Managing the riskiness of defined contribution pension funds in a fair-valuation context

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Abstract With reference to a defined contribution pension scheme, this paper investigates the computation of suitable risk indicators in a fair-valuation context. This subject involves theoretical issues about the choice of the models for the dynamics of interest and mortality rates. The risk analysis is performed by computing the expected tail loss in a stochastic financial and demographic scenario. Numerical applications illustrate the impact of such evaluations on the reserve quantification in a Monte Carlo simulation framework.

Keywords: *defined contribution pension funds, fair value, expected tail loss, mathematical reserve*

INTRODUCTION

Over the last decade, the stability of the insurance industry has been affected not only by the negative economic climate, but also such factors as the crash in the stock market, a steady fall in bond yields and increasing longevity. Regulators have therefore focused their attention on the need for accounting rules and solvency requirements. In particular, at the end of March 2004, The International Accounting Standard Board (IASB) issued the International Financial Reporting Standard 4 Insurance Contracts (IFRS 4), providing, for the first time, guidance on accounting — a first step in the IASB project to achieve the convergence of a large variety of insurance accounting practices.[1] On the one hand, the IFRS 4 'permits insurers to change their accounting policies for insurance contracts only if, as a result, their financial statements present information that is more relevant and no less reliable, or more reliable and no less relevant'; on the other hand, 'it permits the introduction of an accounting policy that involves remeasuring designated insurance liabilities so as consistently to reflect current interest rates and, if insurers so elect, other current estimates and assumptions'.[1] It has been noted that IFRS 4 requires significantly increased disclosure of accounting information, but only relatively limited changes in the accounting methodology. Today, a proper fair value accounting framework needs to be implemented.[2]

As the nature of the old-age pension is very close to the life insurance business, the same phenomenon can be observed in pension fund regulation as well. Indeed, some regulators (UK, Switzerland and The Netherlands) have already taken steps to introduce funding requirements to pension systems.

In general, the world of pension provision is currently undergoing two dramatic transitions: a shift from unfunded social security towards private funding and, within the private sector, a shift from defined benefit (DB) plans towards defined contribution (DC) ones. The reason for the second transition is the poor portability of DB schemes, so that when a worker moves jobs, he can end up with a much lower pension in retirement. Undoubtedly, this is a considerable hindrance to labour market flexibility.

By contrast, DC plans can be easily transferable between jobs, but they impose a wide range of risks on the plan member: the risk that inadequate contributions might be made into the plan, the risk of losses in the value of the pension fund due to falls in asset values, the risk of retiring when interest rates are low, and so on. Obviously, in the presence of a minimum benefit promised by the fund, such risks are transferred from the workers to the fund. In addition, the fund bears the risk of lower than expected mortality due to any underestimated improvements in life expectancy.

In such a framework, one of the key factors that must be carefully taken into account is the development of a suitable valuation model that incorporates financial and demographic risk. Generally, pension liabilities are not fully traded in the secondary market. Suitable models should therefore be developed which incorporate both financial and demographic risks, allowing valuations consistent with the information about market conditions available at each time of valuation. Thus, the financial risk should be quantified using the current observation of market prices and interest rates and the insurance risk should be valued by applying a mark-to-model approach that captures the market future trend of mortality.[3] On this basis, the estimate of the market price of insurance liabilities can be finally extracted.

In terms of solvency, instead, the market values of assets and liabilities need to be used to calculate the amount of capital required to cover all obligations over a defined time horizon at a given confidence level.

In the actuarial literature, a wide range of contributions deal with the fair valuation of insurance liabilities; in particular Milevsky and Promislow propose a stochastic approach to model the future mortality hazard rate in insurance contracts with the option to annuitise;[4] Bacinello deals with the problem of pricing a guaranteed life insurance participating policy;[5] Ballotta and Haberman analyse the fair-valuation problem of guaranteed annuity options in a stochastic mortality environment;[6] and Ballotta *et al.* compare the fair value approach with the traditional accounting system for the mathematical reserves.[7]

As far as the subject of pension fund liabilities is concerned, Olivieri and Pitacco study the effect of the longevity risk on the probability distribution of pension annuities,[8] while Blake *et al.* use the value-at-risk (VaR) methodology to assess how closely DC plans can replicate the benefits deriving from DB plans during the accumulation phase.[9] The same authors also consider the choices available to a member of a DC pension plan at the time of retirement for conversion of his pension fund into a stream of retirement income.[10]

The present paper focuses on risk management for DC schemes. In particular, it addresses the calculation of quantile-based risk measures (VaR and expected tail loss) of the liability of the fund. Quantile-based risk measures computation is founded on the predicted worst-case loss at a specific confidence level. If one considers the liabilities, worst cases coincide with their increase because these rises are additional costs which may result in either a profit shrinkage or a proper loss. Therefore, in this case, one must look at the liability cost distribution centred on the expected cost, where the critical values lie in the right-hand tail. The expected cost can be linked to the expected value of the liabilities without any modification of the relevant risk factors, that is using the informative set available at the beginning of the risk horizon.

On the other hand, as far as the risk factors are concerned, a full quantile-based risk measures estimation should consider both financial and demographic risk factors. If one concentrates only on the financial risk factors, the expected value of the liabilities, in a fair-valuation context, is the value calculated by means of the forward rates implied by the current spot rates. The expected cost is then given by the algebraic summation of the initial value of the liabilities and the expected value at the end of the risk horizon. In any case in which the expected cost is lower than the effective one, there is an accounting loss.

In this framework, the present study estimates, by means of a simulation procedure, the distribution of the potential periodic loss connected to the fund. Through this one can compute the maximum additional cost due to a change in the interest rates at a specific confidence level (VaR) and the average of losses exceeding VaR (expected tail loss, ETL).

The estimation of quantile-based risk measures can have multiple applications, both internal and external.[11] From a managerial point of view, this measure can be used as a benchmark for the quantification of the fund's riskiness. From an institutional perspective, these measures can be effectively applied for comparison among different funds, given that a fair-valuation system could increase the comparability of results across different entities.

The next section of this paper introduces analytical developments. The following section defines the financial and mortality scenario. After this, the paper discusses the simulation procedure and offers numerical evidence. Concluding remarks are presented in the final section.

THE MODEL

Consider a DC pension fund with an individual funding method, which pays off a capital amount resulting from a contribution accumulation process to the subscriber in case of predecease, disability or old age. In the presence of a guarantee of yield, the liability b_h undergone by the fund in the year h with respect to a generic subscriber is given by

$$b_h = \max\{W_h^A, W_h^{GAR}\} \qquad (1)$$

where W_h^A is equal to the share of the equivalent assets constituting the fund and where W_h^{GAR} denotes the minimum guaranteed benefit. In particular

$$W_h^A = \sum_{s=0}^{\xi} n_s a_\xi = \sum_{s=0}^{\xi} \frac{c_s}{a_s} a_\xi \qquad (1.1)$$

and

$$W_h^{GAR} = \sum_{s=0}^{\xi} n_s a_\xi (1+i)^{\xi-s} \qquad (1.2)$$

where n_s is the amount of the fund's share subscribed in the year s, a_ξ is the unit value of the share in the year ξ, c_s is the contribution paid to the fund in the year s, i is the minimum yield guaranteed and ξ denotes the maximum number of years during which the generic subscriber is a fund member before reaching the pension age. In the presence of a minimum benefit, contributions have to be calculated in such a way as to reach the level b_h of benefit promised by the fund. In this case, the contribution determination, by Equation (1.1), depends both on the unit value of the share in the year s and on the minimum guaranteed yield i.

Now, define $\{r_h;\ h = 0, 1, 2, \ldots \xi\}$ and $\{\mu_{x+h};\ h = 0, 1, 2, \ldots \xi\}$ as the random spot rate process and the mortality intensity process respectively, both of them measurable with respect to the filtrations $\mathbf{F}^r$ and $\mathbf{F}^\mu$. The above-mentioned processes are defined in a unique probability space $(\Omega, \mathbf{F}^{r,\mu}, \mathrm{P})$, such that $\mathbf{F}^{r,\mu} = \mathbf{F}^r \cup \mathbf{F}^\mu$.

In this context in case of death (d), disability (i) or old age (v), the liability of the fund towards a scheme member can be expressed in this manner:

$$W^L(0) = \sum_{h=0}^{\xi} b_h \left({}_{h-1/1}Y_x^{(d)} + {}_{h-1/1}J_x^{(i)} \right) + b_\xi {}_\xi Z_x \qquad (2)$$

where

$${}_{h-1/1}Y_x^{(d)} = \begin{cases} e^{-\Delta(h)} & h-1 < T_x \le h \\ 0 & otherwise \end{cases},$$

$${}_{h-1/1}J_x^{(i)} = \begin{cases} e^{-\Delta(h)} \\ 0 \end{cases}$$

if disability occurs, otherwise

$${}_\xi Z_x = \begin{cases} 0 & 0 \le T_x < \xi \\ e^{-\Delta(\xi)} & T_x \ge \xi \end{cases}$$

In the previous expressions, T_x is a random variable which represents the remaining lifetime of a person aged x and $\Delta(h) = \int_0^h r_u du$ is the accumulation function of the spot rate. More clearly, $W^L(0)$ can be expressed as follows:

$$W^L(0) = \sum_{h=0}^{\xi} b_h \left({}_{h-1/1}q_x^{(d)} + {}_{h-1/1}q_x^{(i)} \right) e^{-\Delta(h)} + b_\xi {}_\xi p_x e^{-\Delta(\xi)} \qquad (3)$$

where ${}_{h-1/1}q_x^{(d)}$ and ${}_{h-1/1}q_x^{(i)}$ respectively indicate the probability of death and disability in the time interval $[h-1, h]$, while ${}_\xi p_x$ is the probability of reaching the pension age.

FINANCIAL AND MORTALITY SCENARIOS

The valuation of the financial instruments involving the fund will be made assuming a two-factor diffusion process obtained by joining a

Cox-Ingersoll-Ross model[12] for the interest rate risk and a Black-Scholes model[13] for the stock market risk; thus the two sources of uncertainty are correlated.

The interest rate dynamics $\{r_t;\ t = 1, 2\ldots\}$ are described by means of the diffusion process

$$dr_t = f^r(r_t, t)dt + l^r(r_t, t)dZ_t^r \qquad (4)$$

where $f^r(r_t, t)$ is the drift of the process, $l^r(r_t, t)$ is the diffusion coefficient, Z_t^r is a standard Brownian motion; in particular, in the Cox-Ingersoll-Ross model, the drift function and the diffusion coefficient are defined respectively as[12]

$$f^r(r_t, t) = k(\theta - r_t)l^r(r_t, t) = \sigma_r\sqrt{r_t}$$

where k is the mean-reverting coefficient, θ the long-term period 'normal' rate, and σ_r the spot rate volatility.

In the model, the Cox-Ingersoll-Ross structure[12] describes the accumulation function of the spot rate $\Delta(h)$ (Equation (3)). For pricing interest rate derivatives, the Vasicek model[14] is widely used. However, this model assigns positive probability to negative values of the spot rate; for long maturities this can have a relevant effect and the model therefore appears to be inadequate for the valuation of life insurance policies.

Clearly, for the fair pricing of the fund, the specification of the reference portfolio dynamics is very important. The diffusion process for this dynamics is given by the stochastic differential equation

$$dS_t = f^S(S_t, t)dt + g^S(S_t, t)dZ_t^S \qquad (5)$$

where S_t denotes the price at time t of the reference portfolio, Z_t^S is a standard Brownian motion with the property

$$Cov(dZ_t^r, dZ_t^S) = \varphi dt \quad \varphi \in R$$

As a Black-Scholes type model[13] is assumed, one obtains

$$f^S(S_t, t) = \mu_S S_t \quad g^S(S_t, t) = \sigma_S S_t$$

where μ_S is the continuously compounded market rate, assumed to be deterministic and constant and σ_S is the constant volatility parameter.

The Black-Scholes model[13] is used to price the option described in relation (1).

For the dynamics of the process $\{\mu_{x+t:t}; t = 1, 2, \ldots\}$, consider the Renshaw-Haberman version of the Lee Carter model: this version develops the Lee Carter model in order to forecast the possible future behaviour of mortality by means of suitable reduction factors.[15] Moreover, for the reduction factor being estimated, one can explicitly compute the survival probabilities (see also Ballotta and Haberman[6]) to obtain the mortality table used in the proposed numerical application.

In particular, according to the traditional actuarial approach, the survival function of the random variable T_x is given by

$$\begin{aligned} {}_sp_x &= \Pr(T_x > s/F_0^{\mu}) \\ &= E[e^{-\int_0^S \mu_{x:t}dt}/F_0^{\mu}] \qquad (6) \end{aligned}$$

where F_0^{μ} represents the mortality informative structure available at time 0.

At this point use the reduction factor model to project mortality rates:

$$\mu_{y:t} = \mu_{y:0}RF(y, t) \qquad (7)$$

with $y = x + t$ and $RF(y, 0) = 1$ $\forall y \cdot \mu_{y,0}$ is the mortality intensity of a person aged y in the base year 0, $\mu_{y,t}$ is the mortality intensity for a person attaining age y in the future year t and the reduction factor $RF(y, t)$ is the ratio of the mortality intensity. Taking the logarithm of Equation (7), Renshaw and Haberman[15] obtain

$$\log \mu_{y:t} = \log \mu_{y:0} + \log RF(y, t) \quad (8)$$

subcondition $\log RF(y, 0) = 0$. Defining

$$\alpha_y = \log(\mu_{y:0}) \log\{RF(y, t)\} = \beta_y k_t$$

the Lee Carter structure is reproduced.[16]

In fact, the Lee Carter model for death rates is given by

$$\ln(m_{yt}) = \alpha_y + \beta_y k_t + \varepsilon_{yt} \quad (9)$$

where m_{yt} denotes the central mortality rates for age y at time t, α_y describes the shape of the age profile averaged over time, k_t is an index of the general level of mortality, while β_y describes the trend of mortality at age y to change when the general level of mortality k_t changes. ε_{yt} denotes the error.

APPLICATION

The numerical estimate

This section describes the methodology for computing the VaR and the expected tail loss (ETL) connected to the fund liabilities.

The ETL risk measure is widely used by actuaries. It is also known as expected shortfall, tail conditional expectation, conditional VaR and so on.

Artzner *et al.* include ETL among the coherent risk measures, meaning that it satisfies the three axioms of translation invariance, monotonicity and subadditivity.[17] Moreover, they show that VaR, being not subadditive, cannot be regarded as a 'true' risk measure at all.

On the other hand, by requiring subadditivity only for comonotonic variables, Heyde *et al.* find a new set of axioms which include VaR and quantiles eliminating its inconsistency.[18] The ETL is an attractive risk measure not only because of its coherence but also for many other reasons; in particular, it is useful if interest is focused on the expected size of losses exceeding a given threshold.[19]

To compute these risk measures, one must describe the behaviour of the financial and demographic risk sources. These will be described according to the specific stochastic hypothesis recalled in the previous sections.

The financial aspect of a life insurance liability flow is completely fair value, but the same does not apply to demographic components. According to Buhlmann,[20] their existence in the economic reality makes the fair valuation by means of a portfolio of financial instruments acceptable even if the demographic component is not tradable in an existing market and gives rise to a cash flow that matches the underlying liability flow. On the other hand, as reported in the literature,[6] the systematic and accidental components of the demographic uncertainty can be solved using the best estimate of the survival assumptions.

Consistently with this approach, and with reference to the mortality scenario in a fair-valuation estimating framework, a mark-to-model system can be considered for the demographic quantities. To describe the survival trend, assume that the best prediction for the time evolution of the surviving phenomenon is represented by a fixed set of survival probabilities. Their estimate takes into account the improving trend of mortality rates obtained by using the

Renshaw and Haberman version of the Lee Carter model.

With reference to the financial scenario, as mentioned previously, consider a Cox-Ingersoll-Ross model to describe the evolution in time of the spot rate.

In particular, consider the stochastic differential equation:

$$dr_t = -k(r_t - \theta)dt + \sigma_r\sqrt{r_t}dZ_t^r \qquad (10)$$

with α and σ positive constants, μ the long-term mean and Z_t^r a Wiener process.

As the computation of VaR and ETL requires the knowledge of the distribution of the potential periodic loss function, a Monte Carlo simulation procedure should be used for the financial quantities. To perform the simulation procedure it is necessary to consider the discrete time equation for the chosen stochastic differential equation (SDE) describing the evolution in time of the interest rates.[12] On the basis of the first-order Euler discretisation of Equation (10), with a time interval $[0, T]$ one obtains:

$$r_{t_K+1} = r_{t_k} + \alpha(\mu - r_{t_k})\Delta + \sigma\sqrt{r_{t_k}\Delta}\cdot\varepsilon_k$$
$$k = 1, 2, \ldots, n-1 \qquad (11)$$

where $\Delta = T/n$ is the sampling interval, with ε_k being the increment ΔZ_k of the Wiener process between $t_{k+1} = (k+1)\Delta$ and $t_k = k\Delta$. The increments ΔZ_k are $n(0, \Delta)$ distributed random variables. The discretised process is then represented by the sequence $\{r_{t_1}, r_{t_2}, \ldots, r_{t_n}\}$.

When pricing under risk-neutral measures, one typically approximates the discounting term as:[21]

$$\exp[-\int_t^T r(k)dk)] \qquad (12)$$

with

$$\exp\left(-\sum_{k=1}^{n-1} r_{t_k}\cdot\Delta\right) \qquad (13)$$

To have an accurate approximation, Δ must be small. A weekly sampling interval is selected.

The following simulation procedure is carried out:

(1) generation of a sequence of $n \cdot T$ pseudo random numbers $\{\varepsilon_k\}_{k=1,2,\ldots,n\cdot T}$, $n(0, \Delta)$ distributed;
(2) computation of one simulated path for the stochastic interest rate on the basis of Equation (11);
(3) computation of the discounting factors by means of Equation (13);
(4) computation of the potential periodic loss using the estimated mortality table.

The simulation procedure is repeated n times to gain n values for the potential periodic loss.

At this point, in a year-by-year valuation, one can estimate the VaR and ETL values at time zero, corresponding to the date on which the generic subscriber enters the fund.

Some results

Below, the paper shows the results of ETL and VaR connected to the fund in two different cases:

- in the first, the paper considers three different ages x for the generic subscriber entering the fund (30, 35 and 40) and the outgoing age is fixed as $x + \xi = 60$;
- in the second, the paper fixes $\xi = 20$, varying the age x when the generic subscriber enters the fund: 30, 35 and 40 respectively.

Let $T = \xi$.

The purpose is the computation of VaR and ETL at 95 per cent and 99 per cent confidence levels as absolute measures of the riskiness connected to the fund. From a methodological point of view, it is worth noting that the error term sets for the simulation procedures have been kept constant across different alternatives in order to compare the results. This is the input dataset:

- The risk-adjusted parameters of the Cox-Ingersoll-Ross process[12] are $k = 0.8$, $\sigma = 0.0053$, $\theta = 0.0452$ and $r_0 = 0.0279$. The time series related to the interest rates has been extracted from statistics published by the Bank of Italy and consists of annualised net interest rates of the Government's three-month treasury-bill rate covering the period from January 1996 to January 2006.
- The survival probabilities are calculated by applying the Renshaw and Haberman model.[15] The input data are deduced from the Italian mortality tables for the period 1947–1999.
- Referring to the time evolution of the reference fund, let $\mu_s = 0.03$ and $\sigma_s = 0.20$. For the correlation coefficient φ a slightly negative value is adopted in line with the literature concerning the Italian stock market.

Figure 1 shows the asymptotic behaviour of the empirical distribution of the random variable 'potential periodic loss', considering an entering age $x = 30$ and $n = 10{,}000$. As one can observe, it approximates a normal distribution. In fact, kurtosis takes a value of about 3 and skewness of about 0. Moreover, the study has made the Jarque-Bera test,[22] which is a well-known statistical procedure to test normality. In this case, with the

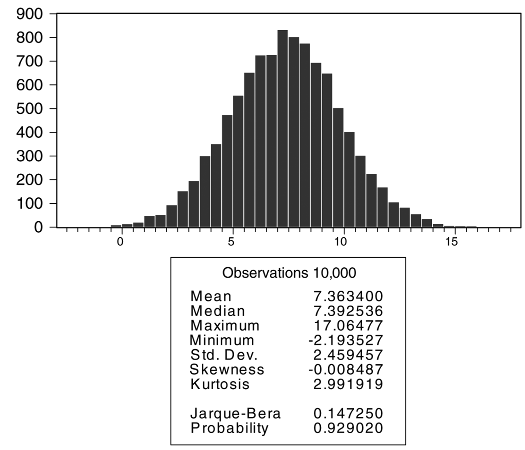

Figure 1: Empirical distribution of the potential periodic loss $t = 0$, $x = 30$

probability about equal to 0.92, one can accept the null hypothesis of normal distribution of $L(t)$. The same behaviour is observed with $x = 35$ and $x = 40$.

Table 1 clearly shows that by fixing the outgoing age $x + \xi = 60$, the risk level borne by the fund decreases as the entering age increases.

One can therefore observe a reduction in the maximum additional cost depending on the minimum benefit promised, which is posed as equal to 1,000.

From an actuarial point of view, this phenomenon is due to the decreasing risk exposure as the number of years the generic subscriber is a fund member decreases.

Table 2 reports the behaviour of the risk level as the entering age increases

Table 1: VaR and ETL at 99 per cent and 95 per cent confidence levels fixing the outgoing age $x + \xi$

Age	n	VAR 99%	ETL 99%	VAR 95%	ETL 95%
30	30	22.1	23.9	18.6	20.8
35	25	17.4	18.5	14.9	16.4
40	20	13.0	13.8	11.4	12.4

Table 2: VaR and ETL at 99 per cent and 95 per cent confidence levels fixing the maximum period of permanence ξ

Age	n	VAR 99%	ETL 99%	VAR 95%	ETL 95%
30	20	14.3	15.2	12.6	13.6
35	20	13.9	14.8	12.2	13.3
40	20	13.0	13.8	11.4	12.4

when fixing ξ — as one can see, the risk level decreases. This effect is a consequence of the well-known longevity phenomenon which is due to improvements in mortality trends. Indeed, the probability of reaching pension age prevails over the probability of death and disability, thus causing a decreasing risk profile.

CONCLUDING REMARKS

In conclusion, the implementation of IFRS 4 gives rise to many reporting issues. Among the most relevant issues is the use of quantile-based measures for the quantification of the riskiness in a fair-valuation context. In this respect, the main target of the paper was to determine the ETL of the DC pension fund for risk management purposes in a fair-valuation context. The suggested model allows one to determine an absolute index of riskiness borne by the fund. Moreover, it can also be applied for measuring the stochastic impact of the interest and mortality rates on the mathematical reserve determinants.

Further research on this subject will involve the use of different stochastic models for interest rates and equity returns to depict a certain number of scenarios. This could lead to the quantification of a proper fair-valuation risk model.

References

1 International Accounting Standard Board (2004) ‘International Financial Reporting Standard 4 Insurance Contracts’, Basel, Switzerland.

2 Jorgensen, P. L. (2004) ‘On accounting standards and fair valuation of life insurance and pension liabilities’, *Scandinavian Actuarial Journal*, Vol. 5, No. 1, pp. 372–394.

3 Grosen, A. and Jorgensen, P. L. (2002) ‘Life insurance liabilities at market value: An analysis of investment risk, bonus policy and regulatory intervention rules in a barrier option framework’, *Journal of Risk and Insurance*, Vol. 69, No. 1, pp. 63–91.

4 Milevsky, M. A. and Promislow, S. D. (2001) ‘Mortality derivatives and the option to annuitise’, *Insurance: Mathematics and Economics*, Vol. 29, No. 3, pp. 299–318.

5 Bacinello, A. R. (2001) ‘Fair pricing of life insurance participating policies with a minimum interest rate guaranteed’, *Astin Bulletin*, Vol. 31, No. 2, pp. 275–297.

6 Ballotta, L. and Haberman, S. (2006) ‘The fair valuation problem of guaranteed annuity options: The stochastic mortality environment case’, *Insurance: Mathematics and Economics*, Vol. 38, No. 1, pp. 195–214.

7 Ballotta, L., Esposito, G. and Haberman, S. (2006) ‘The IASB Insurance Project for life insurance contracts: Impact on reserving methods and solvency requirements’, *Insurance: Mathematics and Economics*, Vol. 39, No. 3, pp. 356–375.

8 Olivieri, A. and Pitacco, E. (2003) ‘Solvency requirements for pension annuities’, *Journal of Pension Economics and Finance*, Vol. 2, No. 1, pp. 127–154.

9 Blake, D., Cairns, A. and Dowd, K. (2001) ‘Pensionmetrics: Stochastic pension plan design and value at risk during the accumulation phase’, *Insurance: Mathematics and Economics*, Vol. 29, No. 2, pp. 187–215.

10 Blake, D., Cairns, A. and Dowd, K. (2003) 'Pensionmetrics 2: Stochastic pension plan design and value at risk during the distribution phase', *Insurance: Mathematics and Economics*, Vol. 33, No. 1, pp. 29–47.
11 Cocozza, R., Di Lorenzo, E., Orlando, A. and Sibillo, M. (2008) 'The value at risk of the mathematical provision: Critical issues', *Journal of Risk Management in Financial Institutions*, Vol. 1, No. 3, pp. 311–319.
12 Cox, J., Ingersoll, J. and Ross, S. (1985) 'A theory of the term structure of interest rates', *Econometrica*, Vol. 53, No. 2, pp. 385–408.
13 Black, F. and Scholes, M. (1973) 'The pricing of option and corporate liabilities', *Journal of Political Economy*, Vol. 81, No. 3, pp. 637–654.
14 Vasicek, O. (1977) 'An equilibrium characterization of the term structure', *Journal of Financial Economics*, Vol. 5, No. 4, pp. 177–188.
15 Renshaw, A. E. and Haberman, S. (2003) 'On the forecasting of mortality reduction factor', *Insurance: Mathematics and Economics*, Vol. 32, No. 3, pp. 379–401.
16 Lee, R. (2000) 'The Lee Carter method of forecasting mortality with various extensions and applications', *North American Actuarial Journal*, Vol. 4, No. 1, pp. 80–93.
17 Artzner, P., Delbaen, F., Eber, J. M. and Heath, D. (1999) 'Coherent measures of risk', *Mathematical Finance*, Vol. 9, No. 3, pp. 203–228.
18 Heyde, C. C., Kou, S. G. and Peng, H. (2006) 'What is a Good Risk Measure: Bridging the Gaps Between Data, Coherent Risk Measure and Insurance Risk Measure', Columbia University, New York, NY.
19 Dowd, K. (2006) 'After VaR: The theory, estimation, and insurance application of quantile-based risk measures', *Journal of Risk & Insurance*, Vol. 73, No. 2, pp. 193–229.
20 Buhlmann, H. (2002) 'New math for life actuaries', *Astin Bulletin*, Vol. 32, No. 2, pp. 209–211.
21 Brigo, D. and Mercurio, F. (2004) 'Interest Rate Models. Theory and Practice. With Smile, Inflation and Credit' (2nd edn), Springer Finance, Heidelberg.
22 Bera, A. and Jarque, C. (1980) 'Efficient test for normality, heteroskedasticity and serial independence of regression residuals: Monte Carlo evidence', *Economic Letter*, Vol. 6, No. 3, pp. 255–259.