Trade Credit as a Markovian Decision Process With an Infinite Planning Horizon

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Abstract

The trade credit process is modelled as a series of stochastic cash flows with recursive decisions over an infinite planning horizon. A closed form solution is obtained for the value of extending credit, and a function is derived for the upper bound on acceptable default risk. Conditions for optimal myopic credit decisions are examined, and the value of extending trade credit is analyzed for sensitivity to function parameters.

Introduction

The extensive use of trade credit to support corporate sales places a burden of almost continuous decision making on credit analysts charged with treading a fine line between losing sales and exposing the firm to excessive risk of losses from customer defaults. These decisions—a major factor in support of the sales effort—are among the most important operating decisions made because they directly affect the survival and profitability of the firm.

In a recent survey of more than 500 members of the Credit Research Foundation, Besley and Osteryoung (1985) observe that almost 96 percent of the firms responding reported that at least 80 percent of total sales are on credit. Less than 2 percent of the firms reported that cash is received for a majority of sales. This is consistent with aggregate statistics showing that accounts receivable make up more than 25 percent of the assets of firms in the S&P 400 Industrials Index (Standard & Poor, 1990).

Similarly, Mian and Smith (1990) analyze the Compustat database and find that receivables make up 21 percent of the assets of U.S. manufacturing firms. The conclusion to be drawn from these studies is that the use of vendor financing is highly important in the conduct of business transactions.

A symbiotic relationship exists between vendors and customers that in many ways goes beyond merely meeting the financing needs that financial institutions otherwise could provide. The provision of trade credit by a vendor often increases the level of influence in determining customer purchases and in the marketing and display of inventory for sale to final users. It also enhances the vendor's access to information concerning the customer's business. This infor-

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mation is valuable for improving the efficiency of materials purchasing, production scheduling, and product delivery.[1]

A further advantage to the vendor may arise in using trade credit as an effective means of price discrimination that circumvents enforcement of the Robinson-Patman Act, a potential that first was described by Schwartz and Whitcomb (1979) and elaborated by Brennan, Maksimovic, and Zechner (1988) using an example from agriculture.

Because trade credit is part of a larger business commitment from the customer point of view, there is some reason to expect flexibility by the vendor during periods of adversity.[2] In addition, the use of vendor financing reduces alternative financing search costs and avoids the restrictive loan covenants and personal guarantees often required by financial intermediaries. It also avoids the problems of inconvenience and loss of control over payments that are presented by having shipping and receiving personnel in charge of funds or arranging for payment immediately upon the arrival of goods. Ferris (1981) also notes that trade credit reduces the aggregate need for transaction balances by separating the timing of cash flows from the uncertain timing of physical delivery of goods.

The rationale for the use of vendor financing is the focus of several recent papers in financial economics. Emery (1987) analyzes trade credit as an optimal response by vendors to variable demand conditions. The use of trade credit as a stimulant to sales, according to his analysis, can arise from imperfections in financial markets combined with advantages that vendors have over financial intermediaries in granting credit. These advantages include enhanced access to credit information that is available through the sales force and vendor advantages in obtaining security interests on goods sold, in repossessing goods where a security interest is held, and in efficiently reselling such goods after repossession is accomplished.

Emery also notes that the buyer often has storage cost advantages that produce an incentive to order goods and take delivery in advance of needs. This reduces manufacturer and distributor storage costs and vendor financing is provided as a *quid pro quo* during the interval. Taking advance orders helps the manufacturer with production scheduling. It also provides the buyer with a hedge against price and quantity uncertainty. In addition, having goods on hand at a

[1] An example of the importance of customer relationships that are enhanced by vendor financing is related by Sloan (1963). The use of trade credit financing to enhance access to information on business customers also was discussed recently by Smith (1987).

[2] This is exemplified by the reaction of Deere & Company to dealer and customer financial distress during the agricultural depression of the mid-1980s. At the end of 1985 Deere reported that 22 percent of dealer obligations were over one year old. During that year the company forgave $53 million in interest through the extension of interest free periods and reduced interest rates. This amount exceeded Deere's net income from all sources that year and indicates a considerable flexibility in dealing with trade credit customers in distress.

known cost facilitates the advance preparation of marketing materials such as catalogues and price lists that are based on the actual quantities, costs, and characteristics of the goods in storage. Smith (1987) also has observed that trade credit is an efficient method of bonding a vendor when the buyer is at risk of receiving inferior quality goods or service.

From an agency and information theory standpoint, the use of trade credit financing may serve as an inexpensive indicator to financial markets of continuing financial health, as investors in senior securities may rely on trade creditors to monitor the financial status of their credit customers. Adams, Wyatt, and Kim (1990) cite the Campeau Corporation bankruptcy case as a compelling example in which trade creditors were apparently the first to detect financial distress and alert senior security markets through their reluctance to provide seasonal inventories on credit. Thus, trade credit may have a derived demand because firms using it more readily can attract investors to buy senior securities by implicitly offering reduced monitoring costs.

Finally, Mian and Smith (1990) point out the tax advantage that arises from vendor financing through installment loans. In such cases, the seller can defer recognition of taxable income in the form of profits to correspond with amortization of the receivable.[3]

In surveying the approach that firms take in making trade credit decisions, Besley and Osteryoung (1985) find that the primary focus of attention is the riskiness of the individual account. Few vendors indicate that they limit accounts receivable in aggregate. Rather, accounts receivable usually are constrained only by the combination of the individual customer credit limits. Thus, a portfolio approach to setting credit policy is not typical, nor is capital rationing often considered a factor in granting credit. Instead, practitioners act as if they face an efficient capital market, using an approach that is consistent with the goal of maximizing shareholder wealth and the principle of value additivity. The decision function, therefore, should be based on the present value of expected cash flows resulting from individual credit decisions. It also should consider the contingent nature of the decisions and the potential for multiple transactions over a long-run future.

The purpose of this paper is to analyze the trade credit-granting problem and to develop such a decision function. As a matter that affects present and future business, the problem is modelled as a recursive decision process with stochastic

[3]Brick and Fung (1984) claim that a tax asymmetry can be used to advantage by sellers to transfer the corporate interest tax shelter to individuals unable to claim an interest expense in calculating personal taxes. If there is such a tax asymmetry to be exploited, however, financial intermediaries have the same tax advantage as vendors (except as indicated by Mian and Smith, 1990). The additional revenue charged by the seller to compensate for the time value of money is taxed in the hands of the vendor in the same way that interest income is taxed in the hands of financial intermediaries.

cash flows extending over an infinite planning horizon. The formulation most readily is solved for a criterion function if the cash flows are assumed to follow a Markov process with stationary transition probabilities. This approach yields a closed form solution that is amenable to direct use in applied situations. An example is presented later in the paper and then extended to analyze the sensitivity of the model to variations in parameter values specific to vendors and customers, respectively. The formulation abstracts from the policy of the vendor concerning credit investigation, effective credit terms, and collection policy.[4]

Previous Research

Multiperiod Credit Models

Bierman and Hausman (1970) were the first to formulate the credit acceptance or rejection decision in a multiperiod framework. Their paper concerns the same decision as the current research: whether or not a vendor should grant credit for an individual transaction with a potential for repeat business over an unlimited planning horizon. Bierman and Hausman assume in their model that established credit terms require payment of the outstanding balance at the end of each credit period and that a failure to receive payment is a default with the loss of the entire receivable. They also implicitly assume that the true conditional probability of receiving payment is stable, but unknown, with the probability estimate being uncertain and distributed according to the beta density function. The parameters determining the mean and higher moments of the probability estimate each period are assumed to be a function of the number of previous customer transactions and payments received. The distribution of the estimate thus is updated according to the outcome for each transaction.

Unfortunately, the Bayesian approach to estimating the true probability of receiving payment is analytically intractable for all but special limiting cases of distribution functions. Therefore, although it continues to be the basis for illustration of the multiperiod nature of the credit decision in texts such as Brealey and Myers (1991), Ross and Westerfield (1988), and Bierman and Smidt (1986) 21 years after its publication, the model presents opportunities for improvement in the interests of generality and ease of application. In particular, implementation of the analysis requires the numerical solution of a unique dynamic programming problem for each credit application. In discussion of the multiperiod credit decision, as in the texts cited above, this limitation remains as an unresolved applications problem.

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[4]For optimal credit investigation and collection strategy see Mehta (1968, 1970, 1974) and Stowe (1985). For a recent review and discussion of alternative approaches to credit analysis, see Srinivasan and Kim (1987).

In the same year Mehta (1970) produced a formulation of the multiperiod credit problem that yields a closed form solution over an unlimited horizon. Following earlier work by Cyert, Davidson, and Thompson (1962), Mehta employs Markov transition probability matrices to estimate and compare estimated acceptance cost and lost sales cost with further investigation cost in decisions to grant credit, deny credit, or investigate further. Unlike the Bierman and Hausman formulation, the estimates of transition probabilities between credit states are assumed to be stationary.

Overall, the formulation presented in the paper and a later monograph (Mehta, 1974) make a seminal contribution to the credit decision literature regarding the optimal level of credit investigation and the optimal amount of collection effort and expense. Improvements can be achieved, however, by focusing on incremental cash flows rather than on accounting numbers. For example, Mehta discounts the book entry to bad debt expense when a receivable is written off as uncollectible. Writing a receivable off as a bad debt, however, simply recognizes that collection of the account is not expected and that its value as a receivable is zero for balance sheet purposes. The only cash flow resulting from such action is a secondary one associated with the tax consequence of reversing the entry recognizing the sale in the first place. Mehta also allocates the variable cost of sales to the period in which collection of the receivable occurs. This allocation thus is discounted for the time value of money as if it were a cash flow at that time. Because economic value is represented most objectively by cash flows, the variable cost of sales should be represented in the valuation function by the cash flow involved and discounted according to its actual timing. Moreover, the cash flows associated with collections should be the amounts collected or the expected amounts *ex ante*.

Atkins and Kim (1977), Dyl (1977), Hill and Riener (1979), Kim and Atkins (1978), Sachdeva and Gitman (1981), Sartoris and Hill (1981), Stowe (1985), and Weston and Tuan (1980) all deal with valuation in the specific context of trade credit. The consensus of this literature is unambiguous in recommending incremental cash flow valuation principles for credit decisions.

Having said all of this in order to describe one of the differences between Mehta's analysis and that which follows, it should be pointed out that the problem in failing to model accurately the timing of cash flows probably has only a small effect on the resulting credit valuations and decisions taken. Because trade credit is usually of short duration, the discount rate is of relatively small importance in such a decision. This is substantiated just prior to the concluding section of this paper, where the value of extending trade credit is shown to be insensitive to changes in the discount factor. This also means that the value of trade credit is also relatively insensitive to the cash flow timing problem in Mehta's formulation, for practical purposes. Mehta's analysis continues to be a seminal work, and small problems of formulation do not reduce the stature of that work and its significance in the literature.

Contingent Claims Models

Taking a different approach, Lam and Chen (1986), Miles and Varma (1986), and Adams, Wyatt, and Kim (1990) all have attempted to use option pricing models to value an account receivable. Lam and Chen employ the Brennan (1979) and Stapleton-Subrahmanyam (1984) models for pricing multivariate contingent claims in discrete time. They consider uncertainty in both the demand for the product by the potential buyer and the ability of the buyer to honor the resulting contractual claim at the maturity of the receivable. Their formulation shows that the effect of pricing policy on the value of receivables depends on the cash flow characteristics of the customer, the price elasticity of product demand, the variability of the demand, and the systematic risk of the revenue. The analysis demonstrates that the contingent nature of the account receivable claim changes the neoclassical marginal cost and revenue conditions for setting product prices. In addition to the price elasticity of demand and the marginal cost of production, optimal product pricing also depends on:

- The probability of default by the buyer;
- The correlation between demand and market returns;
- The variability of demand;
- The correlation between the buyer's cash flow and the credit obligation; and
- The probability of the vendor defaulting on obligations to suppliers due to a default by the customer.

The analysis is an interesting extension to the neoclassical theory of the firm. The formulation lacks sufficient focus on the credit decision, however, to be useful in analyzing actual trade credit decisions, per se.

Miles and Varma's (1986) analysis of the credit decision problem is oriented differently. They use the Black-Scholes (1973) and Merton (1974, 1977) option valuation models to calculate the value of a contractual trade credit obligation of a customer without other debts. They also extend the analysis to consider cases in which existing debt claims are either subordinate, equal in priority, or superior to the credit being valued, respectively. In a similar analysis, Adams, Wyatt, and Kim (1990) extend these results to the case of a firm paying dividends and present a graphical analysis of the effects of relative priority of other debt obligations on the value of the trade credit obligation.

These three studies offer important insights into the credit problem and are useful in defining issues for additional inquiry. The models are only applicable, however, to trade credit decisions involving single transactions. In addition, the equity of the customer must be publicly traded in an efficient market, and many of the other standard assumptions of option pricing models must hold. Unfortunately for trade credit analysts, few instances occur when all of these conditions apply. When the buyer is a privately held company, neither the market value of equity nor its return variance are observable for use as required in contingent claim pricing. Also, the problem requires the use of compound option pricing methods if the customer has risky debt obligations that mature before the receivable due date. Other complications often arise such as the uncertainty of the payment of dividends on equity and of interest on debt. Moreover, the precedence, the size, and the uncertainty of bankruptcy costs incurred in a default effectively rule out the use of option pricing models that require the absence of such costs for analytical tractability.

Miles and Varma (1986, p. 516) appear to cast doubt on the use of discounted cash flow techniques in valuing trade credit obligations. Their discussion centers on the problem of determining the appropriate risk-adjusted discount rate, as option pricing models show that default risk changes as the equity value of the customer firm changes. Miles and Varma, however, mistake their calculation of the promised yield on the credit obligation for the risk-adjusted return required to discount the expected value of the asset. The promised yield includes the expected loss rate due to default, while the risk-adjusted discount rate applies after the probability of default is taken into account. Only the systematic risk of the receivable enters the calculation of the discount rate. Finally, even if concern with the difficulty of estimating appropriate discount rates were justified, the matter is of little consequence in application. The value of extending credit is insensitive to wide variations in the discount rate because the discount period for a given trade credit transaction usually is limited to a small fraction of a year. The sensitivity analysis presented in a later section confirms this.

The Single Transaction Credit Decision

Consider a credit application involving one transaction. Assume for now there is no consideration of future business potential. The decision maker assesses the application and determines the probability of the customer defaulting if credit is granted.[5] The economic costs and benefits of offering credit are measured by the incremental cash flows that result from credit approval, as depicted in Figure 1. The notation is as follows:[6]

[5]Eighty-seven percent of the Besley and Osteryoung survey respondents indicated their firms could measure default probability at least adequately (1985, p. 79).

[6]For notational convenience, tax consequences of all cash flows are assumed to be incorporated into the variable definitions. That is, all variables where taxes apply are defined on an after tax basis, and the definition includes an adjustment, by the appropriate time value factor, for the timing of the tax cash flow.

m = Contribution margin per dollar of sale;
y = Down payment required per dollar of sale;
h = Invoice discount factor if the invoice is paid by the discount date (i.e., $h = 99$ percent if terms are 1 percent/10, net 30);
S = Sale amount (the amount due if the invoice is discounted);
x = Fraction of the sale amount that is expected to be collected after a default, including accrued interest after the default, less collection costs;[7]
p_0 = Probability of invoice discounting;
p_1 = Probability of payment at the due date, conditional on no invoice discounting;
p_c = Probability of a cash sale if credit is denied;
q_i = $1 - p_i$, where p_i is defined above for $i = 0, 1, c$;
α_1 = Discount factor for the time value of money for the period up to the invoice discount date, adjusted for the systematic risk of accounts receivable (i.e., $\alpha_1 \approx 99.5$ percent for ten days if the annual percentage discount rate is 18 percent) (Copeland and Khoury, 1978);
α_2 = Discount factor for the period after the invoice discount date (i.e., $\alpha_2 \approx 99$ percent for 20 days).[8,9]

If credit is offered, the first result is a cash outflow for the variable cost of the sale, $(1 - m)S$, less the down payment, yS, if any.[10] After a credit sale, the customer may elect to discount the invoice with probability p_0 by paying the sale amount less the initial down payment or $(1 - y)S$. If the invoice is not discounted, a payment of the net invoice amount $(1 - y)S/h$ is expected on the due

[7]If the partial balance collected after a default is measured as a fraction of the sale amount plus interest accrued after the regular credit period expires, the interest accrued at a competitive interest rate maintains the present value xS/h regardless of the time it takes to collect the balance in default. Thus, x is defined as the fraction expected to be collected after accrued interest and collection expenses are fully recovered.

[8]Note that α_1 and α_2 do not reflect a yield component to cover the expected default probability, as in the case for the yield to maturity often quoted on risky corporate bonds, for example. Rather α_1 and α_2 are defined specifically as the factors appropriate for discounting the expected value of the payments anticipated on an invoice. These are less than the promised amounts when the probability of default is positive.

[9]The problem of actually estimating an appropriate discount factor for the account receivable from each customer based on the systematic risk involved is one of some theoretical importance. As will be demonstrated in a later section of this paper, however, the value of extending credit is remarkably insensitive to the discount rate employed over a wide range of values, and the issue is not important for practical decision-making purposes.

[10]The payment date for the variable cost of goods sold may be different from the date of sale due to the use of trade payables, the accrual of other costs, or the use of existing inventory. If existing inventory is used, the analysis assumes that immediate replacement is necessary to maintain adequate inventory levels. If trade payables and accruals are used, the cost of doing so is assumed to be the market-determined cost of capital. See Kim and Chung (1990) for a review of the related literature and an analysis of one aspect of credit policy (specifically, the cash discount to offer) in conjunction with inventory policy.

date with probability p_1. A default may occur, however, with probability q_1. In that case, the expected net collection is xS/h. Finally, if credit is denied, a cash sale still will occur with probability p_c.

The net present value of the credit decision for the single transaction just described (and depicted in Figure 1) therefore is:

$$(1)\ V = -(1 - m)S + yS + \alpha_1 p_0(1 - y)S$$

$$+ \alpha_1\alpha_2[q_0p_1(1 - y)(S/h) + q_0q_1x(S/h)] - p_c mS.$$

The first two terms in equation (1) are the variable cost and the down payment, respectively. The third is the expected receipt from a prompt payment by the customer to discount the invoice. The term in square parentheses reflects the expected receipt at the due date contingent on no invoice discounting, allowing for the probability of default and the collection of a partial default balance, respectively, The term $p_c mS$ is the expected contribution to profit and fixed costs from a cash sale if credit is not offered. It appears in the equation with a negative sign because it is the opportunity cost of granting credit.

Shareholder wealth is maximized by approving credit if the net present value is nonnegative. Setting $V \geq 0$ in equation (1) and solving for q_1, the upper bound on acceptable default probability is:

$$(2)\ q_1^* = \frac{mq_c - (1 - y)\{1 - \alpha_1[p_o + \alpha_2 q_0/h]\}}{\alpha_1\alpha_2 q_0(1 - y - x)/h},\ y + x < 1,\ q_0 > 0.$$

This result can be simplified for interpretation purposes by assuming jointly optimal product and credit pricing. Competitive financial markets and jointly optimal pricing of product and trade credit, as discussed in Kim and Atkins (1978), are conditions under which the prompt payment discount factor, h, and the present value discount factor, α_2, have the same value. In that case, $\alpha_2/h = 1$ and equation (2) simplifies to:

$$(3)\ q_1^* = \frac{mq_c - (1 - \alpha_1)(1 - y)}{\alpha_1 q_0(1 - y - x)},\ y + x < 1,\ q_0 > 0.$$

The first term in the numerator of equation (3) reflects the importance of credit in closing the sale because q_c is the probability that denying credit will lose the sale. Therefore, mq_c is the expected value of the incremental contribution margin from granting credit, conditional on ultimate payment. The second term in the numerator reflects the fairly small time value cost (≈ 0.5 percent) of extending credit from the sale date to the invoice discount date. The denominator reflects the present value of the exposure to loss from default, conditional on the buyer foregoing the invoice discount. The exposure to loss is ameliorated by the down payment and the expected collection success if a default occurs.

Equation (3) extends the earlier results of Smith (1964) and Davis (1966) by considering the probability of a cash sale if credit is denied, by allowing for a down payment requirement, and by considering the partial collection of balances in default. Equation (2), being more general, also allows for any difference between the return on accounts receivable and the cost of capital involved in carrying the receivable if a customer foregoes the invoice discount.

While a credit decision is sometimes necessary with only a single sale in mind, this is a special case of the general problem when the probability of a repeat sale is not assumed to be zero. Relaxing this assumption means extending the analysis to a dynamic formulation of the potential for repeat business in a multiperiod context.

The Multiperiod Credit Decision

The problem presented by multiperiod trade credit is one of sequential decisions and stochastic outcomes where each decision depends on the expected value of future decisions. Figure 2 depicts the nature of the problem. It contains all the aspects of a single period decision plus the potential for repeat business which may grow over time. The following additional notation is employed:

g = Growth rate of a customer's future business each period (month);
r_c = Reorder probability following a cash sale;
r_1 = Reorder probability following payment by a credit customer.

The formulation assumes one order per credit period and a credit policy that requires payment of the balance before the next credit sale.[11] If a default occurs, the customer and the vendor are assumed not to entertain further orders. The vendor then expects to collect a fraction of the sale amount, including accrued interest, net of collection expenses. The true probabilities of receiving payments and reorders are not assumed to be functions of experience. This is consistent with the analysis of Bierman and Hausman (1970) and Mehta (1970 and 1974).[12]

[11]This is a matter of definition of credit period and transaction amount so that one turnover of the credit balance occurs per credit period.

[12]Dirickx and Wakeman (1976, p. 1230) note that even though Bierman and Hausman (1970) update the estimated probability of default according to collection experience, their analysis assumes that the true underlying probability of collection is a stationary constant. The difference in treatment here is that the vendor is assumed capable of making an unbiased estimate of the true probability at the outset and does not face the analytically complicating prospect of having to update a prior estimate according to collection experience.

Based on the single transaction analysis in equation (1) above, define the value of credit approval per dollar of sale in a single transaction as $w = V/S$. From equation (1), after rearranging:

(4) $w = mq_c - (1 - y)\{1 - \alpha_1[p_o + \alpha_2 q_0/h]\} - \alpha_1\alpha_2 q_0 q_1(1 - y - x)$.

The first term is the increase in contribution from granting credit to avoid the risk of losing the sale by requiring cash. The second term is a small value reflecting the time value of money up to the invoice discount date each period. The third term is the expected loss from exposure to uncollectible balances due to default. The net result is the expected periodic reward if credit is extended.

The incremental probabilities of receiving payment and a repeat order from a customer granted credit versus the same customer denied credit are shown in Figure 2 and equation (5):

(5) $R = p_0 r_1 + q_0 p_1 r_1 - p_c r_c$.

The first term in equation (5) is the joint probability of the customer paying promptly for an invoice discount and reordering. The second term is the joint probability of payment of the net invoice and a reorder. The third term is the joint probability of a cash sale and a reorder if credit is denied.[13] Therefore, the incremental value of a policy (perhaps suboptimal) of granting credit to support a sale in period t and in all future periods is given by the recursive relation:

(6) $V_t = wS_t + \alpha_1\alpha_2 R V_{t+1}$, $t = 1, \dots, T$

where the horizon is defined by arbitrarily letting $V_{T+1} = 0$.

With an infinite future and convergence, the effect of the horizon in the transition from period t to t+1 vanishes for all t. Therefore, if g is the periodic growth rate of reorder amount, $V_{t+1} = V_t(1 + g)$ and equation (6) can be written as (Hadley, 1964, p. 454-460):

(7) $V_t = wS_t + \alpha_1\alpha_2 R(1 + g)V_t$, $\alpha_1\alpha_2 R(1 + g) < 1$.

Solving for V_t, a policy of granting credit for each order received over an infinite planning horizon has a value of:

[13] R is a summary value that can be interpreted as a function of the overall importance of trade credit to a customer in terms of the probability of repeat business.

$$(8)\ V_t = \frac{wS_t}{[1 - \alpha_1\alpha_2 R(1+g)]},\ \alpha_1\alpha_2 R(1+g) < 1.$$

This is the value of a geometric progression of periodic rewards where the first term is the value of extending credit in period one. The decision to grant credit is optimal if the numerator of equation (8) is nonnegative because the denominator must be positive for convergence. The numerator of equation (8) is nonnegative if w is nonnegative. Assuming stationarity, if granting credit is optimal in period t, extending credit for reorders is expected to be optimal in all future periods unless a default occurs. Setting $w \geq 0$ and solving equation (4) for q_1, the upper bound on the acceptable probability of default is:

$$(9)\ q_1^* = \frac{mq_c - (1-y)\{1 - \alpha_1[p_0 + \alpha_2 q_0/h]\}}{\alpha_1\alpha_2 q_0(1-y-x)/h},\ y + x < 1,\ q_0 > 0.$$

This is the same as the upper bound established for the single transaction case above in equation (2). It is derived from the solution to the dynamic formulation, however, and therefore it establishes the myopic decision criterion as applicable to the multiperiod trade credit decision under stationarity assumptions. As in the single transaction case, if prices and trade credit terms are jointly optimal in efficient markets, then $\alpha_1/h = 1$, and equation (9) simplifies to equation (3) above for easier interpretation as before.

Conditions for Optimal Myopic Decisions

Equation (8) allows for optimal credit decisions that are myopic because the decisions are not dependent on the size or probability of reorders. It also depends upon the stationarity assumptions made in formulating the model. Because the functionality of the model is determined by the range of conditions in which the assumptions apply or the degree to which the results are robust to their relaxation, this is addressed in the remainder of the section.

The requirement for optimal myopic decisions is stationarity in payment probabilities and in operating conditions that determine parameters such as the fractional contribution margin on sales, the fraction of each sale prepaid, if any, and the rate of collection success expected after a default. Stationary payment probabilities mean that the true reliability of credit customers is not expected to increase or decrease over time and that the credit analyst initially is capable of assessing the true probabilities. A stationary contribution margin means there are no fixed costs in filling the current order that are absent or reduced in later orders and no returns to scale that may be realized in future orders. Stationary down payment means that customers are faced with the same sale terms over time, regardless of their history of repeat business. Stationary collection expectation means there are no anticipated changes due to factors such as fixed costs or returns to scale that may mean higher or lower fractional collection costs or rates.

Bierman and Hausman (1970) consider the credit-granting problem with probability estimates that are not assumed to be stationary. They are unable to achieve a general closed form solution for their formulation. They observe that when the probability of payment is expected to improve with time, however, it is optimal to take larger risks than advisable if considering only the short run. This increases the chance for obtaining future business when payment probabilities are expected to improve. The same reasoning also applies if there is an expected increase in future contribution margins or future expected collection success. It can be optimal to extend credit that is unprofitable in the short run to increase the likelihood of future, more profitable, reorders under improved operating or collection conditions. The value of extending credit and the upper bound on acceptable default risk, however, can be found only under conditions of improving parameters by numerical analysis of the dynamic problem using either value iteration or policy iteration methods.

If payment probabilities, contribution margins, prepayment rates, or collection success are expected to deteriorate in future periods, however, the myopic credit decision remains optimal. In that case, each transaction should be considered on its own merits with no obligation to continue granting credit in the future. Credit should be denied as soon as conditions deteriorate enough to make the transaction under consideration inadvisable on its own merits.

In summary, improving operations or credit risk should lead to greater risk taking than myopic criteria indicate, while deteriorating conditions mean that myopia remains optimal.

An Example With Parameter Sensitivity Analysis

The Basic Example Parameters

Equations (1) and (8) give the value of extending credit for single transactions and for recurring business with stationary parameters, respectively. Suppose a textile manufacturer receives a \$1 million order from a distributor together with a credit application. The credit department knows that the marginal cost of the sale, excluding investigation expense, is 85 percent of the total, making the potential contribution margin 15 percent. Assume that the vendor's experience with collection on the basis of the security interest usually taken on this product leads to an expectation that only 30 percent of the sale amount, plus accrued interest on the overdue balance, will be collected if a default occurs. To help limit exposure to loss and to ensure that orders are firm before a production schedule is set, the company has a policy of requiring a 20 percent prepayment on orders. The terms of sale are stated concisely as 20 percent down, 1 percent/10, net 30, indicating the prepayment policy, the invoice discount available, and the ultimate payment date, respectively.

After an investigation the credit analyst concerned believes that the customer will pay in ten days with a probability of 10 percent. If the invoice is not discounted, the customer is expected to pay in 30 days with a probability of 90 percent. If credit is denied, the analyst believes that a cash sale will occur with a probability of 30 percent. Finally, the systematic risk of the receivable indicates a cost of capital of about 1.5 percent per month. This means discount factors of about 99.5 percent and 99.0 percent apply for periods of ten days and 20 days, respectively.

At this point, the value of extending credit can be calculated and a credit decision can be made for a one time transaction. Employing equation (1), as illustrated in Figure 3, the value of extending credit for a single transaction with a down payment of 20 percent, as opposed to requiring all cash, is 6 percent of the amount of the sale. Figure 3 also shows that the maximum acceptable default risk is 22.5 percent. At that level of default risk, the value of extending credit would be zero.

If there is a potential for recurring business, equation (8) gives the value of extending credit. Assume that repeat orders are expected with a probability of 80 percent each period, conditional on a previous credit sale and receipt of payment. The probability of a repeat order is only 60 percent if credit is denied and a cash sale occurs. If repeat orders are received, they are expected to grow at a rate of 6 percent per year or about 0.5 percent per month.

With these parameters, the value of extending credit can be obtained from equation (8). Figure 3 shows this to be 12 percent of the initial sale or about twice as much as for the single transaction, assuming the conditional probability of default is 10 percent. The maximum acceptable probability of default is 22.5 percent, the same as for the single transaction.

Sensitivity Analysis

Figures 4 through 6 show the sensitivity of the value of extending credit to changes in vendor-specific parameters.[14] The value is most sensitive to changes in the contribution margin, as shown in Figure 4. The example assumes the vendor has a 15 percent contribution margin. If this is reduced to 5 percent, the

[14]The single transaction case is omitted from the sensitivity analysis to conserve space.

tolerable default risk is reduced from 22.5 percent to 7 percent. Extending credit to the customer with an initial order of $1 million and a 10 percent probability of default would have a value of minus $20,000, down from about $120,000 on the plus side. In the opposite direction, an increase in the contribution margin from 15 percent to 25 percent increases the acceptable default risk to 39 percent and the value of offering credit to the 10 percent default risk rises to $280,000. Figure 4 shows that the high sensitivity to changes in the contribution margin extends across the entire range of default risk. This makes the contribution margin a variable of major importance in the credit decision.

Figure 5 shows the sensitivity to variations in the expected collection success after defaults. In the example, the expected collection after a default is assumed to be 30 percent or $300,000 of the $1 million initial sale amount. If this is increased to 60 percent (perhaps by establishing an improved security interest), the value of granting credit rises from 12 percent or $120,000 to 18 percent or $180,000. The acceptable default risk jumps to 57 percent. In the other direction, an expectation of zero collection success after a default reduces the value of extending credit to 6 percent or $60,000. Figure 5 also confirms the intuition that collection success is less important in granting credit to highly reliable customers.

Figure 6 illustrates the sensitivity of equation (8) to the down payment provision. There is no down payment required for most recurring trade credit arrangements, so the effect of reducing it to zero in the example is of most interest. Waiving the $200,000 down payment on the $1 million initial sale reduces the value of granting credit to this customer from $120,000 to $80,000.[15] The acceptable limit on default risk decreases from 22.5 percent to 16 percent. Increasing the down payment has a similarly large effect in the opposite direction. Comparing Figures 5 and 6, the effect of changing a down payment provision is similar in nature to the effect of a change in collection rates after a default. Its impact is greater for customers with high default risk, and it has little effect on the value of extending credit to customers who are highly reliable.

Figures 7 through 10 show the sensitivity of the value of granting credit to changes in parameters that are customer-specific. Only two of these parameters are important to the credit decision. They are the probability of default and the probability that the customer will pay cash if credit is denied. The probability of default appears in every graph as the independent variable, and it is easy to see its central role in the credit decision. Figure 7 specifically shows the importance of

[15]There is, however, the potential for an offsetting effect. Reducing the down payment required may attract additional business in the form of larger orders or additional customers. It also may change the probability of losing a sale if credit is denied. Investigating the magnitude of these second order effects is an empirical problem and is beyond the scope of the present paper.

the probability of a cash sale if credit is denied. Intuitively, the most to be gained is from extending credit to reliable customers who would refuse to pay cash. This is shown clearly to be the case in Figure 7. There is also a large effect on the maximum acceptable default risk. In the example, if the analyst were certain the customer would refuse to pay cash, the acceptable default risk would increase from 22.5 percent to about 33 percent. The effect is equally large in the opposite direction if the probability of a cash sale is increased. This is an intuitive result because it is less important to extend credit to customers who willingly would pay cash.

Figures 8 through 10 show the surprisingly small effects of changes in discount rates, reorder growth rates, and reorder probabilities. Figure 8 shows that reducing the time value of money discount factor for a 30 day period from 99.1 percent to 97.0 percent only reduces the acceptable default risk from a high of 23.5 percent to a low of 20.5 percent.[16] The effect of the time value of money on the value of granting credit thus is small and proportionally the same for customers across all levels of default risk.

Figure 9 shows that the anticipated growth rate of reorders is even less important to the credit decision. The same is true for variations in the probability of reorders (as shown in Figure 10), as long as the difference in the probabilities for cash versus credit customers is held constant. The effect of either of these parameters on acceptable default risk is zero, and their effect on the value of granting credit is only appreciable for reliable customers.

Conclusion

This paper analyzes the trade credit problem as a Markov process of cash flows over an infinite planning horizon. A closed form solution was derived for the value of granting credit and for the upper bound on acceptable credit risk. The case of a single transaction credit decision was formulated initially and then extended to the multiperiod case. The model represents a significant extension to the literature concerning trade credit because it yields a closed form solution, comes from an improved formulation based on incremental cash flows, and incorporates several facets of the decision not previously considered. The latter include the potential for cash sales if credit is denied, down payment requirements, invoice discounting potential, partial collection of balances in default, the probability of repeat orders, and the growth rate of repeat business. The criterion function also includes parameters representing the product contribution margin, the probability of default, and the opportunity cost of capital.

[16]This is a change in the annualized cost of capital rate from a low of 10 percent to a high of 42 percent. The small effect of such a large change clearly shows that the risk-adjusted time value of money is relatively unimportant in the credit decision process.

The parameters for a typical transaction were used to illustrate an application of the model, and the value functions were plotted with the probability of default as the independent variable. A sensitivity analysis was conducted to determine the relative importance of the input parameters. The results show that the value of extending credit is highly sensitive to the vendor's contribution margin and to the vendor's exposure to default losses. The latter is reduced through security interests to increase collection success and by requiring down payments as a condition of extending credit. The results of the sensitivity analysis also show that the most important customer parameters are the probability of default and the probability of a cash sale if credit is denied. Other parameters such as the risk-adjusted rate for discounting payments, the probability of repeat orders, and the growth rate of orders were found to be unimportant to the credit decision because they had little or no effect on the value of granting credit or on the maximum acceptable default risk.

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Figure 1
Single Transaction Decision Tree

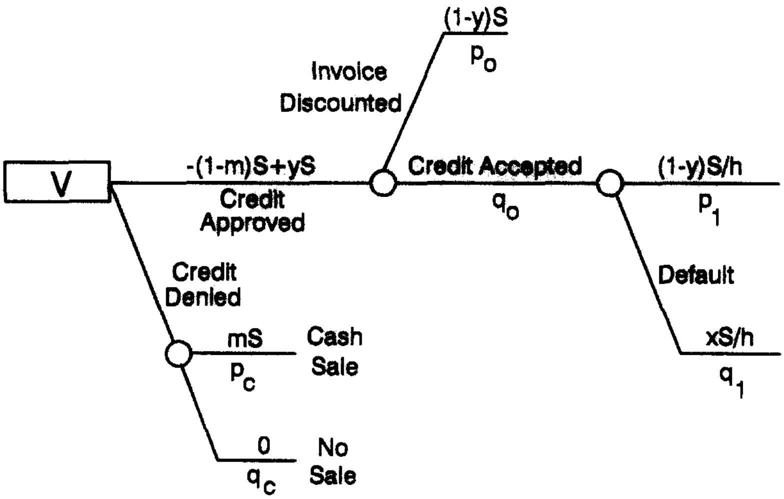

Figure 2
Recurring Transaction Credit Decision Tree

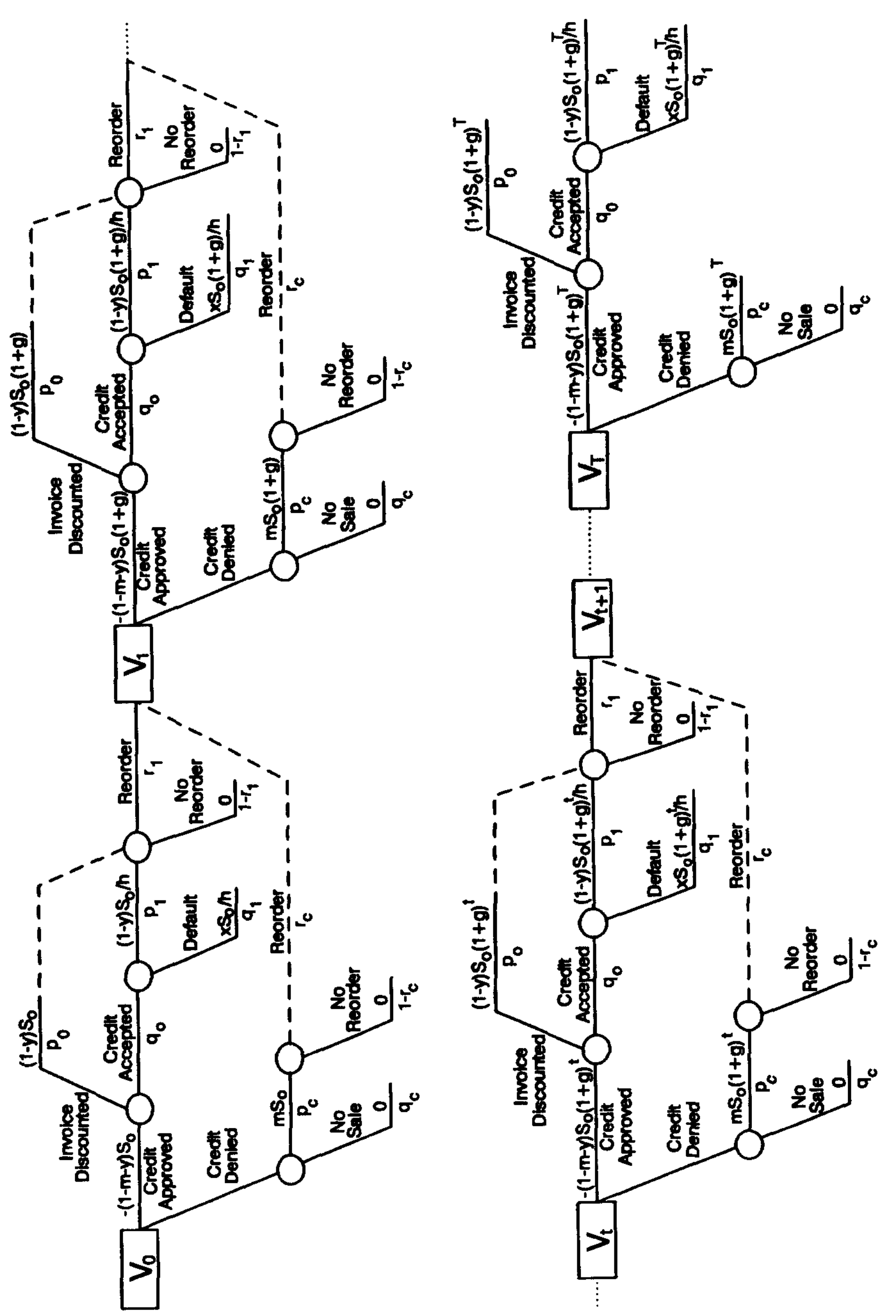

Figure 3
Value of Granting Credit as a Percentage of Initial Sale
Case Example Parameter Values

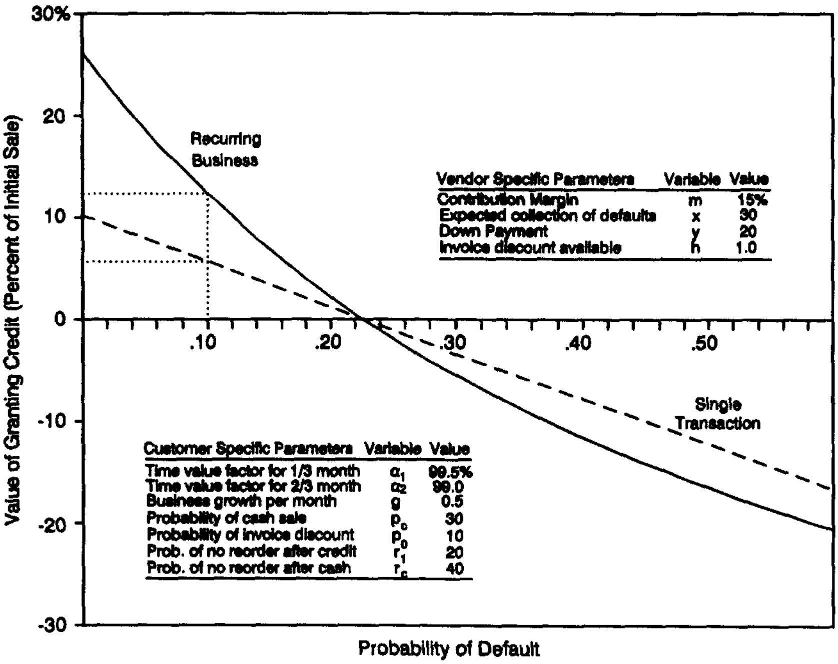

Figure 4
Sensitivity of the Value of Granting Credit to Contribution Margin

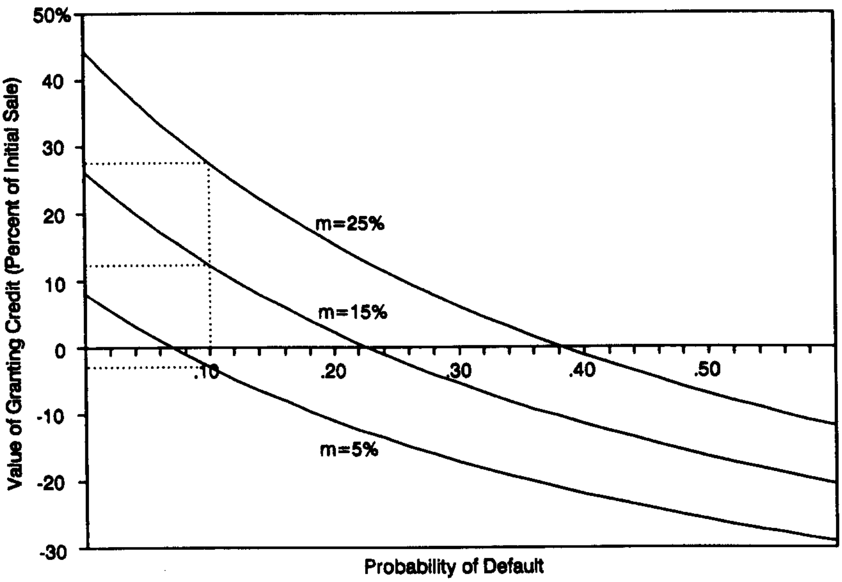

Figure 5
Sensitivity of the Value of Granting Credit to Collection of Defaults

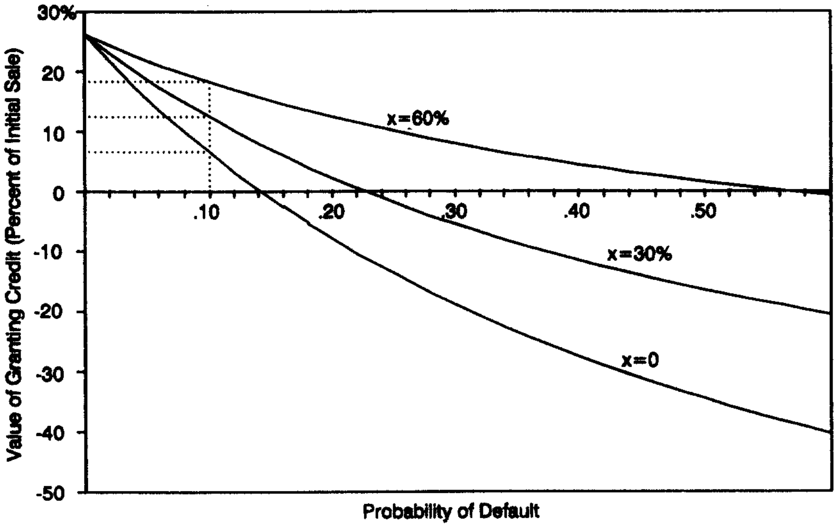

Figure 6
Sensitivity of the Value of Granting Credit to Down Payment

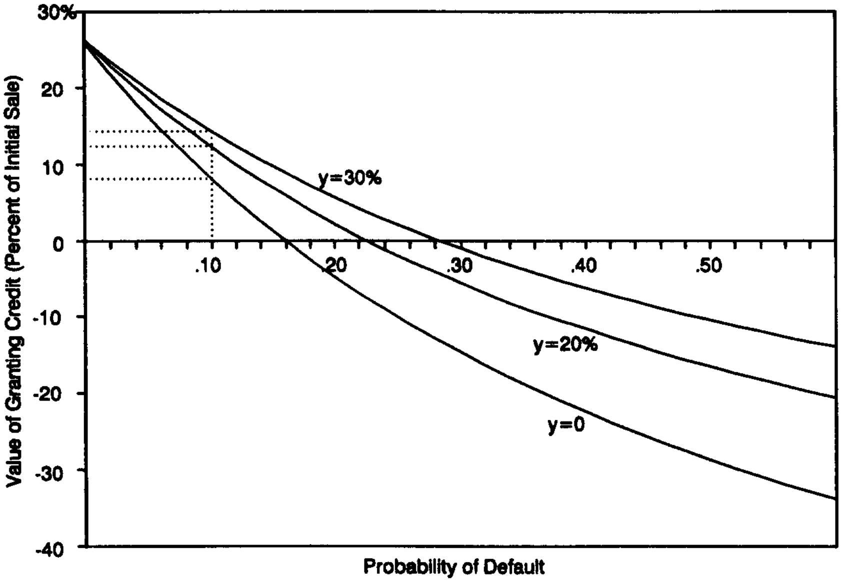

Figure 7
Sensitivity of the Value of Granting Credit to Probability of Cash Sale

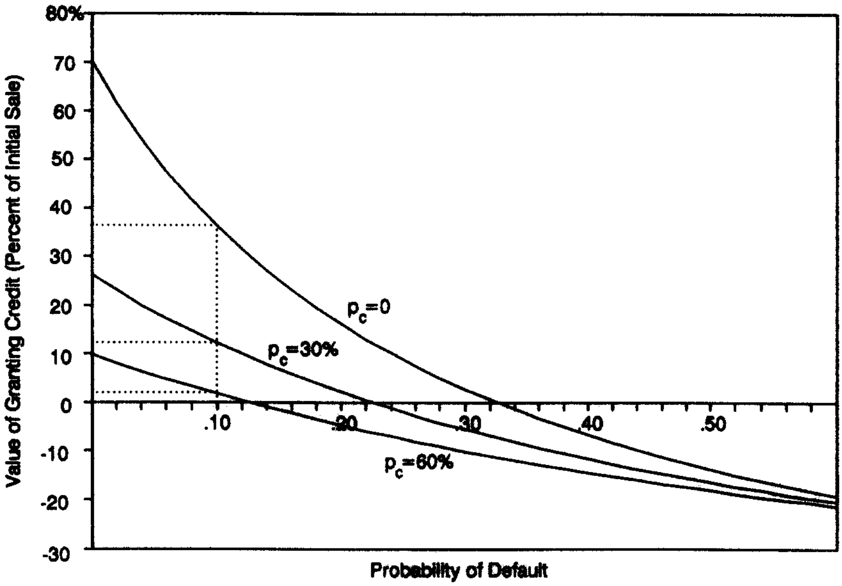

Figure 8
Sensitivity of the Value of Granting Credit to Time Value Discount Factor

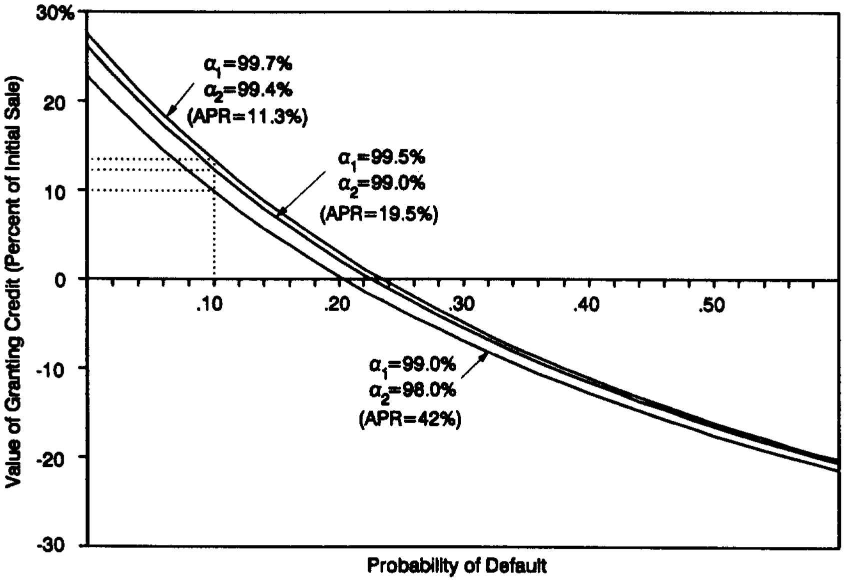

Figure 9
Sensitivity of the Value of Granting Credit to Growth Rate of Reorders

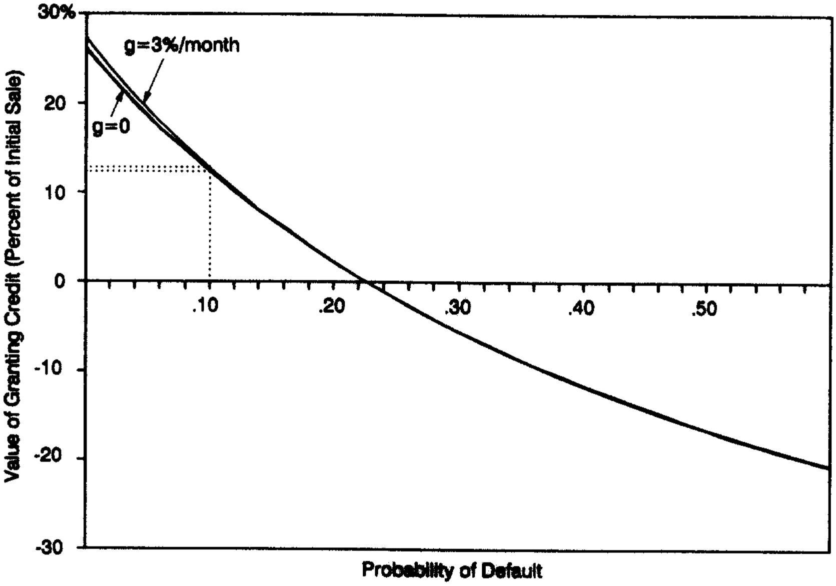

Figure 10
Sensitivity of the Value of Granting Credit to Reorder Probability