Economies of Scale and Scope in Banking: Evidence From a Generalized Translog Cost Function

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This paper provides new evidence on the extent of economies of scale and scope in the banking industry. A generalized translog statistical cost function is used to avoid a number of substantive problems associated with the estimation and use of the conventional translog statistical cost function.

The presence of statistically significant, relatively large economies of scale for all size classifications is reported. These economies of scale remain well beyond a bank size of $1 billion of total assets. No evidence is found of statistically significant global scope economies. There appear to be economies from the joint production of selected product pairs, but these efficiencies appear to be offset by diseconomies in the production of others. Thus, the results do not provide support for the existence of natural monopoly in the commercial banking industry. They do suggest, however, that many smaller banks can improve their productive efficiency through increases in size.

INTRODUCTION

Most of the recent attempts at measuring scale and scope economies for financial institutions rely on parameter estimates obtained from a transcendental logarithmic statistical cost function (TCF).[1] The TCF generally is chosen because it explicitly allows for jointness in production by multiple product firms, its parameters can be estimated easily using standard statistical methods, and the parameter estimates have straightforward economic interpretations. A logarithmic transformation only can be applied to positive values, however. Thus, the TCF cannot be estimated if financial institutions included in the sample fail to produce one or more of the defined products.[2] Further, several of the measures conventionally used to capture economies of scale and scope require a comparison of the costs from both joint and independent production. As a consequence, these

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[1] See Clark (1988) for a recent review of this literature.

[2] This problem often leads to arbitrary or highly aggregated definitions of bank products or the exclusion of banks that fail to produce all of the defined products.

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measures only can be approximated, and the approximations appear to be sensitive to the procedure used to obtain them.[3]

In their path-breaking book, Baumol, Panzar, and Willig (1982) suggest a modification of the TCF. This modification applies a Box-Cox transformation to all products included in the TCF. This provides a statistical cost function that not only allows jointness in production, but also permits zero outputs of defined products. Despite the widespread awareness and obvious appeal of their suggested modification, this hybrid or generalized translog cost function (GTCF) has not been adopted widely in the banking literature. Papers that have used the GTCF have employed a highly restricted version, applying the same transformation parameter to a highly aggregated set of products.[4]

The purpose of this paper is to estimate the parameters of a more generalized and disaggregated GTCF that is better able to distinguish scale- and scope-related economies in the banking industry.

THE CONVENTIONAL TRANSLOG COST FUNCTION

Hypotheses concerning the existence of (dis)economies of scale and scope have been analyzed using parameter estimates obtained from the estimation of a statistical cost function assumed to take the TCF functional form. A general version of this cost function is:

$$(1)\ \ln TC = B_0 + \sum_{i=1}^{m} B_i \ln y_i + \sum_{k=1}^{n} C_k \ln p_k + \left(\frac{1}{2}\right) \sum_{i=1}^{m} \sum_{j=1}^{m} D_{ij} \ln y_i \ln y_j$$

$$+ \left(\frac{1}{2}\right) \sum_{k=1}^{n} \sum_{l=1}^{n} E_{kl} \ln p_k \ln p_l + \sum_{i=1}^{m} \sum_{k=1}^{n} F_{ik} \ln y_i \ln p_k + e$$

where:

ln	=	Natural logarithm;
$y_i (i = 1, \ldots, m)$	=	Output of the ith product;
$p_k (k = 1, \ldots, n)$	=	kth input price;
B_0, B_i, C_k, D_{ij}, E_{kl}, and F_{ik}	=	Parameters to be estimated; and
e	=	Random error term.[5]

[3] As a technical solution to this problem, most authors replace zeroes with arbitrarily chosen or computed small, nonzero values. The sensitivity or fragility of these measures to the approximation procedure has been illustrated in a recent paper by Pulley and Humphrey (1991).

[4] See papers by Berger, Hanweck, and Humphrey (1987), Buono and Eakin (1990), Cebenoyan (1990), and Lawrence (1989).

[5] The parameters of the TCF are constrained to impose the conditions of homogeneity of input prices and symmetry. (See Zardkoohi, Rangan, and Kolari, 1986.)

All of the variables appearing in equation (1) are entered in their logarithmic form. Therefore, the parameters of the TCF cannot be estimated if some of the firms included in the sample fail to produce one or more of the defined products.[6] To deal with this problem, most authors choose one of two courses of action. They either delete from the data any firms that fail to produce one or more of the defined products or they aggregate products to the degree necessary to eliminate any zero outputs from all observations in the data set.[7] A third technical solution consists of replacing any zero outputs in the data set with arbitrarily chosen positive values prior to the estimation of the function's parameters. The procedures employed to estimate and utilize the parameters from the TCF potentially invalidate the resulting estimates of scale and scope economies. Restricting the sample to include only those institutions that produce all defined products introduces an unknown sample selectivity bias to the parameter estimates. Excessive aggregation of products potentially aggregates possible (dis)economies arising from joint production. The replacement of zeroes prior to estimation may bias the parameter estimates in unknown ways. In addition, all three courses of action result in a set of data comprising banks that jointly produce all products included in the TCF. Further, using the TCF as a basis for measuring economies of scale and scope presents other substantive problems.

EVALUATING TOTAL COST WHEN THE OUTPUT OF ONE OR MORE PRODUCTS IS ZERO

In order to utilize the parameter estimates obtained from the TCF to produce approximate values for several conventional scale and scope measures, several authors have obtained approximations using arbitrarily chosen small but nonzero values as replacements wherever the measure calls for a zero output for one or more of the products. This approach presents two problems. First, if the arbitrary replacement value is outside the range of the sample data used to estimate the parameters of the TCF, the estimated values for these measures will be unreliable.[8] Second, even where the selected value is not outside the range of the sample data, such a procedure does not allow a comparison of joint versus independent production as required by these measures because all products explicitly are assigned nonzero values. Thus, as noted by Fuss and Waverman (1981), the TCF only can be used to "provide a sufficient, but not necessary, (local) test for economies of scope. However, because of the degeneracy of the cost function at zero outputs, it is a relatively weak test."

The parameters of the TCF cannot be estimated when the sample includes observations that fail to produce one or more of the defined products. Further, several of the conventional measures of economies of scale and scope cannot be computed directly when the statistical cost function is TCF. This occurs because these measures require that the

[6] The $\ln y_i$ is undefined when $y_i = 0$.

[7] For example, the vector of products might be defined as total assets and total liabilities, where each is the sum of distinct classes of assets and liabilities.

[8] Benston *et al.* (1983) discuss this problem and demonstrate that the measure for global scope economies can be made to yield scope diseconomies through the choice of a sufficiently small replacement value.

statistical cost function be evaluated with one or more of the included products being assigned a value of zero. Finally, the sensitivity of the resulting approximations to sample censoring, overly broad product definitions, and approximation procedures made necessary by the use of the TCF functional form remain an issue.

THE GENERALIZED TRANSLOG COST FUNCTION AND MEASURES OF ECONOMIES OF SCALE AND SCOPE

The GTCF has been proposed as a method of modifying the translog functional form to permit zero outputs. This modification is important because it potentially eliminates the problems associated with both estimation of the TCF and its use in the evaluation of scale and scope economies. The GTCF substitutes the Box-Cox metric for all logged independent products appearing in the TCF. For example, in place of $\ln y_i$, the following substitution is made:

$$(2)\ Y_i = \frac{(y_i^{\theta_i} - 1)}{\theta_i}$$

If $y_i = 0$, then $Y_i = (-1/\Theta_i)$. If this substitution is made for all $\ln y_i$, the GTCF becomes:

$$(3)\ \ln TC = B_0 + \sum_{i=1}^{m} B_i Y_i + \sum_{k=1}^{n} C_k \ln p_k + \left(\frac{1}{2}\right) \sum_{i=1}^{m} \sum_{j=1}^{m} D_{ij} Y_i Y_j$$

$$+ \left(\frac{1}{2}\right) \sum_{i=1}^{n} \sum_{j=1}^{n} E_{kl} \ln p_k \ln p_l + \sum_{i=1}^{m} \sum_{k=1}^{n} F_{ik} Y_i \ln p_k + e$$

Note that the substitution of the Box-Cox metric not only allows the GTCF to be identified for zero outputs, it also has the property that :

$$(4)\ \lim_{\theta_i \to 0} Y_i = \ln y_i.$$

Therefore, the TCF is nested within the GTCF.[9]

Estimation of the parameters of the GTCF and its use in testing hypotheses concerning scale and scope economies present problems not encountered in the use of the TCF. Unlike the TCF which is linear in its parameters, the GTCF cannot be estimated using OLS. An alternative estimation procedure such as the maximum likelihood method must be used to estimate the parameters of this functional form. In addition, the various

[9] Less general versions of the GTCF employed here have been utilized in earlier papers by Berger, Hanweck, and Humphrey (1987), Buono and Eakin (1990), Cebenoyan (1990), and Lawrence (1989). These papers utilize a smaller set of products (five or fewer) and apply a single transformation parameter, θ, in transforming all defined products. This research employs a larger set (eight) of more narrowly defined products and estimates a separate transformation parameter, θ_i, for each of the eight products included in the GTCF cost function.

measures used to assess economies of scale and scope must be revised to allow for the nonlinearities introduced when the Box-Cox metric is employed.[10]

MEASURING ECONOMIES OF SCALE

The parameter estimates obtained from estimating the TCF have been used to measure overall economies of scale for multiproduct firms as well as for each of the specifically defined products (product-specific economies of scale) produced by these firms. To capture overall economies of scale, the following measure generally has been used:

$$(5)\ OSA = \sum_{i=1}^{m} \partial \ln TC / \partial \ln y_i = \sum_{i=1}^{m} \varepsilon_i,$$

where:

ε_i = Output cost elasticity for product i.

Where OSA < 1, there are overall economies of scale. Where OSA > 1, there are overall diseconomies of scale.[11] To capture overall scale economies for the GTCF, the output-cost elasticities must be stated to incorporate the Box-Cox metric. The output-cost elasticity of product i should be written as:

$$(6)\ \varepsilon_i = y_i^{\theta_i}\ [B_i + \sum_{j=1}^{m} D_{ij} Y_j + \sum_{k=1}^{n} \ln p_k]$$

where:

Y_j = Defined as in equation (2).

The measure of product-specific economies of scale most frequently utilized in the literature is average incremental cost. Average incremental cost is measured in the following way:

$$(7)\ PSSA(AIC) = [(IC_i/TC)/\varepsilon_i],$$

where:

IC_i = $[C(y_1, \ldots, y_m) - C(y_1, \ldots, y_{i-1}, 0, y_{i+1}, \ldots, y_m)]$;

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[10] Because of these nonlinearities, the test statistics and restrictions used to test hypotheses concerning economies of scale and scope based upon an underlying TCF are no longer straightforward and easily implemented.

[11] OSA often is expressed as OSA' = $1/\Sigma\varepsilon_i$. Expressed in this form, overall economies (diseconomies) of scale exist where OSA' > 1 (< 1).

TC = $C(y_1, \dots, y_m)$; and

ε_i = Measured as in equation (6) above.

A value of PSSA(AIC) > 1 (< 1) indicates product-specific economies (diseconomies) of scale. Note that the measurement of product-specific economies of scale using equation (7) requires computation of the incremental cost of producing y_i (IC_i). This is problematic when the TCF is utilized because the computation of IC_i requires that this statistical cost function be evaluated under the assumption of a zero output for product i. For the GTCF, PSSA(AIC) can be computed without approximation, in principle, because IC_i can be evaluated using equation (3) at $y_i = 0$.[12]

MEASURING ECONOMIES OF SCOPE

As with economies of scale, the alternative measures of global and product-specific economies of scope require some modification when the cost function is GTCF. Global economies of scope are measured as follows:

$$(8)\ GSO = \frac{\{[C(y_1, 0, \dots, 0) + \dots + C(0, \dots, y_m)] - C(y_1, \dots, y_m)\}}{C(y_1, \dots, y_m)},$$

where:

C() = The GTCF statistical cost function.

If GSO > 0 (< 0), then there are global economies (diseconomies) of scope. This will occur when the costs of independent production of each product are greater (less) than the costs of jointly producing these products. Product-specific economies of scope have been evaluated using several alternative measures. One measure of product-specific economies of scope that requires the determination of total and incremental costs is computed as:

$$(9)\ PSSO(1) = \{[C(y_1, \dots, y_{i-1}, 0, y_{i+1}, \dots, y_m) + C(0, \dots, 0, y_1, 0, \dots, 0)] - C(y_1, \dots, y_m)\} / C(y_1, \dots, y_m),$$

where:

C() = The GTCF statistical cost function.

If PSSO(1) > 0, this implies product-specific economies of scope. Conversely, PSSO(1) < 0 implies product-specific diseconomies of scope.

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[12] C() in equation (7) now denotes the GTCF as presented in equation (3).

It is important to note that both GSO and PSSO(1) require the computation of the independent cost of production of each defined product. Thus, neither GSO nor PSSO(1) are measures that can be evaluated directly when the statistical cost function is of the TCF functional form. The independent and incremental costs included in GSO and PSSO(1), however, can be evaluated correctly at zero outputs, in principle, using the Box-Cox metric.[13]

An alternative approach to the evaluation of product-specific economies of scope focuses on the necessary and sufficient conditions for a cost complementarity between any two products. The sufficient condition for the existence of a cost complementarity can be stated as:

$$(10)\qquad PSSO(2) = \partial^2 TC/\partial y_i \partial y_k < 0$$
$$= (TC/y_i y_k)[(\partial^2 \ln TC/\partial \ln y_i \partial \ln y_k) + \varepsilon_i \bullet \varepsilon_k] < 0; \text{ and}$$
$$\partial^2 \ln TC/\partial \ln y_i \partial \ln y_j = y_i^{\theta_i} \bullet y_j^{\theta_j} \bullet D_{ij}$$

A cost complementarity (product-specific economy of scope) exists between y_i and y_k when PSS0(2) < 0. If PSS0(2) > 0, a product-specific diseconomy of scope exists between products y_i and y_k

ESTIMATION OF THE GTCF

THE DATA

Information reported by 402 banks located in the Chicago CMSA for the year 1989 is obtained to construct the variables used to estimate the parameters of the GTCF. This information is taken from the FDIC's *Report of Income and Condition*. For each bank in the sample, the variables employed represent averages for the year 1989 and are computed by averaging the values from the December 31, 1988 and December 31, 1989 reports.

Limiting the sample to these banks avoids several problems and provides several benefits. In general, the estimation of both the TCF or the GTCF require that proxies for the per unit prices of the relevant inputs be included. The input price of input k theoretically represents the marginal cost of employing an additional unit of this input, where this input is purchased in a competitive market. The balance sheet and income statement data reported by banks to the FDIC are in the form of book values and historical costs. Thus, any measures constructed using these data are potentially misleading as proxies for the per unit prices to be paid for these inputs.[14]

Several examples illustrate this problem. The FDIC data provide information only on total wages and salaries paid. Therefore, simply dividing by the total number of employ-

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[13] C() in equations (8) and (9) now denotes the GTCF as presented in equation (3).

[14] To illustrate this problem, the input price proxies employed by most researchers that have used the FDIC data were computed for the 402 banks in the Chicago CMSA (1989). The computed per unit price of capital ranged from a high of $5,667 to a low of $0, with a mean of $403. The computed per unit price of labor ranged from a high of $55,578 to a low of $284, with a mean of $26,409. Finally, the computed per dollar cost of loanable funds ranged from a high of 7.99 percent to a low of 0 percent, with a mean of 4.60 percent. A similar result was obtained when these proxies were computed for all 12,597 banks.

ees results in an unsatisfactory measure of the unit price of labor, in part because it is based upon historical accounting costs and in part because it confuses hours worked and labor quality with wage rates. Similarly, the FDIC data do not allow the construction of an adequate proxy for the price of bank capital. All call report information on bank premises and equipment are stated at historical accounting values. Further, the reported expense information reflects the depreciation or amortization methods used by each bank during the period. Finally, the call report data consolidate expense and balance sheet information for all facilities under the home office. For banks that operate large branching networks, the per unit price and quality of inputs utilized at the various branches may differ significantly from an unweighted bankwide average cost that would be computed from available balance sheet and income information.

In attempting to improve the proxies constructed using call report data, other researchers have utilized more general market level data. Under the assumption that banks purchase inputs competitively on local markets, the per unit price of labor frequently has been proxied by the hourly manufacturing wage rate in the respective market. Similarly, the per unit price of capital often has been proxied by the rental cost of bank and office buildings in the relevant market.[15] Because all banks in the Chicago CMSA are located in the same market, use of such proxies is equivalent to assuming that, at the margin, all sample banks face the same per unit prices for labor and capital. Further, because the state of Illinois severely limits branch banking, distortions that might arise as a result of differences in input prices across geographic markets that may result when a bank operates branch facilities are minimal.[16]

Under the assumption that all banks in the Chicago market face the same per unit prices for labor and capital at the margin, interbank differences in costs cannot be attributed to interbank differences in input prices. As a consequence, these variables can be deleted from the GTCF.[17] Two benefits from this deletion follow directly. First, the measurement error that would be induced by the use of call report data to proxy input prices is avoided. Second, the ability to delete input prices from the statistical cost function significantly reduces the large number of parameters that must be estimated.[18] The significant reduction in the number of parameters should help to reduce the potential for multicollinearity that has been reported in other attempts at estimating the TCF.

VARIABLE DEFINITIONS

Banks produce a variety of services for their loan and deposit customers. At a minimum, these services include payment, safekeeping, intermediation, and accounting services. For purposes of analysis, banks are assumed to minimize the total costs associ-

[15] Benston *et al.* (1983) take this approach. Gilligan, Smirlock, and Marshall (1982) assume that all banks face the same cost of capital, even when not located within the same market or region.

[16] Illinois branch banking laws permit the establishment of a facility within the county where the home office is located or outside the county if within ten miles of the main office (Amel, 1988).

[17] A similar procedure has been utilized in a recent study conducted by Kolari and Zardkoohi (1990).

[18] Including three inputs in equation (12) would increase the number of parameters that would have to be estimated from 64 to 100.

ated with the production of these services. Thus, the dependent variable, total costs, is defined to include both total operating costs and interest costs. This definition of costs is supported further in a recent work by Humphrey (1990), who has noted that the relative proportions of operating and interest costs to total costs appear to vary systematically with size.[19]

In defining the products to appear in the statistical cost function, an attempt was made to adhere as closely as possible to the definitions employed in constructing the summary data published in conjunction with the functional cost analysis (FCA) program.[20] The 1989 FCA program defines and allocates costs for the following banking functions: demand or transactions deposit function; time deposit function; nondeposit function or purchased funds function; investment function; real estate and mortgage loan function; installment loan function; credit card function; commercial and other loan function; safe deposit function; and trust function.[21] Using the FCA functional definitions and the FDIC *Report of Income and Condition* data, eight products were constructed for inclusion in the GTCF. These products include transaction deposits (TRNDEP), purchased funds (PURFDS), time deposits (TMEDEP), investments (INVEST), real estate and mortgage loans (RELN), installment loans (INSTLN), credit card loans (CCARDLN), and commercial and other loans (COMLN). In addition, the sample data indicates that a large number of institutions operated at least one additional domestic facility. Therefore, an additional independent variable is defined to capture the number of domestic facilities operated by each bank.[22]

MODELS AND ESTIMATION

Three versions of the GTCF are estimated. Under the assumption that all banks in the Chicago market face the same input prices for labor and capital, model 1 comprises the eight products defined above. Its form is:

$$(11)\ \ln TC = \beta_0 + \sum_{i=1}^{m} \beta_i Y_i + \left(\frac{1}{2}\right)\sum_{i=1}^{m} \sum_{j=1}^{m} \gamma_{ij} Y_i Y_j + e,$$

[19] It would be desirable to include only those direct and allocated operating expenses explicitly associated with the products appearing in the GTCF. Unfortunately, the form in which each bank reports its expenses to the FDIC is too highly aggregated to permit this approach. It should be acknowledged that even those papers that are able to utilize FCA data must rely on arbitrary allocation rules and the judgment of the participating banker in attributing operating cost to the appropriate bank service.

[20] The FCA program defines the following list of bank functions: transaction deposit function; time deposit function; nondeposit funds functions; investment function; real estate mortgage loan function; installment loan function; credit card loan function; commercial and agricultural loan function; safe deposit department; trust department; computer customer service department; and nonbanking departments.

[21] The FCA analysis program also defines an international function and nonbanking and data services functions. The FCA program does not provide cost allocation information for the international function because of "insufficient data." This research does not include the trust, safe deposit, or data services functions because it was not possible to construct measures of these variables from the FDIC balance sheet and income statement data. For banks participating in the FCA program these services are relatively minor, generating income as a percentage of available funds between 0.25 percent (smallest banks) and 0.62 percent (largest banks).

[22] The mean is 2.45, the median is 2, the mode is 1, and the standard deviation is 2.75.

where:

Y_i and Y_j (i, j = 1, ... , 8) = Box-Cox metric as defined in equation (2).

Symmetry requires that $\gamma_{ij} = \gamma_{ji}$.

In model 2, a variable defined as the log of the number of domestic facilities (NF) is allowed to enter the cost function. Following Mester (1987), this variable is permitted to interact with the other explanatory variables in the cost function. Thus, model 2 is given the following form:

$$(12)\ \ln TC = \beta_0 + \sum_{i=1}^{m} \beta_i Y_i + \left(\frac{1}{2}\right) \sum_{i=1}^{m} \sum_{j=1}^{m} \gamma_{ij} Y_i Y_j + \beta_b \ln NF + \left(\frac{1}{2}\right) \gamma_{bb} \ln NF^2$$

$$+ \sum_{i=1}^{m} \gamma_{bi} \ln NF\ Y_i + e$$

where:

Y_i and Y_j (i, j = 1, ... , 8), = Box-Cox metric as defined in equation (2).

Symmetry again requires that $\gamma_{ij} = \gamma_{ji}$ and $\gamma_{ib} = \gamma_{bi}$.

The data also indicate that many of the banks in the Chicago market are affiliated with bank holding companies. Thus, to determine whether affiliation with a bank holding company may influence the parameter estimates, a dummy variable (A) with a value of *one if the bank is affiliated with a* BHC *and* O *otherwise is introduced into model* 2. The third model estimated is:

$$(13)\ \ln TC = \beta_0 + \sum_{i=1}^{m} \beta_i Y_i + \left(\frac{1}{2}\right) \sum_{i=1}^{m} \sum_{j=1}^{m} \gamma_{ij} Y_i Y_j + \beta_b \ln NF + \left(\frac{1}{2}\right) \gamma_{bb} \ln NF^2$$

$$+ \sum_{i=1}^{m} \gamma_{bi} \ln NF\ Y_i + \phi A + e$$

where:

Y_i and Y_j (i, j = 1, ... , 8), = Box-Cox metric as defined in equation (2).

Symmetry again requires that $\gamma_{ij} = \gamma_{ji}$ and $\gamma_{ib} = \gamma_{bi}$.

Because the GTCF is nonlinear in its parameters, maximum likelihood estimation is employed. To estimate the parameters of the three models, equations (11), (12), and (13) are used to form the appropriate likelihood functions. The numerical optimization routines from GQOPT are used to maximize the respective log-likelihood functions with respect to the included parameters.[23]

RESULTS

Estimation of the three models produces several interesting results. First, the parameter restrictions imposed in models 1 and 2 can be tested against the more general model 3. To test the restrictions imposed by models 1 and 2, a likelihood ratio test is performed. The likelihood ratio test statistic is given as

$$-2[\log L(B_r) - \log L(B_{ur})] \sim \chi^2_v,$$

where:

$\log L(B_r)$ and $\log L(B_{ur})$ = Maximum values of the restricted and unrestricted log likelihood functions; and

v = Number of restrictions and the number of degrees of freedom.

Model 1 restricts the parameters ϕ, β_b, β_{bb}, and τ_{bi}, (i=1, ... , 8) to be jointly equal to zero. Model 2 restricts only the parameter ϕ to be zero. The maximum log likelihood values for the three models are 328.967035, 380.710367, and 380.739368, respectively. The computed χ^2 for the likelihood ratio test of the restrictions implied by model 1 is 103.54467, while the computed χ^2 for model 2 is 0.05800. Critical χ^2 for $\alpha = 0.01$ and 11 (model 1) and 1 (model 2) degrees of freedom are 24.725 and 6.6035, respectively. Thus, the parameter restrictions imposed by model 1 can be rejected at the $\alpha = 0.01$ level of significance, while the single restriction imposed by model 2 cannot. These two results taken together imply that, for banks in the Chicago market, the number of branch banking facilities appears to have a statistically significant impact on bank costs while affiliation with a bank holding company does not.

A second interesting result can be seen in the estimated Θ_is (i = 1, ... , 8) in Table 1. All of the estimated Θ_is are positive and statistically significant at $\alpha = .01$.[24] The importance of this result derives from the fact that the TCF is a restricted version of the GTCF in which $\Theta_1, \dots, \Theta_8$ are jointly assumed to be zero. Thus, the implicit assumption

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[23] The log-likelihood function is maximized w.r.t. the 53, 63, and 64 parameters appearing in the respective cost functions and σ^2 from the likelihood function using the David-Fletcher-Powell algorithm (Powell, 1971). No attempt is made to estimate the cost functions together with factor share equations. The decision to exclude factor share equations arose from the complexity of the estimation problem posed by the GTCF and recent evidence that systems methods do not yield significantly superior results over single equation methods. (See Guilkey and Lovell, 1980.)

[24] It is also important to note that the estimated θ_is range in size from 0.0156 to 0.1786. Thus, assumption of a single transformation parameter for all defined products, as proposed in several related studies, does not appear to be supported here.

imposed by the TCF would appear to be contradicted, and previous results obtained using the TCF must be viewed with skepticism.[25]

Before turning to a review of the computed measures of scale and scope economies, a third finding requires some discussion. Though the parameters of GTCF were estimated successfully and all θ_is were positive and statistically significant, the use of zeroes in computing GSO, PSSA(AIC), and PSSO(1) resulted in values that, while they could be computed, do not appear to have legitimate economic meaning. Several possible sources, both conceptual and technical, may be offered for this result. One possible source of this result arises from attempts to compute measures that require extrapolation beyond the bounds of the data. Few banks produce zero levels of most of the defined products, and all banks are multiple product firms. For the 402 commercial banks in the current sample, only two of the eight individually defined products (INSTLN and CCARDLN) are not produced by all banks. Nine banks did not produce INSTLN, and only eight banks produced neither INSTLN nor CCARDLN.[26] Thus, the substitution of zeroes into the GTCF to evaluate the various measures of scale and scope economies assumes levels of production that are generally outside the bounds of the data used to estimate the parameters of the cost function. As a result, little confidence can be placed in any extrapolated cost measures computed in this fashion. The present results suggest that this problem is most severe when an attempt is made to use zeroes for output levels of products produced by all banks. Cost estimates appear reasonable when using zeroes for the output levels of the two products, INSTLN and CCARDLN, for which zeroes are observed in the data set, provided within sample output values are employed for the remaining six products.

A second and closely related problem is the conventional measures of scale and scope economies. Measures of scale and scope economies such as GSO, PSSA(AIC), and PSSO(1) require computation of the independent cost of one or more of the products included in the cost function. Because all banks are multiple product firms, it is not clear that these measures have any relevance to the banking industry.

The third and fourth potential sources of problems are of a technical nature. TCF is degenerate at $y_i = 0$, for any i ($\theta_i = 0\ \forall i$, is implicitly assumed in TCF).[27] Therefore, it also may be the case that the GTCF will begin to degenerate at $y_i = 0$ as $\Theta_i \rightarrow 0$ for any i. If the GTCF does degenerate with increasingly smaller values of the θ_is, little is known about what is small or about the rate at which the GTCF degenerates as the θ_is decrease.

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[25] Ideally it would be useful to statistically test the parameter restrictions implied by the TCF against the unrestricted GTCF. The form taken by TCF also imposes the restriction that all included products be produced jointly ($y_i > 0, \forall i$), however. Thus, it is not possible to estimate the parameters of TCF with this sample and the large set of more narrowly defined products. A technical solution might be to replace all zeroes in the data with an arbitrarily chosen positive number prior to estimating the TCF. After replacing all zeroes with a value of 10 (in thousands) and estimating the TCF using an analogous maximum likelihood procedure, a likelihood ratio test was performed on the restrictions. The null hypothesis that all θ_is (i = 1, ... , 8) are jointly zero can be rejected at $\alpha = 0.01$. Replacement of zeroes by positive values larger than 10 resulted in even larger χ^2 values and thus even stronger rejection of the null. It should be noted that the smallest nonzero output observed for any product in the sample is \$79 (in thousands) and thus is considerably greater than the selected value of 10.

[26] The sample includes 124 banks that did not produce CCARDLN.

[27] See Fuss and Waverman (1978) and Baumol, Fischer, and Nadiri (1978).

Table 1—Maximum Likelihood Estimates for Generalized Translog Cost Function

Parameter	Estimate	Standard Error
Θ_1	0.149649*	0.001124
Θ_2	0.136597*	0.001064
Θ_3	0.099223*	0.002378
Θ_4	0.015591*	0.002009
Θ_5	0.140095*	0.001214
Θ_6	0.083735*	0.006741
Θ_7	0.178621*	0.001068
Θ_8	0.060782*	0.001183
β_0	6.045095*	0.538717
β_1	0.118571*	0.041438
β_2	0.222383*	0.036909
β_3	0.120659**	0.047804
β_4	0.057719	0.071271
β_5	0.085109*	0.017634
β_6	0.009147	0.009939
β_7	0.029139	0.019471
β_8	-0.839509*	0.136369
γ_{11}	0.009266*	0.002499
γ_{22}	0.011598*	0.001838
γ_{33}	0.004223*	0.002164
γ_{44}	0.000547	0.000675
γ_{55}	-0.000007	0.000437
γ_{66}	0.000940*	0.000253
γ_{77}	-0.000359	0.000454
γ_{88}	0.142008*	0.019714
$\gamma_{12} = \gamma_{21}$	-0.006879*	0.001612
$\gamma_{13} = \gamma_{31}$	-0.004163	0.002532
$\gamma_{14} = \gamma_{41}$	0.000751	0.002220
$\gamma_{15} = \gamma_{51}$	-0.001052	0.000775
$\gamma_{16} = \gamma_{61}$	-0.001146*	0.000428
$\gamma_{17} = \gamma_{71}$	-0.000671	0.000766
$\gamma_{18} = \gamma_{81}$	-0.002474	0.004698
$\gamma_{23} = \gamma_{32}$	-0.001007	0.002218
$\gamma_{24} = \gamma_{42}$	0.003426	0.003154
$\gamma_{25} = \gamma_{52}$	0.000048	0.000567
$\gamma_{26} = \gamma_{62}$	0.000343	0.000351
$\gamma_{27} = \gamma_{72}$	0.000762	0.000644
$\gamma_{28} = \gamma_{82}$	-0.018198*	0.004168
$\gamma_{34} = \gamma_{43}$	0.000286	0.003656
$\gamma_{35} = \gamma_{53}$	-0.000522	0.000932
$\gamma_{36} = \gamma_{63}$	0.001752*	0.000675
$\gamma_{37} = \gamma_{73}$	0.003988*	0.001070
$\gamma_{38} = \gamma_{83}$	-0.012731**	0.006318
$\gamma_{45} = \gamma_{54}$	0.001232	0.001906
$\gamma_{46} = \gamma_{64}$	0.000997	0.00106
$\gamma_{47} = \gamma_{74}$	0.000104	0.00139
$\gamma_{48} = \gamma_{84}$	-0.011537***	0.006756
$\gamma_{56} = \gamma_{65}$	-0.000283	0.000198
$\gamma_{57} = \gamma_{75}$	0.001379*	0.000364
$\gamma_{58} = \gamma_{85}$	-0.006059*	0.002304
$\gamma_{67} = \gamma_{76}$	-0.000023	0.000154
$\gamma_{68} = \gamma_{86}$	-0.001509	0.001331
$\gamma_{78} = \gamma_{87}$	-0.007632*	0.002561
β_b	0.512514*	0.174287
γ_{bb}	0.104618*	0.030936
$\gamma_{b1} = \gamma_{1b}$	-0.005521	0.007202

Table 1—Maximum Likelihood Estimates for Generalized Translog Cost Function (contnued)

Parameter	Estimate	Standard Error
$\gamma_{b2} = \gamma_{2b}$	0.010822**	0.005496
$\gamma_{b3} = \gamma_{3b}$	-0.007794	0.008153
$\gamma_{b4} = \gamma_{4b}$	-0.026417	0.016504
$\gamma_{b5} = \gamma_{5b}$	-0.001196	0.003103
$\gamma_{b6} = \gamma_{6b}$	-0.000301	0.001377
$\gamma_{b7} = \gamma_{7b}$	0.004254***	0.002569
$\gamma_{b8} = \gamma_{8b}$	-0.018022	0.021227
σ	0.008661*	0.000562
LLF	380.710367	

* Statistically significant at $\alpha = 0.01$ (two tailed test)
** Statistically significant at $\alpha = 0.05$ (two tailed test)
*** Statistically significant at $\alpha = 0.10$ (two tailed test)

A final issue raised in a recent paper by Pulley and Humphrey (1991) concerns the use of both the TCF and the GTCF to measure scope economies. They interpret the intercept term in either specification as representing fixed costs. Therefore, they note that the assumption of specialized production of m separate products assumes that these fixed costs, as represented by the intercept term, will be saved m - 1 times as a result of their joint production. Thus, they conclude that GSO will be overstated systematically, and the degree of overstatement will increase with the number of products included in the TCF statistical cost function. Despite the problems discussed above, for comparison with the results reported in other related studies, approximations to the conventional measures of scale and scope economies are computed and reported. To provide within sample approximations for scale and scope measures that require the use of zeroes, a technical solution is used that has been employed in these studies. Wherever the computation requires a zero, it is replaced with a value equal to 10 percent of the sample median. The median is used as a measure of central tendency because the presence of a relatively small number of very large banks causes the mean of the size distribution to be skewed substantially.[28]

ECONOMIES OF SCALE

Computed measures of global and product-specific scale and scope economies using the parameter estimates obtained from the preferred model 2 are presented in Tables 2 through 7. The parameter estimates appear in Table 1. Table 2 reports the computed measures of overall scale economies (OSA) for the sample mean, sample median, and median quartile levels of all eight products. In each case, the computed OSA reflects firm level scale economies because the number of facilities, NF, was adjusted to reflect the median value for each respective size grouping as suggested in Noulas, Ray, and Miller (1990).

[28] Inspection of Tables 10 and 11 illustrates the extent of the skewedness of the distribution of total assets. As can be seen in these tables, the median and mean are similar for quartiles 1 through 3. The mean and median differ significantly in the fourth quartile, however.

Table 2—Overall Economies of Scale, OSA: Model 2

Quartile	$OSA = \Sigma_i \varepsilon_i$	Standard Error
1	0.851559*	0.013999
2	0.896746*	0.016580
3	0.936774*	0.010972
4[a]	0.968560**	0.015478
4[b]	0.933320**	0.038246
Median	0.921390*	0.013396
Mean	0.924388*	0.020526

[a] Evaluation at quartile 4 product medians
[b] Evaluation at quartile 4 product means
* Statistically significant at $\alpha = 0.01$ (two tailed test)
** Statistically significant at $\alpha = 0.05$ (two tailed test)

Table 3—Global Economies of Scope, GSO: Model 2

Size Category	GSO[a]	Standard Error
Variable Medians	1.440226	1.265698
Variable Means	1.546407	1.319270
Quartile 1	1.525536	-----------[b]
Quartile 2	1.507463	1.301280
Quartile 3	1.368647	1.237406
Quartile 4	1.523309	1.309550

[a] To compute GSO, zeroes were replaced with values equal to 10 percent of the product's sample median
[b] Could not be computed: division by zero encountered in the calculation of the gradient because ln(NF) = 0 when NFAC = 1

Table 4—Expansion Path Subadditivity, EPSUB

Size Category	EPSUB	Standard Error
Quartile 1	-----------	
Quartile 2	0.077986*	0.004224
Quartile 3	0.032269**	0.017238
Quartile 4	0.004038	0.022152
S1[a]	-----------[b]	
S2[a]	-----------[b]	
S3[a]	0.118342*[c]	0.012135
S4[a]	0.090373*	0.005594
S5[a]	0.070314*[c]	0.013313
S6[a]	0.022306[c]	0.012662
S7[a]	0.031862*[c]	0.015526
S8[a]	0.016748[c]	0.024717
S9[a]	0.022945	0.028532
S10[a]	-----------[b]	

[a] S1 through S10 represent size classifications as presented in Table 8
[b] Number of facilities i less than one greater than the previous size class so that the number of facilities for the hypothetical bank is adjusted to one
[c] EPSUB is not computed when output does not increase for all products
* Statistically significant at $\alpha = 0.01$ (two tailed test)
** Statistically significant at $\alpha = 0.05$ (two tailed test)

Table 5—Product Specific Scale Economies, PSSA(AIC)[a]

Product	PSSA(AIC)	Standard Error
TRNDEP	0.4437*	0.0362
PURFDS	0.0376*	0.0376
INVEST	3.0896*	0.4832
RELN	2.4282	3.0869
INSTLN	1.9947	2.2278
CCARDLN	1.6798	3.1509
COMLN	2.7732	4.2682
TMEDEP	0.1262*	0.0041

* Statistically significant at $\alpha = 0.01$ (two tailed test)

Table 6—Product-Specific Scope Economies, PSSO(1)

MEDIAN	TRNDEP	PURFDS	INVEST	RELN	INSTLN	CCARDLN	COMLN	TMEDEP
Panel A Sample Medians								
PSSO(1)	0.31173	0.35260	0.55706	0.18933	0.21177	0.14205	0.12495	0.70841
Std. Dev.	0.12351	0.07337	0.08381	0.03092	0.01898	0.00697	0.03127	0.22944
T-Stat	2.52388	4.80553	6.64622	6.12209	11.1570	20.3533	3.99606	3.08744
Panel B Sample Means								
PSSO(1)	0.80351	0.47533	0.32323	0.12011	0.13801	0.12000	0.0925	1.54620
Std. Dev.	0.27234	0.16377	0.07514	0.07192	0.02512	0.01314	0.06011	0.50613
T-Stat	2.95033	2.90234	4.30139	1.67000	5.49224	9.12791	1.53893	3.05489
Panel C Quartile 1 Medians								
PSSO(1)	0.24747	0.30790	0.44631	0.18745	0.19549	0.18150	0.18047	0.75434
Std. Dev.	0.08927	0.05585	0.06022	0.03131	0.05569	0.01028	0.02059	0.21768
T-Stat	2.77200	5.51273	7.41014	5.98659	3.50979	17.6412	8.76449	3.46537
Panel D Quartile 2 Medians								
PSSO(1)	0.28138	0.35614	0.60872	0.20311	0.23080	0.15458	0.13616	0.67279
Std. Dev.	0.11115	0.06803	0.09370	0.03226	0.01879	0.00765	0.02723	0.21191
T-Stat	2.53138	5.23476	6.49619	6.29554	12.2832	20.1975	5.00035	3.17486
Panel E Quartile 3 Medians								
PSSO(1)	0.38989	0.34518	0.41336	0.17089	0.16750	0.13163	0.12123	0.83753
Std. Dev.	0.15022	0.08451	0.06527	0.03783	0.02290	0.00807	0.03568	0.27564
T-Stat	2.59543	4.08424	6.33267	4.51680	7.31354	16.3085	3.39709	3.03843
Panel F Quartile 4 Medians								
PSSO(1)	0.60391	0.41726	0.48813	0.16666	0.19025	0.11270	0.10214	0.99410
Std. Dev.	0.22543	0.11320	0.09223	0.05915	0.02983	0.00903	0.06152	0.34940
T-Stat	2.67882	3.68609	5.29254	2.81746	6.37776	12.4727	1.66024	2.84510

Table 7—Product-Specific Scope Economies PSSO(2)

Product Pair	PSSO(2)	Std. Dev.	T-Stat
TRNDEP - PURFDS	-1.306752E-06	1.066727E-06	-1.225
TRNDEP - INVEST	-8.143367E-07	3.317896E-07	-2.454
TRNDEP - RELN	1.243479E-09	1.647586E-07	0.008
TRNDEP - INSTLN	-1.010807E-06	5.947517E-07	-1.700
TRNDEP - CCARDLN	-1.866988E-05	1.110063E-05	-1.682
TRNDEP - COMLN	-7.898123E-07	1.211739E-06	-0.652
TRNDEP - TMEDEP	7.002495E-07	2.506781E-07	2.793
PURFDS - INVEST	-5.529568E-07	7.159748E-08	-7.723
PURFDS - RELN	2.436307E-07	6.366705E-08	3.827
PURFDS - INSTLN	-2.311973E-07	1.182673E-07	-1.955
PURFDS - CCARDLN	2.428575E-05	3.427007E-06	7.087
PURFDS - COMLN	1.862345E-06	1.801491E-07	10.338
PURFDS - TMEDEP	-8.360316E-08	9.573826E-08	-0.873
INVEST - RELN	3.065960E-08	2.051037E-08	1.495
INVEST - INSTLN	-1.020269E-07	4.591993E-08	-2.222
INVEST - CCARDLN	1.724541E-05	2.039577E-06	8.455
INVEST - COMLN	2.906848E-06	1.916225E-07	15.170
INVEST - TMEDEP	-6.364900E-07	7.203862E-08	-8.835
RELN - INSTLN	2.708469E-07	2.013261E-08	13.453
RELN - CCARDLN	6.200108E-06	8.437931E-07	7.348
RELN - COMLN	2.705121E-08	1.898016E-08	1.425
RELN - TMEDEP	-2.085587E-07	5.282264E-08	-3.948
INSTLN - CCARDLN	-2.435883E-05	1.879098E-06	-12.963
INSTLN - COMLN	6.353143E-06	2.685021E-07	23.661
INSTLN - TMEDEP	-1.113747E-06	1.166952E-07	-9.544
CCARDLN - COMLN	-1.312355E-07	2.348198E-06	-0.056
CCARDLN - TMEDEP	4.040026E-06	2.687673E-06	1.503
COMLN - TMEDEP	-1.977494E-06	2.022262E-07	-9.779

Table 8—Size Distribution, ($000s)

TOTASSETS		Number of Banks
$0 to $10,000	(S1)	3
$10,000 to $25,000	(S2)	21
$25,000 to $50,000	(S3)	72
$50,000 to $75,000	(S4)	60
$75,000 to $100,000	(S5)	44
$100,000 to $200,000	(S6)	104
$200,000 to $300,000	(S7)	39
$300,000 to $500,000	(S8)	34
$500,000 to $1,000,000	(S9)	17
over $1,000,000	(S10)	8

The OSA measures reported in Table 2 indicate that large and statistically significant economies of scale exist through all four quartiles for the sample of banks utilized in this

Table 9—Quartile Ranges, ($000s)

Quartile 1 TOTASSETS < $51,620	Mean = 33,683	Median = 35,347
Quartile 2 $51,620 < TOTASSETS < $103,530	Mean = 74,094	Median = 73,857
Quartile 3 $103,530 < TOTASSETS < $194,193	Mean = 141,112	Median = 136,510
Quartile 4 TOTASSETS > $194,193	Mean = 939,290	Median = 326,563

Table 10—Variable Means, ($000s)

Products	Sample Means	Quartile One	Quartile Two	Quartile Three	Quartile Four
TRNDEP	70,572.9	9,365.0	19,624.6	34,707.3	219,463.0
PURFDS	112,603.3	5,930.7	12,300.3	27,869.9	406,163.0
INVEST	57,340.9	10,914.5	26,664.0	46,454.6	145,746.0
RELN	48,964.4	8,813.0	18,530.8	34,919.7	134,039.0
INSTLN	15,271.1	2,822.0	7,081.9	12,509.7	38,780.3
CCARDLN	1,436.9	226.5	424.3	1,181.2	3,928.6
COMLN	79,823.2	1,007.8	1,696.9	22,992.4	294,945.0
TMEDEP	112,926.1	18,387.3	42,169.3	78,534.8	313,665.0
NF	2.46	1.5	1.79	2.43	4.13

Table 11—Variable Medians, ($000s)

Products	Sample Medians	Quartile One	Quartile Two	Quartile Three	Quartile Four
TRNDEP	24,946.5	8,397.7	19,600.0	34,178.5	79,659.8
PURFDS	15,747.5	5,193.5	10,957.5	24,817.0	69,221.3
INVEST	31,473.1	9,593.5	23,611.5	41,534.0	89,743.0
RELN	23,301.8	7,740.0	17,463.5	35,030.0	83,705.0
INSTLN	6,201.5	2,123.0	5,129.5	8,947.5	18,104.3
CCARDLN	79.0	15.7	57.0	157.0	451.0
COMLN	3,888.7	773.5	1,379.0	18,833.0	56,479.3
TMEDEP	55,873.5	17,599.5	41,931.0	76,696.0	185,093.0
NF	2	1	2	2	3

paper. In the smallest quartile the computed OSA is 0.8516. Statistically significant overall scale economies decrease in size, but continue throughout the latter three quartiles. Even in the fourth quartile OSA remains less than one and statistically significant, with computed values of 0.9332 or 0.9686 depending on whether it is evaluated at the fourth quartile product mean or median.

It is also interesting to note one additional finding not reported in the tables. The fourth size quartile includes banks of roughly $300 million or more of total assets. The OSA measure was computed for three additional size categories in the fourth quartile: $300 million to $500 million of total assets, $500 million to $1 billion of total assets, and greater than $1 billion of total assets. For these respective categories using product medians, the computed OSAs are 0.9266, 0.8883, and 0.9415, respectively. Thus, for our sample of banks, there appears to be a noticeable improvement in efficiency that can be obtained by banks able to increase their scale beyond $500 million of assets.

Table 5 presents the computed measures of product-specific scale economies, PSSA(AIC), for all eight products with their respective standard errors. The results indicate that statistically significant ($\alpha = 0.01$) product-specific scale economies appear to be strongest for investment securities, INVEST. Four other products (RELN, INSTLN, CCARDLN, and COMLN) also yield values for PSSA(AIC) that are greater than one, suggesting product-specific scale economies, but none are statistically significant at $\alpha = 0.10$. Statistically significant ($\alpha = 0.01$) product-specific scale diseconomies are reported for transactions deposits (TRNDEP), time and savings deposits (TMEDEP), and purchased funds (PURFDS).

ECONOMIES OF SCOPE

Global scope economies, GSO, is computed for the sample means, the sample medians, and quartile medians of all eight products. These computed values are presented in Table 3. This table provides some indication of global economies of scope between 130 percent and 150 percent. Because the approximate standard errors for these measures are relatively large, however, in no instance could the null hypothesis of no global scope economies be rejected at $\alpha = 0.10$. Thus, these results do not support the view that more specialized banking institutions operate at a competitive disadvantage.

The absence of global scope economies also is supported by the cost complementarity results presented in Table 7. The PSSO(2) measures of product-pair cost complementarities indicate that although there appear to be statistically significant efficiency gains from the joint production of some product pairs, there also appear to be other product pairs for which there are statistically significant inefficiencies from joint production.

Some additional evidence on scale and scope economies is reported in Tables 4 and 6. Table 6 presents the results for PSSO(1) computed using the sample means and medians as well as the quartile medians for all products in the output vector. Throughout the first three quartiles there appear to be statistically significant ($\alpha = 0.05$) product-specific scope economies associated with the addition of each of the eight products to a product mix containing the other seven products. In the fourth quartile only the addition of commercial loans fails to produce statistically significant efficiency gains.

Table 4 presents an additional computed measure of productive efficiency that has been labelled expansion path subadditivity, EPSUB.[29] The conventional measure of global scope economies assumes that a given mix of n products can be separated into n firms, each firm responsible for producing one of the products. A more relevant possibility is that the output bundle, Q_L, produced by large firm L could be produced by two smaller firms, A and B, with output bundles Q_A and Q_B such that $Q_A + Q_B = Q_L$. If the cost of producing output bundle Q_L is greater than the sum of the costs of producing Q_A

[29] See Berger, Hanweck, and Humphrey (1987) for the development of this measure.

and Q_B, then large firm L will not be competitively viable. The expansion path subadditivity of Q_L can be assessed by computing the following measure:

$$EPSUB(Q_L) = [C(Q_A) + C(Q_B) - C(Q_L)] / C(Q_L) ,$$

where:

C() = The cost of producing the subscripted output bundle.

That is, EPSUB measures the proportional cost increase or decrease associated with two firm (rather than one firm) production of output bundle Q_L. Therefore, if EPSUB > 0, output bundle Q_L can be produced more efficiently by large firm L than by two smaller firms A and B.

Table 4 reports the EPSUB measure for a number of successively larger size categories. In each case, the large bank is assumed to produce an output bundle equal to the median levels of all eight products produced in the size classification, Q_L. The cost is compared to the output bundle comprising the median levels of the eight products in the adjacent smaller size classification, Q_A, and the cost of a hypothetical bank producing an output bundling given by $Q_B = Q_L - Q_A$. The results indicate that a larger bank has a cost advantage over two smaller banks (one of which is located at the median of the previous size category) across all size categories considered (i.e., EPSUB > 0). EPSUB is not statistically significant beyond size category S7, which corresponds to asset size of $200 million to $300 million.

COMPARISON TO OTHER STUDIES

It is interesting to compare the results reported above with those reported in other related papers, particularly in light of use of the GTCF. In a recent review of the literature on banking productivity, Evanoff and Israilevich (1991) separate the studies they summarize into two groups: small bank cost studies (typically < $500 million in total deposits) and large bank cost studies (> $1 billion in total deposits). The reported range for the scale elasticity measure, OSA, among the small bank cost studies they review is typically 0.95 to 1.05, with either no significant economies or significant diseconomies beyond $100 million to $200 million of deposits. These small bank cost studies appear to indicate that although small banks are inefficient, the extent of the inefficiency is not large and can be eliminated with relatively small increases in scale.

In contrast, Evanoff and Israilevich (1991) report that the results from large bank cost studies indicate that scale economies exist beyond the range reported in the small bank studies. As in the small bank studies the typical range for the OSA measure is .95 to 1.05. The deposit size at which scale economies are exhausted, however, ranges from $4.2 billion to as much as $37.5 billion of deposits. They also cite two studies that report that economies of scale are not exhausted through $50 billion and $140 billion of deposits.

In a sense, the current study helps to bridge the gap between the large and small bank results. The bank sizes in the sample range from $4.1 million to $19.5 billion of total assets, with a mean of $296.1 million. We report a range of scale elasticities of 0.8516 at the output medians of the smallest quartile to 0.9415 at the output means for those banks exceeding $1 billion of total assets. The findings suggest efficiency gains from increased size are largest for the smallest banks, but are not exhausted through a size of $1 billion of total assets. Further, the scale elasticities reported for the means and medians of the largest categories are consistent with the range of scale elasticities reported in most of the large bank cost studies reviewed by Evanoff and Israilevich.

The results for both global and product-specific scope economies are roughly comparable to those reported in other studies that utilize a similar approximation procedure. As was the case in six of 11 studies reviewed by Clark (1988), we do not find statistically significant global scope economies. As in seven of 11 studies reviewed by Clark, however, we report evidence of product-specific scope economies among a number of product pairs. These economies are at least partially offset by product-specific diseconomies of scope among other product pairs.

It is important to note that it is difficult to compare the actual point estimates of GSO and PSSO(1) across studies. The procedure of replacing zeroes with arbitrarily chosen positive values will produce estimates for GSO and PSSO(1) that are sensitive to the choice of the replacement value; therefore, these estimates can vary widely across studies. Further, the sensitivity of GSO and PSSO(1) also will depend on the parameter estimates of the statistical cost function. Pulley and Humphrey (1992) note that the parameter estimates on the squared output terms (i.e., γ_{ii} of equations (11), (12), and (13)) are particularly important in determining the resulting scope estimates. They report that positive parameter estimates on these variables generally produce large scope economies, while negative parameter estimates tend to produce large scope diseconomies.[30]

Finally, unlike Berger, Hanweck, and Humphrey (1987) who report evidence of diseconomies from banking industry consolidation, evidence is found of statistically significant expansion path subadditivity up to a size of $200 million to $300 million of total assets. The current estimates of EPSUB, however, are significantly smaller than those reported by Hunter, Timme, and Yang (1990) obtained using a sample dominated by banks that exceed $1 billion in total assets.

SUMMARY AND CONCLUSIONS

The purpose of this paper is to estimate a generalized translog statistical cost function for commercial banks and to utilize the resulting parameter estimates to evaluate economies of scale and scope. The selection of the generalized translog functional form is motivated by a desire to estimate a cost function that will permit a more highly disaggregated vector of defined products, allow banks to be included in the sample used to estimate the parameters of the cost function even if they do not produce one or more of the included products, and permit the computation of several measures of economies of scale and scope that require the assumption that one or more of the defined products be assigned an output value of zero.

We are successful in estimating the parameters of the generalized translog cost function. The adoption of this functional form allows a more significant disaggregation of the vector of products for which the statistical cost function can be estimated. Given the

[30] Röller (1990) refers to this effect as the *flip-flop property*.

disaggregated vector of outputs, it is also possible to include many banks that did not produce positive amounts of all of the included products. The GTCF has obvious advantages over the TCF.

Unfortunately, we are unable to directly compute economically meaningful estimates of the conventional measures of scale and scope economies that require an evaluation of the cost of independently producing one or more of the defined products. Several possible explanations for this failure are discussed. The most likely explanation is that these conventional measures are inappropriate for the banking industry because their computation requires extrapolation beyond the bounds of the data.

By following the conventional technical procedure of replacing zeroes with values set at 10 percent of the sample medians, we are able to compute within sample approximate point estimates of the various measures of economies of scale and scope. These approximate point estimates indicate that banks can achieve efficiencies from both increased size and joint production.

Overall economies of scale appear to be much larger than those reported in many small bank studies and consistent with the results in several large bank studies. Statistically significant economies of scale are reported for all size classes and remain beyond a size of $1 billion of assets.

We did not find evidence of statistically significant global economies of scope. There appear to be economies from the joint production of selected product pairs, but these efficiencies are offset by diseconomies in the production of others. These results are supported by our computed estimates of expansion path subadditivity.

The results reported in this paper do not provide support for the existence of natural monopoly in the commercial banking industry. They do suggest, however, that many smaller banks can improve their productive efficiency through increases in size.

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