Cost of Equity Capital Redefined

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Abstract

The Modigliani-Miller (MM) theory of corporate finance has been subject to considerable debate and interest for over 30 years. Today it is the dominant theory in the field. This paper highlights an intuitively unappealing implication of the MM model that has remained unnoticed or at least has not received due attention. An alternative definition of the cost of equity is presented that does not have this drawback.

Introduction

Modigliani & Miller (hereafter MM) revolutionized corporate finance. The ideas presented in their major articles (1958, 1963, 1966) have become central to the capital structure and cost of capital theories.

This paper highlights a troublesome property implicit in the MM theory and proposes a new approach that offers a satisfactory solution to this trouble spot.

The basic assumptions of the original MM papers (perfect capital markets, rational investor behavior, no tax differentials, and the implicit assumption of no bankruptcy costs) are retained.

The Modigliani-Miller Theory

According to MM (1963), the value of a levered firm (V_L) with a permanent level of debt (D) in its capital structure is given by:

$$(1)\quad V_L = V_U + \tau D = \frac{(1-\tau)E(X)}{\rho} + \frac{\tau R}{r}$$

where:

V_U = The value of an unlevered firm;
τ = The corporate tax rate;

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$E(X)$ = The expected level of average annual earnings generated by the assets of the firm;
$1/\rho$ = The market capitalization rate for an unlevered firm in the firm's risk class;
r = The rate of interest, assumed to be constant and independent of the size of debt; and
R = The size of the interest bill = rD.

Further, the value of a firm naturally must be equal to the sum of the values of equity (S) and debt (D), so that:

(2) $V_L = S + D.$

The line of reasoning leading to the central formula of the MM theory is broadly as follows. The after tax return (earnings after interest and taxes, plus interest), denoted by the random variable X^τ, can be expressed as:

(3) $X^\tau = (1 - \tau)(X - R) + R = (1 - \tau)X + \tau R.$

MM (1963) argue that from the investor's point of view, the long-run average stream of after tax returns appears as a sum of two components: an uncertain stream, $(1 - \tau)X$, and a sure stream, τR. This suggests that the equilibrium market value of the combined stream can be found by capitalizing each component separately.

The Inconsistency Implicit in the MM Model

The cost of equity capital, i*, is defined as the rate of return required on a firm's equity by the market. In the MM framework, i* can be derived as follows. Utilizing (3):

(4) $E(X^\tau) - \tau R = (1 - \tau)E(X).$

Therefore, equation (1) can be expressed equivalently as:

$$(5)\quad V_L = \frac{E(X^\tau) - \tau R}{\rho} + \tau D = \frac{E(X^\tau) - \tau r D + \rho\tau D}{\rho}$$

$$= \frac{E(X^\tau) + \tau(\rho - r)D}{\rho}.$$

X^{τ} also can be considered to consist of the following two streams (see equation (3)):

- The net profit after interest and taxes accruing to common shareholders. Deducting from earnings before interest and taxes, (EBIT), X, the amounts of taxes, $(X\text{-}R)\tau$, and interest charges, R, yields the net profit, π, which belongs to the shareholders:

  (6) $\pi = X - (X - R)\tau - R = (1 - \tau)(X - R).$

  The expected size of the annual profit stream will be:

  (7) $E(\pi) = (1 - \tau)[E(X) - R].$

- The second part of X^{τ} is the amount of interest charges, $R = rD$.

The value of the firm now can be expressed as:

$$(8)\quad V_L = \frac{E(\pi) + rD + \tau(\rho - r)D}{\rho},$$

and the value of equity is:

$$(9)\quad S = V_L - D = \frac{E(\pi) + rD + \tau(\rho - r)D - \rho D}{\rho}$$

$$= \frac{E(\pi) - (1 - \tau)(\rho - r)D}{\rho}$$

whereby:

$$(10)\quad \rho = \frac{E(\pi)}{S} - \frac{(1 - \tau)(\rho - r)D}{S}.$$

As growth is excluded from the model, the expected rate of return to the shareholders (the cost of equity), i*, is obtained directly by dividing the expected net profit by the market value of equity (e.g., Hamada, 1969; Rubinstein, 1973, and Copeland and Weston, 1988). Consequently, rearranging equation (10) yields the MM (1963) expression of i*:

$$(11)\quad i^* = \frac{E(\pi)}{S} = \rho + (1 - \tau)(\rho - r)\frac{D}{S}.$$

One can argue, however, that it is basically the required rate of return, i*, that determines S in the market, not vice versa. Thus, a more specific expression for

i^* is needed than is provided by equation (11) above. Although not given by MM, this can be accomplished in the MM framework as follows.

Based on equations (1) and (2), the value of equity, S, can be written as:

$$(12)\quad S = V_L - D = \frac{(1-\tau)E(X)}{\rho} + \frac{\tau R}{r} - \frac{R}{r}$$

$$= \frac{(1-\tau)E(X)}{\rho} - \frac{(1-\tau)R}{r}.$$

By noting that the expected net profit to the shareholders is $E(\pi) = (1 - \tau)(E(X) - R)$, i^* can be expressed as:

$$(13)\quad i^* = \frac{E(\pi)}{S} = \frac{(1-\tau)(E(X)-R)}{\frac{(1-\tau)E(X)}{\rho} - \frac{(1-\tau)R}{r}} = \frac{E(X)-R}{\frac{E(X)}{\rho} - \frac{R}{r}}.$$

To illustrate, assume:

$E(X) = 1000$
$r = .05$
$p = .10$
$\tau = 0$ or, alternatively, $\tau = .50$.

Table 1 shows how increasing leverage affects key valuation variables in the MM framework. Figures for the following variables are tabulated:

- Expected annual earnings, $E(X)$;
- The assumed amounts of interest charges, R;
- Required rate of return on equity i^*, obtained from equation (13); [1]
- The value of the whole firm, V (from equation (1));
- The amount of debt, D (= R/r);
- The value of equity, S (given, e.g., by $S = E(\pi)/i^*$, by equation (9), or, simply, $S = V - D$);
- Net profits to common shareholders after interest and taxes, $E(\pi) = (1 - \tau)(E(X) - R)$.

[1] It is impossible to calculate i^* directly from the MM equation (11), as equation (11) requires S to be known. Only after i^* is calculated from equation (13) can S be determined. (After having obtained the value of S (e.g., by $S = E(\pi)/i^*$), MM's equation (11) can be applied, but only to confirm that it yields the same solution as equation (13).)

The tax rate is assumed to be zero in Table 1. Table 2 is based on otherwise identical assumptions except that the tax rate is 50 percent ($\tau = .50$).

Table 1 shows that in the absence of taxes, the value of the firm (V) does not depend on leverage. With taxes, the value of the firm increases due to the tax saving induced by leverage. Introducing taxation does not cause differences in the rates of return for stockholders, if leverage is measured in terms of total earnings and interest payments. (Note that if leverage is measured in terms of market values (for instance by D/S), the i*- values would differ.) And while the tax shield increases the value of the firm with taxation, it does not affect the rate of return on common stock, i*.

The troublesome aspect of the two tables is that the value of equity becomes zero in both cases when the interest bill (R) is only half of the total earnings. The MM theory causes the value of equity to become worthless too soon.[2] (The figures for the example are taken from MM (1958, p. 271, footnote 12), where they restricted the illustration to a single case in which R = 200, implying i* = .133.) If the figures in Table 2 are in millions of dollars, the shares of this firm, which is expected to earn $250,000,000 annually, are worthless.

Note that a shareholder has limited liability. In the worst possible case of bankruptcy, the shareholder will receive nothing. The shareholder, however, does not have to pay any of the firm's losses or costs associated with bankruptcy.

The reason for the inconsistency in the MM model becomes evident if equation (13) is examined. The equity becomes zero (and i* infinite) when:

$$\frac{E(X)}{\rho} - \frac{R}{r} = 0$$

or

$$\frac{E(X)}{\rho} = \frac{R}{r}. \tag{14}$$

Because ρ is is expected to exceed the risk free rate (r), both sides of equation (14) become equal before E(X) = R. How much before depends on the relation of ρ to r.

Thus, one can argue that the model does not give realistic solutions. (Even the well-known extreme corner solution, which suggests that a firm should have nearly 100 percent debt, occurs when the firm may have normal levels of R/E(X).)

[2] It is important to note that the MM theory and the CAPM are shown to be equivalent (e.g., by Hamada, 1969). Hence, the criticisms presented here concern equally the traditional formulation of the CAPM.

The remaining part of this paper is devoted to developing a theoretically justifiable model that does not have the undesirable property of making a stream of positive expected cash flows to the shareholder worthless.

The Proposed New Approach
The Model for the Valuation of the Firm

Consider the expected earnings after taxes, X^{τ}, which is the sum $E(\pi) + R$. This can be expressed, as already noted in equation (3), as:

$$(15)\quad E(\pi) + R = (1 - \tau)E(X) + \tau R = (1 - \tau)(E(X) - R) + R.$$

To obtain the value of debt capital, the amount of interest charges (R) is discounted at the market-determined rate of interest for the firm, r_r. Hence, $D = R/r_r$ (i.e., perpetual risky debt is assumed.[3] The amount of interest payments is assumed to be fixed, however, so that R can be considered a constant). The remaining part of the earnings stream after taxes belongs to the shareholders and should be discounted at the appropriate market determined rate, i*. The formula for the value of the firm becomes:

$$(16)\quad V_L = S + D = \frac{(1 - \tau)(E(X) - R)}{i^*} + \frac{R}{r_r},$$

but this is exactly the same as the formula under the traditional view strongly criticized by MM (1958). But the traditionalists assumed that i* is practically constant, at least up to some conventional level of leverage, then rises rapidly. In the opinion of the author, there is nothing wrong in equation (16) as such. Instead, the key to valuation lies in the capitalization rate, i.e., how the market-required rate of return, i*, is determined by the investors in common stock. (Besides, the MM model also can be expressed so that it is equivalent in appearance to equation (16).) The differences between the traditional view, the MM theory, and the approach to be presented below are only due to differences in i*.

The Basic Definition of Risk

The starting point of the proposed new approach is based on the treatment of risk by MM (1963). They showed how the distribution of after tax earnings is affected by leverage. Their analysis proceeds basically as follows:

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[3] Assuming risk free debt does not alter the structure of the model for the cost of equity that will be developed below.

X is the (long-run average) earnings before interest and taxes generated by the currently owned assets of a given firm in some stated risk class, k. From the definition of risk class,[4] it follows that X can be expressed in the form E(X)Z, where E(X) is the expected value of X and the random variable Z = X/E(X), having the same value for all firms in class k, is a drawing from a distribution, say $f_k(Z)$. Hence, the random variable X^τ, measuring the after tax return, can be expressed as:

$$(17)\quad X^\tau = (1-\tau)(X-R) + R = (1-\tau)X + \tau R$$

$$= (1-\tau)E(X)Z + \tau R.$$

Because

$$(18)\quad E(X^\tau) = (1-\tau)E(X) + \tau R,$$

$E(x^\tau) - \tau R$ can be substituted for $(1-\tau)E(X)$ in equation (17) to obtain:

$$(19)\quad X^\tau = (E(X^\tau) - \tau R)\,Z + \tau R = E(X^\tau)\left(1 - \frac{\tau R}{E(X^\tau)}\right)Z + \tau R.$$

Thus, if $\tau > 0$, the shape of the distribution of X^τ will depend not only on the scale of the stream $E(X^\tau)$ and on the distribution of Z, but also on the tax rate and the degree of leverage (one measure of which is $R/E(X^\tau)$). For example, if $Var(Z) = \sigma^2$:

$$(20)\quad Var(X^\tau) = \sigma^2\,(E(X^\tau))^2\left(1 - \frac{\tau R}{E(X^\tau)}\right)^2.$$

The reason for repeating the MM demonstration is that implicit in it is an extremely useful definition of risk. Consider the case of an unlevered firm. The variance of its after tax earnings is:

$$(21)\quad Var(X^\tau) = Var((1-\tau)X) = (1-\tau)^2\,Var(X)$$

$$= \sigma^2\,((1-\tau)E(X))^2.$$

[4]The risk class (or *risk-equivalent class*) was defined as a collection of firms such that the elements of the (uncertain) future earnings stream of each firm is proportional to (and hence perfectly correlated with) those of every other member of the class (MM, 1966, p. 337, footnote 3).

Var(Z) is expressed using before tax earnings:

$$(22)\quad \mathrm{Var}(Z) = \sigma^2 = \mathrm{Var}\left(\frac{X}{E(X)}\right) = \frac{\mathrm{Var}(X)}{(E(X))^2},$$

and the corresponding variance for an unlevered firm after taxes is:

$$(23)\quad \mathrm{Var}\left(\frac{(1-\tau)X}{(1-\tau)E(X)}\right) = \mathrm{Var}\left(\frac{X}{E(X)}\right) = \sigma^2.$$

Thus, the variance of the ratio (earnings/expected earnings) for an unlevered firm is unaffected by the tax rate. And for a given risk class, all the firms in that class must have equal variance of Z, by definition. Because the capitalization rate for an unlevered firm depends on the riskiness of its risk class, one can assume that the capitalization rate is related to the variance of Z. Because risk more conveniently is expressed in terms of standard deviation, however, Std(Z) will be used as the measure of risk.[5] Denoting Var(Z) (= σ^2) by $\delta_X{}^2$, and the variance of total earnings by $\sigma_X{}^2$:

$$(24)\quad \mathrm{Std}(Z) = \delta_X = \frac{\sigma_X}{E(X)}.$$

Therefore, the appropriate measure of risk for an unlevered firm is the coefficient of variation (coefficient of dispersion) of its earnings (before or after taxes).[6]

[5]In portfolio theory, each portfolio that plots along the efficient frontier in EV (expected return - variance) space also plots along the efficient frontier when portfolio risk is measured in terms of the standard deviation. When lending and borrowing portfolios are introduced, using standard deviation has the advantage of making the efficient set linear. (See, e.g., Phillips & Ritchie, 1983.)

[6]Although measuring risk with the coefficient of variation is implicit in the famous MM (1963) article, it has been overshadowed by the rate of return approach in the portfolio theory and CAPM literatures. There are, however, a few exceptions. Osteryoung, Scott, and Roberts (1977) develop a capital budgeting decision rule based on coefficient of variation. (Their analysis proceeds strictly in the standard CAPM context, however.) DeThomas (1985) and Worley and Green (1989) present models for the required rate of return that are similar in intuition to the model proposed in this paper. Their models are not based on detailed theoretical considerations and, however, consequently do not avoid several deficiencies. Both models ignore the issue of systematic risk.

The Cost of Equity Capital for an Unlevered Firm (Systematic Risk not Considered)

Define the cost of capital for an unlevered company to be equal to the risk free rate plus a premium paid on the business risk, measured by the coefficient of variation of its earnings (before or after taxes). In more specific terms:

$$(25)\quad i^*_U = r_f + a\,\delta_X$$

where:

i^*_U = Required rate of return (the subscript U denotes an unlevered firm);
r_f = Risk free rate of interest;
δ_X = Coefficient of variation of the earnings of the firm; and
a = Measure of market risk aversion.

If $a > 0$, the market (investors in the aggregate) are risk averse. If $a = 0$, the market can be considered risk neutral. If $a < 0$, the market loves risk.

Cost of Equity Capital for a Levered Firm (Systematic Risk not Considered)

Leverage intensifies the variability of the earnings stream accruing to shareholders. The additional variability caused by leverage, called financial risk, increases total risk. The next step in determining the required rate of return is to see how the variability of after tax net profits increases with leverage.

Net profit after interest and taxes (π) is equal to $X(1 - \tau) - R + \tau R$. In accordance with equation (17):

$$(26)\quad \pi = (1 - \tau)X - R + \tau R = (1 - \tau)E(X)Z - R + \tau R.$$

The expected value of this profit stream is

$$(27)\quad E(\pi) = (1 - \tau)E(X) - R + \tau R,$$

and the variance of Z is $\delta_X 2$. The squared coefficient of variation of π, $\delta_\pi{}^2$, is given by:

$$(28)\quad \mathrm{Var}\left(\frac{\pi}{E(\pi)}\right) = \delta_\pi^2 = \frac{[(1 - \tau)E(X)]^2}{[(1 - \tau)(E(X) - R + \tau R)]^2}\,\delta_X^2$$

$$= \frac{[(1-\tau)E(X)]^2}{[(1-\tau)(E(X)-R)]^2}\delta_x^2 = \frac{[E(X)]^2}{[E(X)-R]^2}\delta_x^2,$$

and π has a coefficient variation of:

$$(29)\quad \delta_\pi = \frac{E(X)}{E(X)-R}\delta_x.$$

The formula for the required rate of return, equation (25), now can be amended to include the effect of total risk (business and financial risks) of the firm. For a levered firm:

$$(30)\quad i_L^* = r_f + a\frac{E(X)}{E(X)-R}\delta x.$$

If the firm is unlevered, equation (30) reduces to equation (25). As leverage increases, however, the amount of the interest bill (R) becomes larger, thereby lowering the denominator (E(X) - R), causing a corresponding increase in i^*_L.

It is important to note that i^*_L becomes infinite (equity value becomes zero) only when the interest payments equal expected earnings before interest and taxes. The MM theory, in contrast, implicitly assumes that the value of equity becomes zero before (and sometimes well before) the interest payments equal the expected earnings.

The Cost of Equity Capital Under the Proposed New Approach (Systematic Risk Considered)

The analysis to this point has abstracted from portfolio considerations. Investors can eliminate a considerable amount of risk by diversifying their investments. According to the capital asset pricing model (CAPM), the only risk that the investors should consider is systematic or market risk that cannot be diversified. According to Sharpe and Cooper (1972), "the appropriate measure of risk for a security or portfolio is the covariance of its rate of return with that of a portfolio composed of all risky assets, each held in proportion to its total value."

The division of a security's total risk into systematic (nondiversifiable) and nonsystematic (diversifiable) components is given in the CAPM framework by (see, e.g., Levy & Sarnat, 1986, pp. 333-335):

$$(31)\quad \sigma_i = \sigma_i^{NS} + \beta_i\sigma_m$$

where:

σ_i = The total risk of security i;
σ^{NS}_i = The nonsystematic (diversifiable) risk of security i;
σ_m = The standard deviation of return on the market portfolio; and
β_i = σ_{im}/σ_m^2, the covariance between the returns on the security and the market portfolio, divided by the variance of the market portfolio.

Systematic risk also can be expressed as:

$$(32)\quad \beta_i\sigma_m = \frac{\sigma_{im}\,\sigma_m}{\sigma_m\sigma_m} = c_{im}\sigma_i$$

where:

c_{im} = The correlation coefficient between the returns of security (i) and the market portfolio.

Admittedly, the CAPM approach is concerned with returns and variability of returns on securities, whereas the derivation of the cost of equity capital model here is in terms of earnings and variability of earnings of firms. Equation (32), however, can be applied analogously in the present context. (This is demonstrated in Appendix A, where the proposed model is rederived in the CAPM framework.)[7]

For some firms, the earnings or net profits may be correlated closely with the aggregate market earnings or net profits, while for others the correlation will be more modest. The higher the correlation, the higher is the systematic risk associated with the firm's net profits. Note that because R and $(1 - \tau)$ are assumed constant, the same result is obtained if net profits or earnings before interest and taxes (EBIT) are correlated.

Using the CAPM and equations (24) and (29), the systematic risk of net profits (SRNP) of a firm can be expressed in a number of equivalent ways, e.g.:

$$(33)\quad SRNP_i = c_{\pi_i\pi_m}\,\delta_{\pi_i} = c_{\pi_i\pi_m}\frac{\sigma_{\pi_i}}{E(\pi_i)} = c_{\pi_i\pi_m}\frac{\sigma_{X_i}}{[E(X_i) - R_i]}$$

$$= c_{\pi_i\pi_m}\frac{E(X_i)}{[E(X_i) - R_i]}\delta_{X_i} = c_{X_iX_m}\frac{E(X_i)}{[E(X_i) - R_i]}\delta_{X_i}\,.$$

[7]See especially equations (A13) through (A15) in Appendix A.

SRNP thereby can be defined as the correlation between net profits (or earnings before interest and taxes (EBIT)) of the firm and the aggregate market net profits (or EBIT), multiplied by the coefficient of variation of net profits of the firm.

It is now possible to write a formula for the market-determined cost of equity. Denoting the correlation coefficient simply c, the required rate of return i* can be expressed as:[8]

$$(34)\quad i^* = r_f + a\frac{E(X)}{E(X) - R}\, c\, \delta_X$$

where:

r_f	=	The risk free rate;
a	=	A measure of risk aversion by the market (assumed to be constant at a point of time across all firms in the market);
E(X)	=	Expected earnings before interest and taxes, EBIT;
R	=	The amount of interest charges;
c	=	The correlation between net profits or EBIT of the firm and the aggregate net profits or EBIT of all the firms in the market; and
δ_X	=	$\sigma_X/E(X)$, the coefficient of variation of the EBIT of the firm.

The value of a firm can be determined using the i* (obtained by equation (34)) in equation (16):

$$(16)\quad V_L = S + D = \frac{(1-\tau)(E(X) - R)}{i^*} + \frac{R}{r_r},$$

and the value of a firm's equity is:

$$(35)\quad S = \frac{(1-\tau)(E(X) - R)}{i^*} = \frac{E(\pi)}{i^*}.$$

It is instructive to contrast this approach with the MM theory using an example. For illustrative purposes, maintain the assumptions of the previous example (compare to Table 2), where:

$E(X) = 1000$
$r = .05\ (=r_f)$[9]

[8]In Appendix A, the coefficient of variation-based model for the required rate of return is rederived using the CAPM tools. The resulting model is equivalent to equation (34).

[9]As only the cost of equity has been specified in this paper, the example uses the simplifying assumption that the firm's debt is obtained by discounting the interest stream R at

$\tau = .50.$

To make the i* for unlevered case equal the ρ of the MM theory (in the example, $\rho = .1$), the values for the parameters a, c, and δ_X are constructed to yield $i^* = .1$, when $R = 0$. The selected values are not necessarily typical for real companies. The following values are assumed:

$$
\begin{aligned}
a &= .1 \\
c &= .625 \\
\delta_X &= .8.
\end{aligned}
$$

Comparing the results in Table 3 with those in Table 2 shows that the value of equity (S) does not become zero until the expected net profit after taxes to shareholders E (π) reaches zero.

Neither the new approach nor MM's theory identifies an optimal capital structure. Both imply (bankruptcy costs aside) that the firm should finance with nearly all debt.

Using a CAPM-type approach eliminates need for the cumbersome concept of a risk class.[10] Parameter a, the measure of risk aversion (market price of risk), could be estimated cross-sectionally. It is expected to affect all firms in the market equally. All the other variables are unique to the firm. The economy-wide net profit or EBIT series would be needed to calculate the correlation coefficient (c) for the firm (similar to beta estimation).

Conclusion

In MM theory, the value of a firm's common stock always becomes worthless before (and often well before) the expected net profits are zero. The proposed model does not have this flaw.

In the proposed approach, the rate of return is a function of the risk free rate, all relevant risks (business, financial, and systematic risks), and the attitude of the market toward risk. There is no need for the cumbersome concept of risk class.

This model is, like MM's, simplified and should be expanded in future research.

the certain rate (i.e., r_r is replaced by r_f in equation (16)). This procedure maintains the values of debt equal in Tables 2 and 3. This simplification has no effect on i* or S under the proposed approach.

[10]If the MM model is expressed in the CAPM context, the rigid risk-class assumption also has been shown to be unnecessary (e.g., Hamada, 1969; Rubinstein, 1973).

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Table 1
Impact of Increasing Leverage
(τ = .0)

E(X)	R	i*	V	D	S	E(π)
1,000	0	.1	10,000	0	10,000	1,000
1,000	100	.113	10,000	2,000	8,000	900
1,000	200	.133	10,000	4,000	6,000	800
1,000	300	.175	10,000	6,000	4,000	700
1,000	400	.300	10,000	8,000	2,000	600
1,000	500	∞	10,000	10,000	0	500
1,000	600	neg.	10,000	12,000	-2,000	400

Table 2
Impact of Increasing Leverage
(τ = .50)

E(X)	R	i*	V	D	S	E(π)
1,000	0	.1	5,000	0	5,000	500
1,000	100	.113	6,000	2,000	4,000	450
1,000	200	.133	7,000	4,000	3,000	400
1,000	300	.175	8,000	6,000	2,000	350
1,000	400	.300	9,000	8,000	1,000	300
1,000	500	∞	10,000	10,000	0	250
1,000	600	neg.	11,000	12,000	-1,000	200

Table 3
Impact of Increasing Leverage
Under the Suggested New Approach
(τ = .50)

E(X)	R	i*	V	D	S	E(π)
1,000	0	.1	5,000	0	5,000	500
1,000	100	.106	6,263	2,000	4,263	450
1,000	200	.113	7,556	4,000	3,556	400
1,000	300	.121	8,882	6,000	2,882	350
1,000	400	.133	10,250	8,000	2,250	300
1,000	500	.150	11,667	10,000	1,667	250
1,000	600	.175	13,143	12,000	1,143	200
1,000	700	.217	14,692	14,000	692	150
1,000	800	.300	16,333	16,000	333	100
1,000	900	.550	18,091	18,000	91	50
1,000	1,000	∞	20,000	20,000	0	0

Appendix A
Derivation of the Coefficient of Variation Risk Model in the CAPM Framework

In the derivation of the proposed model, the issues of systematic risk were treated using analogues from the CAPM literature. To show that the shortcuts were justified, the coefficient of variation (CV) risk model will be rederived here in the capital markets context.[11] This analysis also provides additional insight into the content of the risk aversion coefficient, a, in the model.

The fundamental assumption of the model is that the investors perceive the portfolio risk in terms of the aggregate coefficient of variation of the stream of net profits associated with the assets they hold. Their opportunities can be described in an expected return—coefficient of variation of profits space. Assuming that a risk free rate of return, r_f, exists, the prices of securities keep adjusting in this E-CV space until the line drawn from the risk free rate of return tangent to the efficient opportunity set has the market portfolio at the tangency point. In equilibrium, all assets are held according to their market value weights.

The market portfolio naturally contains all the firms in the market. The aggregate net profit of all firms in the market is π_m, with coefficient of variation $Std(\pi_m)/E(\pi_m)$. The value of the aggregate market portfolio, S_m, is the sum of the values of all shares in the market. Hence, the expected rate of return on the market portfolio will be $E(r_m) = E(\pi_m)/S_m$.

A portfolio consisting of b percent invested in the risk free asset and 1 - b percent in the market portfolio has an expected rate of return, $E(r_p)$, of:

$$\text{(A1)} \quad E(r_p) = br_f + (1 - b)E(r_m).$$

Because the CV of a risk free asset is zero, the squared CV of the profit stream produced by this combined portfolio is:

$$\text{(A2)} \quad \left(\frac{Std(\pi_p)}{E(\pi_p)}\right)^2 = (1 - b)^2 \left(\frac{Std(\pi_m)}{E(\pi_m)}\right)^2,$$

and the CV of the portfolio (CV_p) is simply:

$$\text{(A3)} \quad CV_p = \left(\frac{Std(\pi_p)}{E(\pi_p)}\right) = (1 - b)\left(\frac{Std(\pi_m)}{E(\pi_m)}\right) = (1 - b)\, CV_m.$$

[11]The derivation of the model uses many of the tools developed in earlier work on portfolio theory and capital asset pricing (see especially Sharpe, 1964; Fama, 1968; and Copeland and Weston, 1988 for comparison). The concept of risk, however, is completely different in the present model.

As $(1 - b) = CV_p/CV_m$ and $b = (1 - CV_p/CV_m)$, equation (A1) can be rewritten as:

$$\text{(A4)}\quad E(r_p) = \left(1 - \frac{CV_p}{CV_m}\right) rf + \frac{CV_p}{CV_m} E(r_m) = rf + [E(r_m) - rf]\frac{CV_p}{CV_m}.$$

The slope of the capital market line in the (E-CV)-space is obtained by:

$$\text{(A5)}\quad \frac{dE(r_p)}{dCV_p} = \frac{E(r_m) - rf}{CV_m}.$$

In order to obtain the expected return on security i, the CAPM methodology is used (see Copeland and Weston, 1988). A portfolio consisting of b percent invested in risky asset i and 1 - b percent in the market portfolio has the following expected return and CV_p:

$$\text{(A6)}\quad E(r_p) = bE(r_i) + (1 - b)E(r_m)$$

$$\text{(A7)}\quad CV_p = \left[b^2CV_i^2 + (1 - b)^2C\,V_m^2 + 2b(1 - b)Cov\left(\frac{\pi_i}{E(\pi_i)}, \frac{\pi_m}{E(\pi_m)}\right)\right]^{1/2}.$$

The change in expected return and CV of the portfolio with respect to the percentage invested in i is determined as:

$$\text{(A8)}\quad \frac{dE(r_p)}{db} = E(r_i) - E(r_m)$$

$$\text{(A9)}\quad \frac{dCV_p}{db} = \frac{1}{2}\left[b^2CV_i^2 + (1 - b)^2\,C\,V_m^2 + 2b(1 - b)Cov\left(\frac{\pi_i}{E(\pi_i)}, \frac{\pi_m}{E(\pi_m)}\right)\right]^{-1/2}$$

$$\times\left[2bCV_i^2 - 2CV_m^2 + 2bCV_m^2 + (2 - 4b)Cov\left(\frac{\pi_i}{E(\pi_i)}, \frac{\pi_m}{E(\pi_m)}\right)\right].$$

The market portfolio already contains i. Evaluating (A8) and A9) where b = 0 yields:

$$\text{(A10)}\quad \left.\frac{dE(r_p)}{db}\right|_{b=0} = E(r_i) - E(r_m)$$

$$\text{(A11)}\quad \left.\frac{dCV_p}{db}\right|_{b=0} = \frac{1}{2}(CV_m^2)^{-1/2}\,[-2CV_m^2 + 2Cov\left(\frac{\pi_i}{E(\pi_i)}, \frac{\pi_m}{E(\pi_m)}\right)].$$

The slope of the return-risk tradeoff at the capital market line tangency point is:

$$\text{(A12)}\quad \frac{dE(r_m)/db\big|_{b=0}}{dCV_p/db\big|_{b=0}} = \frac{E(r_i) - E(r_m)}{[\mathrm{Cov}\left(\frac{\pi_i}{E(\pi_i)}, \frac{\pi_m}{E(\pi_m)}\right) - CV_m^2]/CV_m}.$$

Equating (A12) and (A5) (the slope of the capital market line) and solving for $E(r_i)$ yields:

$$\text{(A13)}\quad E(r_i) = r_f + \frac{[E(r_m) - r_f]}{CV_m^2}\mathrm{Cov}\left(\frac{\pi_i}{E(\pi_i)}, \frac{\pi_m}{E(\pi_m)}\right),$$

which is a coefficient of variation risk capital asset pricing model.

The correlation between CV_i and CV_m is:

$$\text{(A14)}\quad \mathrm{CORR}(CV_i, CV_m) = \frac{\mathrm{Cov}\left(\frac{\pi_i}{E(\pi_i)}, \frac{\pi_m}{E(\pi_m)}\right)}{\frac{\mathrm{Std}(\pi_i)}{E(\pi_i)}\ \frac{\mathrm{Std}(\pi_m)}{E(\pi_m)}} = \frac{\mathrm{Cov}(\pi_i, \pi_m)}{\mathrm{Std}(\pi_i)\ \mathrm{Std}(\pi_m)}.$$

$\mathrm{CORR}(CV_i, CV_m)$ thereby reduces to the correlation between the net profits of the firm and the aggregate net profits of all firms in the market. Denoting the correlation coefficient simply by $c_{i,m}$, equation (A13) can be expressed equivalently as:

$$\text{(A15)}\quad E(r_i) = r_f + \frac{[E(r_m) - r_f]}{[\frac{\mathrm{Std}(\pi_m)}{E(\pi_m)}]} c_{i,m} \frac{\mathrm{Std}(\pi_i)}{E(\pi_i)}.$$

(Note that $c_{i,m}(\mathrm{Std}(\pi_i)/E(\pi_i))$ equals the systematic risk of net profits measure ($SRNP_i$) in equation (33), which justifies using the CAPM analogy in defining the SRNP.) The coefficient of variation of net profits also can be expressed as $\mathrm{Std}(X_i)/(E(X_i) - R_i)$ (see equations (26) and (29)), whereby:

$$\text{(A16)}\quad E(r_i) = r_f + \frac{[E(r_m) - r_f]}{[\mathrm{Std}(\pi_m)/E(\pi_m)]} c_{i,m} \frac{\mathrm{Std}(X_i)}{[E(X_i) - R_i]}.$$

The proposed model for the cost of equity (34) is:

$$\text{(A17)}\quad i^* = r_f + a\frac{E(X)}{E(X) - R}c\,\delta_X = r_f + a\,c\frac{Std(X)}{E(X) - R}.$$

As $E(r_i)$ and i^* both denote the cost of equity (required rate of return), equations (A16) and (A17) are equivalent when the measures of market price of risk in the two models are equated:

$$\text{(A18)}\quad a = \frac{[E(r_m) - r_f]}{\dfrac{Std(\pi_m)}{E(\pi_m)}}.$$

Thus, the content of coefficient a has been specified.