The Capacity Problem in the Measurement of Cost Curves

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Abstract

This paper quantifies the errors that may occur in the econometric estimation of long-run average cost curves. The results are consistent with econometric estimates of optimal size being smaller than estimates using other approaches. Once the measurement error has been identified, corrective adjustments can be made. Because of the important implications of scale effects on industrial and international trade policies, it is imperative that the measurement errors be recognized and corrected.

Introduction

The exact shape of the long-run average cost curve has important implications for industrial and international trade policies. Optimal size and the cost disadvantage incurred by firms of less than minimum efficient size are considered to be prime determinants of industry concentration. Adam Smith (1776) was the first to argue that gains from the division of labor are limited by the extent of the market. The importance of scale effects as a determinant of industrial concentration was documented by Bain (1956) and later confirmed in studies such as Rosenbluth (1957) which established that concentration in Canadian industries exceeds that of the U.S. because of the smaller Canadian market. Panzer (1989, p. 51) stresses the importance of empirical estimates of long-run cost functions for regulatory purposes in industries that have natural monopoly characteristics such as telecommunications, airlines, and railroads. He also notes that empirical cost functions are becoming increasingly important considerations in antitrust cases such as the U.S. Department of Justice antitrust suit seeking divestiture of the Bell System and in entry regulation such as the application by MCI for entry into AT&T's long distance monopoly markets.

Prior to the 1960s the theory of international trade was based on the principle of comparative advantage, whereby countries would export goods whose production was intensive in the factors of production with which they were

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endowed. Empirical work in the 1960s, however, did not confirm the theory; the majority of world trade was between countries with similar factor endowments. Scale economies were offered as an explanation. In a pioneering and much acclaimed study, Harris and Cox (1983) incorporated scale effects into a general equilibrium model to assess the impact of various types of trade liberalization and industrial policies on the Canadian economy. Their results indicate that the benefits from free trade are substantial, but sensitive to the existence of scale effects. The unfortunate weakness of the Harris and Cox results is the "unresolved empirical issues as to the numerical significance of scale economy effects, especially those produced by regression studies" (Whalley, 1984, p. 391). Furthermore, as pointed out by Fuss and Gupta (1981), regression estimates of optimal size are generally much smaller than are estimates using other approaches.

Because of the importance of scale effects for industrial and international trade policies, the accurate measurement of cost curves is a matter of immense practical significance. This paper examines the errors that may occur in the econometric estimation of long-run average cost curves. Specifically, because firms generally do not operate at the point of tangency of the short-run and long-run average cost curves, empirical estimates of the long-run average cost curve produce a curvilinear error such that the cost disadvantage of nonoptimally sized firms is overstated and the optimal size is underestimated (Kushner, 1975). This phenomenon, of course, is reminiscent of the issue that separated Jacob Viner from his draftsman some 50 years ago. This paper quantifies the errors using common mathematical specifications for the short-run average cost function (SRAC). Although the number of possible functional forms for the SRAC is infinite, four models have been selected that yield a quadratic, a decreasing linear, a decreasing hyperbolic, and an L-shaped long-run average cost curve (LRAC). These functions yield LRAC curves that are qualitatively similar to those obtained empirically, but are nevertheless mathematically tractable. Each model reflects different characteristics of the total cost function.

It should be noted that the results explain why econometric estimates of optimal size are smaller than estimates using other approaches. Once the measurement error has been quantified, corrective adjustments can be made. This is illustrated with a simple numerical example.

MODEL 1—Quadratic LRAC

This research begins with the traditional U-shaped LRAC curve postulated in microeconomic theory. Consider the generic model that represents families of SRAC curves, each of which has a nondegenerate minimum for some parameter value k of the output X. This one parameter family of SRAC curves is given by:

(1) $Y = C_k(X) = A_1(k - k_0)^2 + A_2(X - k)^2 + A_3,$

where:

$C_k(X)$	=	SRAC for a firm of size k, as a function of the output X;
k	=	Parameter for the family; and
k_0, A_1, A_2 and A_3	=	Positive constants.

Note that since $d_{/dX}\, C_k = 0$ when $X = k$, the curve $Y = C_k(X)$ has a minimum at $X = k$. The output X yielding this minimum increases with the size k of the firm.

The long-run average cost curve LRAC is the envelope of the family of SRAC curves given by equation (1). It is obtained by differentiating with respect to k:

$d_{/dk}\, Y = 2A_1(k - k_0) - 2A_2(X - k) = 0,$

which implies:

(2) $A_1(k - k_0) = A_2(X - k);$

(3) $k = \dfrac{A_1 k_0 + A_2 X}{A_1 + A_2}\,;$

(4) $k - k_0 = \left[\dfrac{A_2}{A_1}\right](X - k)$; and

(5) $X - k = \left[\dfrac{A_1}{A_1 + A_2}\right](X - k_0).$

Substitution of equations (4) and (5) into equation (1) yields the equation of the envelope:

(6) $\text{LRAC} = \left[\dfrac{A_1 A_2}{A_1 + A_2}\right](X - k_0)^2 + A_3\,.$

In estimating the theoretical long-run average cost curve, it is assumed that for any given plant size, firms are operating at full capacity, which is defined as the lowest point on the short-run average cost curve.[1]

From equation (1), one sees that the minimum SRAC for a firm of size k is given by (setting X = k):

$$(7)\ C_k(k) = A_1(k - k_0)^2 + A_3.$$

It follows that (replacing k by X) the estimated long-run average cost curve that passes through these minima ($LRAC_{FC}$) is given by:

$$(8)\ LRAC_{FC} = A_1(X - k_0)^2 + A_3.$$

The difference $\Delta Y_{FC} = LRAC_{FC} - LRAC$ obtained subtracting equation (6) from equation (8) is the error in estimation:

$$(9)\ \Delta Y_{FC} = A_1(X - k_0)^2 - \left[\frac{A_1A_2}{A_1 + A_2}\right](X - k_0)^2 = \left[\frac{A_1^2}{A_1 + A_2}\right](X - k_0)^2.$$

The error is zero when $X = k_0$. The error depends indirectly on the size k because X = k; the smaller the firm size, the greater is the estimation error. A particular case is illustrated in Figure 1.

The assumption that firms are operating at full capacity now is relaxed.[2] For utilization rates of less than 100 percent, let 100u, $0 \leq u \leq 1$, denote the utilization rate. Then, from the SRAC for a firm of size k, the cost at an output of X = u.k is:

$$(10)\ C_k(u.k) = A_1(k - k_0)^2 + A_2(u.k - k)^2 + A_3 = A_1(k - k_0)^2 + A_2(1 - u)^2k^2 + A_3.$$

It should be noted that when firms operate at full capacity in the short run, then u = 1 and equation (10) reduces to equation (7). If excess capacity exists, for example, u = 0.9, this expression represents the average cost for a firm of size k operating at 90 percent of designed capacity. Replacing k by X/u yields $LRAC_{LFC}$, the long-run average cost when firms operate at less than full capacity:

[1] An alternative but less common definition of full capacity is the point of tangency of the short-run and long-run average cost curves. Although not presented in this paper, similar estimation errors occur.

[2] Multiple regression techniques sometimes include percentage of capacity utilization as an explanatory variable. The errors, however, are unaffected by such an inclusion.

$$(11)\ LRAC_{LFC} = A_1\left[\frac{X}{u} - k_0\right]^2 + A_2(1-u)^2\left[\frac{X}{u}\right]^2 + A_3.$$

The difference $\Delta Y_{LFC} = LRAC_{LFC} - LRAC$ is the error in estimating LRAC by $LRAC_{LFC}$:

$$\Delta Y_{LFC} = A_1\left[\frac{X}{u} - k_0\right]^2 + A_2(1-u)^2\left[\frac{X}{u}\right]^2 - \left[\frac{A_1A_2}{A_1 + A_2}\right](X - k_0)^2.$$

This is a quadratic in the output X that is factored by completing squares to yield:

$$(12)\ \Delta Y_{LFC} = \frac{[(A_1 + A_2 - uA_2)X - uk_0A_1]^2}{u^2(A_1 + A_2)}.$$

It is interesting to note that this error is 0 at the output $X = uk_0A_1/(A_1 + A_2 - uA_2)^2$, which produces a point of tangency between the SRAC and LRAC curves. (Note: For $u = 1$, this is equivalent to saying that the error is 0 when $X = k_0$.)

Next the LRAC is estimated for a family of firms in which the utilization rate is correlated positively with the size k; this means that the firms approach full capacity as the firm size k increases. The estimated long-run average cost for this case is denoted by $LRAC_{LFCP}$. An expression for it is obtained from equation (11) by assuming that u is an appropriate function of the firm size k having the following properties:

- u(k) tends to 0 as k tends to K (from the right), where $K > 0$ is a constant that may be arbitrarily small; and
- u(k) tends to 1 as k tends to ∞.

A simple (but admittedly arbitrary) function with these properties is $u(p) = (k - K)/p$, $k \geq K$ (where large k are of interest). Because $k = X/u$, $u = X/(X + K)$, $X \geq 0$. Substitution into equation (11) yields:

$$(13)\ LRAC_{LFCP} = A_1(X + K - k_0)^2 + A_2K^2 + A_3.$$

The corresponding error in estimating LRAC is found to be:

$$(14)\ \Delta Y_{LFCP} = LRAC_{LFCP} - LRAC = \frac{[A_1X + K(A_1 + A_2) - k_0A_1]^2}{A_1 + A_2}.$$

Note that this error is 0 when $X = (-K(A_1 + A_2) + k_0A_1)/A_1$, indicating tangency of exactly one of the SRAC curves to the LRAC.

Similarly, an expression for the long-run average cost at actual output is obtained when the utilization rate is correlated negatively with size k. This means that the firms diverge from full capacity as the firm size k increases; denote this by $LRAC_{LFCN}$. In this case u must be a function of k such that:

- u(k) tends to 1 as k tends to 0; and
- u(k) tends to 0 as k tends to ∞.

A simple (but arbitrary) function with these properties is $u(k) = K'/(k + K')$, where K' is a constant. Because $k = X/u$, $u = (K' - X)/K'$, for $X \le K'$ (note that K' may be arbitrarily large). From equation (11) one now obtains:

$$(15)\ LRAC_{LFCN} = \frac{A_1K'^2(X - k_0)^2 + A_1k_0X(kX + 2K'X - 2k_0K') + A_2X^4}{(X - K')^2 + A_3}$$

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The corresponding error in estimating LRAC is seen to be:

$$(16)\ \Delta Y_{LFCN} = LRAC_{LFCN} - LRAC$$

$$= \left[\frac{1}{A_1 + A_2}\right]\left[\frac{A_2X^2 + A_1X(K' + k_0) - K'k_0A_1}{X - K'}\right]^2 .$$

Once again, this error is 0 at a point of tangency.

MODEL 2—Linear LRAC

A LRAC curve that decreases linearly next is examined. Consider the one parameter family of SRAC curves given by:

$$(17)\ Y = C_k(X) = a^2X + (b - mk) + \frac{c^2k^2}{X},$$

where:

$k > 0$	=	Parameter;
X	=	Output; and
a, b, c, and m	=	Positive constants.

These curves correspond to families of total cost functions that have:

- Unit costs that decrease linearly with the size of the firm; and
- Fixed costs which increase with the size k.[3]

In fact, it is legitimate to refer to k as the size of the firm because Y in equation (17) achieves its minimum at X = ck/a, which increases with k. By rescaling Y and k, however, it is possible to absorb the constants a and c and, without loss of generality, consider the family:

$$(18)\ Y = C_k(X) = X + (b - mk) + \frac{k^2}{X},$$

with the property that the firm of size k has its minimum cost at X = k, as in the first model, equation (1). A particular case is illustrated in Figure 2.

To obtain the theoretical LRAC, recall from equation (18) that $d/_{dk}Y = -m + 2k/X = 0$ which implies $k = mX/2$. Therefore, the envelope has the equation:

$$(19)\ \text{LRAC} = \left[1 - \frac{m^2}{4}\right]X + b,$$

which is linear in the output X.

To estimate the LRAC, the minimum SRAC from equation (18) first is determined:

$$C_k(k) = k + (b - mk) + \frac{k^2}{k} = (2 - m)k + b,$$

or, letting X be the variable,

$$(20)\ \text{LRAC}_{FC} = (2 - m)X + b,$$

which is also linear in the output X. The error is therefore:

$$(21)\ \Delta Y_{FC} = \left[1 - m + \frac{m^2}{4}\right]X = \left[1 - \frac{m}{4}\right]^2 X.$$

It is interesting to note that when the unit costs are constant in k (i.e., m = 0), the error becomes:

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[3]Similar analyses hold for other functional forms such as the generalized Leontief (e.g., Fuss 1977, 1978) and the translog (e.g., Christensen, Jorgenson, and Lau, 1973; and Caves, Christensen, and Tretheway, 1984). For a discussion of the properties of the different functional forms, see Caves and Christensen (1980).

(22) $\Delta Y_{FC/m=0} = X.$

It should be noted that, unlike the previous case, the error grows with the size of the firm.

Once again the assumption that firms are operating at full capacity is relaxed. For a utilization rate of 100u percent, where $0 < u \leq 1$:

$$(23)\ C_k(u.k) = uk + (b - mk) + \frac{k^2}{uk} = \left[u - m + \frac{1}{u}\right]k + b.$$

Consequently (replacing k by X/u),

$$(24)\ LRAC_{LFC} = \frac{\left[u - m + \frac{1}{u}\right]X}{u} + b = \left[1 - \frac{m}{u} + \frac{1}{u^2}\right]X + b\,.$$

The corresponding error is:

$$(25)\ \Delta Y_{LFC} = \left[\frac{m}{2} - \frac{1}{u}\right]^2 X.$$

It is again interesting to note that this is 0 when u = 2/m. In this case, tangency to the LRAC curve is achieved by all SRAC curves in the family.

Finally, when the utilization rate is correlated positively with the size k, one obtains:

$$(26)\ \Delta Y_{LFCP} = \frac{[X(m-2) - 2K]^2}{4X},$$

and when the utilization rate is correlated negatively with size,

$$(27)\ \Delta Y_{LFCN} = \frac{X[K'(m-2) - mX]^2}{4(K' - X)^2}.$$

MODEL 3—Hyperbolic LRAC

This section examines a hyperbolic, decreasing LRAC curve. Consider the family of SRAC curves:

$$(28)\ Y = C_k(X) = \frac{b^2}{k} + \frac{k}{X^2}.$$

These curves, of course, have no minimum for finite X and decrease asymptotically to $Y = C_k(\infty) = b^2/k$ with increasing X.[4] In this model it is assumed that physical size determines full capacity in the following sense: the output producing full capacity is that value of X for which $[C_k(X) - C_k(\infty)]/C_k(\infty) = r^2$, where r is a constant. Solving this equation for X in terms of k yields $X = k/rb$, so that:

$$(29)\ LRAC_{FC} = \frac{b\left[\frac{1}{r} + r\right]}{X}.$$

On the other hand, the envelope is found to be:

$$(30)\ LRAC = \frac{2b}{X}.$$

Therefore, the error is:

$$(31)\ \Delta Y_{FC} = LRAC_{FC} - LRAC = \frac{b\left[\frac{1}{r} + r - 2\right]}{X}.$$

This figure is 0 when $r = 1$, indicating that all SRAC curves are tangent to LRAC for this value of r.

A particular example is illustrated in Figure 3.

In this model excess capacity can be quantified by simply increasing r^2, as this has the effect of increasing the minimum of the SRAC curves. Therefore, ΔY_{LFC} is given by equation (31) with the appropriate value of r.

MODEL 4—L-Shaped LRAC

Finally a model similar to Model 1, but with the property that the LRAC curve remains constant after decreasing to its minimum value, is considered. Perhaps the simplest mathematical formulation results by adapting the SRAC equations from Model 1, as follows:

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[4]According to Johnston (1960, p. 13) the declining long-run average cost is "the one suggested as the most plausible by the accumulated empirical evidence." Wiles (1956), examining 44 sets of data, concurs: "decreasing costs are almost universal. Sharply increasing costs with size are practically unknown and even slight increases are rare."

$$(32)\ Y = C_k(X) = \begin{cases} A_1(k - k_0)^2 + A_2(X - k)^2 + A_3, & k \leq k_0 \\ A_2(X - k)^2 + A_3, & k > k_0 \end{cases}$$

A particular case is shown in Figure 4. Note that the curves $Y = C_k(X)$ have the same minimum value A_3 for $k > k_0$. As these curves agree with those of Model 1 for k less than the hypothesized limiting value k_0, the analysis of Model 1 applies to yield the equations:

$$(33)\ LRAC = \begin{cases} \left[\dfrac{A_1A_2}{A_1 + A_2}\right](X - k_0)^2 + A_3, & k \leq k_0 \\ A_3, & k > k_0 \end{cases}$$

and

$$(34)\ LRAC_{FC} = \begin{cases} A_1(X - k_0)^2 + A_3, & k \leq k_0 \\ A_3, & k > k_0 \end{cases}$$

The difference produces the same error in estimation $\Delta Y_{FC} = LRAC_{FC} - LRAC$ given by equation (9) for $k \leq k_0$, and 0 otherwise.

Although not shown in this paper, similar measurement errors occur when firms produce beyond full capacity. Excess capacity, however, is of major concern in the literature. Scherer (1990, pp. 376, 468) contends that firms overinvest in order to serve customers in times of peak demand, with the hope of maintaining their loyalty permanently. Furthermore, during demand growth, companies invest heavily in additional capacity in order to maintain market share. Excess capacity also serves as an entry-deterring strategy. Established firms may threaten an entrant with a predatory increase in output. To make the threat credible, the established firms must commit themselves to an investment such that excess capacity would be observed (Spence, 1977; Eaton and Lipsey, 1980).[5]

[5]For a review of the literature on excess capacity and examples of excess capacity as an entry deterring strategy, see Scherer (1990, pp. 381-384).

Example

This paper has presented deterministic models illustrating measurement errors in the estimation of long-run cost curves. The question arises as to the magnitude of these errors in existing econometric studies. The data necessary to obtain the actual measurement errors are a full range of short-run cost output information. Unfortunately, published studies do not provide such data. An example, however, can show the method of determining the magnitude of the errors should the data be available.

Suppose that the long-run average cost curve is estimated to be:

$$(35)\ LRAC_{est} = 0.01X^2 - 0.05X + 0.6625 = 0.01(X - 2.5)^2 + 0.6.$$

Output is in millions of units, and costs are in millions of dollars. The coefficients chosen here for $LRAC_{est}$ are consistent, for example, with those obtained by regression in Johnston's (1960) study of electrical generation in the United Kingdom. To determine the possibility of measurement errors, utilization rates first must be determined. Suppose it is determined that firms operate at 100 percent of designed capacity, which this research will interpret to be the minimum of the SRAC curves.[6] Assuming Model 1, this means that $LRAC_{est}$ given by equation (35) is the $LRAC_{FC}$ given by equation (8). Consequently, the parameters may be identified as $A_1 = 0.01$, $k_0 = 2.5$ and $A_3 = 0.6$ pertaining to the corresponding family of SRAC curves by comparing equations (8) and (35). In order to specify the correct LRAC curve, statistical estimates for one SRAC curve must be obtained to identify the last parameter A_2. Accordingly, by way of example, the following SRAC curve is considered:

$$(36)\ C_1(X) = 0.6425 - 0.04X + 0.02X^2.$$

Comparison of equation (36) with equation (1) shows that $A_2 = 0.02$; note that in Model 1 all curves in the family must have the same coefficient of X^2. Consequently, the curve (36) is the member (with $k = 1$) of the parametric family given by (compare equation (1)):

$$(37)\ Y = C_k(X) = 0.01(k - 2.5)^2 + 0.02(X - k)^2 + 0.6.$$

[6]Full capacity usually is interpreted as the firm operating at 100 percent of designed capacity (for example, see Scherer 1990, p. 287). On the other hand, the McGraw-Hill annual survey of manufacturing companies indicates that, regardless of size, the desired rate of utilization ranges from 90 percent to 100 percent of capacity, suggesting that the minimum of the average cost occurs at between 90 percent and 100 percent utilization. One therefore would have to determine the level of utilization at full capacity for the specific industry being studied.

Therefore, the true LRAC is given by equation (6) as:

(38) $LRAC = 0.00667(X - 2.5)^2 + 0.6$.

From equation (9) it now is possible to calculate the error in estimation as:

(39) $\Delta Y_{FC} = 0.00333(X - 2.5)^2$.

For example, an output of $X = 6.5$ results in an error in the measurement of cost of approximately $53,300. In other words, the cost disadvantage of being nonoptimal is overstated by 7.5 percent. For other levels of output, corrections would be made in a similar manner.

As indicated earlier in the paper, utilization rates can be less or greater than 100 percent of designed capacity. For example, if business conditions become depressed and firms operate at 80 percent of capacity, the error would be estimated using equation (12), and a corresponding corrective adjustment would be made.

Summary and Conclusions

This research has quantified the errors when the long-run average cost curve is estimated using common mathematical formulations for the short-run average cost. For example, if the short-run average cost curve is specified in a generalized quadratic functional form, the estimation errors can be summarized as follows:

- If firms operate at full capacity in the short run, statistical cost output estimates accurately will measure the optimal size but will overstate the cost disadvantage of nonoptimally sized firms (c.f., equation (9)).

- If firms operate with excess short-run capacity, statistical cost output estimates will underestimate the optimal size and overestimate the cost disadvantage of nonoptimally sized firms. The estimation error will be correlated positively with firm size (c.f., equation (12)). (The errors are especially acute when utilization rates are correlated negatively with firm size (c.f., equation (16)).)

Given the quantification of the errors, the estimated long-run average cost curve then can be adjusted to give a proper cost-output relationship. To do so requires a statistical estimation of the short-run average cost curves to determine the parameters for the functional form. Because of the important implications of scale effects on industrial and international trade policies, it is imperative that the measurement errors be recognized and that corrections be made. Such corrections will reconcile the difference between econometric and engineering estimates of optimal size.

References

1. Bain, J.S., *Barriers to New Competition* (Cambridge: Harvard University Press, 1956).

2. Caves, D.W., L.R. Christensen, and M.W. Tretheway, "Flexible Cost Functions for Multiproduct Firms," *Review of Economics and Statistics*, 62, no. 3 (August 1980), pp. 477-481.

3. Caves, D.W., L.R. Christensen, and M.W. Tretheway, "Economics of Density Versus Economies of Scale: Why Trunk and Local Airlines Differ," *Rand Journal of Economics*, 15, no. 4 (Winter 1984), pp. 471-489.

4. Christensen, L.R., D.W. Jorgenson, and L.I. Lau, "Transcendental Logarithmic Production Frontiers," *Review of Economics and Statistics*, 55, no. 1 (February 1973), pp. 28-45.

5. Eaton, B.C., and R.G. Lipsey, "Exit Barriers are Entry Barriers: the Durability of Capital," *Bell Journal of Economics*, 11, no. 2 (Autumn 1980), pp. 721-729.

6. Fuss, M., "Factor Substitution in Electricity Generation: A Test of the Putty-Clay Hypothesis," in M. Fuss and D. McFadden (eds.), *Production Economics* (Amsterdam: North Holland, 1978).

7. Fuss, M., " A Survey of Recent Results in the Analysis of Production Conditions in Telecommunications," in L. Courville, A. de Fontenay, and R. Dobell (eds.), *Economic Analysis of Telecommunications* (Amsterdam: North Holland, 1983).

8. Fuss, M. and V.K. Gupta, "A Cost Function Approach to the Estimation of Minimum Efficient Scale, and Suboptimal Capacity: with an Application to Canadian Manufacturing," *European Economic Review*, 15 (1981), pp. 123-135.

9. Harris, R.G. with D. Cox, *Trade, Industrial Policy and Canadian Manufacturing* (Toronto: Ontario Economic Council, 1983).

10. Johnston, J., *Statistical Cost Analysis* (New York: McGraw-Hill Book Company, 1960).

11. Kushner, J., "The Estimation Bias in the Measurement of Cost Curves," *Engineering Economist*, 20, no. 2 (Winter 1975), pp. 151-155.

12. Panzer, J.C., "Technological Determinants of Term and Industry Structure," in R. Schmalensee and R.D. Willig (eds.), *Handbook of Industrial Organization* (Amsterdam: North Holland, 1989).

13. Rosenbluth, G., *Concentration in Canadian Manufacturing Industries* (Berkeley: University of California Press, 1957).

14. Scherer, F.M., *Industrial Market Structure and Economic Performance*, 3rd ed. (Chicago: Rand McNally, 1990).

15. Smith, A., *The Wealth of Nations*, (New York: The Modern Library, 1937).

16. Spence, A.M., "Entry, Capacity, Investment and Oligopolistic Pricing," *Bell Journal of Economics*, 8, no. 2 (Autumn 1977), pp. 534-544.

17. Whalley, J., review article on "Trade, Industrial Policy and Canadian Manufacturing" by Richard G. Harris with David Cox, *Canadian Journal of Economics*, XVII, no. 2 (May 1984), pp. 386-398.

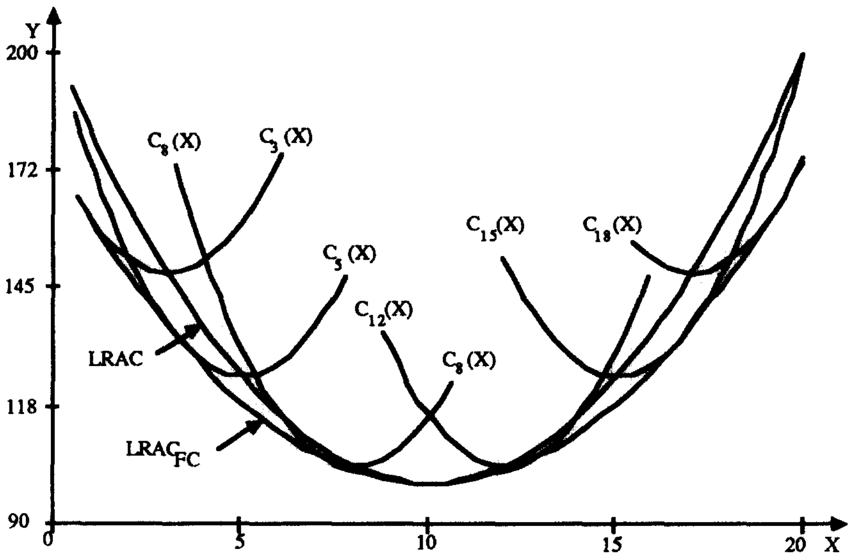
18. Wiles, P.J.D., *Prices, Cost and Output* (Oxford: Basil, Blackwell and Mott, 1956).

Model 1
Quadratic LRAC

Example

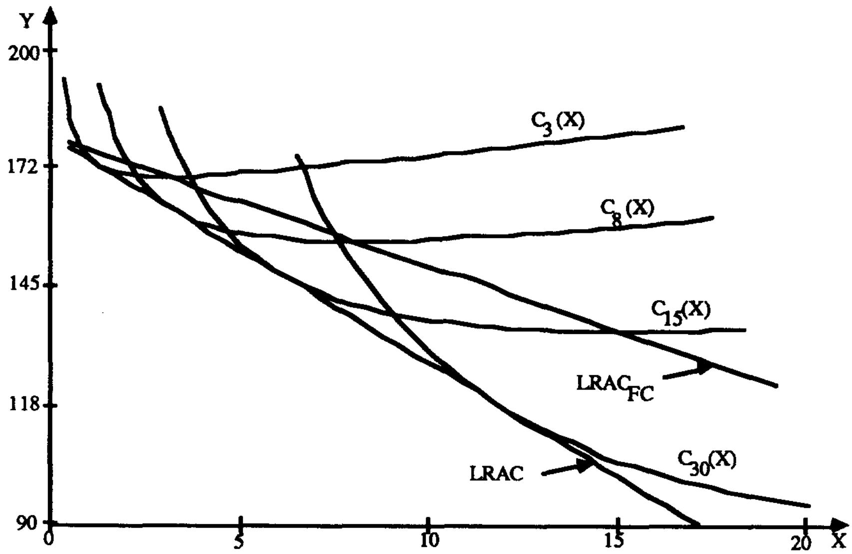
$Y = C_k(X) = (k - 10)^2 + d(x - k)^2 + 100$

$(A_1 = 1, A_2 = 3, A_3 = 100, k_0 = 10)$



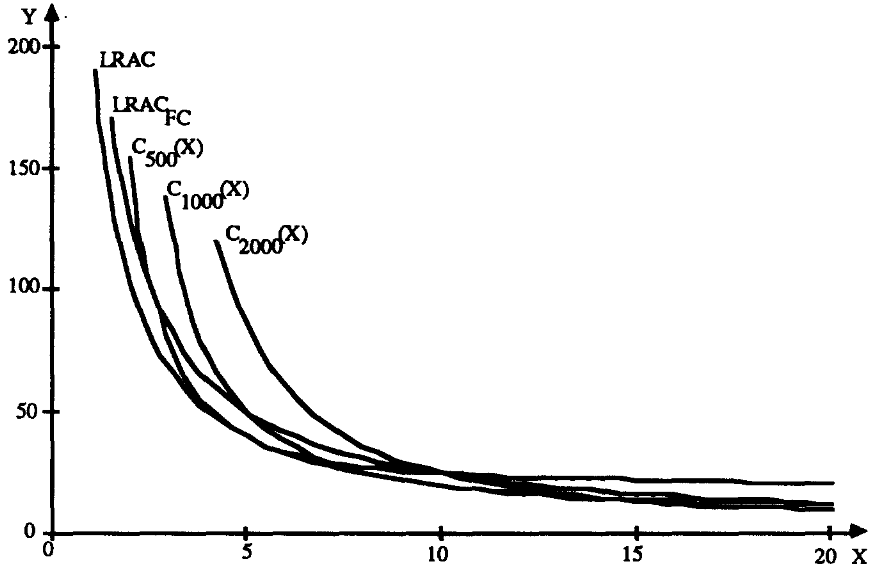
Model 2
Linear LRAC

Example
$Y = C_k(X) = X + (180 - 5k) + k^2/X$
$(b = 180, m = 5)$



Model 3
Hyperbolic LRAC

Example
$Y = C_k(X) = 10{,}000/k + k/X^2$
$(b = 100, r = 0.5)$



Model 4
L-Shaped LRAC

Example

$$Y = C_k(X) = \begin{cases} (k-10)^2 + 3(X-k)^2 + 100, & k \le k_0 \\ 3(X-k)^2 + 100, & k > k_0 \end{cases}$$

$(A_1 = 1, A_2 = 3, A_3 = 100, k_0 = 10)$