The Valuation of American Calls on Futures Contracts: A Comparison of Methods

Trevor W. Chamberlain
McMaster University

Richard W. T. Chiu*
McMaster University

Abstract

This study extends the analysis of approximation methods for pricing American call options on futures contracts. Whereas previous studies have adopted a specific method as giving the true values of American options and then used those values as a basis for evaluating the European model, the present study develops binomial as well as both explicit and implicit finite difference approximations and then compares their predictions for a range of parameter values and futures prices.

Introduction

In recent years, there has been a proliferation of traded securities whose values are derived directly from the values of other assets. Among these securities are options written on futures contracts. Although such options have been in existence in Europe for many years, they only recently have become available in North America. In 1982, the United States Commodity Futures Trading Commission allowed each commodity exchange to trade options on one of its futures contracts. Since then trading in futures options has been initiated on every major exchange. The underlying spot assets include financial instruments such as bonds, Eurodollars, stock indices, foreign currencies, precious metals, and agricultural products.

The growth in interest in futures options within the investment community has been matched by a number of recent efforts to derive models for pricing them. The first such effort was that of Black [2], who derives a model for pricing European options on forward contracts that is identical in form to the well known Black-Scholes [3] formula. Under the assumption of a nonstochastic risk free rate of interest, Cox, Ingersoll, and Ross [8] have shown that Black's model can be applied to the pricing of options on futures as well.[1]

*The authors wish to thank three anonymous referees of the QJBE for their helpful comments and suggestions.

[1]The nonstochastic risk free rate assumption is required to establish the proper hedge ratio between forward and futures contracts. Otherwise the hedge

0747-5535/90/1300-0003 $01.00
copyright University of Nebraska-Lincoln

As Brenner, Courtadon, and Subrahmanyan [6] have pointed out, however, it may be optimal to exercise an American call on a futures contract before expiration. If so, the use of Black's pricing model will give inexact solutions.

The importance of the early exercise feature in pricing American options also has been studied by Courtadon [7], who analyzes Treasury bond options on both the spot and futures instruments, and Ramaswamy and Sundaresan [14], who compare options on stock indices with options on stock index futures. In the absence of an exact analytic solution to the pricing of the futures options, the latter authors employ the implicit finite difference approximation method. Whaley [16, 17] similarly has calculated the prices of American calls on futures using the Geske-Johnson [10] compound option approach and a quadratic approximation approach derived by Barone-Adesi and Whaley [1]. Finally, Shastri and Tandon [15] recently have compared the pricing performance of Black's European option model with that of a variation of Geske and Johnson's approximation formula.

Ramaswamy and Sundaresan infer, on the basis of their application of the implicit finite difference technique, that the value of the opportunity to exercise early was small. For out-of-the-money options, Whaley also finds that the prices obtained using the Black model were similar to those obtained using numerical methods, whereas for in-the-money-options, "the early exercise privilege contribute[d] meaningfully to the futures option value" [17, p. 133]. Similarly, Shastri and Tandon conclude that Black's model can be used to price American options on futures "if the option is not deep in the money and is near maturity" (p. 613). They also note that it performs well when the volatility of the futures price and the level of the risk free interest rate are low.

The present paper extends the analysis of approximation methods for pricing American call options on futures contracts. Whereas previous studies have adopted a specific method as giving the true values of American options and then used those values as a basis for evaluating the European model, the present study develops alternative approximation methods and then compares their respective predictions for a range of parameter values and futures prices. The underlying premise in considering alternative approaches is that as approximations, no single method can be adopted unequivocally as providing true American call prices. The paper concludes with a brief summary of the study's results and their implications.

---

factor will be uncertain and it will be impossible to construct an arbitrage portfolio. With stochastic interest rates, the futures price will depend on factors such as the correlation between changes in the price of the underlying asset and changes in the risk free rate.

The Valuation Methods
European Calls

The first attempt to derive a pricing equation that could be applied to options on futures contracts was undertaken by Black, whose model is identical in form to the Black-Scholes model for stock options when dividends are paid continuously. Because the model does not allow for the possibility of early exercise, however, it only can be relied upon to provide exact prices for European futures options.

In deriving his model, Black relied upon the following assumptions:

1. Futures prices follow a lognormal distribution with a known and constant variance;
2. The risk free rate of interest is nonstochastic and constant during the life of the option contract;[2]
3. Taxes and transactions costs are zero.

Under these assumptions, he showed that the partial differential equation governing the value of the call option is as follows:

$$(1) \quad C_t - rC + (1/2)F^2 \sigma^2 C_{FF} = 0,$$

where,

C	= the value of the option;
F	= the value of the futures;
r	= the risk free rate of interest;
σ	= the standard deviation of logarithmic asset return per period; and
C_{FF}	= the second derivative of C with respect to F.

The boundary condition for the partial differential equation in equation (1) is that on the expiration date, T,

[2] The nonstochastic risk free rate assumption may be challenged on the grounds that interest rate uncertainty has a bearing on the volatility of the underlying asset prices. Apart from the fact that this assumption is common to most option pricing models, as Whaley [17] points out, the validity of a model should not be evaluated on the basis of its assumptions, but on the quality of its predictions. Nonetheless, it is probably fair to say that models developed under this assumption are more appropriate for pricing options on commodities and commodity futures than for pricing options on financial assets such as debt instruments.

(2) $C_T = \max\,[0, F_T - E]$,

where,

E = the exercise price of the option.

Equation (1) can be solved under the assumption that the option cannot be exercised before the expiration date. If the rate of interest is constant, the spot price, S, can be written as a function of the futures price; that is, as $F = Se^{r\tau}$. Substituting into the Black-Scholes pricing expression for an European option:

(3) $C = e^{-r\tau}\,[FN(d_1) - EN(d_2)]$,

where

$$d_1 = \log\,(F/E)/\,(\sigma(\tau)^{1/2}) + (1/2)\sigma(\tau)^{1/2},$$

$$d_2 = d_1 - \sigma(\tau)^{1/2}, \text{ and}$$

$$N(d_1) = \int_{-\infty}^{d_1} (1/(2\pi)^{1/2})e^{-(1/2)x^2}dx,$$

which is a solution to equation (1).

American Calls

For American call options on stock, it can be shown that the option holder should not consider exercising the option except just before an ex dividend date. For American calls on futures contracts, in contrast, it may be advantageous to exercise the option even though the underlying asset does not pay a dividend. To see this, assume that the futures price is much larger than the exercise price. The two cumulative probability functions $N(d_1)$ and $N(d_2)$ in equation (3), therefore, will be close to one, and the option will have a value of $e^{-r\tau}(F - E)$. The immediate exercise value is F - E, however, so that given the choice, the option holder will choose to exercise immediately. As such, the opportunity for early exercise will make an American call option on a futures contract more valuable than an otherwise identical European call. Unfortunately, for American calls there is no known analytic solution to equation (1). Thus, it is necessary to find an approach for approximating the value of the call.[3]

The Binomial Method

The first approximation method to be examined is the well known binomial approach that was described first by Cox, Ross, and Rubinstein [9]. Though its primary importance is as a pedagogical tool, the binomial method also provides an efficient numerical procedure for valuing complicated option contracts. The application of the method to pricing futures options involves approximating the process according to which futures prices are determined. In particular, it is assumed that futures prices follow a multiplicative binomial process, with the price at time t+1 equal to:

$$(4)\quad F_{t+1} = \begin{cases} F_t e^u \text{ with probability } q \\ F_t e^d \text{ with probability } 1 - q \end{cases}$$

where

u, d, and q = constants.

What this statement says is that at time t+1, one of two events will occur: The futures price either will increase to $F_t e^u$, with a probability of q, or decrease to $F_t e^d$, with a probability of 1 - q. Similarly, at time t+2, the possible futures prices are $F_t e^{2u}$, $F_t e^{u+d}$, and $F_t e^{2d}$, with probabilities of q^2, $2q(1 - q)$, and $(1 - q)^2$, respectively, and, in general, at some future time T,

$$(5)\quad F_T = F_t e^{ju+(N-j)d} \text{ with probability } \binom{N}{j} q^j (1 - q)^{N-j},$$

where

N = the number of periods from t to T; and
j = the number of price increases between t and T.

[3]For a complete survey of option value approximation methods, see Geske and Shastri [11].

The next step is to find an expression for the value of a call option on the futures contract. To do this, either the risk free arbitrage technique or the expected value technique can be adopted.

The Risk Free Arbitrage Technique

The risk free arbitrage technique involves construction of a duplicating portfolio consisting of the futures contract and a bond. This portfolio is balanced in each time period in order to duplicate the cash flow pattern of the call option over its entire life [t,T]. If such a portfolio can be constructed, then at time t, the value of the call option should be equal to that of the portfolio.

It is assumed that at time t the portfolio consists of n_t futures contracts and m_t bonds. The bonds have a maturity of one period. Because the futures contracts do not represent anything of value, the value of the portfolio is:

$$(6)\quad V_t = m_t e^{-r\tau/N}$$

At time t+1, the n_t futures contracts will generate a cash flow of $n_t(F_{t+1} - F_t)$, and the m_t bonds will mature. The value of the portfolio is therefore:

$$(7)\quad V_{t+1} = n_t(F_{t+1} - F_t) + m_t$$

Because the futures price either can advance to $F_t e^u$ or drop to $F_t e^d$, one of two values for V_{t+1} is possible:

$$(8)\quad V^u_{t+1} = n_t F_t\,(e^u - 1) + m_t, \text{ or}$$

$$V^d_{t+1} = n_t F_t\,(e^d - 1) + m_t$$

Solving equations (6), (7), and (8) and eliminating terms, the following expression is obtained for V_t:

$$(9)\quad V_t = e^{-r\tau/N}[\phi V^u_{t+1} + (1 - \phi)V^d_{t+1}],$$

where

$$\phi \quad = (1 - e^d)/(e^u - e^d).$$

Proceeding in the same fashion until time T is reached gives the general binomial option pricing formula:

(10) $C_t = e^{-r\tau} \sum_{j=0}^{N} \binom{N}{j} \phi^j (1 - \phi)^{N-j} \max [0, F_t e^{ju + (N - j)d} - E]$

Except for the definition of ϕ, the form of this expression is identical to the binomial formula for common stocks derived by Jarrow and Rudd [12].

The Expected Value Technique

A more straightforward way of getting equation (9) is as follows: At time t+1, the futures price will have a value of $F_t e^u$, with probability q, and a value of $F_t e^d$, with probability 1 - q, so that

(11) $E(F_{t+1}) = qF_t e^u + (1 - q)F_t e^d$

As the expected future cash flow from the futures contract is zero, the futures price should remain unchanged; that is,

(12) $E(F_{t+1}) = F_t$

Substituting equation (12) into equation (11) gives

(13) $q = (1 - e^d)/(e^u - e^d) = \phi$

The next step is to find the appropriate values for u and d, keeping in mind that as $N \rightarrow \infty$, the binomial process should approximate a lognormal process. Under the assumption that futures prices follow a lognormal distribution with a known and constant variance:

(14) $\log (F_T/F_t) = -(\sigma^2/2)\tau + \sigma(\tau)^{1/2}Z$,

where

Z = a standard, normal random variable, with a mean equal to zero and a standard deviation equal to one.

If j is the number of upward movements in the futures price during the interval [t,T]:

(15) $\log (F_T/F_t) = Nd + (u - d)j$,

because, from equation (12), $F_T = F_t e^{ju + (N - j)d}$. Equating equation (14) and equation (15) gives

(16) $-(\sigma^2/2) + \sigma(\tau)^{1/2} Z = Nd + (u - d)j$

From the DeMoivre-Laplace theorem,[4] it can be shown that for large N,

(17) $(j - Nq)/(Nq(1 - q))^{1/2} \approx Z$

If j is set equal to one-half, then:

(18) $j \approx ((N)^{1/2}/2)Z + N/2$

Substituting into equation (16) yields

(19) $-(\sigma^2/2) + \sigma(\tau)^{1/2} Z = Nd + (N/2)(u - d) + (u - d)((N)^{1/2}/2)Z$

Equating coefficients,

(20) $u = \sigma(\tau/N)^{1/2}[1 - (\sigma/2)(\tau/N)^{1/2}]$; and

$d = \sigma(\tau/N)^{1/2}[1 + (\sigma/2)(\tau/N)^{1/2}]$

Jarrow and Rudd derive expressions paralleling equation (20) for a binomial stock process. Their expressions, unlike the present ones, include an interest rate term. This difference reflects the fact that under conditions of risk neutrality, which they assume, a stock is expected to earn a return equal to the risk free rate of interest, whereas a futures contract is expected to earn a return equal to zero.

The boundary condition described by equation (2) gives the option value at time T. Using equation (9), one can work backward to obtain the value of the option for any t. Because it is an American option that is being valued, however, the early exercise feature also has to be taken into consideration. In each time period, the intrinsic value of the option--that is, F_t - E--must be examined. If the exercise value is greater than the calculated present value, the option should be exercised, and the exercise value should be used in subsequent calculations.

---

[4]See Jarrow and Rudd [12, p. 189].

Finite Difference Methods

Instead of approximating the price process for the futures contract, one can solve the partial differential equation governing the value of the futures option. In those instances in which a closed form solution does not exist, there are a number of numerical techniques for approximating the differential equation.

Some of these techniques involve replacing the partial derivatives by finite differences and, as such, are referred to as finite difference methods. There are various ways in which the partial derivatives can be stated as finite differences. All of them, however, lead to solutions that can be classified as either explicit or implicit. Whereas the explicit solution procedures involve solving for option prices in terms of previously known prices, the implicit procedures require that a set of simultaneous equations be solved. In the discussion that follows, both explicit and implicit finite difference approximations for futures options will be considered.

From equation (1), the partial differential equation to be approximated can be written as

(21) $C_t = rC - (1/2)F^2\sigma^2C_{FF}$

As shown by Brennan and Schwartz [4], when using finite difference methods it is possible to simplify the solution by first log transforming the equation. To obtain the log transform of equation (21), let $y = \log F$ and $W = C$, so that $C_t = W_t$ and $C_{FF} = [W_{yy} - W_y]e^{-2y}$. Substituting into equation (21), one obtains the log transformed equation

(22) $W_t = rW + (1/2)\sigma^2W_y - (1/2)\sigma^2W_{yy}$

The transformed equation has two desirable characteristics: it simplifies numerical analysis because its coefficients are constant; and, unlike the explicit finite approximation to equation (21), the approximation to equation (22) is generally stable in the sense that during the approximation process the errors become progressively smaller.

The Explicit Finite Difference Approximation

If h and k are discrete increments in the futures price and time, respectively, and $i = 0 \ldots n$ and $j = 0 \ldots m$ are index variables,[5] one can begin by defining

[5]For the index variable ranges indicated, the stock price will lie between the minimum and maximum values, and the time variable will evolve from the current time to the expiration of the option.

(23) $W(y, t) = W(ih, jk)$

An explicit finite approximation to the partial derivatives of W(y, t) can be written as follows:

$$(24)\quad W_y = (W_{i+1,\, j+1} - W_{i+1,\, j+1})/(2h)$$

$$W_{yy} = (W_{i+1,\, j+1} - 2W_{i,j+1} + W_{i-1,\, j+1})/h^2$$

$$W_t = (W_{i,j+1} - W_{i,j})/k$$

Substituting into equation (23) yields

$$(25)\quad W_{i,j} = (1/(1 + rk))\, [aW_{i-1,\, j+1} + bW_{i,\, j+1} + cW_{i+1,\, j+1}],$$

where

$a = (\sigma^2 k/(2h))\, [1/h + 1/2];$

$b = 1 - (\sigma/n)^2 k$; and

$c = (\sigma^2 k/(2h))\, [1/h - 1/2].$

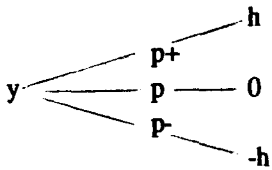
Using equation (25), one is able to solve for $W_{i,j}$ in terms of $W_{i,j+1}$. Because the terminal values of $W_{i,j}$ are given by the boundary conditions, the system of equations represented by equation (25) can be solved recursively for all values of $W_{i,j}$.

As explained by McCracken and Dorn [13], in order to obtain a stable explicit solution, the coefficients must be nonnegative. To ensure that this condition is met, it is necessary to set $h < 2$ and $k < h^2/\sigma^2$. If the equation is not transformed, the corresponding coefficients will be negative for sufficiently large values of i, and the explicit finite difference approximation generally will be unstable.

It also should be noted that the coefficients add to one; with the nonnegativity condition, the coefficients can be interpreted as probabilities:

$$(26)\quad W_{i,j} = (1/(1+r)^k)\, [p^- W_{i-1,j} + pW_{i,j} + p^+ W_{i+1,j}]$$

The value of the option in any period j may be regarded as its expected next period value discounted to the present at the risk free rate of interest. The expected value in the next period is obtained by assuming that y, the logarithm of the futures price, follows a jump process:



Finally, because the option, as an American option, can be exercised at any time prior to expiration, the exercise value in all time periods has to be examined. If the exercise value is greater than any of the discounted future values, the option should be exercised.

The Implicit Finite Difference Approximation

Using the implicit finite difference approximation method, the partial derivatives of W(y,t) in (23) become:

$$(27)\quad W_y = (W_{i+1,j} - W_{i-1,j})/(2h)$$

$$W_{yy} = (W_{i+1,j} - 2W_{i,j} + W_{i-1,j})/h^2$$

$$W_t = (W_{i,j+1} - W_{i,j})/k$$

Substituting into equation (22) yields:

$$(28)\quad W_{i,j+1} = (1/(1 - rk))\,[aW_{i-1,j} + bW_{i,j} + cW_{i+1,j}],$$

where

$a = -(\sigma^2 k/(2h))\,[1/h + 1/2],$

$b = 1 + (\sigma/h)^2 k;$ and

$c = (\sigma^2 k/(2h))\,[1/2 - 1/h]$

For any value of j, equation (28) represents a system of n equations with n + 2 unknowns. Thus, two more conditions are required to solve the system. The first condition can be obtained by noting that when the futures price is very low, the probability of exercising the option is slight, and the value of the option approaches zero; that is,

$$(29)\quad \lim_{F \to 0} C = 0$$

In order to approximate condition (29), it is assumed that for sufficiently small $F_{i,j}$,

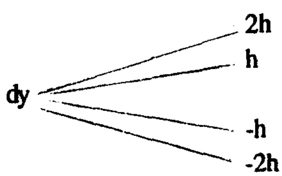
$$(30)\quad W_{0,j} = W_{1,j}, \quad j = 0 \ldots m$$

Using (30), equation (28) can be transformed into a new set of equations

$$(31)\quad g_i = e_i W_{i-1,j} + f_i W_{i,j}$$

If it now is assumed that for high futures prices the option will be exercised immediately, the value of the option will be its immediate exercisable value. In other words, let $W_{n,j} = E_j$ and solve for $W_{n-1,j}$. If $W_{n-1,j}$ is larger than E_j, one can solve for the other values of $W_{i,j}$. If $W_{n-1,j}$ is smaller than E_j, let $W_{n-1,j} = E_j$ and solve for $W_{n-2,j}$. Continuing in this manner, the entire set of equations can be solved. The process is repeated for all other periods in time.

As shown by Brennan and Schwartz [5], at any time j, the option value, $W_{i,j}$, also can be viewed as the expected option value in the next period discounted by the risk free rate of interest. Unlike the finite difference technique, however, it is assumed that the logarithm of the futures price follows a generalized diffusion process:



The futures price thus is allowed to jump to an infinite number of possible future values. As such, the method would appear to allow reasonably accurate approximations.

Simulation Results

The preceding section surveyed and, as required, developed the binomial, explicit finite difference, and implicit finite difference methods for approximating the values of American calls on futures contracts. In this section, the results of using these methods, represented by equations (10), (25), and (28) are compared with those obtained for the Black model, represented by equation (3). The comparison is done for various combinations of the following futures prices and option parameters: futures price (F) = \$80 to \$120 in increments of \$10; exercise price (E) = \$100; time to maturity (T) = 90 days and 180 days; risk free rate of interest (r) = 0.08 and 0.12; and futures price volatility (σ) = 0.20 and 0.40. Simulation analysis not only permits an examination of the comparative statics properties of futures options, but also provides some insight into whether the opportunity to exercise an American option early makes an important contribution to its value.

The value of the early exercise feature should be reflected in differences between the European model's estimates and those obtained using the three approximation methods. For all models, however, similar changes in the option price estimates for a given change in either the futures price or an option parameter are expected. Specifically, in all cases, the option price should be an increasing function of the futures price, the term to maturity, and the futures price volatility and a decreasing function of the risk free rate of interest.

As for the simulation procedure, a maximum of 200 iterations was allowed in each case for the three approximation methods. If convergence (defined to be two successive iterations with no change in price, to the nearest cent) had not occurred by that time, the simulation was terminated. This was done in order to impose an upper limit on the variation in computational efficiency between the three methods.

The results of the analysis are presented in Table 1. For all of the models, one observes, not surprisingly, that as the life of the futures contract increases, the option becomes more valuable. This, of course, reflects the fact that given more time, the futures will have a higher probability of appreciating in value. Similarly, an increase in the standard deviation of the futures price increases the value of the option. As is the case in spot markets, the greater the volatility of the underlying asset, the higher the probability that the futures price will exceed the exercise price.

An increase in the interest rate, on the other hand, has an adverse effect on the value of the option. This, of course, is just the opposite of what one would expect for a call option on an asset trading in the spot market. In the case of the latter, an increase in the interest rate will reduce the present value of the amount that the option holder will have to pay when he or she exercises the option in the future and, as such, will make the option more valuable. For the holder of a call option on a futures contract, however, "increases in [the interest rate] are equivalent to a higher 'implicit dividend' on the futures price" [14, p. 1327].

When the indicated option values are examined on a model by model basis, one finds that application of either the binomial or the explicit finite difference method suggests that the opportunity to exercise before maturity increases the value of the option. In contrast, and contrary to expectation, the approximations offered by the implicit finite difference method are generally lower than the European option values. As between the binomial and explicit difference methods, the former's estimates consistently fall below those provided by the latter and, as such, suggest a smaller early exercise premium.

In order to highlight the differences in option values using the various techniques, the percentage variation of each American call value approximation from its European counterpart is presented in Table 2. What these variation statistics indicate is that according to the binomial and implicit difference approximations, the early exercise feature, as a percentage of the option value, becomes more important as the futures price increases relative to the exercise price. To this extent, the present results parallel those obtained by Ramaswamy and Sundaresan, Whaley, and Shastri and Tandon. Such, however, is not the case when explicit difference estimates are employed. The percentage variations of these estimates from the corresponding European option values suggest that the early exercise premium falls dramatically as the option moves into the money.

If alternative approximation methods give substantially different results, particular care must be taken in inferring whether and, if so, under what conditions Black's model can be used to approximate the prices of American calls. Although previous evidence has suggested that, in Whaley's words, "it may be appropriate to apply the computationally less expensive Black model to approximate the values of out-of-the-money American futures options" [16, p. 57], if the estimates offered by the explicit finite difference method are taken to be the true values, one may conclude that only the prices of deep-in-the-money options should be estimated using the European formula.

For all of the approximation methods, one observes that the percentage variations of the American call values from their European counterparts increase when the risk free rate of interest increases. This reflects the fact that when the interest rate is low, the opportunity to exercise early is of little value because the probability of occurrence is small. As the interest rate rises, early exercise becomes more likely, and the value of an American call will increase relative to that of an European call.

In Table 3, the percentage variations of the explicit difference and implicit difference estimates from the corresponding binomial estimates are presented. For both finite difference methods, the percentage variation declines as the futures price rises relative to the exercise price. This apparent tendency for the three American call estimates to converge may be interpreted as suggesting that in the absence of any reason for favouring a particular method, the estimates obtained using any one of them will be increasingly reliable as the futures price moves into the money.[6]

Summary

This paper began by reviewing the theory underlying the valuation of European calls on futures contracts and then presented alternative approximation methods when early exercise is allowed. Simulations involving the resulting models, in conjunction with plausible option parameters, suggested that the indicated value of the early exercise premium was contingent upon the American call value approximation method employed.

The results of previous studies indicated that this premium was only important in the pricing of in-the-money options. The present study's results for the explicit finite difference method, however, seem to suggest that the premium as a percentage of the call price contributes more to the pricing of out-of-the-money options. As such, it is difficult to specify the conditions under which Black's European option model may be relied upon to provide useful estimates of American futures option values.

Finally, the estimates obtained using the binomial explicit finite difference and implicit finite difference methods appear to converge as the futures price rises above the exercise price. Hence, even if it is not apparent which of the three methods provides the most accurate approximations, for deep-in-the-money options, at least, any one of them may be acceptable.

---

[6] The same inference may be drawn from an examination of the percentage variations of the implicit finite difference estimates from the explicit finite difference estimates.

References

1. Barone-Adesi, G. and R.E. Whaley, "Efficient Analytic Approximation of American Option Values," Working Paper No. 15, Institute for Financial Research, University of Alberta (1985).

2. Black, F., "The Pricing of Commodity Contracts," *Journal of Financial Economics*, 4 (January-March 1976), pp. 167-179.

3. Black, F. and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81 (May-June 1973), pp. 637-659.

4. Brennan, M. and E. Schwartz, "The Valuation of American Put Options," *Journal of Finance*, 32 (May 1977), pp. 449-462.

5. Brennan, M. and E. Schwartz, "Finite Difference Methods and Jump Processes Arising in the Pricing of Contingent Claims: A Synthesis," *Journal of Financial and Quantitative Analysis*, 13 (September 1978), pp. 461-474.

6. Brenner, M., G. Courtadon, and M. Subrahmanyan, "Options on the Spot and Options on Futures," *Journal of Finance*, 40 (December 1985), pp. 1303-1317.

7. Courtadon, G., "Options on Default Free Bonds and Options on Default Free Bond Futures: A Comparison," Working Paper, Graduate School of Business, New York University (1983).

8. Cox, J.C., J.E. Ingersoll, and S.A. Ross, "The Relation between Forward and Futures Prices," *Journal of Financial Economics*, 9 (December 1981), pp. 321-346.

9. Cox, J.C., S.A. Ross, and M. Rubinstein, "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, 7 (September 1979), pp. 229-263.

10. Geske, R. and H.E. Johnson, "The American Put Valued Analytically," *Journal of Finance*, 39 (December 1984), pp. 1511-1524.

11. Geske, R. and K. Shastri, "Valuation by Approximation: A Comparison of Alternate Option Valuation Techniques," *Journal of Financial and Quantitative Analysis*, 20 (March 1985), pp. 45-71.

12. Jarrow, R.A. and A. Rudd, *Option Pricing* (Homewood, Illinois: Richard D. Irwin, 1983).

13. McCracken, D. and W. Dorn, *Numerical Methods and Fortran Programming* (New York: John Wiley & Sons, 1969).

14. Ramaswamy, K. and S.M. Sundaresan, "The Valuation of Options on Futures Contracts," *Journal of Finance*, 40 (December 1985), pp. 1319-1349.

15. Shastri, K. and K. Tandon, "Options on Futures Contracts: A Comparison of European and American Pricing Models," *Journal of Futures Markets*, 6 (Autumn 1986), pp. 593-618.

16. Whaley, R.E., "On Valuing American Futures Options," *Financial Analysts Journal*, 42 (May-June 1986), pp. 49-59.

17. Whaley, R.E., "Valuation of American Futures Options: Theory and Empirical Tests," *Journal of Finance*, 41 (March 1986), pp. 127-150.

Table 1

Call Values Using Alternative Valuation Methods (Exercise Price (E) = 100)

Option Parameters[a]	Futures Price (F)	European	Binomial	Explicit Finite Difference	Implicit Finite Difference
T = 0.25	F = 80	0.04	0.04	0.05	0.00
r = 0.08	F = 90	0.70	0.70	0.77	0.03
σ = 0.20	F = 100	3.90	3.93	4.18	1.07
	F = 110	10.73	10.82	11.21	10.00
	F = 120	19.75	20.03	20.32	20.00
T = 0.25	F - 80	0.04	0.04	0.05	0.00
r = 0.12	F = 90	0.69	0.69	0.77	0.03
σ = 0.20	F = 100	3.87	3.90	4.13	1.07
	F = 110	10.63	10.76	11.11	10.00
	F = 120	19.55	20.01	20.17	20.00
T = 0.50	F = 80	0.29	0.29	0.40	0.00
r = 0.08	F = 90	1.69	1.70	1.96	0.03
σ = 0.20	F = 100	5.40	5.47	5.91	1.07
	F = 110	11.72	11.90	12.90	10.00
	F = 120	19.90	20.36	20.51	20.00
T = 0.50	F = 80	0.29	0.29	0.39	0.00
r = 0.12	F = 90	1.66	1.68	1.92	0.03
σ = 0.20	F = 100	5.30	5.39	5.81	1.07
	F = 110	11.49	11.77	12.68	10.00
	F = 120	19.51	20.24	20.29	20.00

Table 1 (continued)
Call Values Using Alternative Valuation Methods (Exercise Price (E) = 100)

Option Parameters[a]	Futures Price (F)	European	Binomial	Explicit Finite Difference	Implicit Finite Difference
$T = 0.25$	$F = 80$	1.15	1.17	1.37	0.00
$r = 0.08$	$F = 90$	3.49	3.53	3.92	0.03
$\sigma = 0.40$	$F = 100$	7.77	7.85	8.47	1.13
	$F = 110$	13.97	14.09	15.25	10.00
	$F = 120$	21.68	21.86	22.21	20.00
$T = 0.25$	$F = 80$	1.13	1.16	1.36	0.00
$r = 0.12$	$F = 90$	3.45	3.50	3.89	0.03
$\sigma = 0.40$	$F = 100$	7.69	7.79	8.41	1.13
	$F = 110$	13.83	14.00	15.16	10.00
	$F = 120$	21.46	21.74	22.04	20.00

[a] T = the time to expiration, r = the risk free rate of interest, and σ = the standard deviation of the logarithmic futures return

Table 2

Percentage Variations of the American Call Approximations from the European Call Values
(Exercise Price (E) = 100)

Option Parameters[a]	Futures Price (F)	Binomial	Explicit Finite Difference	Implicit Finite Difference
$T = 0.25$	F = 80	0.00	25.00	-100.00
$r = 0.08$	F = 90	0.00	10.00	-95.71
$\sigma = 0.20$	F = 100	0.76	7.18	-72.56
	F = 110	0.84	4.47	-6.80
	F = 120	1.42	2.89	1.27
$T = 0.25$	F = 80	0.00	25.00	-100.00
$r = 0.12$	F = 90	0.00	11.59	-95.65
$\sigma = 0.20$	F = 100	0.78	6.72	-72.35
	F = 110	1.22	4.52	-5.93
	F = 120	2.35	3.17	2.30
$T = 0.50$	F = 80	0.00	37.93	-100.00
$r = 0.08$	F = 90	0.59	15.98	-98.22
$\sigma = 0.20$	F = 100	1.30	9.44	-80.19
	F = 110	1.54	10.07	-14.68
	F = 120	2.31	3.07	0.50
$T = 0.50$	F = 80	0.00	34.48	-100.00
$r = 0.12$	F = 90	1.12	15.66	-98.19
$\sigma = 0.20$	F = 100	1.70	9.62	-79.81
	F = 110	2.44	10.36	-12.97
	F = 120	3.74	4.00	2.51

Table 2 (continued)
Percentage Variations of the American Call Approximations from the European Call Values
(Exercise Price (E) = 100)

Option Parameters[a]	Futures Price (F)	Binomial	Explicit Finite Difference	Implicit Finite Difference
$T = 0.25$	F = 80	1.74	19.13	-100.00
$r = 0.08$	F = 90	1.15	12.32	-99.14
$\sigma = 0.40$	F = 100	1.03	9.01	-85.46
	F = 110	0.86	9.16	-28.42
	F = 120	0.83	2.44	-7.75
$T = 0.25$	F = 80	2.65	20.35	-100.00
$r = 0.12$	F = 90	1.45	12.75	-99.13
$\sigma = 0.40$	F = 100	1.30	9.36	-85.31
	F = 110	1.23	9.62	-27.69
	F = 120	1.30	2.70	-6.80

[a] T = the time to expiration, r = the risk free rate of interest, and σ = the standard deviation of the logarithmic futures return

Table 3
Percentage Variations of the Explicit Finite Difference and Implicit Finite Difference Call Values from the Binomial Call Values (Excercise Price (E) = 100)

Option Parameters[a]	Futures Price (F)	Explicit Finite Difference	Implicit Finite Difference
$T = 0.25$	F = 80	25.00	-100.00
$r = 0.08$	F = 90	10.00	-99.96
$\sigma = 0.20$	F = 100	6.36	-73.54
	F = 110	3.60	-7.58
	F = 120	1.45	-0.15
$T = 0.25$	F = 80	20.00	-100.00
$r = 0.12$	F = 90	11.59	-95.65
$\sigma = 0.20$	F = 100	5.90	-72.56
	F = 110	3.25	-7.06
	F = 120	0.80	-0.05
$T = 0.50$	F = 80	27.50	-100.00
$r = 0.08$	F = 90	15.29	-98.24
$\sigma = 0.20$	F = 100	8.04	-80.44
	F = 110	8.40	-15.97
	F = 120	0.74	-1.77
$T = 0.50$	F = 80	34.48	-100.00
$r = 0.12$	F = 90	14.29	-98.21
$\sigma = 0.20$	F = 100	7.79	-80.15
	F = 110	7.73	-15.04
	F = 120	0.25	-1.19

Table 3 (continued)

Percentage Variations of the Explicit Finite Difference and Implicit Finite Difference Call Values from the Binomial Call Values (Excercise Price (E) = 100)

Option Parameters[a]	Futures Price (F)	Explicit Finite Difference	Implicit Finite Difference
$T = 0.25$	$F = 80$	17.09	-100.00
$r = 0.08$	$F = 90$	11.05	-99.15
$\sigma = 0.40$	$F = 100$	7.90	-85.61
	$F = 110$	8.23	-29.03
	$F = 120$	1.60	-8.51
$T = 0.25$	$F = 80$	17.24	-100.00
$r = 0.12$	$F = 90$	11.14	-99.14
$\sigma = 0.40$	$F = 100$	7.96	-85.49
	$F = 110$	8.29	-28.57
	$F = 120$	1.38	-8.00

[a] T = the time to expiration, r = the risk free rate of interest, and σ = the standard deviation of the logarithmic futures return

Copyright © 2002 EBSCO Publishing