector of ones). He subscripts his variables by c for the geographic cluster and suppresses identification of the various goods; here the variables are subscripted for the goods group G with the sample cluster notation suppressed.

Estimates of the price elasticity of demand as defined in the Hicks composite commodity model can be derived through a simple extension of Deaton's method for estimating price elasticities of physical quantity. While in his theoretic presentation Deaton stops short of assuming that base prices are the same in all clusters, he assumes that variation in relative prices between clusters must be limited sufficiently (p. 10) so that unit value deflated by a cluster price index can be taken to represent a measure of quality. As he omits cluster-specific price relatives from his empirical specifications of physical quantity and quality choice, the resulting estimation method is identical to one which could be derived using the Hicks composite commodity model. Presentation of the method in this section is based almost entirely on Deaton's (1986) work, even though references are given to Hicks' composite commodity model equations. The extension of his work here, justified by use of aggregation theory, is simply the identification of the price elasticity of demand with the *sum* of the price elasticity of physical quantity *and that of quality*, rather than with the price elasticity of physical quantity alone.

In the following exposition, Deaton's assumptions that there is no real price variation within clusters (1986, p. 17) and that preferences are weakly separable over composite commodities (1986, p. 8), are carried over. The addition of the assumption of weak separability of preferences[3] allows the elementary good demand functions to be simplified from

(6) $x_i = x_i(p, Y)$

functions of all prices and income to

(7) $x_i = x_i(p_G, E_G)$

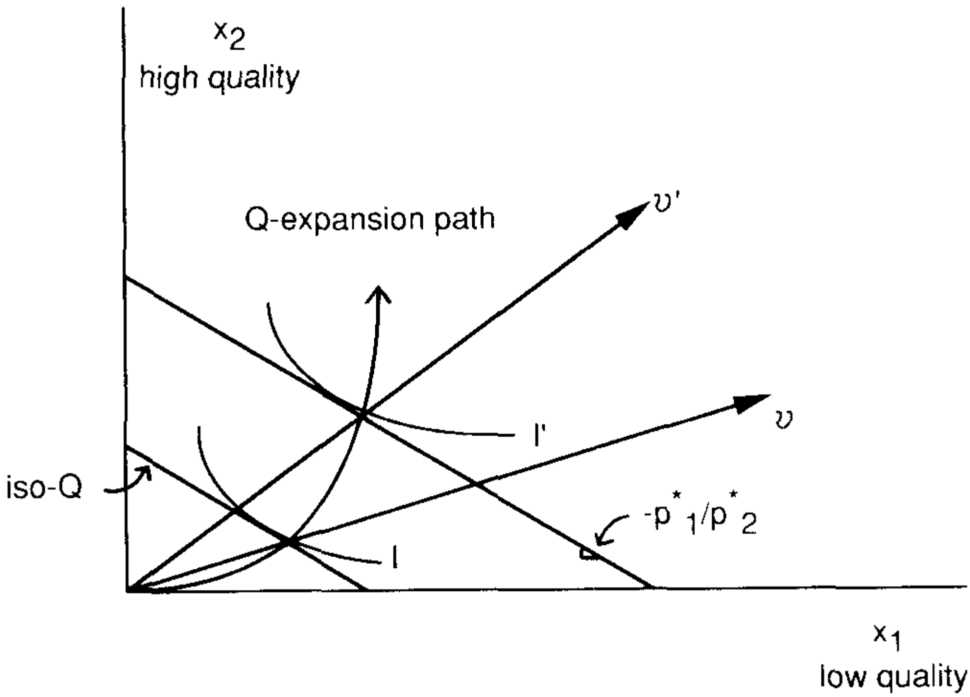
functions of only within-group prices p_G (i.e., the vector of prices with $i \in G$) and group expenditure. Because demand functions are homogeneous of degree zero, and base prices are constant, these can be simplified further to:

(8) $x_i = x_i(p_G P_G^{-1}, E_G P_G^{-1}) = x_i(p_G^*, Q_G) = x_i(Q_G)$.

This second stage budgeting problem is illustrated in Figure 2. By weak separability, there exists a well-defined preference ordering over x_1 and x_2, represented by subutility indifference curves I and I'. Iso-Q lines with slope $-p_1^*/p_2^*$ perform like budget lines, and Q-expansion paths are analogous to the income-expansion paths of first stage budgeting. In this diagram, quality is shown as increasing (from v to v') with Q. If Q increases with income, then in this case both quality and quality-adjusted quantity would be normal. (Iso-q lines—representing constant physical quantity—would be represented by lines with slope of -1.)

[3] That is, there exist some subfunctions u_G and function U such that $U(x_1, x_2, \ldots, x_R)$ can be rewritten as $U(u_1(x_1), u_2(x_2), \ldots, j_G(x_G), \ldots, u_M(x_M))$, where x_G is the vector of x_i's, $i \in G$. Deaton begins with such a model in his 1987 work.

Figure 2—Second Stage Choice of Quality When Preferences are Weakly Separable



In the interest of providing a replication (as well as extension) on U.S. data of Deaton's study, which estimated own-price (physical quantity) elasticities for four food commodities using data from the Ivory Coast, the econometric model specification has been borrowed from Deaton (1986) in all but one aspect. The only difference between the specifications is that, due to data processing constraints, this paper does not attempt to exploit intertemporal price variation. As in Deaton's (1986) work, only the demand for food items will be examined; cross-price effects will be ignored; and a simple log-linear specification will be used.[4]

With relative prices fixed, the demand equation for elementary commodity i (a member of group G) for household h (located in geographical cluster c) can be obtained by combining equations (8) and (2):

[4] Later works by Deaton include cross price effects (1987) and share-form instead of log-linear functional forms (1988). Extension of Deaton's methodology to the Hicksian case should not be complicated much by such refinement. Application of these and other modifications to the U.S. data in the empirical example that follows was precluded by the author leaving the employment of the Bureau of Labor Statistics and hence losing access to the physical quantity and geographical data. The performance of the model on U.S. data (discussed below) also implies that application of refined techniques to this particular data source may be of somewhat dubious usefulness.

(9) $x_{ih}(Q_{Gh}) = x_{ih}(Q_{Gh}(P_c,Y_h)) = x_{ih}(P_c,Y_h)$

where:

$P_c \equiv$ The $M \times 1$ vector of group price levels in cluster c.

Because q_G is a function of only the x_i's ($i \in G$, equation (5)), and V_G is a function of only the x_i's ($i \in G$), the fixed base prices and the group price level = g. These too can be written as functions of only group price levels and income.

Deaton's (1986) equations relating physical quantity and unit value for each commodity to the independent variables can be written as (dropping the G subscripts for simplicity)

(10) $\ln q_{hc} = \alpha_q + \beta_q \ln Y_{hc} + \gamma_q \cdot z_{hc} + \theta_q \ln P_c + f_c + u_{qhc}$

(11) $\ln V_{hc} = \alpha_V + \beta_V \ln Y_{hc} + \gamma_V \cdot z_{hc} + \theta_V \ln P_c + f_c + u_{Vhc}$

where:

q_{hc} and V_{hc} = The observed physical quantity and unit value of purchases of the commodity by household h in cluster c;
Y_{hc} = The per capita income of household h in cluster c;
z_{hc} = A vector of demographic variables describing the household;[5]
P_c = The price level of the commodity in the cluster to which household h belongs;
f_c = A cluster fixed effect; and
u_{qhc} and u_{Vhc} = Random error terms.

The cluster fixed effect in the physical quantity equation represents unobservable taste variation across clusters and is assumed to be orthogonal to both P_c and z_{hc}. Such a fixed effect term is not included in the unit value equation, because if variation in unit values across clusters is due to systematic cross-cluster differences in tastes, the use of cross-cluster comparisons for price elasticity estimation would be precluded.

Equations (10) and (11) cannot be directly estimated, as the variable P_c is not observed. Estimation, according to Deaton's method, proceeds in four stages. First, within-cluster estimation[6] of equations (10) and (11) yields estimates of the α's, β's, γ's, and variances and covariances of the error terms σ_{qq}, σ_{VV}, and σ_{qV}. Second, cluster mean physical quantities and unit values are purged of the effects of income and demographics by subtracting

[5] The use of per capita income and demographic variables are attempts to (crudely) adjust for the differences in standards of living achievable from a given household income and differences in the form of household preferences across households of different composition. Note that if there is relative price variation within clusters, these equations are misspecified due to lack of terms for the within-group price relatives.

[6] That is, estimation with all variables expressed as deviations from cluster means, or equivalently, with dummy variables for clusters added to each equation. The P_c term drops out.

$$\tilde{\beta}_q \ln Y_{.c} + \tilde{\gamma}_q \cdot z_{.c} \text{ and } \tilde{\beta}_V \ln Y_{.c} + \tilde{\gamma}_V \cdot z_{.c}$$

from the respective left side variables. Third, $\tilde{\phi}$, a consistent estimate of θ_q/θ_V, is calculated from the estimated variance of the new cluster mean unit value measure, $\tilde{m}_{VV}$, the estimated covariance between the new cluster mean q and V measures, $\tilde{m}_{qV}$, the estimated within-cluster error variance and covariance, and ψ, a measure of average cluster size, according to the formula

$$(12)\ \tilde{\phi} = \frac{\tilde{m}_{qV} - \tilde{\sigma}_{qV}/\psi}{\tilde{m}_{VV} - \tilde{\sigma}_{VV}/\psi}$$

The σ's are subtracted from the m's because any covariance between q and V within a cluster is presumed to be due to measurement error.[7] If unit value moved one-to-one with market price (i.e., $\theta_V = 1$), $\tilde{\phi}$ would be a suitable measure of the price elasticity of physical quantity, adjusted for measurement error. To allow for variation of quality with price, a fourth stage must be accomplished. The estimate from the various stages must be combined with an equation that can be derived from the earlier identities:

$$(13)\ \theta_V = 1 + \frac{\beta_V\,\theta_q}{\beta_q}$$

(A repetition of Deaton's derivation of this equation, in the notation of the present paper, is given in the appendix.) The following equations, derived from equation (13) and solved using the estimates of income elasticities at stage one and $\tilde{\phi}$ in stage three, yield the desired price elasticities:

$$(14)\ \tilde{\theta}_q = \frac{\tilde{\phi}}{1 - \tilde{\phi}(\tilde{\beta}_V/\tilde{\beta}_q)}$$

$$(15)\ \tilde{\theta}_V = 1 + \tilde{\theta}_q(\tilde{\beta}_V/\tilde{\beta}_q)$$

Deaton (1986) regards $\tilde{\theta}_q$ as the estimated price elasticity of the commodity, and stops here.

The use of the Hicksian composite commodity assumptions (which we have seen are implicit in Deaton's specification) extends the estimation procedure one additional step. Instead of interpreting $\tilde{\theta}_q$ as the price elasticity of the commodity, it is seen to be the price elasticity of the *physical* quantity of the commodity (dependent on the physical metric chosen). The estimated price elasticity of demand for the *commodity* is, by equation (3) in elasticity form,

[7] Errors-in-variables estimators of this type are discussed and formulas for their asymptotic variance are derived in Deaton (1985, 1986) and Fuller (1975).

(16) $\tilde{\theta}_Q = \tilde{\theta}_q + \tilde{\theta}_V$

An expression for the approximate asymptotic standard error for this elasticity can be derived using Taylor expansions.[8]

AN EXAMPLE USING U.S. DATA

The diary component of the *U.S. Consumer Expenditure Survey* is used to gather data on expenditures and quantities of foods purchased for home use. The survey is conducted by the Census Bureau for the Bureau of Labor Statistics. Respondents are asked to record the dollar value of each purchase, the number of cans, bottles, etc. and the net weight or volume per bottle, can, etc. Respondents are asked to record purchases for a two week period. Information on household composition, income, employment, and expenditures on other items purchased on a daily or weekly basis also is gathered. The survey has been conducted on a continuing basis since October 1979; however, in order to ease computation only data collected during the second quarter of 1985 are used in this study. Prices are assumed to be constant within each geographical primary sampling unit (PSU) over this three month period.

The quantity-related fields in the continuing survey data base are currently available only internally within the BLS. Until this study, they have been used only to check reasonableness of the expenditure data. Nelson (1987) describes these data and the creation of the data set used in this study. Seven groups of goods were not used for the present study because of severe problems with nonreporting or nonstandard reporting of quantities of items purchased. (For example, fresh fruits and vegetables often are recorded as being bought by the bag, head, etc.) Quantities of the goods analyzed here have been recoded into pounds, except for milk (quarts) and eggs (dozens). The absence of strong data quality control procedures for gathering and coding the physical quantity data suggests that measurement error may be a sizable problem.

The database for the second quarter of 1985 covers 1470 consumer units, but only 937 were used in the analysis. Consumer units were omitted from the sample if they only completed one week of the survey (282 households), did not report values for major sources of income (243 households), or reported negative incomes (eight households). The 937 remaining consumer units were clustered in 102 PSUs representing both urban and rural areas.

The number of households observed making quantifiable purchases of any composite commodity, however, ranges from a low of 269 for poultry to a high of 696 for milk. One approach to dealing with the fact that many households do not report expenditures on a commodity could be to treat no response as a true zero response and to estimate the within equations using a Tobit procedure and a different functional form. If consumer units who do not report such expenditure during the two week survey period probably do

[8] An expression for var $(\tilde{\theta}_q)$ can be found in Deaton (1986). The following expressions were used in calculation of other variances, where $\rho \equiv \beta_V / \beta_q$:

$$\text{var}(\tilde{\theta}_V) = (\tilde{\rho}\tilde{\theta}_V^2)^2 \, \text{var}(\tilde{\phi}) + (\tilde{\phi}\tilde{\theta}_V^2)^2 \, \text{var}(\tilde{\rho}),$$

$$\text{var}(\tilde{\theta}_Q) = [(1 + \tilde{\rho})\tilde{\theta}_V^2]^2 \, \text{var}(\tilde{\rho}) + [\tilde{\phi}(\tilde{\phi} + 1)\tilde{\theta}_V^2]^2 \, \text{var}(\tilde{\rho}).$$

purchase the good at some time (as is likely the case here), however, this formulation is not immediately convincing. Complete analysis of the selection problems generated by using only expenditure reporters for analysis would require a model not only of goods demand, but of frequency of purchase as well. Rather than reflecting the influence of a latent variable, zero/nonzero reporting of expenditure may reflect a certain probabilistic element in observing consuming households doing their purchasing in any given time period. Deaton (1986, p. 30) suggests that to the extent that this is the case and the probabilistic element is uncorrelated with the other variables in the model, the simple double log model will yield consistent estimates (with only an adjustment to the intercept term). While the possible problems involved can not be trivialized, this paper replicates Deaton's study in using in analysis only households with positive purchases.

The within-cluster estimation of equations (10) and (11) was accomplished using measures of physical quantity, unit value, per capita income, dummy variables representing each PSU, and 14 demographic variables representing the number of females and males in seven different age classes. The income elasticities derived from these regressions are reported in the top section of Table 1. (The income elasticity of expenditure is equal to the sum of the other two elasticities.) The income elasticity of quality is positive for all commodities but milk and eggs and, in some cases, is highly statistically significant. In contrast, the income elasticity of physical quantity is negative for all commodities but beef and fish. U.S. consumers apparently substitute away from physical quantity and toward quality with increases in income. The interaction of the two factors leads to negative income elasticities of demand for six of the 11 commodities. While this may seem odd, other cross-section studies of income elasticities in the U.S. for food at home also have resulted in low (Blaylock and Smallwood) or negative (Haidacher *et al.*,) estimates of the income elasticity of expenditure on some commodities. While factors such as dieting may contribute to these negative income elasticities, the availability of food away from home probably plays a large part; simple income elasticities of expenditure (controlled for demographics) of .18 for food at home, .47 for food away from home, and .31 for total food were estimated on roughly the same data set.

The effects of the demographic variables and PSU dummies can be summarized briefly.[9] Demographic variables tend to be highly statistically significant as a group in the physical quantity and expenditure equations, as might be expected. The magnitudes and signs of the individual demographic coefficients frequently show a larger positive impact of an additional male on household expenditures on the food item, all else constant, than for an additional female. The coefficients in the milk equation are particularly interesting in light of recent concern about the effect of calcium intake on women's health; adult women ages 18 to 54 seem to make no marginal impact on household expenditures, in contrast to significant positive impacts for members of all other groups. Demographics seem to make a statistically significant difference in quality choice only for poultry, milk, and (marginally) sweets.[10] The PSU dummies are statistically insignificant

[9] Tables presenting the full results of the regressions and F-tests are available from the author.

[10] The degree of variation of the unit value of milk purchases with demographics is somewhat surprising given the relatively high degree of physical homogeneity (no pun intended) within this commodity group. (Cream purchases seem to constitute only a small percentage of purchases in this group.) The existence of

as a group, in the unit value equation, for six of the 11 commodities. Considering that interarea price variation is the key to estimation of price elasticities in this model, the low value of this F-test found in the majority of the unit value equations is not encouraging.

The remainder of Table 1 presents the results of the calculation of price elasticities.[11] One of the intermediate computations, that of the within-PSU covariance to variance ratio (or $\tilde{\sigma}_{qV}/\tilde{\sigma}_{VV}$ in the seventh row of numbers), is of particular interest. If one were to assume that unit value represents exogenous price and that the sum of physical quantities is the appropriate measure of quantity demanded, then this estimate would give one the desired measure of the price elasticity.[12] These price elasticities are appealing; they are all of the hypothesized sign, all exhibit plausible values (varying from moderate inelasticity to moderate elasticity, and are all highly statistically significant. Unfortunately, they take no account of quality issues. And if the assumptions of the Deaton model are correct, these elasticities of physical quantities with respect to local unit values would be entirely the result of measurement error.

Final estimates of price elasticities of demand are statistically significant for only three of the eleven commodities; the estimate of the corrected price elasticity of physical quantity is statistically significant for only one commodity, and for no commodity is the estimated price elasticity of quality statistically significantly different from zero.[13] Imprecision is clearly a major problem with this model and data.

The results for beef are the most intuitively appealing in sign and magnitude, though only the final demand elasticity is (marginally) statistically significant. The estimated income elasticities of physical quantity, quality, and demand have positive signs. The within-PSU error covariance to variance ratio (-.751) is the naive price elasticity estimated under the assumption of no quality variation and may represent only measurement error. The across-PSU covariance to variance ratio, based on cluster mean physical quantities and unit values purged of income and demographic effects, is -.741. Adjusting this for measurement error (using equation (12)) results in a larger negative price elasticity (-.803), in this case reflecting the relative importance of correction for attenuation bias. Further adjustments for the endogeneity of quality choice gives an estimate of the price

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bulk discounts - lower prices per ounce for gallons than for quarts of milk - could be an added source of variation in unit value. Such discounts would add significant complexity to the analysis if it is assumed that they create piece-wise linear budget constraints. A possible rationale for the current treatment might be provided by an assumption that smaller packages provide greater quality in the dimension of convenience.

[11] Some of the results of Table 1 (the full set of income elasticities and the value of the price elasticities of physical quantity, quality, and demand) previously have appeared in Nelson (1990).

[12] That is, the seventh line of Table 1 also represents what the coefficient on Ln(V) in a regression of Ln(q) on Ln(V), income, demographics, and PSU dummies. Such assumptions have been made by, for example, Timmer and Alderman (1979) and Timmer (1981) for Indonesian data. Of course, it could be that in the Indonesian data quality variation and measurement error are not important.

[13] Asymptotic standard errors with the related significance levels are shown only for $\tilde{\phi}$, $\tilde{\theta}_q$, $\tilde{\theta}_v$, and $\tilde{\theta}_Q$, since $\tilde{\theta}_V$ and $\tilde{\theta}_E$ and differ from $\tilde{\theta}_v$, and $\tilde{\theta}_Q$, respectively, only by a constant.

Table 1—Summary of Estimated Elasticities, United States Consumer Expenditure Survey, Diary Component, April-June 1985

	Cereals	Bakery	Beef
Sample Size	511	682	393
Income Elasticities From "Within" Regressions			
Physical Quantity (β_q)	-.185***	-.042	.047
Unit Value, Quality (β_V,β_v)	.115***	.146***	.064
Expenditure, Demand (β_E,β_Q)	-.070	.104***	.110
Derivation of Price Elasticity of Physical Quantity (θ_q)			
Average PSU Size (ψ)	3.26	3.86	2.76
Within PSUs:			
Covariance ($\tilde{\sigma}_{qV}$)	-.296	-.120	-.210
Spurious Price Variance ($\tilde{\sigma}_{VV}$)	.380	.217	.280
Covariance to Variance Ratio	-.779***	-.551***	-.751***
Across PSUs:			
Covariance ($\tilde{m}_{qV}$)	-.072	-.039	-.063
Price Variance ($\tilde{m}_{VV}$)	.119	.064	.085
Covariance to Variance Ratio	-.606	-.614	-.741
With Measurement Error Correction ($\tilde{\delta}$)	7.24 (57.7)	-1.09 (1.8)	-.803 (.91)
With Quality Correction ($\tilde{\theta}_q$)	1.32 (2.0)	.39 (.6)	-.382 (.46)
Other Price Elasticities			
Unit Value (θ_V)	.182	-.359	.476
Quality (θ_v)	-.818 (1.2)	-1.36 (.96)	-.524 (.58)
Expenditure (θ_E)	1.50	.032	.094
Price Elasticity of Demand ($\tilde{\theta}_Q$)	.499 (.85)	-.968* (.57)	-.906* (.50)

Table 1 (continued)—Summary of Estimated Elasticities

Sample Size	Pork 373	Other Meat 391	Poultry 269	Fish 311
Income Elasticities From "Within" Regressions				
Physical Quantity (β_q)	-.144	-.071	-.260**	.162*
Unit Value, Quality (β_V,β_v)	.093**	.084*	.161**	.040
Expenditure, Demand (β_E,β_Q)	-.051	.013	-.099	.203**
Derivation of Price Elasticity of Physical Quantity (θ_q)				
Average PSU Size (ψ)	2.90	2.64	1.95	2.29
Within PSUs:				
Covariance ($\tilde{\sigma}_{qV}$)	-.259	-.204	-.414	-.091
Spurious Price Variance ($\tilde{\sigma}_{VV}$)	.251	.282	.400	.284
Covariance to Variance Ratio	-1.03***	-.725***	-1.04***	-.321***
Across PSUs:				
Covariance ($\tilde{m}_{qV}$)	-.066	-.136	-.259	-.030
Price Variance ($\tilde{m}_{VV}$)	.081	.140	.301	.118
Covariance to Variance Ratio	-.812	-.927	-.863	-.251
With Measurement Error Correction (ϕ)	-4.31 (9.7)	-1.77** (.88)	-.495 (.43)	-1.57 (5.5)
With Quality Correction ($\tilde{\theta}_q$)	2.43 (3.7)	1.62 (2.4)	-.713 (.90)	1.13 (2.9)
Other Price Elasticities				
Unit Value (θ_V)	-.563	-.917	1.44	.719
Quality (θ_v)	-1.56 (2.0)	-1.92 (1.6)	.441 (.60)	-.281 (.78)
Expenditure (θ_E)	1.86	.707	.728	-.413
Price Elasticity of Demand ($\tilde{\theta}_Q$)	.864 (1.9)	-.293 (1.0)	-.272 (.36)	-1.41 (3.6)

Table 1 (continued)—Summary of Estimated Elasticities

	Eggs	Milk	Processed Vegetables	Sweets
Sample Size	413	696	430	393
Income Elasticities From "Within" Regressions				
Physical Quantity (β_q)	-.093*	-.047	-.346***	-.231**
Unit Value, Quality (β_v,β_v)	-.001	-.000	.194***	.254***
Expenditure, Demand (β_E,β_Q)	-.095*	-.047	-.152**	.022
Derivation of Price Elasticity of Physical Quantity (θ_q)				
Average PSU Size (ψ)	2.78	3.99	3.02	2.62
Within PSUs:				
Covariance ($\tilde{\sigma}_{qV}$)	-.066	-.098	-.426	-.595
Spurious Price Variance ($\tilde{\sigma}_{VV}$)	.120	.071	.455	.681
Covariance to Variance Ratio	-.551***	-1.38***	-.937***	-.874***
Across PSUs:				
Covariance ($\tilde{m}_{qV}$)	-.020	.045	-.127	-.344
Price Variance ($\tilde{m}_{VV}$)	.068	.035	.148	.359
Covariance to Variance Ratio	-.287	-1.27	-.860	-.958
With Measurement Error Correction ($\tilde{\delta}$)	.168 (.47)	-1.15** (.46)	-4.79 (33.0)	-1.18*** (.36)
With Quality Correction ($\tilde{\theta}_q$)	.168 (.47)	-1.14* (.65)	2.85 (11.7)	4.02 (6.7)
Other Price Elasticities				
Unit Value (θ_v)	1.00	.991	-.594	-3.41
Quality (θ_v)	.002 (.05)	-.009 (.40)	-1.59 (6.5)	-4.41 (6.4)
Expenditure (θ_E)	1.17	-.151	2.25	.613
Price Elasticity of Demand (θ_Q)	.170 (.48)	-1.15** (.46)	1.25 (5.2)	.387 (.90)

Asymptotic standard errors in parentheses
*** Significant at the 1 percent level
** Significant at the 5 percent level
* Significant at the 10 percent level

elasticity of physical quantity of -.382. Unit value does not rise one-to-one with price, but rather rises only .476 percent with a 1 percent increase in price. A 1 percent exogenous price increase causes a .524 percent decrease in the indicator of quality and a .382 percent decrease in physical quantity, leading to a .906 percent decrease in the measure of overall demand.

Bakery products and milk also have estimated price elasticities of demand that are statistically significantly different from zero. Though these final elasticities are of the expected sign and not unreasonable in magnitude, the component elasticities exhibit somewhat less intuitively appealing behavior. With the assumption of weak separability, both income and exogenous price can affect the demands for the elementary goods only through the demand for the composite commodity. Certain relationships between the price and income effects hence are locked-in by the underlying model. In the case of bakery goods, an increase in income increases demand for the quality-adjusted commodity through an increase in quality, accompanied by a small reduction in the physical quantity. In precisely the opposite manner, an increase in price reduces demand by decreasing quality and increasing physical quantity, according to the sign of the estimated parameters.

The estimated income elasticities of physical quantity and of unit value are negative for milk (though both are statistically insignificant). This conjunction of signs (in combination with other magnitudes) leads to negative estimated price elasticities for physical quantity, quality, and demand. In the case of milk, it would make little effective difference whether one used the estimate of the price elasticity of physical quantity (-1.14) or of quality-adjusted quantity (-1.15) as the estimate of the price elasticity of demand because the price elasticity of quality is small (as well as insignificant). Unit value is estimated to increase almost one-to-one with price.

For the remaining eight commodities, estimates of the price elasticities of q, v, and Q are not statistically significantly different from zero. Estimates of the price elasticities for fish are not unreasonable (following the same pattern in sign as those for beef), though imprecise. The signs on estimated elasticities for sweets and for other meats follow the same pattern as for bakery products. The signs of the price elasticities of demand for pork and for processed vegetables, if taken seriously, would imply that these are giffen goods. Poultry exhibits a demand elasticity of the expected sign, but a positive elasticity of quality with respect to price. The imprecision of the estimates suggests that the signs of the estimates should, however, not be subjects of undue concern.

In the derivation of estimates for cereals and eggs, the measurement error correction appears to cause the estimate of the price elasticity of physical quantity to change sign (from negative to positive). In these two cases, the within-cluster covariance essentially overwhelms the across-cluster covariance, as can be seen by substituting the values from the table into equation (12). It might be better to report zeros as estimates for the remainder of the table for these two commodities, but the imprecision of the reported calculations also suggests that the estimates be treated as zeros. (The same sign switch occurs with Deaton's (1986) estimates for one commodity group, starches, for rural households, for exactly the same reason.)

DISCUSSION AND CONCLUSION

While Deaton used an ill-defined measure of food quantity in his estimation of price elasticities using household survey data, a simple extension of his method yields a way of estimating demand elasticities for theoretically valid Hicksian composite commodities.

The empirical results from U.S. data suggest that the theoretical distinction between "physical quantity" of a commodity (i.e., the simple sum of physical quantities of elementary goods) and "quantity demanded" (defined as a quality-adjusted quantity measure with reference to aggregation theory), may be empirically important for some commodities. Statistically significant demand elasticities are obtained for two of the commodities, while the corresponding price elasticities of physical quantity are not statistically significant. Price elasticities of quality, while often sizable in magnitude, are estimated too imprecisely, like most of the parameters, for strong conclusions to be drawn. For one commodity with a statistically significant price elasticity of demand, the price elasticity of quality appears to be small in magnitude, and hence the estimated elasticities for physical quantity and quality-adjusted quantity are very close. The low level of precision obtained for most of the price elasticity estimates in the U.S. data probably is related to the structure assumed for price variation. The method developed by Deaton is based on assumptions of an absence of real price variation within geographical clusters and substantial price level variation across clusters. Considering the size of some of the primary sampling units for the *Consumer Expenditure Survey* (e.g., the Los Angeles standard metropolitan statistical area) and the weak performance of the PSUs in the unit value equations, these assumptions are suspect. It may be that real price variation within the U.S. may be less related to geography (due to a well-developed transportation network) and more related to locally varying factors (such as price discrimination or supermarket promotional activities) than is compatible with Deaton's method.

Further research may yield a better database for estimation of price and income elasticities of demand for food, in which the constant relative price assumption underlying the Hicksian formulation is more plausible; the extension of Deaton's method described here should be considered as one way of accomplishing such estimation. Meanwhile, however, the contrast between the conceptually sophisticated, though largely statistically insignificant, results of applying the Hicksian model and the highly statistically significant, but spurious, estimates derivable using a naive view of quality (in Table 1) should serve to remind us that high or low t-statistics can not be taken in themselves as indicators of good or bad empirical research.

REFERENCES

1. Blaylock, James R., and David M. Smallwood, "U.S. Demand for Food: Household Expenditures, Demographics, and Projections," *U.S. Department of Agriculture, Economic Research Service Technical Bulletin No. 1713* (February 1986).

2. Cox, Thomas L., and Michael K. Wohlgenant, "Prices and Quality Effects in Cross-Sectional Demand Analysis," *American Journal of Agricultural Economics* (November 1986), pp. 908-919.

3. Cramer, J.S., "Interaction of Income and Price in Consumer Demand," *International Economic Review*, 14, no. 2 (June 1973), pp. 351-363.

4. Deaton, Angus, "Panel Data From Time Series of Cross-Sections," *Journal of Econometrics*, 30 (1985), pp. 109-126.

5. Deaton, Angus, "Quality, Quantity, and Spatial Variation of Price," Research Program in Development Studies Working Paper 127, Woodrow Wilson School (November 1986).

6. Deaton, Angus, "Quality, Quantity, and Spatial Variation of Price," *American Economic Review*, 78, no. 3 (June 1988), pp. 418-430.

7. Deaton, Angus, "Estimation of Own and Cross-Price Elasticities From Household Survey Data," *Journal of Econometrics*, 36 (1987), pp. 7-30.

8. Fuller, Wayne A, "Regression Analysis for Sample Survey," *Sankhya: The Indian Journal of Statistics*, 37, no. 3, Series C, pp. 117-132.

9. Haidacher, Richard C., John A. Craven, Kuo S. Huang, David M. Smallwood and James R. Blaylock, "Consumer Demand for Red Meats, Poultry, and Fish," U.S. Department of Agriculture, Economic Research Service, National Economics Division, *ERS Staff Report No. AGES820818* (September 1982), pp. 40-41.

10. Hicks, J. R., *Value and Capital* (Oxford: Oxford University Press, 1936), as cited in Green and Deaton and Muellbauer.

11. Houthakker, H.S., "Compensated Changes in Quantities and Qualities Consumed," *Review of Economic Studies*, 19 (1952-1953), pp. 155-164.

12. Muellbauer, John, "Household Production Theory, Quality, and the Hedonic Technique," *American Economic Review*, 64, no. 6 (1974), pp. 977-994.

13. Nelson, Julie A., "Food Quantity Data in the *U.S. Consumer Expenditure Survey*: An Analysis of Usefulness for Economic and Statistical Studies," U.S. Bureau of Labor Statistics Working Paper No. 174 (October 1987).

14. _____, "Quality Variation and Quantity Aggregation in Consumer Demand for Food," *American Journal of Agricultural Economics*, 73, no. 4 (November 1991), pp. 1204-1212.

15. _____, "Quantity Aggregation in Consumer Demand Analysis When Physical Quantities are Observed," *Review of Economics and Statistics*, 72, no. 1 (February 1990), pp. 153-156.

16. Theil, H. "Qualities, Prices and Budget Enquiries," *Review of Economic Studies*, 19 (1952-1953), pp. 129-147.

17. Timmer, C. Peter, and Harold Alderman, "Estimating Consumption Parameters for Food Policy Analysis," *American Journal of Agricultural Economics*, 61 (1979), pp. 982-987.

18. Timmer, C. Peter, "Is There Curvature in the Slutsky Matrix?" *Review of Economics and Statistics*, 63 (1981), pp. 395-402.

Appendix—Derivation of Identifying Equation (Source: Deaton, 1986)

Letting θ denote price elasticity and β denote income elasticity, equation (4) in the text implies

(A1) $\theta_E = 1 + \theta_v + \theta_q$

(A2) $\beta_E = 0 + \beta_v + \beta_q$

Denoting the vector of elementary goods in group G by x_G

(A3) $v_G = v_G(p_G^*, x_G, q_G)) = v_G(x_G) = v_G(E_G P_G^{-1})$

where the third expression follows from constancy of the base prices and the definition of q_G as the sum of the elements in x_G and the last expression follows from equation (8), the definition of Q_G, and the constancy of base prices.

Denoting the elasticity of a variable with respect to group expenditure by η, and noticing that group expenditure itself depends on income, (A3) implies

(A4) $\beta_v = \eta_v \beta_E$

(A5) $\theta_v = \eta_v(\theta_E - 1)$

Using (A2) to substitute for β_E in (A4), the resulting equation to substitute for η_v in equation (A5), (A1) to substitute for θ_E in (A5), and finally (5) (in elasticity form) to convert from quality to unit value terms, gives

(13) $\theta_V = 1 + \dfrac{\beta_V \theta_q}{\beta_q}$

the expression in the text.