Performance Measurement When Return Distributions are Nonsymmetric

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Abstract

This paper argues that to ignore higher moments of utility one must do so on the basis of economic arguments (normal returns or quadratic utility) or empirical evidence demonstrating that higher moments are not important to investors. Extending the work of Prakash and Bear (1986), a generalized performance evaluation model is presented that allows for multiple moments of utility. The evaluation models are applied to the returns of internationally diversified mutual funds. The results indicate that ignoring higher moments of utility has significant impact upon the performance rankings of these funds.

Introduction

Modern portfolio construction often involves attempts by portfolio managers to modify the return distributions of their portfolios. For example, Bookstaber and Clarke (1983) have shown that the returns on portfolios of common stocks can be modified substantially using a variety of option strategies. In addition, international funds with highly skewed return distributions offer their investors risk reduction possibilities through their low covariance of returns with domestic securities (Proffitt and Seitz, 1983).

If return distributions are nonsymmetrical and investors value skewness, as argued by Arditti (1967), then traditional performance measures are inadequate. To address the problem of defining performance in the face of nonsymmetric distributions, Prakash and Bear (1986) (hereafter PB) developed measures that recognized skewness of return distributions. The PB formulas were developed on the basis of the Kraus and Litzenberger (1976) (hereafter KL) skewness preference model.

This paper demonstrates that the KL three moment model is a special case of Rubinstein's (1973) parameter preference model. In the KL model, higher

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moments were ignored, as they were not behaviorally justified. Without entering this controversy of whether more than three moments are behaviorally justified, a performance measure based on n-moments is developed. This general performance measure has, as special cases, the Treynor and PB performance measures.

One of the desirable attributes of the general performance model is that it readily is adaptable to a form that is useful in empirical tests.[1] The application of the performance measures is illustrated with an analysis of internationally diversified mutual funds. Research concerning internationally diversified mutual funds continually encounters problems with the performance measurement of these funds relative to domestic funds using traditional techniques. Because internationally diversified funds have skewed and flattened return distributions, performance problems of international funds may be addressed using the general model of performance measurement. Performance rankings based on three or higher moments are essentially the same for internationally diversified funds, while those based on two moments differ significantly with the ranking achieved using PB measures. Thus, this paper not only empirically applies the PB measures to a set of internationally diversified funds, but also (albeit indirectly) tests the validity of the higher moment CAPM.

Development of the Model

By stating utility as a Taylor series expansion of an individual's expected wealth, Rubinstein (1973) developed the fundamental theorem of parameter preference security valuation. This model, which is a state-independent, state-preference model in which probability data are summarized by central moments, states:

$$(1)\ E_i(R_j) = E_i(R_k) + \sum_{n=2}^{\infty} \theta_{in} \cdot \sigma_{in}(\omega_i, R_j - R_k),\ \forall j,k$$

where:

[1] Again the work of PB has been extended to a general model. In addition, errors in the application of the PB model have been corrected.

$$\theta_{in} = \frac{U_i^n[E_i(\omega_i)]}{(n-1)!E_i[U_i'(\omega_i)]}$$

U_i = an individual's continuously differentiable utility of wealth function;

E_i = an individual's expectation operator;

ω_i = an individual's future wealth = $\Sigma_j X_{ij} \cdot R_j$ where X_{ij} is the ith individual's investment in security j whose return is R;

U_i^n = the nth derivative of U_i evaluated at $E_i(\omega_i)$;

$\sigma_{in}(\omega_i,R_j)$ = $E_i\{[\omega_i - E(\omega_i)]^{n-1}[R_j - E(R_j)]\}$

= joint central moment of order n of the bivariate density function of ω_i and R_j or, more concisely, the nth comoment of (ω_i,R_j).[2]

If instead of measurable utility, individuals are assumed to maximize the ordinal utility function:[3]

$\phi(E_i(\omega_i), M_{i2}, M_{i3}, \ldots, M_{in})$

where:

M_{in} = $[\omega_i - E_i(\omega_i)]^n$ = the ith individual's nth moment of wealth about the mean $([\omega_i - E_i(\omega_i)]^n)$

Rubinstein (1973) showed that in equilibrium, equation (1) may be reduced to

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[2]A complete explanation of the concept of a comoment may be found in Rubinstein (1973).

[3]See Rubinstein (1973), footnote 6.

$$(2)\quad E_i(R_j) = R_f + \sum_{n=2}^{\infty} \frac{dE_i(\omega_i)}{d(M_{in})^{1/n}} \cdot \frac{1}{(M_{in})^{(n-1)/n}} \cdot \sigma_{in}(\omega_i, R_j),\ \forall j.$$

Multiplying the first term in the summation by M_{i2}/M_{i2}, the second term in the summation by M_{i3}/M_{i3}, etc., results in:

$$(3)\quad E_i(R_j) = R_f + \sum_{n=2}^{\infty} \frac{dE_i(\omega_i)}{d(M_{in})^{1/n}} \cdot (M_{in})^{1/n} \cdot \sigma_{in}(\omega_i, R_j),\ \forall j.$$

Applying the conditions for aggregation to establish a model for market equilibrium allows equation (3) to be written as:

$$(4)\quad E(R_j) = R_f + \sum_{n=2}^{\infty} \frac{dE(\omega_i)}{d(M_{mn})^{1/n}} \cdot (M_{mn})^{1/n} \cdot \sigma_{mn}(R_m, R_j),\ \forall j.$$

where:

$M_{mn} = [R_m - E(R_m)]^n$ = nth central moment of the market rate of return.

Following Rubinstein (1973), equation (3) states:

> an individual i will adjust his/her portfolio until the expected rate of return of each security is equal to the risk free rate plus a risk premium equal to the weighted sum of the comoments of the rate of return of the security with his or her future wealth.

Each comoment is weighted by the ratio $[dE_i(\omega_i)/d(M_{in})^{1/n}] \cdot (M_{in})^{1/n}$, which reflects the corresponding individual measure of risk aversion.[4]

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[4]Based on the assertion that higher moments introduce unsuitable utility functions for risk adverse investors, KL truncated the series in equation (4) after the second term. Using KL's notation, equation (4) may be stated as:

The Case for Higher Moments of Utility[5]

Some authors have asserted that the third and higher moments may be neglected if it is assumed that returns are distributed normally. Levy (1969) has pointed out, however, that the fact that little information was contained in the higher moments is different from the assumption that the values are nearly zero. The third and higher odd moments disappear due to symmetry. The fourth and higher even moments, although a predictable function of the second, are positive and far from zero, i.e.,

$$\int_{-\infty}^{\infty} (X-E(X))^4 f(x)\, dx>0$$

To ignore the higher moments at least one of the following conditions must be true:

$$E(R_j) = R_f + b_1(\beta_j) + b_2 \cdot \gamma_j$$

where:

$$b_1 = \frac{dE(\omega)}{d[M_{m2}^{1/2}]} \cdot (M_{m2})^{1/2}$$

$$b_2 = \frac{dE(\omega)}{d[M_{m3}^{1/3}]} \cdot (M_{m3})^{1/3}$$

$$\beta_j = \frac{\sigma_{m2}(R_m, R_j)}{M_{m2}} = \frac{E\{(R_m - E(R_m))\,(R_j - E(R_j))\}}{E\,[R_m - E(R_m)]^2} = \frac{COV(R_m, R_j)}{\sigma_m^2}$$

$$\gamma_j = \frac{\sigma_{m3}(R_m, R_j)}{M_{m3}} = \frac{E[(R_m - E(R_m))^2\,(R_j - E(R_j))]}{E\,[R_m - E(R_m)]^3} = \frac{M_{m2}}{M_{m3}}$$

[5]A portion of this section was taken directly from Smith (1977).

- The distribution has negligible dispersion, thus all moments above the first are zero (Samuelson, 1970);
- The applicable utility function has derivatives that are zero for the third and higher degrees;
- The presence of normal distributions and quadratic utility functions (Rubinstein, 1973).

Even when normal distributions are not assumed, authors generally have argued that the fourth and higher derivatives may be ignored because they are not behaviorally justified (KL). This assertion is based on the assumption that functions with increasing absolute risk aversion or functions with absolute risk aversion decreasing so fast as to cause an increasing relative risk aversion are not behaviorally acceptable.

Such behavioral judgments are based on Arrow (1971) and Pratt (1964). Arrow and Pratt advocated that although individuals would invest more wealth at risk as their wealth increased, individuals would not be likely to invest a larger portion of their wealth at risk as their wealth increased. The Arrow-Pratt measures of risk aversion, however, are local measures of risk aversion. That is, risk tolerance ($-U'/U''$), absolute risk aversion ($-U''/U'$), and relative risk aversion ($-\omega U''/U'$) are all parameters that do not deal with totality of risk. Specifically, these measures deal only with the second moment of the return distribution as a result of the second degree truncation of the Taylor series expansion of utility used in their derivation. Thus, these measures are essentially absolute variance aversion, absolute variance tolerance, and relative variance aversion, as variance is a component of, not the whole of, risk. The extent to which these measures correctly describe investor risk posture is the extent to which the truncated Taylor series expansion correctly describes investor utility of wealth.

An infinite series of derivatives is required by the assumptions of risk-averse utility functions under nonsatiation. A rational utility function must have an infinite stream of nonzero derivatives because a curve that is concave downward must have a positive third derivative if the curve is not to turn downward. Such an event would exhibit a negative marginal utility of wealth. A positive third derivative must be decreasing if the curve is not to turn upward, implying a negative fourth derivative. The fourth derivative must be decreasing negative to prevent the third from becoming negative, implying a positive fifth derivative, ad infinitum.

Thus, the case for rational utility functions containing increasing relative risk aversion with respect to wealth is weaker than generally has been believed. It is expected that wealthy individuals would have a larger percentage of their wealth invested at risk than would poorer individuals. That is, an individual is expected to place larger proportions of additional increments of wealth at risk. Therefore, utility functions with decreasing relative risk aversion also may be admissible. This result implies that although equations such as the general KL skewness preference model are acceptable,[6] so too are the more general models as expressed in equation (4).

A Performance Measure With Higher Derivatives of Utility

Based on equation (2) aggregated over all individuals, the expected return on security j may be expressed as:

$$(5)\quad E(R_j) = R_f + \sum_{n=2}^{\infty} \frac{dE(\omega_i)}{d(M_{mn})^{1/n}} \cdot \frac{1}{(M_{mn})^{(n-1)/n}} \cdot \sigma_{mn}(R_m, R_j)$$

$$= R_f + \sum_{n=2}^{\infty} \lambda_n \cdot \sigma_{mn}(R_m, R_j),$$

where:

$$\lambda_n = \frac{dE(\omega_i)}{d(M_{mn})^{1/n}} \cdot \frac{1}{(M_{mn})^{(n-1)/n}}$$

By truncating equation (5) after the second term in the series, PB (1986) determined the values of λ_1, λ_2 for the nonincreasing risk aversion family of utility functions, i.e., those functions that satisfy the condition:

$$-[U'(\omega)/U''(\omega)] = a + b\omega$$

Recall that these functions are:

logarithmic: $U(\omega) = \ln(a + \omega)$, $b = 1$
exponential: $U(\omega) = -\exp(-\omega/a)$, $b = 0$

[6] See footnote 4.

power: $U(\omega) = [1/(b-1)]\,(a+w)^{(b-1/b)},\ b \neq 0, 1$

When the Taylor series expansion of the logarithmic utility function is not arbitrarily truncated after the third term, then λ_n $(n = 2, 3, \ldots)$ may be expressed as:[7]

$$(6)\quad \lambda_n = \left.\frac{(-1)^n}{(a+R_m)^{n-1}}\right| K_{1m}$$

where:

a = the risk preference parameter in the utility function, and

$$K_{1m} = 1 + \sum_{n=2}^{\infty} \frac{(-1)^n M_{mn}}{(a+R_m)^n}$$

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[7]*For the logarithmic utility function, the values of* λ_2 *and* λ_3*, as described* by PB (1986) are:

$$\lambda_2 = \left.\frac{1}{(a+R_m)}\right| K_{1m}$$

$$\lambda_3 = \left.\frac{-1}{(a+R_m)^2}\right| K_{1m}$$

where:

$$k_{1m} = 1 + \frac{M_{m2}}{(a+R_m)^2} - \frac{M_{m3}}{(a+R_m)^3}$$

Assuming an exponential utility function, the values of λ_n, n = 2, 3, ... are:

$$(7)\quad \lambda_n = \left.\frac{(-1)^n}{(n-1)! \cdot a^{n-1}}\right| K_{2m}$$

$$K_{2m} = 1 + \sum_{n=2}^{\infty} \frac{(-1)^n M_{mn}}{n! \cdot a^n}$$

For the power utility function the values of λ_n, n = 2, 3, ... , are:

$$(8)\quad \lambda_n = \left.\frac{(-1)^n \cdot \prod_{k=0}^{n-2} (kb+1)}{(n-1)! \cdot b^{n-1} \cdot (a+R_m)^{n-1}}\right| K_{3m}$$

$$K_{3m} = 1 + \sum_{n=2}^{\infty} \frac{(a+E(R_m))^{-n}}{n! \cdot b^n} \cdot \prod_{n=2}^{\infty} (n-1)b+1$$

In the sets of equations (6), (7), and (8),

$$(9)\quad \lambda_n \cdot \eta_n = \eta_i \cdot \lambda_i,\ n = 3, 4, \ldots;\ i = 2, 3, \ldots, n$$

where η_n for the logarithmic, exponential, and power utility functions are respectively:

$$\text{(10)} \qquad \eta n = \begin{vmatrix} [-(a+R_m)]^{n-2} \\ \\ (n-1)!\,(-a)^{n-2} \\ \\ (-1)^n \cdot (n-1)! \cdot b^{n-2}\,(a+R_m)^{n-2} \end{vmatrix} \prod_{k=0}^{n-2} (kb+1)$$

By substitution, equation (5) may be written as:

$$\text{(11)} \qquad E(R_j) = R_f + \lambda_n \cdot \eta_n \cdot \sum_{i=2}^{n} \frac{1}{\eta_i} \cdot \sigma_{mi}(R_m, R_j) \; (n=3, 4, \ldots)$$

or

$$\text{(12)} \qquad \lambda_n = \frac{E(R_j) - R_f}{\eta_n \cdot \displaystyle\sum_{i=2}^{n} \frac{1}{\eta_i} \cdot \sigma_{mi}(R_m, R_j)}$$

Alternatively,

$$\text{(13)} \qquad E(R_j) = R_f + \lambda_n \cdot \eta_n \cdot \sum_{i=2}^{n+1} \frac{1}{\eta_i} \cdot \supseteq \sigma_{mi}(R_m, R_j) \; (n=2, 3, \ldots)$$

or

$$\text{(14)} \qquad \lambda_n = \frac{E(R_j) - R_f}{\eta_n \cdot \displaystyle\sum_{n=2}^{n} \frac{1}{\eta_n} \cdot \sigma_{mn}(R_m, R_j)}$$

Equations (11) and (13) show that the parameters λ_n are a function of the market-determined return and the utility function (η). In addition, λ_n, as defined in equations (12) and (14), can be used to rank the performance of portfolios. When the series in equation (5) is limited to a single term, (i.e., without coskewness and higher moments), these performance measures reduce to Treynor's reward-to-volatility ratio.[8]

Empirical Applications

For empirical investigations, the parameter λ_n can be estimated using equation (11) or equation (13). In *ex post* form, equation (11) can be written as:

$$(15) \qquad R_{jt} = \psi_0 + \sum_{n=1}^{k} \psi_n \cdot Z_{jn} + \varepsilon_{it}$$

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[8]For a two moment function, that is equation (5) truncated after the second term in the series, $\lambda_2\eta_2 = \lambda_3\eta_3$, where:

$$\eta_3 = \begin{cases} -(a+R_m) & \text{for logarithmic utility} \\ -2a & \text{for exponential utility} \\ -\dfrac{2b}{b+1}(a+R_m) & \text{for power utility} \end{cases}$$

Therefore equations (12) and (14) become:

$$\lambda_3 = \frac{E(R_j) - R_f}{\eta_3\lambda_3\sigma_{m2}(R_m,R_j) + \eta_3\sigma_{m3}(R_m,R_j)}$$

and

$$\lambda_2 = \frac{E(R_j) - R_f}{\lambda_2\sigma_{m2}(R_m,R_j) + (\lambda_2/\eta_3)\sigma_{m3}(R_m,R_j)}$$

These relations are the PB results.

where:

ε_{it} = error term that follows the usual regression assumptions,

$$Z_{jn} = (1/q) \cdot \sum_{t=1}^{q} [(R_{mt}-\bar{R}_m)^n (R_{jt}-\bar{R}_j)],$$

t = observation number;
j = fund;
q = number of months in the observation period; and
n = 1, 2, or 3 for covariance, coskewness, and cokurtosis respectively.

For ranking purposes, the estimates of ψ_n can be used to estimate η_n. Thus, as noted by PB, one need not worry about the existence of a particular utility function. The statistical significance of the OLS estimate ψ_n will provide the answer to the question of whether investors base their decisions on the nth moment(s).

Performance Test of Internationally Diversified Funds

Internationally diversified mutual funds have been of interest to the investment community in recent years. This interest is reflected by the increased attention given these funds in the popular press and by the growth in both the number and in the average size of the internationally diversified funds offered to the public. The principal reason for this growth has been the high level of return.

International funds offer their investors risk reduction possibilities through their low covariance of returns with domestic securities. The low covariance of returns leads to superior two moment performance measures (such as the Sharpe, 1966; Treynor, 1965; and Jensen, 1968, measures). This performance superiority is noted when the international funds are compared to both traditional domestic funds and to the domestic market (Proffitt and Seitz, 1983). Portfolios should be evaluated utilizing higher moments, however, when returns come from an asymmetrical distribution and when investors' utility is a function of moments in the distribution of returns beyond the second. Thus, researchers using only two moment models must demonstrate that the above conditions are not relevant.

The incorporation of higher moments is particularly appropriate for internationally diversified funds. From 1976 to 1982, the exchange rate between the dollar and most foreign currencies fell consistently. The influence of the dollar's decline in value led to distributions of returns for internationally diversified funds that exhibited positive skewness. As shown by Arditti (1967), there is reason to believe that investors' utility of wealth is increased when skewness is positive.

The Data

The sample consisted of monthly holding period returns of 27 internationally diversified mutual funds from January 1976 through June 1982. The observation period included 66 months. To qualify as an internationally diversified fund, a mutual fund had to hold positions in equity securities traded primarily in overseas markets. Investments in multinational firms located in the U.S. did not qualify as international diversification. Investments in ADRs (American depository receipts, certificates originated by U.S. banks representing shares of foreign equity securities) did qualify as international diversification. The fund had to be invested primarily in stocks; bond and money market funds were not included. A listing of the funds in the sample is included in Table 1.

Data Analysis

Holding period returns for each fund (j) over each month in the sample (t) were calculated as below:

$$R_{jt} = [(P_{jt} - P_{jt-1}) + D_{jt}] / P_{jt-1} \tag{16}$$

The moments of the probability distribution of fund returns were calculated as defined in equation (15). Next, an OLS regression of the form specified in equation (15) was run. Separate runs were made for the case where the set of independent variables consisted of covariance, coskewness, and cokurtosis (n = 3), covariance and coskewness (n = 2), and covariance by itself (n = 1). Results from these regressions are given in Table 2.

As indicated in Table 2, the equation utilizing the fourth moment (n = 3) of the distribution yielded no significant regression coefficients. Following PB, this is interpreted as an indication that the fourth moment of the distribution is not significant to international fund investors. Redefining the equation for the third moment (n = 2), a significant skewness coefficient is obtained. The sign of the skewness coefficient is negative, as predicted by PB.

To address this point, it has been verified that a return-generating model of a form that includes higher moments of the return distribution (specifically, the third) is appropriate and significant. The next step is to evaluate the funds using this model.

PB defined μ as

$$\mu = \psi_1/\psi_2$$

where:

ψ_1 = OLS coefficient for the covariance term; and
ψ_2 = OLS coefficient for the skewness term.

Using μ, ψ_1 and ψ_2, the measures that are used to rank portfolios, are defined as:

$$\psi_1 = \overline{r_j} \;/\; (Z_{j1} + Z_{j2}/\mu)$$

$$\psi_2 = \overline{r_j} \;/\; (\mu \cdot Z_{j1} + Z_{j2})$$

where Z_{jn} is defined in equation (15).

Finally, the funds were ranked according to the traditional Treynor (T) and Sharpe (S) measures, which are:

$$T = \overline{r_j} \;/\; (Z_{j1} \cdot \sigma_m^2)$$

and

$$S = \overline{r_j} \;/\; (\sigma_j^2)$$

Table 1 provides the fund rankings utilizing all measures above. The rankings are the same for ψ_1 and ψ_2.

An issue of obvious interest is whether the various performance measurement techniques calculated in Table 1 ranked the funds in a different manner. Correlations between fund rankings obtained with the different measurement techniques were obtained using both Spearman's rho and Kendall's tau. In general, Spearman's rho is based upon the squares of the differences between ranks, while Kendall's tau is based upon the consistency of relative ranks over the entire sample. Both measures are appropriate for ordinal data, and both utilize only nonparametric assumptions about the associated distributions.

Table 3 depicts the values obtained and their associated significance levels. Except the correlation between the Sharpe and Treynor scores, almost all coefficients differ significantly from zero at the 1 percent significance level. The Sharpe and Treynor correlations are significant at the 5 percent level.

Table 3 shows that the Sharpe and the higher moments fund rankings are positively correlated, while the Treynor and the higher moments fund rankings are negatively correlated. Furthermore, there is a negative correlation between the Sharpe and Treynor fund rankings.

When interpreting the sign of these correlation coefficients, recall that well-diversified funds will not differ significantly in their rankings with either the Sharpe or the Treynor measures. If the fund is well diversified, its nonsystematic risk largely will be eliminated. Because the only difference in the two ranking measures is their incorporation of total risk (Sharpe measure) versus systematic risk (Treynor measure) in the denominator, rankings will not differ much unless the funds are not adequately diversified. The significant negative correlations observed between the rankings obtained with the Sharpe and Treynor measures are viewed as evidence of incomplete diversification on the part of the fund managers.

By incorporating higher moments into the performance measurement of fund returns, some of the errors in model specification contained in the single factor (market) return-generating process model are resolved empirically. The higher moments model incorporates not only mean and variance, but skewness, which seemed to be an important factor in investor evaluation of these securities.

The higher moments model utilizes a more complete description of the distribution of returns. The standard CAPM requires the assumption of either normally distributed returns or quadratic utility functions. With the case of international funds, return distributions are skewed. The possibility of nonquadratic utility functions has been introduced. In either case, the Treynor measure, which incorporates systematic risk, no longer can be used properly. The problem identified above is significant and is present in the return distributions of securities other than international funds. For example, one of the primary motives for hedging with options is to limit potential portfolio losses. Options, therefore, provide skewness to the distribution of returns of their portfolios. Other examples no doubt exist. Researchers and practitioners need to give more attention to the proper measurement of performance under these conditions.

Summary

This paper has shown that a multimoment portfolio evaluation technique based on the work of PB can be extended readily to the evaluation of portfolios that exhibit skewness in the distribution of their returns. Higher than three order moments were not found to be empirically significant in the case of international mutual funds, thus confirming the wisdom that the higher order moments may not (after all) be behaviorally justified.

The PB performance measure was applied to the returns of internationally diversified mutual funds from 1976 to 1982, a period in which these funds exhibited significant positive skewness. Although it was found that kurtosis was not a significant variable in explaining fund performance, skewness was significant. Furthermore, the sign of the skewness coefficient was found to be negative as hypothesized in prior theoretical work. The current empirical work demonstrates that the fund rankings obtained utilizing this measure are different from those obtained using the more traditional two moment measures such as that of Treynor (1965).

The lack of robustness in the mean-variance portfolio measures has forced investigators to develop performance evaluations that consider nonnormal return distributions. International securities, common stock portfolios hedged with options contracts, and other types of portfolios have been shown to demonstrate asymmetric return distributions. Performance measures such as the one utilized in this paper seem to be proper to evaluate correctly the performance of these portfolios.

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Table 1
Fund Rankings

Fund	Ranking Criteria: Sharpe	Treynor	Prakash & Bear's ψ_1 and ψ_2
ASA	13	19	1
Japan Fund	22	10	13
New Perspective	25	9	24
Bullock Fund	23	7	22
Canadian Fund	27	2	26
Delta Trend Fund	12	11	10
Equity Income Fund	19	21	12
Research Capital Fund	3	25	3
International Investors	5	27	4
Keystone International	9	4	2
Newton Growth Fund	18	13	20
Oppenheimer Time Fund	8	5	19
Putnam Growth Fund	17	26	16
Putnam International	20	18	21
Scudder International	24	8	23
Sogen International	15	23	6
American Industry	26	3	27
Stein Roe Cap. Opportunities	10	6	18
Templeton Growth	21	15	25
United Continental Growth	16	14	11
Eaton Vance Growth	14	20	15
Golcanda	11	16	17
Magellan	1	1	5
Oppenheimer Special Fund	6	12	14
Sequoia	7	17	9
Strategic Investors Fund	4	22	7
United Services Gold Fund	2	24	8

Table 2
Regression Results

$$R_{j2t} = \psi_0 + \sum_{n=1}^{k} \psi_n \cdot Z_{jn} + \varepsilon_{jt}$$

	ψ_0	ψ_1	ψ_2	ψ_3
k = 3	.0041	-10.3698 (0.212) df = 23	10.1964 (0.481) df = 23	5,204.697 (0.089) df = 23
k = 2	.0014 df = 24	6.0914 (0.108) df = 24	-239.8115 (0.019)	
k = 1	-.0002	12.2477 (0.004) df = 25		

Significance levels of the coefficients are in parentheses

Table 3
Correlations Among Fund Rankings

	$\rho_{1,2}$	$\rho_{1,3}$	$\rho_{2,3}$
Spearman's rho	-0.326 (0.048)	0.752 (.00006)	-0.449 (0.011)
Kendall's tau	-0.254 (0.0315)	0.578 (.00001)	-0.345 (.0058)

1 = Sharpe measure
2 = Treynor measure
3 = gamma (higher moments measure)

Significance levels of the correlation coefficients are in parentheses