An Equilibrium Pricing Model With Decreasing Marginal Transaction Costs

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Abstract

The theoretical model developed in this study establishes that when marginal transaction costs are decreasing, the expected rate of return on a security is related positively to its systematic risk and its average marginal transaction costs. The empirical part of the study relies on the bid-ask price spread and a number of alternative transaction cost measures based on firm size, share price, and trading activities. The results of the test are consistent with the hypothesis that transaction costs are an important additional factor explaining security returns.

Introduction

One of the underlying assumptions of the Markowitz [19] portfolio theory and the capital asset pricing model (CAPM) derived by Sharpe [28] and Lintner [18] is that security trading occurs with zero transaction costs. Several studies have examined the effect of nonzero transaction costs on the portfolio and the capital asset pricing theory. Brennan [9] finds that introducing transaction costs makes it economically infeasible to hold a perfectly diversified portfolio. Goldsmith [14] demonstrates that the fraction of a portfolio invested in risky securities is an increasing function of wealth when transaction costs exist. Milne and Smith [23] develop a pricing model by incorporating bid-ask price spreads as a measure of transaction costs and show that the equilibrium price of a security is bounded by its bid and ask prices. Levy [17] and Mayshar [20, 21] establish that when transaction costs constrain the number of securities an investor is willing to hold, the variance of returns on a security is the major factor in determining the expected return and that the market price of risk is smaller than that specified by the traditional CAPM.

Recently, the validity of the CAPM has been questioned in light of the results obtained by Banz [3], Reinganum [24], and others showing that small firms have higher risk-adjusted abnormal returns than do large firms. One of the

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hypotheses offered in the literature to explain the small firm effect is the high costs of transacting small firms' securities.[1] In particular, faced with higher transaction costs and the resulting lower marketability, investors would require a higher return in order to trade in small firm securities.

Stoll and Whaley [32], using New York Stock Exchange firms, find that for holding periods between three months and one year, after-transaction-costs abnormal returns are not significantly different from zero. Under these conditions, Stoll and Whaley establish that the small firm effect is eliminated. Schultz [29], however, reports significant net-of-transaction-costs abnormal returns defined over shorter holding periods for American Stock Exchange firms. Although Schultz concludes that transaction costs do not fully explain the small firm effect, Amihud and Mendelson [1] confirm Stoll and Whaley's findings and suggest that firm size is a proxy for liquidity. Recently, Merton [22] develops the following theoretical rationale: When investors know about only a subset of securities due to the existence of information costs, securities with less information available to investors and with smaller investor base will have larger rates of return.

This paper develops an equilibrium pricing model in which decreasing marginal transaction costs are considered explicitly. Although decreasing marginal transaction costs are more realistic, previous studies assume either constant or proportional transaction costs (Milne and Smith [23]). The theoretical analysis of this study is similar in spirit to that of Levy [17], Mayshar [20, 21], and Merton [22] in the broad set of issues bearing upon the pricing effects of transaction costs. Whereas Levy and Mayshar are primarily concerned with portfolio choice decisions, however, this paper addresses the ability of transaction costs in explaining the variations in security returns. Also, Levy, Mayshar, and Merton obtain the market equilibrium condition by aggregating only the investors who decide *ex ante* to hold a set of securities. In contrast, this paper deals with all investors who constitute the security's demand function. The demand function only is constrained by decreasing marginal transaction costs. Also, transaction costs here are defined more broadly than in Milne and Smith [23] to include all direct and indirect costs as well as bid-ask spreads. The empirical part of this study documents evidence that transaction cost-related variables (such as bid-ask price spread, firm size, share price, and trading activities) are important variables in explaining variations in security returns.

[1] An alternative hypothesis presented in the literature to explain the small firm effect is tax-loss selling. For example, see Keim [16], Dyl [12], Branch [8], Roll [26], Givoly and Ovadia [13], Reinganum [25], and Brown, Keim, Kleidon, and March [10]. Also, see Schwert [30] for further discussions.

The equilibrium conditions of the model developed demonstrate that if the transaction cost rate is uniquely security-specific and decreasing, the security's equilibrium return before transaction costs is related positively to both its systematic risk and to its average marginal transaction costs. A corollary, if the transaction cost rate is a decreasing function of firm size, then the before-transaction-costs risk-adjusted return for small firms should be greater than that for larger firms; therefore, transaction costs may explain the small firm effect.[2] But if marginal transaction costs are zero or constant across securities, the traditional zero-transaction-cost CAPM is shown to be not affected.

An Equilibrium Model with Transaction Costs

For the purpose of this study, all the standard assumptions of the CAPM are invoked except for transaction cost of trading.[3] Transactions costs are defined here to include direct costs such as brokerage fees and bid-ask spread for providing trading services as well as indirect costs relating to collecting information and managing portfolios. Further, it is assumed without loss of generality that transaction costs are incurred at the beginning of the investment horizon, i.e., investors can purchase a package of services for a certain fee at that time. Assume that investor i allocates initial wealth, W_i, between riskless security, F_i, and N risky securities S_{ij}, $j = 1, \ldots, N$. Then, the initial wealth constraint can be expressed as:

$$(1)\quad W_i = \sum_j S_{ij} + F_i + \sum_j C_{ij}(S_{ij}) + C_{if}(F_i),$$

where:

C_{ij} and C_{if} = the transaction cost functions for the risky asset j and the riskless asset, respectively.

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[2]Blume and Stambaugh [7] show that the small firm effect is essentially a January effect because most abnormal security returns occur at the turn of the calendar. The current study does not explain the seasonality of security returns, but rather the determination of expected returns during an investment horizon.

[3]The major assumptions are (i) one period model, (ii) no transaction costs, (iii) no taxes, (iv) no restrictions on short selling, and (v) borrowing and lending at a risk free rate.

Transaction costs for asset j may differ across large and small investors if there are transaction economies of scale. The transaction cost functions are twice differentiable everywhere including zero.

End of the investment horizon terminal wealth $\tilde{W}_i$, then, becomes:[4]

$$(2) \quad \tilde{W}_i = \sum_j S_{ij} R_j + F_i R_f,$$

where:

R_j = one plus the random rate of return on security j; and
R_f = one plus the risk free rate.

Suppose a decreasing marginal utility, $U_i(\tilde{W}_i)$, so that $U_i'(\tilde{W}_i) > 0$ and $U_i''(\tilde{W}_i) < 0$ and the investor's objective is to maximize the expected utility of future wealth subject to the initial wealth constraint. That is,

$$(3) \quad \text{Max } Z = E[U_i(\tilde{W}_i)] + L_i[W_i - \sum_j S_{ij} - F_j - \sum_j C_{ij}(S_{ij}) - C_{if}(F_i)],$$

where:

L_i = a Lagrangian multiplier.

Differentiating Z with respect to S_{ij}, F_i, and L_i to obtain the first order conditions results in:

$$(4a) \qquad \frac{\partial Z}{\partial S_{ij}} = E(U_i'R_j) - L_i[1 + C_{ij}'(S_{ij})] = 0$$

$$\text{for } j = 1, \ldots, N,$$

$$(4b) \qquad \frac{\partial Z}{\partial F_i} = E(U_i'R_f) - L_i[1 + C_{if}'(F_i)] = 0,$$

[4]The approach adopted here to derive the equilibrium condition is similar to Rubinstein's [27].

$$(4c) \qquad \frac{\partial Z}{\partial L_i} = W_i - \sum_j S_{ij} - F_i - \sum_j C_{ij}(S_{ij}) - C_{if}(F_i) = 0,$$

where:

C'_{ij} and C'_{if} = the marginal transaction costs for the risky asset j and the riskless asset, respectively.

From equations (4a) and (4b) it can be shown that:[5]

$$(5) \quad E(U'_i)\ [E(R_j) - \frac{1 + C'_{ij}(S_{ij})}{1 + C'_{if}(F_i)} R_f] = -Cov(U'_i, R_j).$$

To simplify this result further: in the specific case where a quadratic utility function of the form $U_i = \tilde{W}_i - a_i\tilde{W}_i^2$, where $a_i > 0$ and $\tilde{W}_i < 1/2a_i$, is employed, the marginal utility becomes:

$$(6) \quad U'_i = 1 - 2a_i\tilde{W}_i.$$

Substitution of (6) into (5) and rearranging terms yields:

$$(7) \quad \theta_i \ [E(R_j) - \frac{1 + C'_{ij}(S_{ij})}{1 + C'_{if}(F_i)} R_f] = Cov(\tilde{W}_i, R_j),$$

where:

$\theta_i = E(U'_i) / 2a_i$.

From equation (7) the equilibrium condition specifies that an investor i would allocate wealth to various securities such that the risk aversion coefficient, θ_i, times the expected excess return on security j over the transaction-costs-adjusted

[5]Note that $E(U'_iR_j) = E(U'_i)E(R_j) + Cov\ (U'_i,R_j)$.

riskless rate is equated to the covariance between the investor's future wealth and the return on security j.

In order to examine an individual investor's trading behavior, equation (7) is rewritten for a given expected return $E(R_j) = E(\tilde{S}_{ij}) / S_{ij}$ as:

$$(8) \quad S_{ij} = \frac{E(\tilde{S}_{ij})}{\dfrac{Cov(\tilde{W}_i, R_j)}{\theta_i} + \dfrac{1 + C'_{ij}(S_{ij})}{1 + C'_{if}(F_i)} R_f} .$$

This result is not of a completely closed form for S_{ij}, but is solvable for certain specifications of the transaction cost function (e.g., a log form) and is sufficient for analyzing an investor's wealth allocation under different scenarios of transaction costs.

First, if $C'_{ij} = C'_{if} = 0$ (no transaction cost case), S_{ij} is determined for given values of $E(\tilde{S}_{ij})$, $Cov(\tilde{W}_i, R_j)$, θ_i and R_f as in the traditional CAPM approach. Second, consider that C'_{ij} is a constant for all j and also $C'_{ij} > C'_{if}$ (fixed transaction cost rate case). Then, equation (8) shows that S_{ij} becomes smaller than that under the zero transaction cost case. The relative size of the decrease in S_{ij} is approximately the same among risky assets, however, because C'_{ij} is constant. In this case if $C'_{ij} = C'_{if}$, then according to equation (8), S_{ij} remains unchanged. This implies that transacting in the risk free asset becomes relatively more costly than transacting in risky assets. Finally, in the presence of differential marginal transaction costs (decreasing rate), the decrease in S_{ij} varies among assets; the larger the marginal transaction cost of asset j, the larger the decrease in S_{ij}.

In brief, the analysis suggests that investors will hold less amount and less number of risky assets under fixed or decreasing marginal transaction cost rates than otherwise would be permitted with no transaction costs. The reason is that relatively more costly assets are not chosen or chosen less, other factors being equal. When capital markets are in equilibrium, however, the condition of homogeneous expectations requires investors to hold assets exactly the same as if there were no transaction costs. Only returns before transaction costs are adjusted in the final equilibrium.

The Market Equilibrium

To obtain the market equilibrium condition, some researchers (e.g., Levy [17], Mayshar [20, 21], and Merton [22]) include in the security's demand function only investors who decide to hold the security. This presupposes that investors who decide to buy a security are known before the equilibrium price is determined. Further, investors who consider a security, given $C_{if}'(F_i)$ and $C_{ij}'(S_{ij})$, but decide not to hold it are excluded from the demand function. It would appear to be more appropriate to include all investors in specifying the security's demand function. Investors who decide to purchase a security are not known *ex ante* and those who consider but decide against holding a security still constitute the lower end of the demand curve.

Aggregating equation (7) then over all investors yields:

$$(9) \quad \sum_i \theta_i \left[E(R_j) - \frac{1 + C_{ij}'(S_{ij})}{1 + C_{if}'(F_i)} R_f \right] = S \, Cov(R_m, R_j),$$

where:

$$\sum_i \tilde{W}_i = \sum_i \sum_j S_{ij} R_j = \sum_j S_j R_m = S R_m;$$

$$S = \sum_j S_j \text{ and}$$

R_m = the average rate of return on all risky securities.
Thus, the equilibrium condition for security j becomes:

$$(10) \qquad E(R_j) = \delta_0 R_f + \delta_1 \beta_j,$$

where:

$$\delta_0 = \frac{\sum_i \theta_i \dfrac{1 + C'_{ij}(S_{ij})}{1 + C'_{if}(F_i)}}{\sum_i \theta_i};$$

$$\delta_1 = \frac{S\sigma_m^2}{\sum_i \theta_i};$$

$$\beta_j = Cov(R_m, R_j) / \sigma_m^2.$$

That is, δ_0 is the weighted average of marginal transaction costs of security j incurred by investors using their risk aversion coefficients as weights, and δ_1 is the market price of risk.

Note that if $C'_{if} = C'_{ij} = 0$ or if $C'_{if} = C'_{ij} = c$, where c is a constant, then equation (10) reduces to the traditional CAPM: $E(R_j) = R_f + \delta_1\beta_j$. Equivalently, if marginal transaction costs are zero or constant across securities, the equilibrium condition specified by the traditional CAPM is maintained.

Alternatively, in the more prevalent case where C'_{if} and $C'_{ij} > 0$ and C''_{if} and $C''_{ij} < 0$ such that the C_{ij} values are not the same across firms because of differences in size, trading activities, and riskiness, equation (10) reveals that δ_0 is an increasing function of C'_{ij} and hence $E(R_j)$. To develop this basic result further, consider a costless trading in the riskless asset, and let $\bar{C}'_j$ be the weighted average of C'_{ij} using investors' risk averse coefficients as weights. Equation (10) then becomes:

(11) $$E(R_j) = R_f(1 + \bar{C}'_j) + \delta_1\beta_j.$$

For the market portfolio, equation (11) shows that δ_1 becomes $E(R_m) - (1 + \bar{C}'_m)R_f$, where $\bar{C}'_m$ is the market value-weighted average of $\bar{C}'_j$ values.[6] Equation (11) thus can be rewritten as:

[6]Because the market return is the market value-weighted average of individual returns, from equation (11) it can be shown that:

(12) $E(R_j) = R_f(1 + \bar{C}_j') + [E(R_m) - (1 + \bar{C}_m')R_f]\ \beta_j.$

Therefore, equation (12) establishes that the expected return on security j in equilibrium depends not only on its systematic risk, but also on its average marginal transaction costs. More specifically, from equation (12) it can be shown, *ceteris paribus*, that,

(i) The greater are the average marginal transaction costs of a security, the greater is its expected rate of return before transaction costs, and
(ii) the market price of risk is smaller than that required for the zero-transaction-costs CAPM; that is,

$$E(R_m) - R_f > E(R_m) - (1 + \bar{C}_m')R_f.$$

Equation (12) can be rewritten in the context of the traditional security market line to analyze the excess return as:

(13) $E(R_j) - R_f = R_f(\bar{C}_j' - \bar{C}_m'\beta_j) + [E(R_m) - R_f]\beta_j.$

$$E(R_m) = R_f\ (1 + \sum_j w_j\ \bar{C}_j') + \delta_1 \sum_j w_j\beta_j,$$

where:

w_j = the market value weight for asset j.

This results in:

$\delta_1 = E(R_m) - (1 + \bar{C}_m')\ R_f,$

where:

$$\bar{C}_m' = \sum_j w_j\ \bar{C}_j' \text{ and } \sum_j w_j\beta_j = 1.$$

Thus, $\bar{C}_m'$ is the market value-weighted average of the average marginal transaction costs of individual assets.

Thus, the excess return, $R_f(\bar{C}_j' - \bar{C}_m'\beta_j)$, is associated positively with $\bar{C}_j'$ and is associated negatively with β_j. In terms of Merton's [22] results, the respective shadow costs for the market portfolio and security j are $R_f\bar{C}_m'$ and $R_f\bar{C}_j'$. Therefore, the condition for the excess return to exist is: $\beta_j < (\bar{C}_j' / \bar{C}_m')$. This indicates that any stocks, not necessarily small firm stocks, can generate positive excess returns given that β_j is smaller than $\bar{C}_j' / \bar{C}_m'$. In general, small firm stocks tend to have greater beta values and greater transaction costs than large firm stocks. For the size effect to exist, it is required that small firm stocks have $\beta_j < (\bar{C}_j' / \bar{C}_m')$, whereas large firm stocks do not.

Figure 1 depicts the functional relationship between the systematic risk and the equilibrium rate of return with and without transaction costs. Security 1 is assumed to be costless to trade, but transaction costs exist for trading other securities, i.e., $\bar{C}_m' > 0$. The line representing security 1 intercepts at the riskless rate and has a smaller slope than the traditional CAPM line. The increase in average marginal transaction costs from security 1 to securities 2 and 3 results in a parallel upward shift in the equilibrium security market line.[7] That is, as the average marginal transaction costs increase, the equilibrium market price is determined at a higher gross expected rate of return for a given beta value.

Empirical Evidence
Prior Studies

Several empirical studies support relation (12) which specifies that stocks requiring larger transaction or information costs generate larger expected gross returns on the risk-adjusted basis. The bid-ask spread often is used as a proxy for transaction costs. Stoll and Whaley [32] report that during the 1960-1979 period, the mean relative bid-ask spread decreases from 2.93 percent for the portfolio of the smallest market value decile of firms to 0.69 percent for the largest decile firms. Also, the mean commission rate decreases from 1.92 percent for the smallest decile firms to 1.01 percent for the largest decile firms. Similar evidence has been documented by Demsetz [11], Tinic [33], Tinic and West [34, 35], Benston and Hagerman [5], Stoll [31], and Grant and Whaley [15]. Stoll and Whaley find that when returns on securities are adjusted for transaction costs, the size effect does not exist.

In addition, Arbel, Carvell, and Strebel [2] show that lesser known neglected stocks yield larger gross expected returns than widely known stocks. Barry and Brown [4] examine the differential information about securities as a factor

[7] The risky security 2 line is similar to Black, Jensen, and Scholes' [6] finding that high (low) risk securities earn lower (higher) returns than what the traditional CAPM predicts.

explaining differential returns. The period of listing used as a crude measure of differential information is found to be associated closely with the firm size effect. Amihud and Mendelson [1] use the same bid-ask spread data as Stoll and Whaley to show that the expected return is an increasing and concave function of the spread. When the size variable is added to the return-spread relation, the size effect is found to be insignificant; hence, the size effect appears to be a consequence of the spread effect. Merton [22] uses investor base as a proxy for information costs and reports that less known stocks with smaller investor bases tend to have relatively larger expected returns than stocks with larger investor bases.

Test of Equation (12) Using the Bid-Ask Price Spread

As discussed earlier, the research to date supports equation (12) as regards the effect of transaction costs and more broadly defined information costs on the equilibrium return. This study similarly uses the bid-ask spread as a measure of transaction costs to test the transaction cost hypothesis. The bid-ask spread is defined as:

$$SP_j = \frac{A_j - B_j}{0.5(A_j + B_j)} \tag{14}$$

where:

SP_j = the bid-ask spread for security j;
A_j = the ask price; and
B_j = the bid price.

The empirical model to test (12) can be shown as:

$$\bar{R}_j - R_f = k_0 + k_1 SP_j + k_2 \hat{\beta}_j + u_j \tag{15}$$

where:

$\bar{R}_j$ = mean rate of return on security or portfolio j;
$\hat{\beta}_j$ = estimate of the beta coefficient;
u_j = random error.

k_0 is the constant, k_1 is hypothesized to be positive, and k_2 is the estimate of

consequence of the spread effect. Merton [22] uses investor base as a proxy for information costs and reports that less known stocks with smaller investor bases tend to have relatively larger expected returns than stocks with larger investor bases.

Test of Equation (12) Using the Bid-Ask Price Spread

day in each year of the study period from *Stock Quotations on the New York Stock Exchange* published by F. E. Fitch. The three month Treasury bill rates were used as the riskless rate and the monthly S & P 500 Index returns as the market returns. A total of 404 NYSE firms that have complete data were examined. Equation (15) was tested at the portfolio level for the total period as well as for two ten year subperiods, 1966-1975 and 1976-1985. Table 1 shows the descriptive statistics for the variables using the 404 sample securities.

In order to reduce the selection bias and measurement errors, portfolios were established according to the method suggested by Black, Jensen, and Scholes [6]. First, firms were ranked in ascending order of beta values during the first five year period, and then they were divided equally into six beta groups. Next, firms within each beta group were ranked in ascending order of SP_j at the beginning of the five year period. Then they were divided equally into six subgroups. Thus, a total of 36 (six beta levels times six SP_j levels) portfolios were established for the five year period. Then, the 36 portfolios were updated similarly for each of the remaining five year periods of a study period. Finally, for each of the 36 portfolios the average return, beta, and average size of the bid-ask spread were computed. When the numbers of firms were not divided evenly by six, the first groups received one more firm. Approximately 11 securities were included in each portfolio. As shown in Table 2, the correlations between the beta coefficient and the SP_j variable are all insignificant at the 10 percent level. Therefore, the multicollinearity problem of the two independent variables in equation (15) is seemingly unimportant.

Table 3 reports the results of the contemporary cross-sectional OLS regressions of equation (15) for the total period (1966-1985) as well as for the two ten year subperiods (1966-1975 and 1976-1985). The table also includes as a benchmark the results of the regressions of the traditional CAPM using the same portfolios. The k_1 coefficients for SP_j are significantly positive at the 1 percent level for both the total period and the second subperiod. The k_1 coefficient for the first subperiod is positive but statistically not significant. The k_2 coefficients for β_j are positive and significant for all the periods. Also, the k_2 values are slightly smaller than those obtained by the traditional CAPM in the total period and the second subperiod. Thus, the overall results support the hypothesis that transaction costs are important in explaining differences in security returns.

Test of Equation (12) Using Alternative Measures of Transaction Costs

In this section, several alternative proxy measures of transaction costs (such as share price and trading activities) are employed to show additional evidence on the hypothesis. In order to derive an empirically testable form of the relationship expressed in equation (12), it is hypothesized that transaction costs are related to a broader set of proxies that represent firm size, price per share, and trading activities. Moreover, the following uniform log cost function is assumed:

(16) $\bar{C}_j = b_0 + b_1 \, Ln(X_j)$,

where:

b_0 = positive constant;
b_1 = positive marginal transaction costs rate; and
X_j = proxy measure of transaction costs for firm j.

The first and second partials of equation (16) are:

(17) $\bar{C}'_j = b_1/X_j > 0$, and

(18) $\bar{C}''_j = -b_1/X_j^2 < 0$.

The function expressed in equation (16) thus satisfies the decreasing marginal transaction costs condition.

Substitution of equation (17) into equation (12) yields:

(19) $E(R_j) = R_f(1 + b_1/X_j) + [E(R_m) - (1 + b_m/X_m)R_f] \, \beta_j$.

Equivalently, equation (19) can be formulated in terms of the following empirically testable regression model:

(20) $\bar{R}_j - R_f = d_0 + d_1(1/X_j) + d_2\hat{\beta}_j + v_j$,

where:

d_0 = constant;
d_1 = estimate of b_1R_f;
d_2 = estimate of $[E(R_m) - (1 + b_m/X_m)R_f]$;
v_j = random error.

Given the above formulation, if X_j is to explain the cross-sectional variations in security returns, d_0 is expected to be zero, and d_1 and d_2 are expected to be positive.

In this study, the following six measures are chosen for the variable X_j for each firm:

- Market value of equity (MV),
- Book value of total assets (BV),
- Net sales (SA),
- Share price (PR),
- Number of shares outstanding (SO),
- Number of shares traded (ST).

Because the size of the firm is associated negatively with the marginal transaction costs, three firm size variables (market value of equity, book value of total assets, and net sales) are included. The market value of equity is used most often as a size variable in various studies on the firm size effect. The share price was included because brokerage commission rates are related negatively to the share price. More actively traded and more widely known firms are associated with less information/transaction costs. The number of shares traded and outstanding during a year were included as indicators of trading activities.

There are two purposes of using these various proxies. First, even if the transaction cost effect is well represented by the market value of equity, alternative transaction cost-related variables would be worth identifying. Second, if the trading activity variables including share price are related to security returns, it can be inferred that these trading-related variables not only explain the variations in security returns, but also the size variables are proxies for transaction costs with the trading variables.

Annual data for the proxy variables for the same sample firms used in the previous test were drawn from the COMPUSTAT tapes for the same test period. Table 1 also includes the descriptive statistics for these proxy variables. The proxy measures across firms are highly positively correlated to each other. Share price appears to be the least correlated variable among the six measures. For each proxy variable, 36 portfolios were established following the same procedure as explained in the previous section. As also shown in Table 2, the correlations between the beta coefficient and the inverse of the proxy variables are all insignificant at the 10 percent level.

Table 4 reports the results of the cross-sectional OLS regressions of equation (20) for each of the six proxy variables for the total period (1966-1985) as well as for the two ten year subperiods (1966-1975 and 1976-1985). For the three periods the coefficients (d_1) for all the proxy variables are significantly positive at the 1 percent level except for SA and PR in the first subperiod. The results therefore indicate that the relationship of security returns with the alternative proxy variables used here is as strong as with the more commonly used market value of equity. This supports the hypothesis that transaction costs, which are considered the common factor of the proxies, are important in explaining differences in security returns.

The market price of risk, d_2, is significantly positive in each of the six regressions for each period. The intercept values (d_0) for the total period for all the proxy variables are all close to zero as hypothesized in equation (18). For the two subperiods, however, the intercept values are either significantly positive or negative. Thus, the intercept term appears to be intertemporally unstable.

Considering that the above results differ from those obtained from the traditional CAPM, the following is noted. Except for SA and ST, the d_2 values are consistently smaller than those obtained from the zero-transaction-cost CAPM. This finding is consistent with the hypothesis shown earlier from equation (12) about the reduction in the market price of risk as a consequence of introducing transaction costs. Moreover, the overall explanatory power, R^2, of the transaction costs-based regressions is substantially higher than the traditional CAPM counterpart. In many cases, the addition of the Xj variable more than doubled the R^2 value. This result lends credence to the importance of transaction costs as an omitted variable in the CAPM, hence a factor related to those costs should be considered in the pricing model.

The results of the tests, overall, indicate that not only the bid-ask spread but also the market value of equity, share price, and trading activity variables strongly explain cross-sectional variations in the expected returns of securities. Thus, it should be noted that this study is not simply to test the firm size effect, but rather to show that (consistent with previous evidence) transaction cost-related variables are found to be important factors in explaining asset pricing.

Concluding Remarks

The equilibrium pricing model derived in this study demonstrates that under the decreasing marginal transaction costs, the expected rate of return on a security or portfolio before transaction costs is related positively to both the average marginal transaction costs and systematic risk. The model indicates that the before-transaction-costs expected rate of return on a security becomes greater as its average marginal transaction costs increase. The introduction of transaction costs also is shown to result in a smaller market price of risk than that obtained from the traditional CAPM. Given that transaction costs are a decreasing function of firm size, the analysis provides a theoretical basis for explaining the relationship between transaction costs and the small firm effect. Additionally, it is shown that the structure of security returns as derived by the traditional CAPM is not altered when marginal transaction costs are either zero or constant across firms.

This study uses the bid-ask spread as a measure of transaction or information costs. Also, based on findings of prior research regarding the average marginal transaction costs of a security being related inversely to the size of the firm, share price, and trading activities, various proxy variables were used to test the model developed here. The results of the test are consistent with the relationship derived from the model. In general, these findings confirm the empirical evidence provided by Stoll and Whaley [32], Barry and Brown [4], Amihud and Mendelson [1], and Merton [22] that return anomalies are related to transaction and information costs as measured in terms of the bid-ask spread, period of listing, and investor base, and that firm size is one of several proxies for these costs. Transaction cost proxy variables tested in this study are found to substantially improve the explanatory power of the pricing model. Although this study explains the transaction cost-related size effect, it does not attempt to explain the seasonality of security returns (January effect). The theoretical explanation of the seasonality is beyond the scope of the current study and may be addressed in future research.

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Table 1
Correlations, Means, and Standard Deviations of Variables (404 Stocks)

	β_i	MV	BV	SA	PR	SO	ST	SP	Mean	SD
1966-1975										
β_i	1								1.112	.354
MV	-.12	1							1045.4	2871.9
BV	-.14	.70	1						1356.7	3606.6
SA	-.10	.74	.74	1					1186.3	2552.9
PR	-.01	.61	.19	.27	1				38.0	26.9
SO	-.15	.82	.87	.82	.26	1			21.8	39.5
ST	.17	.51	.55	.59	.26	.66	1		3.7	3.7
SP	.19	-.37	-.29	-.34	-.57	-.37	-.40	1	.01133	.00486
$\bar{R}_i - R_f$	.17	.01	-.09	-.07	.26	-.05	.01	-.23	.00168	.00616
1976-1985										
β_i	1								.987	.379
MV	-.08	1							1712.2	4169.0
BV	-.08	.73	1						3555.1	8227.7
SA	.03	.74	.78	1					3520.1	7780.8
PR	.11	.50	.24	.30	1				32.2	17.9
SO	-.14	.89	.83	.82	.30	1			44.2	71.2
ST	.02	.77	.70	.72	.34	.87	1		18.4	21.7
SP	.09	-.17	-.13	-.13	-.32	-.17	-.24	1	.01105	.01007
$\bar{R}_i - R_f$	.24	-.13	-.12	-.11	.09	-.20	-.21	.09	.00763	.00656

Table 1 (continued)
Correlations, Means, and Standard Deviations of Variables (404 Stocks)

1966-1985	β_i	MV	BV	SA	PR	SO	ST	SP	Mean	SD
β_i	1								1.055	.327
MV	-.12	1							1378.8	3477.3
BV	-.13	.73							2455.9	5882.9
SA	-.08	.75	.77	1					2353.2	5124.1
PR	.05	.61	.25	.33	1				35.1	20.6
SO	-.18	.88	.86	.84	.37	1			33.0	54.1
ST	.03	.76	.69	.73	.45	.85	1		11.1	12.3
SP	.13	-.28	-.23	-.24	-.47	-.30	-.39	1	.01118	.00633
$\bar{R}_i - R_f$	.34	-.08	-.14	-.10	.14	-.16	-.11	-.02	.00466	.00412

Note: MV = market value, BV = book value, SA = sales, PR = share price, SO = number of shares outstanding, ST = number of shares traded, and SP = bid-ask spread. MV, BV, SA, SO, and ST are in millions of units

Table 2

Correlations Between β_j and Transaction Cost Measures for 36 Portfolios

Measures	1966-1975	1976-1985	1966-1985
1/MV	.21	.11	.15
1/BV	.23	.05	.09
1/SA	-.10	-.12	-.12
1/PR	.12	.14	.13
1/SO	.18	.02	.07
1/ST	-.25	-.22	-.23
SP	.01	.16	.10

Note: See Table 1 for the definitions of the variables

Table 3
Results of Regressions with 36 Portfolios:

$$\bar{R}_j - R_f = d_0 + d_1 SP_j + d_2\hat{\beta}_j + v_j$$

(t-values are in parentheses)

	d_0	d_1	d_2	R^2
1966-1975	-.0015	.0246	.0027	.23
	(-1.27)	(.41)	(3.09)	
	-.0013		.0027	.22
	(1.26)		(3.13)	
1976-1985	.0019	.2437	.0032	.66
	(2.23)	(5.98)	(4.34)	
	.0038		.0039	.29
	(3.41)		(3.77)	
1966-1985	-.0002	.1459	.0031	.57
	(-.23)	(3.82)	(5.02)	
	.0011		.0033	.38
	(1.36)		(4.58)	

Note: SP = bid-ask spread

Table 4

Results of Regressions Using Different Measures of X_j with 36 Portfolios
(t-values are in parentheses)

$$\bar{R}_j - R_f = d_0 + d_1(1/X_j) + d_2\hat{\beta}_j + v_j$$

	1966-1975				1976-1985				1966-1985			
X_j	d_0	d_1	d_2	R^2	d_0	d_1	d_2	R^2	d_0	d_1	d_2	R^2
MV	-.0018	.1248	.0025	.28	.0027	.3803	.0034	.51	.0004	.2361	.0030	.53
	(-1.47)	(2.20)	(2.36)		(2.31)	(4.55)	(3.17)		(.42)	(4.34)	(3.67)	
	-.0016		.0029	.18	.0037		.0040	.20	.0010		.0035	.26
	(-1.23)		(2.72)		(2.54)		(2.98)		(.85)		(3.52)	
BV	-.0015	.1474	.0023	.25	.0025	.9390	.0037	.69	.0003	.5179	.0030	.65
	(-1.24)	(1.88)	(2.17)		(3.21)	(6.41)	(5.35)		(.34)	(5.60)	(5.01)	
	-.0014		.0027	.16	.0037		.0039	.30	.0011		.0034	.32
	(-1.09)		(2.59)		(3.42)		(3.85)		(1.17)		(4.04)	
SA	-.0020	.0977	.0029	.23	.0021	.8726	.0043	.54	-.0001	.3836	.0037	.57
	(-1.63)	(1.37)	(2.96)		(2.11)	(4.13)	(5.09)		(-.13)	(3.73)	(5.89)	
	-.0014		.0028	.19	.0038		.0039	.30	.0010		.0034	.38
	(-1.20)		(2.81)		(3.45)		(3.81)		(1.26)		(4.64)	

Table 4 (continued)
Results of Regressions Using Different Measures of X_j with 36 Portfolios
(t-values are in parentheses)

$$\bar{R}_j - R_f = d_0 + d_1(1/X_j) + d_2\hat{\beta}_j + v_j$$

	1966-1975				1976-1985				1966-1985			
X_j	d_0	d_1	d_2	R^2	d_0	d_1	d_2	R^2	d_0	d_1	d_2	R^2
PR	-.0017	.0130	.0027	.21	.0018	.0545	.0034	.65	-.0001	.0390	.0031	.56
	(-1.34)	(.64)	(2.76)		(2.23)	(5.28)	(4.96)		(-.11)	(3.49)	(5.01)	
	-.0013		.0027	.20	.0038		.0039	.35	.0011		.0034	.40
	(-1.19)		(2.88)		(3.87)		(4.29)		(1.38)		(4.79)	
SO	-.0022	.0090	.0022	.51	.0017	.0256	.0039	.59	-.0005	.0165	.0032	.71
	(-2.39)	(4.53)	(2.83)		(1.59)	(5.50)	(4.12)		(-.66)	(6.94)	(5.42)	
	-.0015		.0028	.20	.0037		.0040	.22	.0010		.0035	.28
	(-1.30)		(2.96)		(2.67)		(3.10)		(.86)		(3.62)	
ST	-.0024	.0009	.0031	.24	.0015	.0045	.0048	.50	-.0006	.0023	.0040	.53
	(-1.80)	(1.82)	(2.99)		(1.22)	(4.23)	(4.79)		(-.64)	(4.17)	(5.36)	
	-.0012		.0026	.16	.0038		.0038	.24	.0012		.0033	.29
	(-1.00)		(2.54)		(2.96)		(3.22)		(1.22)		(3.71)	

Figure 1
The CAPM With and Without Transaction Costs