Decomposition of Market Model Variation in the Presence of Misspecification

Steve A. Johnson*
University of Texas at El Paso

Richard L. Sprinkle
University of Texas at El Paso

This paper examines the prevalence of misspecification in the traditional return-generating form of the market model using daily returns. The return specifications of individual securities and the portfolios constructed from these securities are considered. Comparisons are made between the levels of diversification in portfolios composed of correctly specified securities and those constructed of securities that are specified incorrectly by the market model. The results of these tests suggest that significant differences exist in the level of systematic risk in the two types of portfolios.

INTRODUCTION

In the finance literature, the returns on common equity have been used widely to analyze two effects. First, in an event study, a preevent data set is used to estimate the parameters of a market model. The estimated parameters from this model then are used to generate forecast errors (or abnormal returns) from an event window.[1] A second use of equity returns has been the decomposition of a security's risk or a portfolio's risk into diversifiable (unsystematic) and undiversifiable (systematic) risks.[2] If the market model used in estimating either the decomposition of risk or the residuals in the event window is misspecified, the estimated coefficients may be biased and inconsistent.[3] If so, it may be inappropriate to use the estimated coefficients for the purposes of estimating a portfolio's systematic risk or excess returns. A similar problem also may exist if a misspecified model is used to estimate the cost of capital or the residuals for testing various announcement effects.[4]

To assess the degree of misspecification inherent in the traditional valuation framework, Cheng and Lee (1986) examined three forms of the market model. Cheng and Lee employed the Ramsey version of the RESET test on the monthly returns of 451 individ-

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[1]For additional details on event study methodology, see Thompson (1985), Karafiath (1988), and Klein and Rosenfeld (1987).

[2]For a summary on the decomposition of risk, see Evans and Archer (1968), Fama (1976), and Ben-Horin and Levy (1980).

[3]For an analysis of the effects of model misspecification on tests of the market model, see Brenner (1977).

[4]If the market model is misspecified, then the results from studies like Fama, *et al.* (1969), Ball and Brown (1968), and others may be suspect.

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ual securities from January 1965 through December 1977. Their results indicate that the Sharp-Lintner, the Fischer-Black, and the standard single index form of the market model all yield approximately the same degree of misspecification.[5] These tests indicate that if the estimated model is not specified correctly, the estimated beta coefficient is biased and the accuracy in forecasting the rates of return may be reduced. As a result, traditional market model frameworks may be inappropriate in numerous contexts.

In addition, Brown and Warner (1985) suggest that daily and monthly returns differ in potentially important respects.[6] For example, the use of daily returns rather than monthly returns in event studies increases the likelihood of detecting abnormal performance around a specific event. In general, their results indicate that the power of an event study increases threefold with the use of daily returns. Thus, the use of daily returns has significant advantages in determining abnormal returns. Furthermore, portfolio theory suggests that as the number of securities in a portfolio increases, two related things occur:

- The total risk of the portfolio declines to the market level of risk; and
- The returns of the portfolio begin to mirror the returns of the market portfolio.

If the securities included in the portfolio are misspecified, however, will this affect the degree of (mis)specification of the portfolio? That is, does the inclusion of individual securities that are misspecified affect the single index market model specification of the entire portfolio? If so, the estimated portfolio may yield biased and inconsistent estimates of the level of systematic risk.

This study extends the work of Cheng and Lee (1986) by applying a specification error test to determine whether the single index form of the market model for individual securities as well as portfolios of securities is specified correctly using daily returns. Four related types of evidence on the effects of model specification are presented. First, this paper updates work on whether the single index form of the market model for individual securities is specified correctly. Second, this paper investigates the effect of portfolio size (diversification) on the specification of the single index model. Third, evidence is presented on the effect that portfolio weighting has on model specification. Last, this research tests the effect of misspecification on the estimation of systematic risk.

METHODOLOGY AND SPECIFICATION TEST

The standard single index model used to describe the return-generating process of equity securities through time may be written in the following form:

(1) $R_{jt} = a_j + b_j (R_{mt}) + e_{jt}$

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[5]For each of the forms of the market model, see Sharpe (1964), Lintner (1965), or Black (1972). For each of these models, the misspecification rates ranged from 27 percent to 31 percent. In addition, Huang and Jo (1988), using monthly returns and the Haussman specification test on 452 firms from January 1960 to December 1983, found misspecification rates ranging from 11 percent to 23 percent.

[6]For evidence of these problems, see Scholes and Williams (1977), Schwartz and Whitcomb (1977), Hawawini and Vora (1980), and others.

where:

R_{jt} = Return of an individual security j (or a portfolio of securities) at time t;

R_{mt} = Market return at time t;

b_j = (Beta), measure or index of systematic risk; and

e_{jt} = Serially uncorrelated random errors.

The variance of this relationship is:

(2) $s_j^2 = b_j^2 (s_m^2) + s_e^2$

where:

s_j^2 = Total risk of the security's (portfolio's) returns;

$b_j^2 (s_m^2)$ = Systematic portion of the security's (portfolio's) risk; and

s_e^2 = Unsystematic portion of the security's (portfolio's) risk.[7]

Suppose that the true return-generating process of equity returns through time is of the following form:

(3) $R_{jt} = a_j + b_{1j}(R_{mt}) + b_{2j}(C_{2jt}) + b_{3j}(C_{3jt}) + \ldots + b_{kj}(C_{kjt}) + v_{jt}$,

where:

$C_{2jt}, C_{3jt}, \ldots, C_{kjt}$ = Other macroeconomic factors that should be included in the return-generating process.[8]

If these factors are omitted from the model and equation (1) is estimated, a significant specification error may occur. It is well-known that misspecification in the form of an omitted variable yields biases that depend on the correlation between the omitted variable and the included explanatory variable, $[COV(R_{mt},C_{ijt})]$, and the sign of the omitted vari-

[7]An alternative, normalized measure of the degree of systematic risk of a security is the model's estimated R^2, and the corresponding measure of unsystematic risk is $(1 - R^2)$.

[8]The development of the APT model by Ross (1976) was based on the assertion that the market model should contain factors in addition to the returns of the market. Over the years possible additional accounting and finance factors that may be included in the return-generating process have been explored by King (1966), Cohen and Pogue (1967), Beaver (1972), Downes and Dyckman (1973), Rosenberg and McKibben (1973), Lee and Zumwalt (1981), Pettway and Bradford (1987), and others.

able's regression coefficient, (b_{it}). Even if the correlation between the omitted and explanatory variable is zero, the estimated constant, (a_j), generally is biased; therefore, forecasts generated from the estimated equation also will be biased.

Tests of the omitted explanatory variable hypothesis have been developed by Ramsey (1977), Ramsey and Schmidt (1976), Thursby and Schmidt (1977), and Thursby (1979, 1981). The results of their work indicate that the RESET procedure has substantial power in detecting errors associated with omitted variables or incorrect functional forms in regression analysis.

The RESET procedure amounts to using the standard F test to determine whether $b_{2j} = b_{3j} = \ldots = b_{kj} = 0$ in the augmented regression:[9]

$$(4)\ R_{jt} = a_j + b_{1j}(R_{mt}) + b_{2j}(Z_{2jt}) + b_{3j}(Z_{3jt}) + \ldots + b_{kj}(Z_{kjt}) + v_{jt}.$$

Thursby and Schmidt (1977) and Thursby (1979) indicate that the best set of test variables (Zs) is composed of the squares, cubes, and fourth powers of the explanatory variables used in the initial regression. It is this variant of RESET that is employed in this work.

EMPIRICAL ESTIMATION AND TESTING

To test whether the single index form of the market model (equation (1)) is specified correctly, daily returns with dividends of 284 randomly selected securities were taken from the CRSP tapes from January 1, 1980 through December 31, 1986. In addition, the returns of the S&P 500 index with dividends were used as the market rate of return. First, the single index model for each security was estimated using the following form:[10]

$$(5)\ \ln(1 + R_{jt}) = a_j + b_j \ln(1 + R_{mt}) + e_j.$$

Second, the RESET procedure was applied on the 284 estimated models to determine whether each security's model is specified correctly. The single index model was accepted as being correctly specified for 198 of the 284 securities. The RESET test indicated that the single index model was an incorrect specification for the remaining 84 securities.[11] This model form could yield biased estimates of systematic risk and inconsistent residuals in an event study for these 84 securities, approximately 30 percent of the sample.[12] The RESET procedure also was run with the returns of the CRSP value-weighted index as the

[9] Employing this procedure provides a preliminary test of the validity of the single index model versus multivariable forms such as the APT.

[10] The model is estimated in log form to reproduce the results of Cheng and Lee (1986).

[11] Two securities were eliminated from groups 1 and 2 due to the presence of significant serial correlation in the error structure.

[12] In addition, the daily return data were compounded into monthly return data and the RESET procedure was applied on each security using the newly formed monthly data. The single index form was accepted as being correctly specified for 271 of the 284 securities. Thus, for only 13 securities did the RESET test indicate that the single index model was an incorrect specification. The 5 percent rejection rate on monthly returns compares with a 30 percent rejection rate from Cheng and Lee (1986) and a 10 percent rejection rate from Huang and Jo (1988).

proxy for the market. A similar rejection rate of approximately 30 percent was the result.[13]

Given these preliminary results, three groups of random portfolios were constructed. Group 1 consists of a series of 30 unique portfolios constructed randomly from those securities where the single index model is an incorrect specification of the market model (i.e., those securities that did not pass the RESET test). Group 2 consists of a second series of 30 portfolios constructed randomly from those securities where the single index model is a correct specification of the market model (i.e., those securities that passed the RESET test). Group 3 consists of a third series of 60 portfolios constructed randomly from the total 284 security sample.

In addition, the initial portfolio in each series was constructed containing five securities; then five additional securities were added to each original portfolio. The process of adding five new securities to each portfolio continued until each portfolio contained 50 securities. Finally, each individual portfolio (and series) was constructed using both equal weighting and market value-weighting schemes.

The single index form for each portfolio was estimated using equation (5), and the RESET test was applied on each portfolio's model estimate to determine whether the single index form for each portfolio was specified correctly. The results of this procedure are presented in Table 1. The results generally indicate that portfolios that are composed of securities that did not pass the RESET test (group 1) also have the highest RESET test rejection rate, 88 percent to 94 percent.[14] Portfolios composed of securities that passed RESET (group 2) have the lowest RESET test rejection rate (10 percent to 16 percent), and portfolios composed of a random selection of securities (group 3) have rejection rates ranging from 44 percent to 51 percent.[15] Although these results may not be surprising, they indicate the degree of model misspecification that can occur when estimating the market model for portfolios.

As the number of securities in each portfolio increases, the rejection rates increase for all groups. The RESET rejection rates for groups 1 and 3, however, increased dramatically. Last, the results indicate that portfolios that are constructed with equal weighting given to each security have, in general, a higher RESET rejection rate (5 percent for group 1, 3 percent for group 2, and 6 percent for group 3) than those portfolios that were built using value weighting. This last result is not surprising because the market return used in the study is the S&P 500, a value-weighted index.[16]

To test for the effects of model specification on the estimation of systematic risk, the average percentage level of systematic risk $[b_j^2\ (s_m^2)\ /\ s_j^2]$ for each of the value-weighted portfolios was computed. In addition, the hypothesis that the estimated

[13]The results of these tests are available on request.

[14]The authors recognize that replicative sampling errors may occur as a result of the relatively small number of securities in the sample (284) and in the various test groups.

[15]The daily returns of the group 3 portfolios also were compounded into monthly returns, and the RESET procedure reapplied. The RESET results indicate that the degree of misspecification of this randomly constructed group of portfolios is approximately 2 percent.

[16]Huang and Jo (1988) achieve similar results when comparing the error rates of equal versus value-weighted indexes.

Table 1—Specification Error Rates for Portfolios of Different Sizes

Number of Securities in Portfolio	Weighting Scheme	Portfolios Composed of Securities From Group 1	Group 2	Group 3
5	Equal	73.3%	6.7%	26.7%
	Value	70.0%	6.7%	26.7%
10	Equal	83.3%	13.3%	33.3%
	Value	70.0%	10.0%	31.6%
15	Equal	90.0%	16.7%	40.0%
	Value	83.3%	10.0%	35.0%
20	Equal	100.0%	13.3%	46.7%
	Value	83.3%	20.0%	38.3%
25	Equal	96.7%	16.7%	50.0%
	Value	90.0%	16.7%	51.7%
30	Equal	96.7%	20.0%	53.3%
	Value	96.7%	13.3%	45.0%
35	Equal	96.7%	16.7%	58.3%
	Value	93.3%	20.0%	46.7%
40	Equal	100.0%	20.0%	62.0%
	Value	100.0%	10.0%	57.7%
45	Equal	100.0%	20.0%	70.0%
	Value	96.7%	13.3%	56.7%
50	Equal	100.0%	16.7%	66.7%
	Value	100.0%	6.7%	53.3%
Average	Equal	93.7%	16.0%	50.7%
	Value	88.3%	12.7%	44.3%

level of systematic risk differs between group 1 portfolios and the other two groups was determined with a simple t-test. The results of this test are shown in Table 2.

The results generally indicate that as the number of securities in the portfolio increases, the systematic portion of the portfolio's total risk also increases. Also, there is a significant difference in the level of the estimated systematic risk inherent in portfolios from group 1 and group 2. Because group 1 contains portfolios comprising securities where the single index model is an incorrect specification and group 2 contains portfolios made of correctly specified securities, this result indicates that the estimated level of systematic risk for group 1 portfolios is upwardly biased. This upward bias ranges from approximately 3.5 percent to 6.5 percent. Finally, there is a significant difference in the level of the estimated systematic risk between portfolios from group 2 and group 3 for only portfolios that contain 40 and 45 securities.

CONCLUSIONS

This paper presents evidence on whether the return-generating form of the market model is specified correctly using daily returns. A few tentative conclusions can be drawn

Table 2—Average Systematic Risk of Portfolios of Different Size Based on Value Weights

Number of Securities in Portfolio	Portfolios Composed of Securities From				
	Group 1	Group 2	Group 3	H1	H2
5	50.89%	47.72%	50.02%	1.422	1.343
10	65.39%	64.34%	64.41%	0.653	0.063
15	74.23%	72.01%	72.18%	2.081 **	0.021
20	79.89%	76.52%	76.95%	4.141 **	0.547
25	82.35%	79.57%	80.33%	3.544 **	1.102
30	84.98%	82.36%	83.11%	4.157 **	1.337
35	86.79%	84.00%	84.57%	5.110 **	1.080
40	88.09%	85.08%	85.91%	5.807 **	1.675 *
45	89.47%	86.19%	87.04%	7.467 **	1.956 *
50	89.71%	86.70%	87.29%	7.634 **	1.415

H1 is the t-test for equality of systematic risk between portfolios in groups 1 and 2
H2 is the t-test for equality of systematic risk between portfolios in groups 2 and 3
* Significant at the 0.05 level
** Significant at the 0.10 level

from the results. First, the results for single securities indicate that the single index model is an incorrect specification in approximately 30 percent of the cases.

Second, the results for random portfolios from the total sample indicate that the single index model is an incorrect specification for about half the cases and that the degree of misspecification increases as the number of securities in the portfolio increases. In addition, the degree of model misspecification ranges from approximately 90 percent for portfolios that contain only misspecified securities to approximately 15 percent for portfolios that contain only correctly specified securities.

Third, the results indicate that portfolios that are constructed using equal weights have, in general, a higher degree of misspecification than do portfolios built using value weighting. This result, however, may stem from the use of a value-weighted market index.

Last, the results indicate that there is a bias in the level of estimated systematic risk in the portfolios that contain misspecified securities. Accordingly, this evidence indicates that the single index model can be a misspecified model for portfolios as well as for individual securities. Analysts using this model should proceed with caution by applying a specification test before employing the traditional market model in further research.

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