A Simulation Comparison of Actuarial and Contingent Claims Models for Unfunded Pension Liabilities

G. Lee Willinger*
University of Oklahoma

Abstract

Corporate financial statements must include pension plan disclosures. The disclosures include information that allows users to compute the unfunded pension liability. Willinger (1985) discusses an alternative contingent claims model for the valuation of unfunded pension liabilities. This alternative model may be preferred to the actuarial present value model, as the model reduces the opportunities to manipulate the pension liability. This paper develops simulations that compare the actuarial model to the contingent claims model. The results provide evidence that the contingent claims model reduces variation in the possible pension liability and therefore reduces the possible manipulative qualities of the actuarial model.

Introduction

The Financial Accounting Standards Board (FASB) recently issued Financial Accounting Standards (FAS) No. 87 (1985) and No. 88 (1985) for pension accounting. Some of the provisions of FAS No. 87 provide instructions for employee pension benefit calculation, selection of pension obligation calculation assumptions, pension disclosures, and balance sheet presentation. Unlike previous pension pronouncements, however, FAS No. 87 requires the unfunded accumulated benefit obligation on the balance sheet. This additional minimum pension liability reflects the difference between the present value of the projected accumulated pension benefits and the fair market value of the pension plan portfolio.

The present value computation of the projected accumulated pension benefits requires a discount rate. One of the FASB's objectives in producing pension accounting standards was to provide guidance for the choice of a discount rate. FAS No. 35 (1980, par. 20a) states:

> Assumed rates of return shall reflect the expected rates of return during the periods for which payment of benefits is deferred and shall be

*This paper is a product of the author's dissertation written at Florida State University. The author wishes to thank Elton Scott, Joe D. Iceman, Edward V. McIntyre, Robert A. Marshall, and Kenneth S. Lorek. The author also would like to thank Gary Emery, Sunkook Kwon, Phil Harsha, Bart Ward, Shane Moriarity, Fran Ayres, and Gary Sirak for their comments on previous drafts.

0747-5535/91/1500-0072 $01.00 1300

> consistent with returns realistically achievable on the types of assets held by the plan and the plan's investment policy.

This provided little, if any, guidance and allowed firms to justify a wide variety of discount rates. In an attempt to correct this, FAS No. 87 (1985, par. 44) states:

> Assumed discount rates shall reflect the rates at which the pension benefits could be effectively settled. It is appropriate in estimating those rates to look to available information about rates implicit in current prices of annuity contracts that could be used to effect settlement of the obligation (including information about available annuity rates currently published by the Pension Benefit Guarantee Corporation). In making those estimates, employers may also look to rates of return on high-quality fixed-income investments currently available and expected to be available during the period to maturity of the pension benefits.

FAS No. 87 requires that the discount rate reflects rates required to settle pension obligations. As discussed above, a discount rate fitting the FAS No. 87 description may present a variety of choices. The array of choices and the year-to-year variation in model parameters provide a firm with a chance to manipulate current and historical pension obligations. This paper compares the manipulative possibilities and theoretical aspects of the FAS No. 87 actuarial present value model to a contingent claims model.

Evidence pointing to manipulation of the unfunded pension liability and the pension expense is not readily available. There are some documented instances, however, where changes in interest rates gave the appearance of manipulation. Chrysler was expected to have a loss in the fourth quarter of 1979. Through actuarial adjustment of the interest rate from 6 percent to 7 percent, the pension cost for the quarter was reduced $50 million; the reported profit for the quarter was approximately $43 million (*Wall Street Journal*, 1980). Eastern Airlines first quarter 1981 profit was $4.1 million. This profit, however, may have been the result of increasing the interest rate assumption from 7 percent to 9 percent. This reduced Eastern's pension cost of $25 million, when compared to the previous year's expense (Baker, 1981). Crown Zellerbach increased its interest rate assumption for external reporting in 1980 from 6.25 percent to 8.5 percent, but used 7.5 percent for internal reporting. This reduced the present value of the anticipated pension payout and changed a $77 million shortfall in 1979 to a $85 million surplus in 1980 (Baker, 1981). Finally, Bethlehem Steel provided two reports: one for financial statement purposes and one for internal purposes. The financial statement report using a 10 percent interest rate assumption showed a net liability of $400 million. The internal report using a 7 percent interest rate assumption showed a $1 billion net liability (Baker, 1981). In all of these cases, the corporation's bottom line benefitted from the change in interest rate assumptions. If the interest rate assumption changes were based on increased future returns, the changes might be appropriate. Interest rates were falling, however, or would fall in all of these cases. The changes, therefore, appear to be manipulative.

All of the above instances relate to the manipulation of periodic pension expense. The model developed in this paper is for the unfunded pension liability found on the balance sheet. As shown below, however, the market and users of financial information are interested in the nature and content of the unfunded pension liability.

The FASB considered a mandated discount rate for the computation of pension liabilities. A rate set in this manner would eliminate variation in results and increase comparability. One interest rate, however, is not appropriate for all time spans of pension liabilities. Per the theory of interest, debts of long, intermediate, and short time frames have different appropriate interest rates. In addition, a market-based interest rate represents the risk assessments of a wide range of individuals. The interest rate(s) set by fiat of the FASB may represent only a portion of this wide range.

Although the contingent claims model may have analytical and theoretical advantages, the adoption of the contingent claims model will depend upon the realization of these advantages. Comparison of actual pension liabilities to contingent claims model results may be possible if extensive pension information is available. FAS No. 87 expanded disclosures, but the detailed individual employee information required to verify the liability empirically is still not publicly available. In addition, the possible perturbations in future data may not be present in current data. An alternative that provides relief from these problems is a simulation.

Previous papers by Winklevoss (1982) and Kingsland (1982) use commercially available simulation models to compute projected pension obligations. The simulations presented in this paper assume the values computed for retirement day benefits, but investigate the possible range of results of the present value computations. A range of present values is possible because the structure of interest rates changes on a year-to-year basis. Although a set of required market-based interest rates may produce less variation in results than rates set through a judgment process, the extent of this reduction in variation is not intuitively or analytically evident. The simulations compare the application of an actuarial present value model and a contingent claims model when liability variable variation exists. The simulation results provide evidence that the contingent claims model results in less variation than does the actuarial model. This reduced variation may enhance both investor confidence and predictability of future values while decreasing opportunities to manipulate the unfunded pension liability.

Prior research into the usefulness of the unfunded pension liability disclosures resulted in mixed findings. Oldfield (1976) found that the market treated the unfunded vested benefit, a major component of the total unfunded pension liability, as a surrogate for the true pension obligation. Feldstein and Seligman (1981) found similar results using a different methodology. Daley (1984), using an equity valuation model derived from the Sharpe (1964), Lintner (1965), and Mossin (1966) capital asset pricing model (CAPM), found that the market is most correlated with the income statement's pension expense. Daley does provide a caveat, however, that his research does not indicate that other pension disclosures may not contain marginal information. In addition, a pension liability disclosure based on a contingent claims model may contain greater information content. The results from these researchers are inconclusive about the market's evaluation of pension disclosures.

The Unfunded Pension Liability Model

Accountants and actuaries face the difficult problem of evaluating unfunded pension liabilities. Unfunded pension liabilities, like employee stock options, leases, and purchase commitments, represent executory contracts. An executory contract is a contract where rights and duties offset. The uncertainty involved in the contingent nature of the unfunded pension liability presents a valuation problem. A contingent claims model approach, however, may recognize that uncertainty and provide a model for valuation.

An employee covered by a pension plan earns wages and pension benefits.[1] The employer pays the employee's wages in dollars and establishes a pension fund to provide pension benefits. The employer's yearly contributions to the pension fund may or may not fully fund the future pension liability. An unfunded pension liability results when the employer's contributions are less than the present value of the future liability.

The actuary uses a present value model to evaluate the unfunded pension liability. The actuarial present value model is (Ezra, 1980):

$$(1)\ \mathrm{UPL} = \frac{B}{(1 + RR)^{T}} - M$$

where:

[1]There are two types of corporate pension plans. The first is a defined benefit plan. This type of plan establishes a definite benefit for each employee. The contributions are whatever is necessary to provide the desired future employee benefit. The other type of plan is a defined contribution plan. Under this plan, employer contributions are fixed. The employee's benefits depend upon the contributions made, the plan investment earnings, and the employee's profile. The type of plan that is assumed and discussed in this research paper is a defined benefit plan.

UPL = The unfunded pension liability related to the unfunded accumulated benefit obligation or the unfunded projected benefit obligation;
B = Future retirement date value of accrued benefits as actuarially computed;
RR = An appropriate discount rate;
M = The current market value of the pension plan portfolio; and
T = Time to retirement.

The valuation of the liability is the difference between the present value of the retirement date benefits and the current value of the pension plan portfolio. Pressures to minimize (maximize) pension liabilities, however, may produce manipulation in the assumed FAS No. 87 discount rate and the resulting liability. Accountants and actuaries can avoid this possible manipulation and develop a model analogous to the liability described above. The model providing this opportunity is a put contingent claims model.

A Black and Scholes-type (1973) contingent claims model is applicable to unfunded pension liabilities. The exercise price is the amount of retirement benefits due in the future. After the application of economic assumptions and an actuarial cost method, both the exercise price and retirement benefits are fixed for that specific evaluation date. Both use a discount rate to evaluate the liability in present value terms. To do this, the actuarial model applies an appropriate FAS No. 87 discount rate and the Black and Scholes-type model applies a riskless discount rate. The underlying security is the pension fund. The current unfunded pension liability is the expected present value of the difference between the retirement benefits (exercise price) and the pension plan assets (underlying security). The pension liability is, therefore, a put contingent claim.

A put contingent claim allows the holder to receive a specified payment (exercise price) at some future date in exchange for an asset (underlying security). The put has value contingent on the underlying asset having a value less than the exercise price. The expected present value of the future differences between the exercise price and the underlying asset is the put value.

The put model discussed in this paper is Merton's (1976) European put extension of the Black and Scholes (1973) option pricing model. The model is:

$$\mathrm{UPL} = -\mathrm{M} \bullet \mathrm{N}\left[\frac{-\ln(\mathrm{M}/\mathrm{B}) - [r + (\sigma^2/2)]\mathrm{T}}{\sigma\sqrt{\mathrm{T}}}\right] + \mathrm{B}e^{-r\mathrm{T}} \bullet \mathrm{N}\left[\frac{-\ln(\mathrm{M}/\mathrm{B}) - [r - (\sigma^2/2)]\mathrm{T}}{\sigma\sqrt{\mathrm{T}}}\right]$$

where:

UPL = The unfunded pension liability related to the unfunded accumulated benefit obligation or the unfunded projected benefit obligation;
M = Current market value of the pension fund portfolio;
B = Future retirement date value of accrued benefits as actuarially computed;
T = Time to retirement;
σ^2 = Variance of the return to the market portfolio per unit of time;
N = Cumulative standard normal distribution function; and
r = Risk free market value of interest.

The assumptions of the put option pricing model are:

- There are no penalties for short sales;
- Transaction costs and taxes are zero;
- The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus, over a finite time interval (such as over the life of an option contract), the distribution of stock prices is log normal;
- The options are European type and can be exercised only at maturity;
- The stock pays no dividends;
- The short-term interest rate is known and constant over the life of the option;
- The borrowing and lending rate of interest are the same; and
- The market operates continuously.

An intuitive explanation of the put option model as it applies to unfunded pension liabilities is as follows. The value of the put, UPL, is a function of the expectation of flows from a pension transaction. At the appropriate future retirement date, the corporate sponsor will put the securities on hand with a value of M to the retirees in partial satisfaction of the corporate pension liability payment B. Although early retirement or quitting before retirement may affect B, the going concern assumption requires that all pension liabilities will be realized fully. This assumption also insures that the financial statements will be presented conservatively. The probability that UPL will exist at retirement date is the probability, N(•), that the security's price, M, will be less than B in T time periods from the present. The present value of the expected value of the outflow, $Be^{-rT} \cdot N(\cdot)$, minus the expected value of the inflow, $M \cdot N(\cdot)$, is the value of the put. The variables in N(•) represent the mean of the distribution M/B, ln(M/B), and a location parameter, $(r + \sigma^2/2)T$ and $(r - \sigma^2/2)T$. The form of the normal operator, N(•), and the assurance that the put valuation only considers values below the put exercise prices results from the derivation found in Black and Scholes (1973) and/or Cox and Ross (1976).

The second assumption states that transaction costs and taxes are zero. Pension plans pay no taxes, but must pay transaction costs. Even if trading is frequent, however, the size of the transactions will minimize transaction costs. A small increase to the risk free discount rate will offset these costs.

The third assumption requires that stock prices follow a log-normal process. Cox and Ross (1976) and Merton (1976) develop pricing models for other processes as well. Fama (1976) provides the justification for the cumulative standard normal and the assumption of log-normality.[2]

The assumption of a European type option fits well with pension plans. As described above, the going concern assumption states that assets and liabilities of an organization will have a long life. Early retirements and possible sponsor terminations therefore will not affect the valuation of pension liabilities. In addition, even if a retiree does not elect to receive a lump sum distribution or is to receive supplemental benefits, the assumption necessary to evaluate pension liabilities is that the exercise price of the option is the present value of any future annuity the retiree may receive.

The assumption of no dividends is consistent with the Chicago Board Options Exchange (CBOE) policy. Dividends do not affect the exercise price of a call (or put) option. The dividends paid to the underlying securities of the pension fund are reinvested into the fund. Dividends are not paid to holders of claims against the fund.

The sixth assumption assumes the short-term interest rate is known and constant over the life of an option. The pension plan liability is of long duration. Brennan and Schwartz (1980) and King (1984) provide evidence that contingent claims models appropriately may evaluate contingent claims of long duration.

The unique contribution of the Black and Scholes model comes from its observation of stock and call option prices. Movements in option values reflect stock price movements. Buying either the stock or option long while shorting the other provides a hedge. The hedge is riskless when offsetting purchases and short sales are in correct proportions (and are adjusted continuously). The return to such a hedge must be riskless. It is this riskless return that provided the required discounting factor missing from the models that led to the development of the Black and Scholes model. The riskless return is presumed to be an observable variable.

For further discussion of the contingent claims models for pension costs, see Willinger (1985).

[2]If $1 + R$ is a return on an initial investment of \$1 for a specific time period and $1 + r_i$ is the return for k intermediate periods, then the $\ln(1 + R)$ with k successive intermediate periods is equal to $\ln(1 + r_1) + \ln(1 + r_2) \ldots \ln(1 + r_k)$. The central limit theorem states that as k becomes larger, the distribution of $1 + R$ will approximate a normal. Because $\ln(1 + R)$ is a rate of return with continuous compounding, the k subperiods become very large and the assumption of log-normality is appropriate.

Theoretical and Practical Advantages of the Contingent Claims Model

The contingent claims model for unfunded pension liabilities uses a riskless discount rate. Historically, Treasury bill and Treasury bond rates are surrogates for the available riskless rate of return. As each contingent claims is for a specific length of time, only a narrow range of interest rates should be appropriate. The narrow range of available riskless rates of return, as determined in competitive markets, helps provide a broad agreement of the appropriate discount rate. Although FAS No. 87 provides guidance for discount rate selection, a range of rate choices still exists. The agreement regarding an appropriate discount rate provides a distinct advantage over the judgment-based FAS No. 87 actuarial model discount rates.

The contingent claims model for unfunded pension liabilities is a conservative model. The model only considers the partial distribution of future pension fund values that are less than the future retirement benefits. The emphasis on down-side risk ensures employees, managers, and investors that all possible future risks are impounded in the pension liability. This results in a conservative estimate of the unfunded pension liability, a conservative estimate being one that does not overstate assets and provides for all liabilities. Theoretically, accounting information should possess this conservative characteristic.

This accountant's version of conservatism is subject to other interpretations. In FASB Statement No. 5, "Accounting for Contingencies" (1975), certain loss contingencies require an accrued liability on the balance sheet, however, while only gain contingencies with high probability may be disclosed as a footnote. An overfunded pension liability is a type of contingent gain—even if the contingent gain has a high probability of occurring, it would be disclosed as a footnote and not on the balance sheet. As such, the contingent claims unfunded pension liability model does not consider the overfunding for valuation purposes. In addition, accounting treatment of investments requires the use of lower of cost or market (FASB, 1975). Recognizing the excess of the pension plan investment portfolio relative to the pension liabilities violates this lower of cost or market criterion. Because the pension assets purchase price (cost) is matched to the present value of future expectations of the pension liabilities, zero overfunding occurs if expectations are realized. When the pension assets perform better than expectations, an overfunded position is a possibility. This overfunding, however, is relative to an unrealized gain in the pension portfolio. As such, it should not be recognized until realized which is a basic tenet of lower of cost or market and conservatism.

The contingent claims model also possesses construct validity that equals or exceeds the construct validity of the actuarial model. Accounting models that possess construct validity provide the qualitative characteristic of representational faithfulness (FASB, 1980). The contingent claims model for unfunded pension liabilities contains constructs that appropriately describe the unfunded pension liability. The model incorporates a distribution of possible values for an underlying benchmark pension fund portfolio, the variation in that distribution as a function of time, and an appropriate market-determined discount rate. These aspects of the unfunded plan liability are not evident in the actuarial model. The actuarial models assume a wide range of discount rates and produce a wide range of unfunded pension liabilities. Although actuarial models possess some construct validity, the contingent claims model for unfunded pension liabilities contains more explicit constructs that contribute directly to the valuation process.

Finally, the contingent claims model provides information that enhances both accounting disclosure characteristics of consistency and comparability. The underlying pension fund portfolio is modelled with the return distribution of a market portfolio. Based on the broad set of expectations represented in a market portfolio, the market portfolio fits the requirements of the prudent man rule of the Employees Retirement Income Security Act of 1974 (ERISA). The historical distribution of the returns to a market portfolio provides a consistent and comparable basis to evaluate the unfunded pension liability. The market portfolio becomes a benchmark on a year-to-year, firm-to-firm basis. Without a common benchmark, each unfunded pension liability is relative to the particular pension fund and the specific rate of return. To insure that all unfunded pension liabilities computed are comparable, the option pricing model uses the variance of the return distribution associated with a market portfolio. The actual pension plan portfolio used in conjunction with this variance allows the unfunded pension liabilities to reflect performance that is better or worse than a market portfolio. (Appendix A provides a detailed example of the implementation of the contingent claims option pricing model.)

The use of a market-based discount rate also provides consistency and comparability. Individual employee retirement liabilities with the same duration now can be discounted using the same market-determined discount rate. With actuarial models, a discount rate is applied to all employees, but that rate differs from firm to firm. In addition, the market-determined discount rate may change on a year-to-year basis, but the rate will change for all employees with similar time to retirement. The market-determined discount rate leads to information that is more consistent and comparable.

The Simulations

The Contingent Claims Model

The contingent claims unfunded pension liability model requires fixed values for the risk free market value of interest and the variance of the return to the market portfolio. These requirements come from the Black and Scholes contingent claims model that generally has been used (exceptions: King, 1984; Noreen and Wolfson, 1981) to value contingent claims of short duration (30, 60, or 90 days). The duration of unfunded pension liabilities is often ten years or longer. At any point in time over that long period, annual changes in the parameters or errors in estimating these parameters may occur. The range of values that can be specified annually for the model variables provides a further opportunity for manipulation. Manipulation may occur, as FAS No. 87 does not mandate a specific interest rate. In addition, annual specification of the variables will involve various combinations of values within the ranges of the variables. The sensitivity of the contingent claims model for unfunded pension liabilities to possible manipulation of variables or an annual range of values is evaluated with a set of simulations.

The contingent claims simulations model a range of results that a firm may confront over a series of years or even in one year. Because the parameters change yearly, the same historical liability computed with one set of parameters in the past may change currently to reflect a new set of circumstances. Even if a current change is reasonable, the range of possible results presents a manipulative opportunity. To provide a variety of parameters and results, the contingent claims model simulations draw from three distributions of separate variables of the unfunded pension liability model. These variables are the risk free market value of interest (r); the variance of the return to an assumed underlying market portfolio (σ^2); and the logarithm of the ratio of the current market value of the pension plan portfolio (M) to the retirement date value of accrued benefits (B) ($\ln(M/B)$). In addition, a value for the number of years until the pension is due (T), the current value of the pension plan portfolio (M), and the retirement date value of accrued benefits (B) must be specified. After specification of all parameters, an unfunded pension liability is computed. This process is repeated 1,000 times sampling from the hypothesized distributions for r, σ^2, and $\ln(M/B)$. The outcomes from this process approximate the distribution of results that one firm may confront in any one year for the unfunded pension liabilities.

Upon completion of the contingent claims model simulations, a similar set of simulations for actuarial models occurs. The model that produces outputs with the least variation possesses the desirable trait of reduced potential manipulation. A comparison of the variations in the output of both models provides evidence that the contingent claims model reduces the possible manipulation and variation of the unfunded pension liability.

The variable distributions, the specified parameters, and the possible correlations are the central features of the contingent claims model simulation. In addition, the comparable actuarial model simulations require model and parameter specifications. A development of these features follows.

The Risk Free Market Value of Interest (r)

Annual applications of the model for unfunded pension liabilities require annual specifications of the current assumed available risk free market value of interest. A surrogate for the assumed available risk free market value of interest is the Treasury bond rate. The appropriateness of this rate is discussed below. These estimates of the risk free rate will change on a year-to-year basis due to structural economic shifts and perturbations. Annual specification and possible shifts in the risk free market value of interest are motivations to provide a distribution of possible values for this variable.

The assumed available risk free market value of interest used in this research is the United State Treasury bond rate. The Treasury bond rate is used because it is a virtually riskless long-term obligation. To estimate the shape and the range of the distribution of Treasury bond rates, annual returns (yields to maturity) reported on a monthly basis for the years 1967 through 1986 were collected. Because pension liabilities are, in general, of long duration, a range of rates over a longer period of time is justified. To allow for a more contemporary rate structure, however, the range is held to a 20 year time frame. The logarithms of one plus these raw returns were computed and plotted in the form of a histogram. The range of these estimates of the logarithms of the risk free market value of interest is 4.3 percent to 13.2 percent. The mean is 7.8 percent, and the standard deviation is 2.3 percent.

Brennan and Schwartz (1980) examined contingent claims models for violation of the constant risk free market value of interest assumption. Their research used a ten year time horizon and a range of risk free market values of interest of 3.12 percent to 14.17 percent. King (1984) tested the Brennan and Schwartz model and found that it performed well on convertible debentures with maturities of two to 25 years. The simulation herein uses three different time horizons and a range of 4.25 percent to 13.25 percent. The random sample taken of the risk free market value of interest is accomplished through the use of the empirical distribution function of the Treasury bond rates.

Variance of the Return to a Market Portfolio (σ^2)

An estimate of the future variance is necessary for the put contingent claims model. The model considers the variation in the possible returns to a market portfolio to allow this model to make a risk adjustment in the value of the contingent claims. This estimated future variance is assumed stable over the period of the liability. Even if the variance of the return to the market portfolio does not shift over time, the model requires an *ex ante* estimate of the variance. This estimate is subject to error due to different estimating techniques and changes from year to year as new estimates are made. Therefore, a distribution for this sample variance is included in the simulation.

The variance estimate required for the put contingent claims model is the variance of a return to a market portfolio including dividends. An approximation to the distribution of the estimated variance to a market return including dividends is based on the data tapes from the Center for Research on Security Prices (CRSP) developed at the University of Chicago.

The daily returns to a market portfolio for the years 1963 through 1985 (5,778 returns) were converted to continuously compounded returns and summed on an annual basis. This provided 23 annual returns. The sample variance of these 23 returns was .06270. Because the returns are assumed to be log-normally distributed (Fama, 1976), the theoretical distribution of sample estimates of the annual variance is a chi-square distribution with 22 degrees of freedom. Using the relationship (Chou, 1975):

$$(3)\quad \hat{S}^2 = \frac{x^2_{(v)}\sigma^2}{v}$$

where:

$\hat{S}^2$ = Unbiased estimator of σ^2 (here equal to .06270);

$x^2_{(v)}$ = Chi-square variable with v degrees of freedom;

σ^2 = Population variance.

Fishman's algorithm for the generation of random chi-square variates and estimates of the population variance were computed for each simulation iteration (Kennedy and Gentle, 1980).

The index used in computing the estimate of the future variance is an index of stocks. Many pension funds contain investments in bonds. Tepper (1981) argues (assuming perfect markets and no clientele effect) that pension funding should be all debt for tax reasons. The variance estimate for a fund containing bonds, however, should be less than a variance estimate for a fund containing only stocks. Using a market stock index as a basis for the estimate of the variance provides an upper bound on the variance estimate. The upper bound for the estimate of the future variance translates into an upper bound for the unfunded pension liability. In general, the value of a put option is correlated positively with the magnitude of the variance of the underlying security. The upper bound for the unfunded pension liability provides a conservative estimate of the liability.

The Logarithm of the Current Value of the Pension Plan Portfolio to the Future Retirement Day Value of Accrued Benefits (ln(M/B))

The underlying security for this model of unfunded pension liabilities is the current market value of the pension plan portfolio of securities. At each evaluation date, the value of this portfolio will be different. The relationship between this current value for the pension fund and the future retirement day value of accrued benefits will vary as the market price level varies so that the ratio of the underlying portfolio to the required future value varies randomly. The distribution of the ratio is necessary to test the sensitivity of the unfunded pension liability model to the different levels of funding.

The ln(M/B) is modeled as log-normally distributed. The ln(M/B) is equivalent to the ln(M) – ln(B). Because the ln(B) is a fixed value (the exercise price in the original Black and Scholes model), the distribution of ln(M) drives the distribution of ln(M/B). Osborne (1959) presented evidence that stock price movements are similar to geometric Brownian motion. Geometric Brownian motion implies log-normality. Consistent with the Black–Scholes–Merton model, the simulations assume log-normality.

The distributions specified for the ln(M/B) have a range that is matched to the specific time to retirement (T). There are three separate simulations with one using a ten year, 20 year, and 30 year T. (See explanation below.) As shorter time spans, on average, will be funded in a greater amount than will longer time spans, three different ranges are appropriate. The ranges approximate the present value of a dollar at the highest and lowest value of the risk free interest range for the specific T. Although there is no direct relationship between M and B, theoretically the current market value of the pension plan portfolio will grow to meet the retirement day liabilities. M theoretically is the present value of the future retirement benefits. Because the ratio M/B is the ratio of the current value of the pension fund to the proposed future value, using the present value of $1 is appropriate. For example, for T of 20 years, the present value of $1 to be received 20 years from today at the lowest simulation interest rate of 4.25 percent is .43. The present value of $1 to be received in 20 years at the highest rate of 13.25 percent is .08. Hence, the range for M/B for the 20 year T simulation is .08 to .43. The ranges of M/B for the other two simulations are .29 to .66 for the ten year T and .02 to .29 for the 30 year T.

The mean assumed for each distribution is the midpoint of the range, .475, .255, and .155. At higher range values most circumstances will result in overfunding, while values at and below the mean generally will result in underfunding. This is confirmed subsequently in the simulation using actuarial models. The variances assumed for each distribution in terms of the M/B ratio are .0021, .00191, and .001139. The variances are an attempt to include, per Chebychevv's inequality, 93.75 percent of all possible ratio drawings in the specified ranges. To meet this criterion, eight standard deviations must subsume the ranges. For example, in the 20 year T simulation, the difference between the highest M/B (.43) and the lowest M/B (.08) is .35. This value divided by eight equals .04375. The .04375 equals the standard deviation, and the square of .04375, .00191406, equals the variance of M/B for the 20 T year simulation. The shape of the distributions specified in the simulations is log normal. Appendix B contains a derivation of the parameters for the ten year T log-normal distribution. The 20 year and 30 year simulations contain log-normal distribution parameters computed in the same manner.

Time to Retirement Date (T), Current Value of the Pension Plan Portfolio of Securities (M), and Future Retirement Day Value of Accrued Benefits (B)

Pension liabilities may vary in duration. Individuals at various stages of their work life may not collect pension benefits for up to 40 years. In an attempt to simulate a variety of work life situations, the time to retirement date (T) is fixed at ten, 20, and 30 years in three separate simulations.

Values for ln(M/B) are drawn from a normal distribution. Using each drawing as a new combination of M and B while stating either M or B *ex ante* allows computation of the remaining value. For this research it is assumed that B is fixed at $100,000.[3] If a random drawing from the distribution of ln(M/B)

[3]The simulation does not consider explicitly a range of retirement benefits (B). The choice of the ratio, M/B, however, implicitly considers a range of B. Because the simulation chooses a ratio, it is the relationship of M to B that is important. If, after choosing a ratio, a B also is chosen, M is deterministic Providing a fixed B in this research allows a simpler computation and a comparable set of results without losing the generality of the variable B. In addition,

equals –2.0, for example, the antilog of –2.0 equals .1353353. This means the raw ratio, M/B, equals .1353353. Because B equals 100,000, substituting and solving for M provides the answer $13,533.53. For this drawing and all drawings from the distribution of the ln(M/B), the values for M are computed in the manner above.

The Correlations

There are three combinations of variables that may display correlations: the logarithm of the ratio of the current market value of the pension plan portfolio to the retirement date accrued benefits (ln(M/B)) and the risk free market value of interest (r), the current market value of the pension fund portfolio (M), and the variance of the return to a market portfolio (σ^2), and the risk free market value of interest (r) and the variance of the return to a market portfolio (σ^2).

Conventional financial theory states that common stocks are a hedge against inflation. As such, changes in nominal interest rates due to expected or unexpected inflation should be correlated positively with common stock returns. Brody (1976), Jaffe and Mandelker (1976), Nelson (1976), and Fama and Schwert (1977), however, present empirical evidence that negative correlation exists between common stock returns and the expected and unexpected inflation components of interest rates. In an extended investigation of this paradox, Fama (1981) analytically and empirically described the relationship between real activity (i.e., capital expenditures) and inflation. Fama found that positive correlation between stock returns and real activity and negative correlation between inflation and real activity resulted in a spurious negative correlation between stock returns and inflation. Given this spurious relationship, the simulation will not correlate the ln(M/B) and r.

The current market value of the pension fund portfolio (M) and return volatility (σ^2) are seemingly unrelated items. Theoretically, the CAPM implies that the price level of a security should not affect the variance of the return to that security. Some empirical investigations, however, suggest a relationship may exist under special circumstances. Reilly and Drzycimski (1978), Reilly and Gustavson (1985), and Ohlson and Penman (1985) provide evidence of negative correlation between price volatility and price level. These works investigate variance changes in periods before and after splits. Although the investigations found negative correlation, these empirical aberrations relate only to stock splits. For this reason, the simulation did not correlate M and σ^2

A possible correlation between the risk free market value of interest (r) and the variance of the return to the market portfolio (σ^2) results from the Capital

pension liabilities of previous years are fixed values that change only with plan amendments. In the context of pension liabilities, a fixed value for B is reasonable.

Market Line (CML) (1972). As the risk free rate rises or falls due to expected or unexpected inflation, the CML will pivot on the efficient frontier. This pivoting will change the market portfolio and the market variance. The inflation that results in a change of the risk free market value of interest also will result, however, in a rise of the efficient frontier. These offsetting changes may produce an insignificant correlation. As in the previous pairs of variables, the simulation did not correlate r and σ^2.

The Actuarial Model Simulation

To gain insights into the range of unfunded pension liabilities that actuarial assumptions produce, two actuarial model simulations are developed. The simulations use distributions of the highest and lowest set of interest rates appropriate under FAS No. 87. The set of highest rates is the AAA corporate bond rates. These rates are appropriate as "high quality fixed income investments" required by FAS No. 87. The lowest set of rates is the distribution of Treasury bond rates. Treasury bonds represent a virtually riskless investment, thereby making Treasury bonds the highest quality fixed income investment. The settlement rates published by the PBGC and required by FAS No. 87 had a 1986 range of 6.75 percent to 11.00 percent, with a mean of 8.65 percent. In general, these rates fall between the AAA rates and the Treasury bond rates. Correlations between the PBGC rates and the AAA and the Treasury bond rates are high and significant. Because the PBGC settlement rates fall somewhere between the AAA corporate bond rates and the Treasury bond rates, these two simulations provide the boundaries for and subsume all the appropriate rates under FAS No. 87.

The collection of the data for both distributions is for years 1967 to 1986. The range of AAA corporate bond rates is 5.03 percent to 15.49 percent, with a mean of 9.31 percent and a standard deviation of 2.538 percent. The range of the Treasury bond rates is 4.4 percent to 14.14 percent, with a mean of 8.136 percent and a standard deviation of 2.559 percent. Although for any one year the range of rates is a subset of the ranges outlined above, the purpose of the simulations is to describe the entire set of possible outcomes that may occur over the pension liability life. For that reason, the simulations use a 20 year range of rates.

Using the empirical distribution function, 1,000 random drawings of risky returns develop the sample. Each drawing of risky return produces an unfunded pension liability value through the use of the following computation:

$$(4)\ UPL = \frac{\$100{,}000}{(1 + RR)^{T}} - M$$

where:

UPL = The unfunded pension liability related to the unfunded accumulated benefit obligation of the unfunded projected benefit obligation;
RR = The Treasury bond or AAA corporate bond rate;
M = The current value of the pension plan portfolio; and
T = Time to retirement.

As in the contingent claims model simulation, the future retirement benefit is $100,000 to be received ten, 20, or 30 years from the present evaluation date. The antilog of the ln(M/B) multiplied by $100,000 produces the value of M. The same values of ln(M/B) used in the contingent claims model simulation are used in the actuarial model simulation. Negative output values from this simulation represent overfunding.

The results of these simulations are compared to the contingent claims model simulation results. Corporations faced with a wide range of possible outputs may manipulate the unfunded pension liability. The model that minimizes those possible manipulations may be more desirable.

The Results and Analysis

Table 1 presents the results of the nine simulations. Each time frame (ten, 20, 30) is presented separately. Complete and 90 percent ranges permit comparisons of the breadth of the distributions.

When computing an unfunded pension liability, an actuary uses a rate of return that encompasses any risk and the long-term earning possibilities. For the largest contingent claims model unfunded pension liabilities of $34,396 (ten year, pension plan portfolio of 41,090), $29,122 (20 year, pension plan portfolio of 19, 076), and $21,065 (30 year, pension plan portfolio of 10,690), the equivalent actuarial interest assumptions would be 2.85 percent, 3.72 percent, and 3.90 percent, respectively. These actuarial rates of return are less than the available riskless rate of return of 4.79 percent and 4.29 percent used in the observation that generated the largest unfunded liability. For the smallest values of $1,594 (ten year, pension plan portfolio of 47,846), $262 (20 year, pension plan portfolio of 25,666), and $32 (30 year, pension plan portfolio of 15,551), the actuarial interest assumptions are 7.30 percent, 6.98 percent, and 6.39 percent, respectively. The actuarial rate of return is much less than the available riskless rate of return of 12.41 percent that generated the smallest unfunded liability.

The above comparison points out that when an alternative model uses a more comprehensive set of input variables, an appropriate discount rate may be less than an anticipated actuarial rate of return. The range of the rates of return used currently, however, is greater than the simulation suggests is prudent. Surveys of post–FAS No. 87 corporate discount rates provide evidence that the range of discount rates used is 6.00 percent to 11.25 percent ("News, Pension Discount Rates," 1989; Stone and Ingram, 1988). Given the simulation situation, the required range of rates of return under actuarial assumptions is smaller than the range corporations currently use. For example, the ten year range of 2.85 percent to 7.30 percent, the 20 year range of 3.72 percent to 6.98 percent, and the 30 year range of 3.90 to 6.39 percent are all less than the range of survey rates of 6.00 percent to 11.25 percent.

The contingent claims model simulations produced 90 percent of its total values in a $22,000, $20,000, and $14,000 range. The Treasury bond rate simulations using the actuarial model produced 90 percent ranges of $36,000, $33,000, and $24,000. Ranges for the AAA corporate bond rate simulations were similar. The actuarial assumptions may allow a range more than 1.5 times as large as the contingent claims model. In addition, the range allows overfunding and underfunding. A limited range of values may reduce the potential for manipulation of the unfunded pension liability while raising the statistical confidence in the reported value.

The contingent claims model provides little possibility of an overfunding situation. In the actuarial model simulations, an average of 628 values indicated overfunding. The contingent claims model in these cases considers the probability of the pension fund not meeting return expectations and thereby gives a more conservative estimate of the unfunded pension liability (conservative from an accounting perspective being a larger liability or, in some cases, a liability rather than an asset).

Although T varies in these simulations, its effect is minimal. In all three cases of T, all three models perform in a relative manner. Time to retirement, therefore, seems not to affect the various model possibilities.

The highest and lowest output values of contingent claims model simulation seem to indicate that the interest rate is the most important factor. The interest rate for the lowest value is 12.41 percent, while the interest rates of 4.79 percent and 4.29 percent produced the largest value. These values are near the extremes of the empirical distribution used in the simulation. The highest and lowest values appear to be related strongly to the interest rate and less affected by the other variables.

Limitations and Extensions

This research uses simulation to test the properties of a contingent claims model. Simulation, however, provides little, if any, external validity. Empirical testing using additional information from existing, terminated, or bankrupt corporations will provide this external validity.

The most important of the contingent claims model assumptions is the stability of the parameters. The instability of the parameters and the possibility of stochastic variables may lessen the value of this contingent claims model. A valuable extension to this research would develop a model that included the alternative specification of variables and stochastic variables over long durations.

Although the arguments presented led to the use of uncorrelated model variables, the nonexistence of correlation is not shown. Consideration of the existence of correlation and its effect on the simulations would provide a valuable extension to this study.

The model development in this research provides construct validity related to the prudent man requirement of ERISA. As the definition of the prudent man rule changes, however, so will the constructs of this model. The choice of a benchmark of performance depends on this definition and may require an adjustment.

The contingent claims model for unfunded pension liabilities provides information for reporting purposes. The model, however, does not make a normative statement regarding funding of pension plans. Further research is needed to determine an acceptable level of unfunded pension liabilities.

The simulations made comparisons of individual unfunded pension liabilities. Group comparisons with varying profiles may enhance the analysis. An extension to this paper could include group comparisons created with aggregates of individual contingent claims model liabilities.

In order to compute an unfunded pension liability, the model must specify a retirement day benefit claim (B) and time to retirement (T). Although the simulations used varying times to retirement, the model is unable to deal with a stochastic B or T. As the B and T may vary from individual to individual and within each individual profile, the model may be limited in its applicability.

Finally, this research focuses on the possible manipulations in the discounting process. Other variables, however, may be negotiable between the corporation and the actuary. Some of these are the estimates of future employees earnings, cost of living increases, and the forces of decrement. It may be possible to negotiate these factors and offset the reduced manipulation in the discounting process. An extension to this research would allow the other negotiable factors to vary or provide a model that would control these possible manipulations.

Summary and Conclusion

The unfunded pension liability assumes fixed input parameters. Each year the model is used, however, a range of possible input variable values may exist. The simulation allowed a range of values to permit various combinations of variable values and the possibility of manipulation. These results were compared to actuarial model simulations. The results provide evidence that even with the possibility of a range of values and manipulation, the contingent claims model provides a more restrictive manipulative result than actuarial models. In addition, the theoretical and practical considerations of the contingent claims model may provide the FASB with sufficient support to adopt the model as an appropriate mandated model.

References

1. Baker, Thomas, "Reading the Tea Leaves," *Forbes* (June 22, 1981), pp. 76-78.

2. Black, F., and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* (May/June 1973), pp. 637-654.

3. Brennan, M., and E. Schwartz, "Analyzing Convertible Bonds," *Journal of Financial and Quantitative Analysis* (November 1980), pp. 907-929.

4. Brody, Z., "Common Stocks as a Hedge Against Inflation," *Journal of Finance* (May 1976), pp. 459-470.

5. Chou, Y., *Statistical Analysis* (Holt, Rinehart and Winston: New York, 1975).

6. Cox, J.C., and S.H. Ross, "The Valuation of Options for Alternative Stochastic Processes," *Journal of Financial Economics* (May 1976), pp. 145-166.

7. Daley, L.S., "The Valuation of Reported Pension Measures for Firms Sponsoring Defined Benefit Plans," *The Accounting Review* (April 1984), pp. 177-198.

8. Ezra, D.D., "How Actuaries Determine the Unfunded Pension Liability," *Financial Analysts Journal* (July/August 1980), pp. 43-50.

9. Fama, E.F., "Stock Returns, Real Activity, Inflation, and Money," *American Economic Review* (September 1981), pp. 545-565.

10. __________, *Foundations of Finance* (Basic Books: new York, 1976).

11. Fama, E.F., and G.W. Schwert, "Asset Returns and Inflation," *Journal of Financial Economics* (November 1977), pp. 115-146.

12. Feldstein, M., and S. Seligman, "Pension Funding Share Price and National Saving," *Journal of Finance* (September 1981), pp. 801-824.

13. Financial Accounting Standards Board, *Accounting for Contingencies, Financial Accounting Statement No. 5* (Financial Accounting Standards Board: Connecticut, 1975).

14. __________, *Accounting for Certain Marketable Securities, Financial Accounting Statement No. 12* (Financial Accounting Standards Board: Connecticut, 1975).

15. __________, "Qualitative Characteristics of Accounting Information", *Financial Accounting Statement Board Concepts No. 2* (Financial Accounting Standards Board: Connecticut, 1980).

16. __________, *Accounting and Reporting by Defined Benefit Pension Plans, Financial Accounting Statement No. 35* (Financial Accounting Standards Board: Connecticut, 1980).

17.. __________, *Employer's Accounting for Pensions, Financial Accounting Statement No. 87* (Financial Accounting Standards Board: Connecticut, 1985).

18. __________, *Employer's Accounting for Settlements and Curtailments of Defined Benefit Pension Plans and for Terminating Benefits, Financial Accounting Statement No. 88* (Financial Accounting Standards Board: Connecticut, 1985).

19. Francis, J.C., *Investments: Analysis and Management* (McGraw-Hill: New York, 1972).

20. Hines, W.W., and D.C. Montgomery, *Probability and Statistics* (Ronald: New York, 1972).

21. Jaffe, J., and G. Mandelker, "The 'Fisher Effect" for Risky Assets: An Empirical Investigation," *Journal of Finance* (May 1976), pp. 447-458.

22. Kennedy, W.J., and J.E. Gentle, *Statistical Computing* (Marcel Dekker, Inc.: New York, 1980).

23. King, R.D., "The Effect of Convertible Bond Equity Values on Dilution and Leverage," *The Accounting Review* (July 1984), pp. 419-431.

24. Kingsland, L., "Projecting the Financial Condition of a Pension Plan Using Simulation Analysis," *Journal of Finance* (May 1982), pp. 577-584.

25. Lintner, J., "The Valuation of Risk Assets and Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics* (February 1965), pp. 13-37.

26. Merton, R.C., "Theory of Rational Option Pricing," *Bell Journal of Economics* (Spring 1973), pp. 141-183.

27. __________ , "Option Pricing When Underlying Stock Returns are Discontinuous," *Journal of Financial Economics* (May 1976), pp. 125-144.

28. Mossin, J., "Equilibrium in a Capital Asset Market," *Econometrica* (October 1966), pp. 768-783.

29. Nelson, R.C., "Inflation and Rates of Return on Common Stocks," *Journal of Finance* (May 1976), pp. 471-483.

30. "News, Pension Discount Rates," *Journal of Accountancy* (January 1989), p. 94.

31. Noreen, E., and M. Wolfson, "Equilibrium Warrant Pricing Models and Accounting for Executive Stock Options," *Journal of Accounting Research* (Autumn 1981), pp. 384-398.

32. Ohlson, J.S. and S.H. Penman, "Volatility Increases Subsequent to Stock Splits: An Empirical Aberration," *Journal of Financial Economics* (June 1985), pp. 251-266.

33. Oldfield, G., "Financial Aspects of the Private Pension System," *Journal of Money, Credit and Banking* (March 1976), pp. 48-55.

34. Osborne, M.F.W., "Brownian Motion in the Stock Market," *Operations Research* (March/April 1959), pp. 145-173.

35. Reilly, F.K., and E.F. Drzycimski, "Price Volatility and Price Level," working paper #505, College of Commerce and Business Administration, University of Illinois at Urbana-Champaign, 1978.

36. Reilly, F.K., and S.G. Gustavson, "Investing in Options on Stocks Announcing Splits," *The Financial Review* (May 1985), pp. 121-142.

37. "Slick Accounting Plays Help Many Companies Improve Their Income," *Wall Street Journal* (June 20, 1980), pp. 1-34.

38. Sharpe, W., "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance* (September 1964), pp. 425-442.

39. Stone, M., and R.W. Ingram, "The Effect of Statement No. 87 on the Financial Reports of Early Adopters," *Accounting Horizons* (September 1988), pp. 48-61.

40. Tepper, I., "Taxation and Corporate Pension Policy," *Journal of Finance* (March 1981), pp. 1-13.

41. Willinger, G.L., "A Contingent Claims Model for Pension Costs," *Journal of Accounting Research* (Spring 1985), pp. 351-359.

42. Winklevoss, H., "PLASM: Pension Liability and Asset Simulation Model," *Journal of Finance* (May 1982), pp. 585-594.

Table 1
Descriptive Statistics for Simulation Results

Simulation	Mean*	Standard Deviation	Range* High	Range* Low	90 Percent Range** High	90 Percent Range** Low
T = Ten Year						
Contingent Claim	$15,510	6,975	34,396 (.04796)	1,594 (.1241)	26,000	4,000
Actuarial-Treasury	-333	11,372	26,764 (.04302)	-32,339 (.13509)	16,000	-20,000
Actuarial-AAA	-5,121	10,383	23,762 (.04791)	-35,618 (.149227)	11,000	-22,000
T = 20 Year						
Contingent Claim	10,313	6,480	29,122 (.042995)	262 (.1241)	21,000	1,000
Actuarial-Treasury	-2,210	10,403	25,243 (.04302)	-30,801 (.13509)	14,000	-19,000
Actuarial-AAA	-6,705	8,932	21,393 (.04791)	-32,541 (.149227)	8,000	-21,000
T = 30 Year						
Contingent Claim	6,027	4,663	21,065 (.042995)	32 (.1241)	14,000	114
Actuarial-Treasury	-3,568	7,495	18,451 (.04302)	-23,926 (.13509)	9,000	-15,000
Actuarial-AAA	-6,839	6,156	14,748 (.04791)	-24,619 (.149227)	4,000	-16,000

* Negative numbers denote overfunding
** Approximately 90 percent of all values are contained in the stated range
Values in parentheses are interest rates producing these values

Appendix A

To aid understanding of the contingent claims model, an example is developed below. The example describes the practical application of the model and the computation of unfunded pension liabilities for an individual in a corporation. The model is applied to a one person firm for one year. It is assumed that an employee's pension fund can be distinguished from the totality of the fund. If individual records are not kept, the model is applied only once to the entire employee population. The specifics used in this application are as follows:

- The available riskless rate of return is approximated by the Treasury bond rate. The 20 year rate = .08
- The estimated future variance of the returns to a market portfolio including dividends, as approximated by the historical returns to a market portfolio available on a variety of data tapes, is .06270. (See the simulation section for details on this computation.)
- Accrued expected benefits earned for individual 1 at current age 45 and retirement age 65 is 3000
- Total fund currently on hand for individual 1 at current age 45 and retirement age 65 is 300
- The computation of the unfunded pension liability for individual 1 at the end of year 1 is:

$$P = -300 \cdot N\left[\frac{-\ln\frac{300}{3000} - [.08 + \frac{.06270}{2}]\,20}{.25\sqrt{20}}\right]$$

$$+\ 3000\,e^{-.08(20)} \cdot N\left[\frac{-\ln\frac{300}{3000} - [.08 + \frac{.06270}{2}]\,20}{.25\sqrt{20}}\right]$$

The solution to the above equation is p = \$376. This is the unfunded pension liability at this date for individual 1. This liability is relative to the return to a market portfolio and funding choice. It may be due to prior years' underinvestment, disturbances in the market return, or both. Unless there is gross overfunding, the contingent claims model unfunded pension liability never will be less than or equal to zero. The assumed market distribution for the returns to the pension plan assets contain the possibility of future losses as well as gains. The output of this model provides a conservative point of view regarding possible future losses.

Appendix B

The actual simulation will draw from a normal distribution comprised of the ln(M/B). To convert the mean and variance of the distribution of M/B to the mean and variance of a log-normal distribution of the ln(M/B), the following derivation is employed (Hines and Montgomery, 1972):

(1) $E(X) = e^{\mu + 1/2\,\sigma^2}$

(2) $V(X) = e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2}$

where:

$E(X)$ = Mean of M/B;
$V(X)$ = Variance of M/B;
μ = Mean of ln(M/B); and
σ^2 = Variance of ln(M/B).

Solving for μ and σ^2 cannot be accomplished analytically, so a numerical process is used. First take the natural log of both sides of equation (1). This provides the results:

(3) $\ln E(X) = \mu + 1/2\,\sigma^2.$

Solving equation (3) for μ:

(4) $\mu = \ln E(X) - 1/2\,\sigma^2.$

Substituting equation (4) into equation (2):

(5) $V(X) = e^{2(\ln E(x) - 1/2\,\sigma^2) + 2\sigma^2}$

$- e^{2(\ln E(X) - 1/2\,\sigma^2) + \sigma^2}$.

Using the values of E(X) = .475 and V(X) = .00213906 in equation (5):

$$(6)\ .00213906 = e^{2(-.74444047 - 1/2\,\sigma^2) + 2\sigma^2}$$

$$- e^{2(-.74444047 - 1/2\,\sigma^2) + \sigma^2}.$$

Reducing equation (6):

$$(7)\ .00213906 = e^{-1.4888809 + \sigma^2} - e^{-1.4888809}.$$

Solving equation (7) for σ^2:

$$(8)\ .22776407 = e^{-1.4888809 + \sigma^2}$$

$$\sigma^2 = .00943594.$$

Using $E(X) = .475$ and $\sigma^2 = .00943594$ in equation (1) and solving for μ:

$$.475 = e^{\mu + 1/2\,(.00943594)}$$

$$(9)\quad -.74444047 = \mu + .0047177$$

$$-.74915817 = \mu.$$

The parameters for the log-normal distribution of ln(M/B) for the ten year simulation are :

$\mu = -.74915817$ and $\sigma^2 = .00943594.$