Implied asset correlation in retail loan portfolios

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Abstract Credit risk arises from the interaction of multiple connected factors, but the most frequently-used models designed to measure it assume only one. These models — which, *inter alia*, fit distributions to loss data — are heavily influenced by the common correlation between loan values and the single factor (commonly assumed to be some gauge of economic health). Scarce and shoddy loss data for retail loan classes hamper the estimation of this correlation. A technique is proposed to calculate asset correlations embedded in empirical loss data. These values are then compared with those stipulated by the Basel II Accord for minimum capital requirements.

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INTRODUCTION

To remain solvent under all but the most severe of circumstances, banks dedicate a battery of resources to the accurate, timely calculation of economic capital. This is an internal measure, designed to cushion against calamitous events, and it embraces the catholic array of risks faced by banks as well as any diversification benefits that arise between disparate risks. As such, economic capital is distinct from regulatory capital which is externally imposed by national regulatory bodies, covers only a handful of risks, ignores inter-risk diversification and applies fairly rigid constraints on the way capital reserves should be measured. Regulatory capital and all matters pertaining thereto are governed by two accords, designed and disseminated by the Basel Committee on Banking Supervision (BCBS). The accord of 1988 (Basel I) was an attempt by the BCBS to improve the risk management procedures practised by banks.[1] Basel I addressed only credit risk (and, later, market risk[2]), but this was somewhat coarsely determined across loan quality with little or no distinction made between superior and inferior borrowers. Capital charges determined using the 1988 accord were considered by banks to be punitive and inequitable.[3] The assembly and extensive implementation of the new capital accord (Basel II) has heralded, among other changes (the treatment of operational risk, for example), a much improved treatment of credit risk.[4] In their management of credit risk, banks now have the option of adopting either the standardised approach (in which risk weights for loan exposure amounts are specified by the BCBS — similar to Basel I) or the internal ratings-based (IRB) approach (in which specific capital requirement formulas are specified by the BCBS, but some flexibility regarding the input parameters is allowed).

The IRB approach harnesses quantitative estimates of obligor-level risk (eg the probability of default (PD) and loss given default (LGD)) and is grounded in well-established concepts from modern portfolio-based risk management, thereby providing a sophisticated and more meaningful capital framework than Basel I. The IRB approach employs an asymptotic single-risk factor (ASRF) calculation methodology that allows relatively straightforward analytical solutions, rather than a full-blown multi-factor model typical of internal bank credit economic capital systems. Nevertheless, the IRB approach is based upon credit risk modelling concepts that are broadly consistent with capital models used increasingly by banks to measure portfolio-level risk and to manage and allocate capital across the enterprise.

The single systematic risk factor required by ASRF models can be interpreted as a reflection of the state of the global economy. All borrowers are linked to one another by this single risk factor and the strength of that linkage is measured by the asset correlation. The BCBS has calibrated and set predetermined values for the asset correlation within each of the IRB equations, which are broadly segmented by asset classes specified under Basel II (eg corporates, commercial mortgages, residential mortgages, credit cards and consumer lending).[5] Asset correlations thus determine the shape of the risk weight formulas and, because different borrowers and/or asset classes show different degrees of dependency on the overall economy, asset correlations are asset class dependent.

Banks *must* comply with regulatory rules to sustain capital adequacy and in so doing they *must* use the prespecified BCBS asset correlation values. Economic capital models provide valuable additional information that banks use in their overall assessment of their capital adequacy[6] and they therefore retain an avid interest in the estimation of implied asset correlations embedded in their empirical loss data — independent of the BCBS specifications. These values are of critical import for internal economic capital models. The retrieval of implied asset correlation values from empirical gross loss data — while possible — is non-trivial. This paper examines the extraction of retail asset correlations, assesses their robustness and compares them with those specified by the BCBS.

The next section of this paper provides a brief literature survey of the field of asset correlation in credit-risky portfolios and establishes the relevance of the asset correlation to the Basel II Pillar 1 formulation. The paper goes on to introduce and evaluate the Vasicek distribution, including a robust methodology which can be used to extract relevant parameters from it. The following section introduces and employs the beta distribution as a tool to compare results obtained from the Vasicek distribution. Retail loan loss data spanning several decades are then analysed and the results obtained from this analysis are presented and discussed. The final section concludes the paper.

LITERATURE SURVEY

The BCBS has explained and defended the choice of credit risk framework, equations and correlation values in a special explanatory note.[7] The note does not divulge the analytical reasoning nor the mathematical foundations upon which the IRB approach is based. Instead, only broad conclusions are presented. This was a deliberate attempt by the BCBS to allow pertinent credit modelling concepts to be cemented into the consciousness of a select, non-technical audience (later papers provided much more detail).[8]

The empirical relationship between the average asset correlation, firm PD and firm asset size measured by the book value of assets was examined by imposing the ASRF approach within the KMV methodology for determining credit risk capital requirements.[9] In related work, the empirical examination of asset correlation using portfolios of US publicly-traded real estate investment trusts as a proxy for commercial real estate (CRE) lending more generally was undertaken.[10] CRE lending as a whole was found to have the same calibrated average asset correlation as corporate lending, providing support for the US regulatory decision to treat these two lending categories similarly for regulatory capital purposes.

Asset correlations for automotive lease exposures were measured using a single systematic factor ordered probit model in which the obligor status was limited to two states: default and survival.[11] This model made use of a restricted version of CreditMetrics™.[12] The results of this analysis showed that the empirically estimated correlations were significantly lower than those specified by the BCBS. To ascertain an adequate, empirical asset correlation, the authors suggested taking into account one extra dimension (the volatility of the probability of default).[11]

The gap between the impact of systematic risk on the loss distribution of a credit-risky loan portfolio and the lack of empirical estimates of the default correlation has also been explored.[13] Ten years of monthly default data were used for over 50,000 German corporates. Results from this study suggested that the asset correlation parameter depends on both the probability of default as well as on the obligor firm size.

A comprehensive review of corporate defaults and the role of asset correlation found that asset correlations are only one source of dependence; explicitly modelling dependencies other than unexpected losses (such as dependence between LGD and PD) will be underestimated unless empirical asset correlations (from default data) are increased.[14] The authors concluded that default data are the best source of default correlations as no intermediate process is assumed, but admit that default data are invariably either sparse or unavailable.

Banks have developed sophisticated internal ratings-based models and have collected abundant BCBS input IRB data including correlation parameters.[5] In addition, for corporate loans, much academic research has been published on credit risk modelling.[15] The picture for retail portfolios, however, is very different. Few academic papers have been published on the modelling of retail portfolio risks and the PD, LGD and exposure at default (EAD) data collected by banks are often sparse and lacking in detail.[5]

Banks continue to struggle with systems and procedures required for retail loan portfolios in Basel II; many are not sufficiently sophisticated and some are inundated with other, more pressing implementation issues.[16] While various large banks have performed some retail loan analysis, the majority continue to apply the Basel rules without any focus on whether or not the BCBS-specified parameters produce realistic outcomes.[16]

A real need, therefore, exists for the development of a robust, yet practical, methodology to measure retail loan portfolio implied asset correlations. However, the lack of *individual* exposure default data coupled with the overall dearth of data for retail portfolios conspire to severely constrain detailed asset and default correlation studies for these asset classes. A non-exhaustive presentation, analysis and evaluation of the Vasicek distribution follows in the next section.

THE VASICEK DISTRIBUTION

Vasicek[17,18,19] derived an expression for the distribution of credit portfolio losses using a Merton-type model. In this approach, the portfolio credit risk is quantified due to its potential default rate using a value-at-risk approach. Vasicek achieved analytical tractability by assuming an ASRF framework[20,21] which — apart from assuming only one systematic risk factor influencing the default risk of all loans in the portfolio — also assumes the portfolio is infinitely fine-grained (ie it comprises a nearly infinite number of credits with infinitely small exposures). Vasicek's assertion that the cumulative probability that the portfolio loss, L, will be less than some variable, x, is given by:

$$P[L \leq x] = N\left[\frac{\sqrt{1-\rho}\cdot N^{-1}(x) - N^{-1}(p)}{\sqrt{\rho}}\right], \tag{1}$$

Where ρ is the asset correlation between all loans and the systematic single risk factor; p is the average probability of default for the portfolio; and $N(\ldots)$ and $N^{-1}[\ldots]$ refer to the cumulative standard normal distribution and the inverse standard normal cumulative distribution function, respectively.

This cumulative distribution — which describes the portfolio losses and is driven by two parameters (p and ρ) — is defined over the interval $0 \leq x \leq 1$ and is given by:

$$F(x; p, \rho) = N\left[\frac{\sqrt{1-\rho}\cdot N^{-1}(x) - N^{-1}(p)}{\sqrt{\rho}}\right], \tag{2}$$

With $p > 0$ and $0 < \rho < 1$. As $\rho \to 0$, the distribution converges to a 0, 1 distribution with probabilities p and $1 - p$

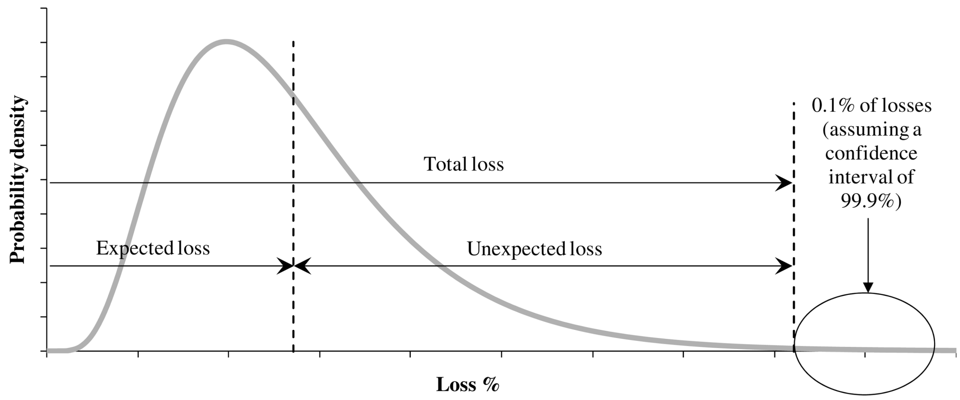

Figure 1: Important features of a typical loss distribution

respectively. When $p \to 0$ or $p \to 1$, the distribution becomes concentrated at $L = 0$ or $L = 1$ respectively. A further important property is $F(x; p, \rho) = 1 - F(1 - x; 1 - p, \rho)$.

It is important to note that Equation 2 defines the 'total loss' shown in Figure 1, ie the cumulative probability of obtaining a loss value less than x.

The highly skewed and leptokurtic loss distribution has the following density:

$$f(x; p, \rho) = \sqrt{\frac{1-\rho}{\rho}} \cdot \exp\left(\frac{1}{2}(N^{-1}(x))^2 - \frac{1}{2}\left(\frac{\sqrt{1-\rho}\cdot N^{-1}(x) - N^{-1}(p)}{\sqrt{\rho}}\right)^2\right), \quad (3)$$

and it is unimodal with the most prevalent loss — the mode — located at:

$$L_{\text{mode}} = N\left[\frac{\sqrt{1-\rho}}{1-2\rho} \cdot N^{-1}(p)\right]. \quad (4)$$

The inverse of this distribution — ie the α-percentile value of L is given by:

$$L_\alpha = F(\alpha; 1-p, 1-\rho). \quad (5)$$

Figure 1 provides the relevant features of a typical, skewed distribution for a collection of loan losses. The expected loss (EL) is the average portfolio loss and the total loss is a *defined* point — in this case, the point below which 99.9 per cent of all losses fall. In Figure 1, the area under the curve to the left of the total loss position represents 99.9 per cent of all portfolio losses. The unexpected loss (UL) depends upon the definition of the total loss point, being the difference between the total loss and the expected portfolio loss.

The procedure for extracting empirical asset correlations from loss data is as follows:

(1) Source gross loss time series data as a percentage of total loan value.
(2) Calculate the mean (p in Equations 2–5) and the mode (L_{mode} in Equation 4). These values are obtained analytically from the simple *mean* gross loss and the most *prevalent* gross loss, respectively, over the time period under scrutiny. These values are easily calculated using standard statistical software or simple spreadsheets.

(3) Knowing p and L_{mode}, the empirical asset correlation may be extracted using Equation 4:

$$\frac{N^{-1}(L_{\text{mode}})}{N^{-1}(p)} = \frac{\sqrt{1-\rho}}{1-2\rho}, \text{ so}$$

$$\left[\frac{N^{-1}(L_{\text{mode}})}{N^{-1}(p)}\right]^2 = \frac{1-\rho}{(1-2\rho)^2}$$

Substituting for $\xi = \left[\frac{N^{-1}(L_{\text{mode}})}{N^{-1}(p)}\right]^2$ gives:

$$\xi(1-2\rho)^2 = 1-\rho$$

$$4\xi\rho^2 + (1-4\xi)\rho + (\xi - 1) = 0$$

which is a quadratic equation in ρ (the asset correlation) with solutions:

$$\rho : \frac{(4\xi - 1) \pm \sqrt{8\xi + 1}}{8\xi}. \qquad (6)$$

Both components of ξ (L_{mode} and p) are known (Step 2), so ρ is easily calculated. The lower of the two possible values for ρ is chosen because the higher ρ results in unrealistic values of UL.

The total portfolio loss measured at a confidence interval of 99.9 per cent may also be measured empirically by combining Equations 5 and 2 (note that a confidence interval of 99.9 per cent implies $\alpha = 0.1$ per cent):

$$F(\alpha; 1-p, 1-\rho) = N\left[\frac{\sqrt{1-(1-\rho)} \cdot N^{-1}(\alpha) - N^{-1}(1-p)}{\sqrt{1-\rho}}\right]$$

Total gross loss

$$= N\left[\frac{N^{-1}(p) + \sqrt{\rho} \cdot N^{-1}(\alpha)}{\sqrt{1-\rho}}\right].$$

The gross total loss is simply the sum of unexpected and expected gross losses ($\text{UL}^{99.9\%}$ + EL; see Figure 1). Subtracting the gross expected loss from the total loss gives:

$$UL^{99.9\%} = N\left[\frac{N^{-1}(p) + \sqrt{\rho} \cdot N^{-1}(\alpha)}{\sqrt{1-\rho}}\right] - EL,$$

But $EL = p$, the portfolio expected loss, and as gross loss data are used, this value is also the portfolio probability of default. Thus:

$$UL^{99.9\%} = N\left[\frac{N^{-1}(p) + \sqrt{\rho} \cdot N^{-1}(\alpha)}{\sqrt{1-\rho}}\right] - p. \qquad (7)$$

No assumptions regarding recoveries have been made or assigned. When the analysis is complete and the values calculated, both sides of Equation 7 are multiplied by the LGD and the result is the familiar UL in the 'net loss' sense (this is also the well-known total loss estimate from the Pillar I equations in the BCBS formulation). No other adjustments need to be made and the inclusion of LGD here serves only to obfuscate an otherwise relatively straightforward explanation — hence its omission from the present (and later) discussions. The $\text{UL}^{99.9\%}$ is thus presented here as a *gross* unexpected loss.

Armed with the analytics to extract asset correlation values from empirical loss data, several aspects of implied retail asset correlations were investigated. The details of these investigations — and the results obtained from them — are presented and discussed in the next section.

DATA AND ANALYSIS

The analysis which constitutes this section explores aspects of the empirical correlations derived from gross loss data.

The data span some 24 years (ie Q1 1985 to Q1 2009) and were compiled from the quarterly Federal Financial Institutions Examination Council Consolidated Reports of Condition and Income (data for each calendar quarter become available approximately 60 days after quarter-end). The data span the turbulent early and late 1990s, the benign credit conditions which characterised the 2003–08 period as well as the recent downturn in the credit environment leading to the credit crunch which began in mid-2007 and has yet to run its course. Charge-offs (the values of loans and leases removed from the books and charged against loss reserves) from the 100 largest US banks are measured by consolidated foreign and domestic assets. The US Federal Reserve uses annualised charge-off rates, net of recoveries and outstanding as of quarter-end. The charge-off rates are calculated from data available in the report of condition and income (call report), filed each quarter by all commercial banks. Charge-off rates for any category of loan are defined as the flow of a bank's net charge-offs (gross charge-offs — recoveries) during a quarter divided by the average level of its loans outstanding over that quarter. These ratios are multiplied by 400 to express them as annual percentage rates.[22]

As net losses = gross losses × loss given default, only knowledge of LGDs is required to convert net losses to gross losses. These average LGDs were obtained from the BCBS's fifth quantitative impact study,[23] using the 'G10 group 1: including US' group (as loss data used in this study were obtained from US commercial banks). The LGD averages for different retail portfolios are as follows:

- *residential mortgage:* 20.3 per cent;
- *qualifying revolving:* 71.6 per cent;
- *other retail:* 48.0 per cent;
- *high volatility commercial real estate (HVCRE):* 35.0 per cent.

First, the influence of the distribution assumption on the asset correlations is explored. The Vasicek and beta distributions are used to fit the loss data, even though losses — which are invariably assumed to be highly skewed and leptokurtic — do not always embrace this pattern. The former distribution is the one chosen by the BCBS for the Pillar 1 regulatory capital calculations and the latter lends itself to robust analysis within a relatively simple mathematical framework. The beta distribution is also used elsewhere in the Basel Accords (eg in the treatment of securitisation). Results from other distributions were also explored, but because the conclusions reached were broadly similar, these have been omitted for clarity (goodness-of-fit tests were applied to all distributions and the beta distribution consistently performed the best).

Next, empirical correlations (extracted from the loss data and deduced from both the Vasicek distribution and the beta distribution) are compared with BCBS *specified* asset correlations conducted using the entire data set as described above, ie spanning some 24 years.

Finally, the time evolution of the empirical correlation and the BCBS specified correlations is compared. Basel specifies that a minimum of seven years of data must be used to estimate the LGD (paragraph 472) and the EAD (paragraph 478).[24] Using seven years of loss data as a benchmark, relevant asset correlations are calculated. The seven-year quarterly loss data window is then rolled forward one quarter (maintaining a seven-year rolling window) to calculate the next asset correlation value. This process is continued until Q1, 2009 (ie March 2009).

Effect of distribution assumption

Employed by the BCBS in the first pillar of the Basel II capital requirement calculations, the Vasicek distribution is used to describe the dispersion of credit losses of many banks whose local regulators have approved them for the IRB approach. However, many fat-tailed, leptokurtic distributions exist and may be used as a 'best fit' to the loss data.

Using several retail loan classes, empirical asset correlations were compared using both the Vasicek and beta distributions. The latter is popular in the BCBS analysis of securitisation and is completely characterised by two parameters, α and β, which can be calculated from the mean (μ) and standard deviation (σ) of the gross losses. These quantities are linked to the beta distribution shape parameters as follows:

$$\alpha = \mu \cdot \left(\frac{\mu \cdot (1-\mu)}{\sigma^2} - 1 \right) \qquad (8)$$

$$\beta = \frac{\alpha}{\mu} \cdot (1-\mu). \qquad (9)$$

It is straightforward to measure the gross mean loss $\mu(=p)$ and σ over the period under scrutiny so α and β are easily calculated. The probability density function for the beta distribution is given by:

$$f(x:\alpha,\beta) = \frac{1}{\mathrm{B}(\alpha,\beta)} \cdot x^{\alpha-1} \cdot (1-x)^{\beta-1},$$

where $\mathrm{B}(\alpha, \beta)$ is the beta function given by $\dfrac{\Gamma(\alpha)\cdot\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ and where $\Gamma(\cdots)$ is the gamma function.

The cumulative beta distribution is fully specified by:

$$P(x) = \frac{\int_0^x (1-t)^{\beta-1} \cdot t^{\alpha-1} dt}{\mathrm{B}(\alpha,\beta)} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} \cdot \int_0^x (1-t)^{\beta-1} \cdot t^{\alpha-1} dt$$
$$1 \geq x \geq 0, \quad \alpha,\beta > 0 \qquad (10)$$

where x is the distribution variable.

The total losses (EL + UL) are simply the value of x when $P(x) = 99.9$ per cent (so chosen to comply with Basel II).

The asset correlation is derived by calculating the correlation value which equates the *BCBS* total loss at a 99.9 per cent confidence interval with the *empirical* total loss at a 99.9 per cent confidence interval (using the beta distribution). Note that all estimates — as with the Vasicek distribution and Equation 7 — are based on gross loss data; no recoveries are taken into account at this stage. The extraction of empirical asset correlations from loss data proceeds as follows:

(1) Source gross loss time series data as a percentage of total loan value.

(2) Calculate the mean, μ, and standard deviation, σ, of these gross loss data. These values are obtained analytically from the simple average and the standard deviation of the gross losses, respectively, over the time period under scrutiny.
(3) Using Equations 8 and 9, calculate α and β.
(4) Using Equation 10, determine the value of x when $P(x) = 99.9$ per cent. This is the total gross loss at the 99.9th percentile ($L_{\text{total}}^{99.9\%}$) of the fitted beta distribution.
(5) Substitute the $L_{\text{total}}^{99.9\%}$ value obtained from Step 4 into the BCBS equation for the total gross loss value (measured at a 99.9 per cent confidence interval), ie:

$$UL^{99.9\%} = N\left[\frac{N^{-1}(p) + \sqrt{\rho}\cdot N^{-1}(0.999)}{\sqrt{1-\rho}}\right] - EL$$

$$EL + UL^{99.9\%} = \text{Total gross loss} = L_{\text{total}}^{99.9\%}$$

$$L_{\text{total}}^{99.9\%} = N\left[\frac{N^{-1}(p) + \sqrt{\rho}\cdot N^{-1}(0.999)}{\sqrt{1-\rho}}\right] \quad (11)$$

The left-hand side of Equation 11 is known (empirically) from Step 4 above.
Equation 11 becomes:

$$N^{-1}(L_{\text{total}}^{99.9\%}) \cdot \sqrt{1-\rho} = N^{-1}(p) + \sqrt{\rho} \cdot N^{-1}(0.999),$$

with ρ the only unknown in this equation.

Letting $\omega = N^{-1}(L_{\text{total}}^{99.9\%})$, $\pi = N^{-1}(p)$ and $\psi = N^{-1}(0.999)$ and squaring both sides gives:

$$\omega^2 \cdot (1-\rho) = \pi^2 + \psi^2 \cdot \rho + 2\pi\psi \cdot \sqrt{\rho}$$
$$0 = (\psi^2 + \omega^2) \cdot \rho + 2\pi\psi \cdot \sqrt{\rho} + (\pi^2 - \omega^2),$$

which is a quadratic in $\sqrt{\rho}$ with solutions:

$$\rho : \left[\frac{-2\pi\psi \pm \sqrt{(2\pi\psi)^2 - 4\cdot(\psi^2+\omega^2)\cdot(\pi^2-\omega^2)}}{2\cdot(\psi^2+\omega^2)}\right]^2,$$

and which is easily solved as ω, π and ψ are all known quantities.

The results are shown in Figure 2 for US consumer loan gross losses. Figure 2(a) shows the cumulative density function for the respective distributions as well as the cumulative empirical loss data. Figure 2(b) illustrates the density functions for the respective distributions as well as a histogram of empirical losses for comparison. Recall that the Vasicek distribution is unimodal so fitting an (apparent) bimodal distribution (as seen in Figure 2(b)) results — in this case — in a suboptimal fit. While in this example the beta distribution provides the better fit to the empirical data, this is by no means always the case.

Figure 3 shows results obtained for US 'other consumer' loans. In this case, the Vasicek distribution provides a good fit to the main body of loss data but a poor fit to the tail region, while the reverse is true for the beta distribution (Figure 3(a)). For these losses, the unimodal data are well-suited to the Vasicek distribution which effectively models the bulk of the losses (Figure 3(b)).

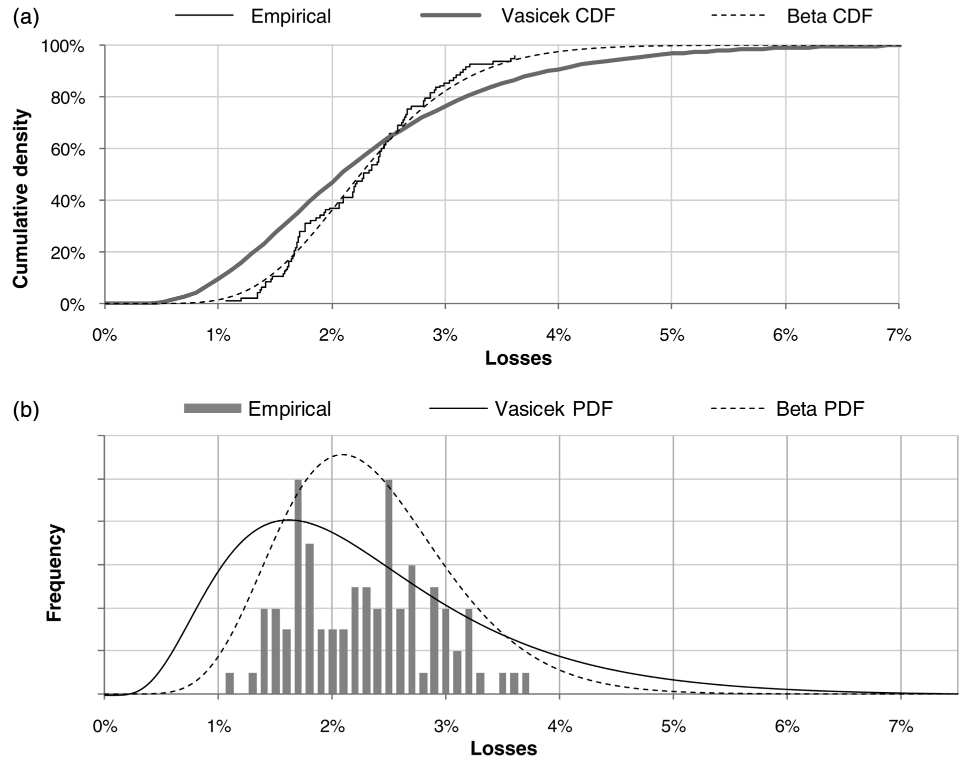

Figure 2: (a) Cumulative and (b) density function for US consumer loan losses from Q1 85 to Q1 09

Comparison with Basel correlations

The BCBS specifies the asset correlations to be used in the IRB Foundation approach — these are summarised in Table 1.

Using the full 24-year span of gross loss data, the empirical asset correlations (using both the Vasicek and beta distributions) were compared with the Basel-specified asset correlations as shown in Figure 4.

The empirically measured correlations are all lower than those specified by the BCBS, but are fairly similar for most retail asset classes. The residential mortgage and commercial real estate asset classes are an exception: in these cases, the BCBS specified value of 15 per cent is considerably higher than the values obtained by either the Vasicek or beta distributions (see Figure 5).

Capital charges for each asset class using the three approaches are given in Figure 5. The BCBS specifies that, to take into account the omission of PD and LGD correlations, a 'downturn LGD' must be used in the IRB approach. A principles-based approach was suggested for the estimation of downturn LGDs, which requires banks to identify appropriate downturn conditions and the adverse dependencies between default and recovery rates. From these, banks must produce LGD parameters for their exposures which are consistent with the identified downturn conditions.[25] The implicit assumption made by the BCBS is that a credit risk model with systematic correlation between PD and LGD using long-term LGD inputs should give comparable capital to a credit risk model without correlated PD and LGD using downturn

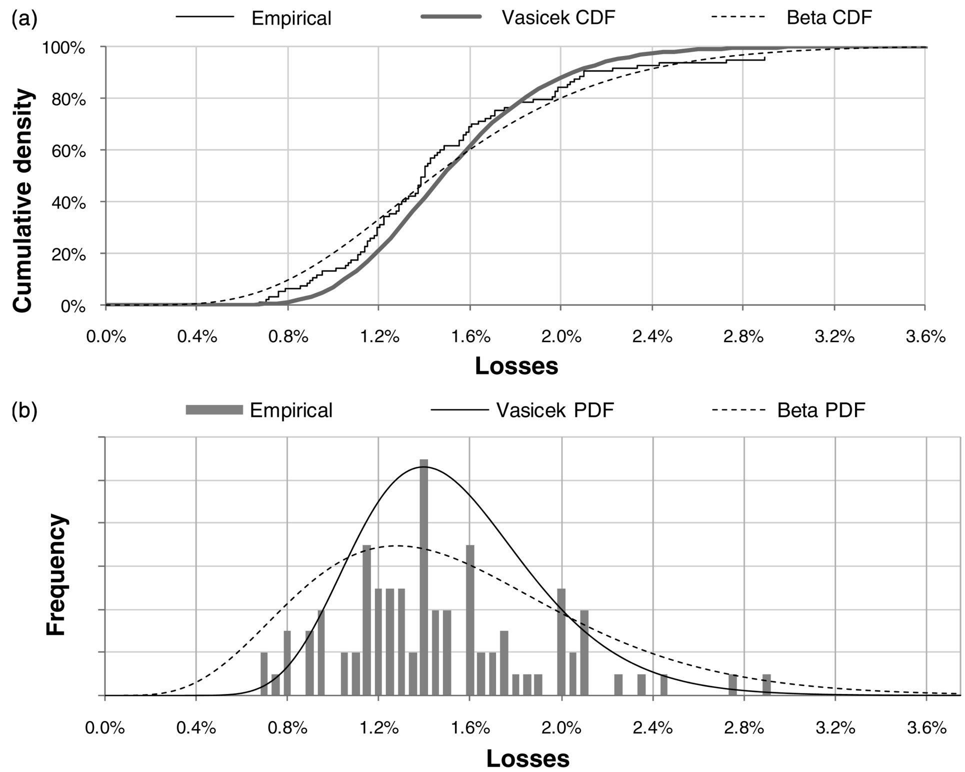

Figure 3: (a) Cumulative and density function for other US consumer losses from Q1 85 to Q1 09; (b) comparison with Basel correlations

LGD inputs. Mean LGDs need to be increased by about 35 per cent to 41 per cent in order to compensate for the lack of correlation. Downturn LGDs were obtained from the LGDs used in this study by increasing the latter by (on average) 38 per cent, ie by multiplying these by 1.38.[25] The downturn LGDs were then used to estimate the capital charges using the BCBS IRB approach. For the Vasicek and beta distribution empirical approaches, unaltered LGDs were used, ie values taken from Table 1.

The capital charges as specified by the BCBS are similar to the charges calculated using the Vasicek distribution for all asset classes except for residential

Table 1: Loan input parameters under Basel II's foundation IRB

Retail loan type	Correlation	
Mortgages	Fixed (15 per cent)	
Qualifying revolving	Fixed (4 per cent)	
Other retail	Varies with PD	$0.03 \cdot \left(\frac{1-e^{-35PD}}{1-e^{-35}}\right) + 0.16 \cdot \left[1 - \left(\frac{1-e^{-35PD}}{1-e^{-35}}\right)\right]$
High-volatility commercial real estate	Varies with PD	$0.12 \cdot \left(\frac{1-e^{-50PD}}{1-e^{-50}}\right) + 0.30 \cdot \left[1 - \left(\frac{1-e^{-50PD}}{1-e^{-50}}\right)\right]$

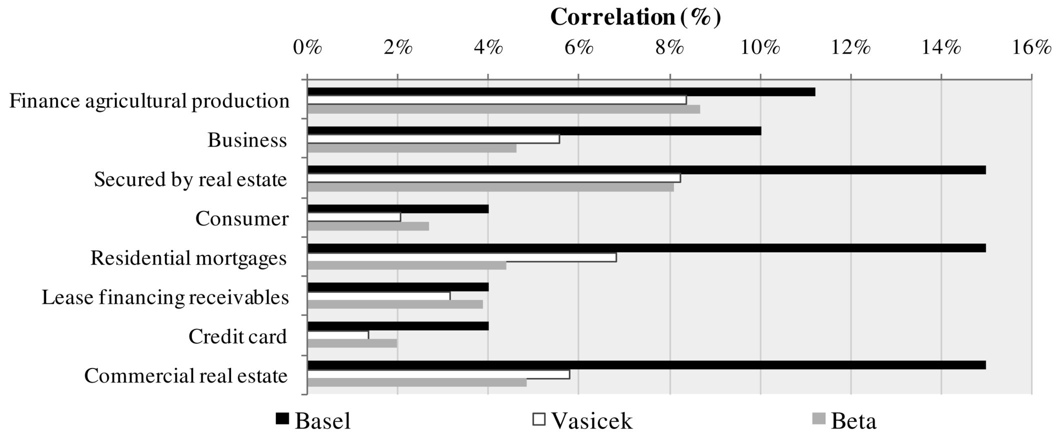

Figure 4: Comparison of empirical asset correlations (derived from the Vasicek and beta distributions respectively) and Basel II specified asset correlations

mortgages and commercial real estate where capital charges are considerably higher using BCBS-specified values.

A comparison of capital charges relative to the BCBS-specified charges is given in Figure 5.

Average capital charges using the beta distribution's implied asset correlation value are 2.8 times the average of the BCBS-specified values while capital charges using the Vasicek distribution's implied asset correlation value are 2.9 times the average of the BCBS-specified values. These results are both interesting and encouraging from an empirical point of view. Given the assumptions made in this analysis, the values agree well for most retail asset classes — particularly if the Vasicek distribution is used. Capital charges using BCBS-specified values for residential mortgages appear somewhat punitive, ie they are 4.0 times those derived using correlations derived empirically from the Vasicek distribution and 5.8 times those derived using correlations derived empirically from the beta distribution. A similar situation exists for capital charges for commercial real estate. These charges are 3.3 times those derived using correlations derived

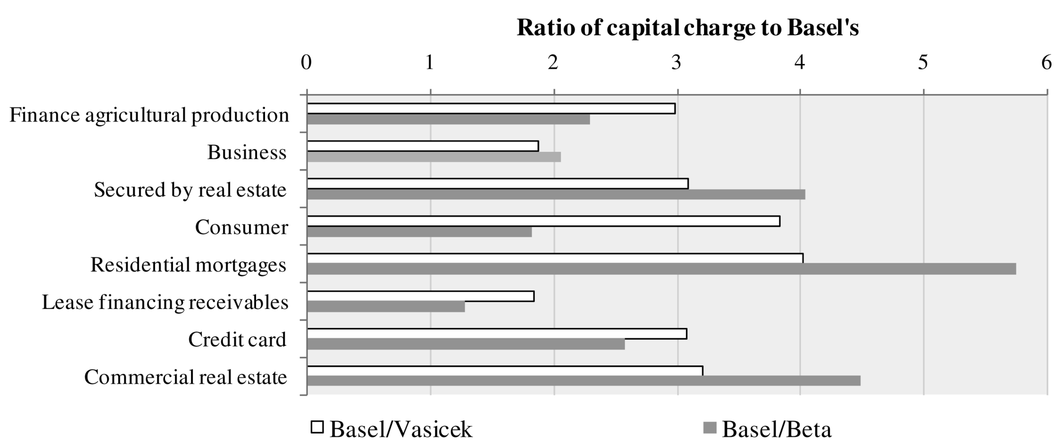

Figure 5: Comparison of capital charge ratios — BCBS-specified/Vasicek derived and BCBS-specified/beta derived

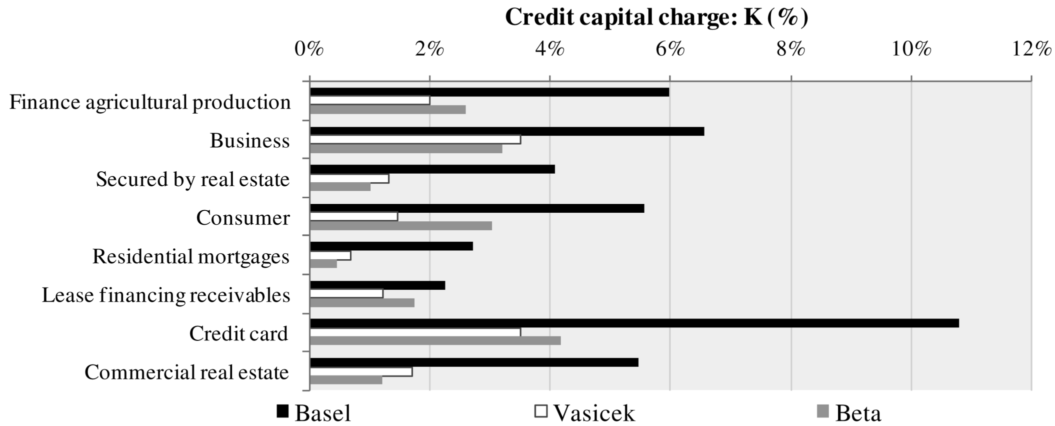

Figure 6: Comparison of capital charges; for the BCBS capital charges, specified correlations and downturn LGDs were used; for Vasicek and beta distribution capital charges, implied correlations and standard average LGDs were used

empirically from the Vasicek distribution and 3.9 times those derived using correlations derived empirically from the beta distribution.

That the Basel capital charges are higher for *all* retail loan types is not an unexpected result. The BCBS has repeatedly stressed the goal of conservatism and a sufficiently large capital cushion to shield banks from insolvency. Using its current formulation, however, the BCBS does not have many parameters that can be adjusted to achieve these aims, particularly if banks use the IRB approach for analysing their credit risk. The LGD is one such adjustable parameter, asset correlation is another. Taking both these parameters into account, the capital charges for most retail asset classes compare favourably. While these values are elevated, they are not punitive. For residential mortgages and commercial property, however, BCBS-specified capital charges are 3–6 times larger than those obtained empirically. It is not immediately clear why this should be the case. It is true that banks employing the IRB approach (rather than Basel I or the Basel II standardised approach) significantly reduce their capital charges for residential mortgage and commercial property portfolios. It may be the case that the BCBS simply wishes to reduce the rate of capital charge reduction for these portfolios at least until some time has passed and the validity of this reduction can be assessed.

Results from the BCBS's fourth quantitative impact study indicate the following decreases in capital charges for Basel II banks: 79 per cent for home equity loans, 73 per cent for residential mortgage loans, and 27 per cent for small business loans. Under Basel I, the minimum capital requirement for mortgage loans is 4 per cent; thus, an average drop of 79 per cent implies that minimum capital requirements for Basel II banks would be >1 per cent for these asset classes. As there is a cost to maintaining capital, lower capital requirements would result in a cost (and correspondingly a *pricing*) advantage, for large bank retail credits under Basel II. Lower capital requirements will also make it easier for Basel II banks to achieve higher returns on equity (ROE).

To compete with the cost advantage and higher ROEs of Basel II banks, smaller, community banks may be forced to make concessions in pricing and underwriting guidelines that could impair their profitability and ultimately their viability.[26] It is possible the BCBS wished to avoid these possibilities and hence adjusted the one variable at its disposal — namely the asset correlation, which ultimately resulted in higher capital charges for Basel II compliant banks.

Effect of economic milieu

The empirical correlation values are highly sensitive to the parameters estimated from portfolio gross loss data. Clearly, losses change over time. Around the middle of 2007, the global economy witnessed the end of a benign, half-decade long period characterised by low interest rates, constrained inflation, universally loose monetary policy and low default rates for all loan types, particularly residential mortgages. This munificent epoch came to an abrupt end in August 2007 and has continued apace. The milieu in January 2009 is circumscribed by surging inflation, stagnant or rising interest rates, tightening monetary policy and increased default rates, delinquencies and foreclosures. Bank losses have surged in the resulting credit crunch with several seemingly invincible institutions, including investment banks and building societies, suffering significant losses. Global losses from the credit crunch are currently (early 2010) estimated to be \$1trn[27] but the number continues to grow. There are also strong indications that this market turmoil will last for some time — estimates vary from early 2010 to several years into the next decade of the new millennium.[26]

The loss profiles of most banks in the current climate, then, are very different from those which preceded August 2007 and, correspondingly, empirical correlations embedded in the loss data could also be substantially different. To investigate this effect, a rolling, seven-year window of quarterly losses was used to estimate the empirical asset correlations (using both Vasicek and beta distributions) and to compare these with the BCBS specified correlations. These results are shown in Figure 7(a–h) for various categories of retail loans. The effect of the credit crunch is only now being recorded in the loss data: increased losses are evident from the last two quarters of 2008 and the first quarter of 2009. The duration and severity of these losses remain unknown.

On the whole, BCBS specified correlations are higher than empirically-derived values — in most cases, considerably so. The derived correlations from the Vasicek and beta distributions show broadly similar results. For the most part, changes in correlations are 'in phase' with one another, although the Vasicek implied correlations are more sensitive to changes in underlying loss data: these correlations experience larger swings than those implied by the beta distribution. In addition, the Vasicek correlation appears to be more sensitive to the changing loss milieu. A possible reason for this is the relative sensitivity of the underlying distribution drivers, ie the mean and the mode of the gross losses for the Vasicek distribution and the mean and the standard deviation of the gross losses for the beta distribution. The standard deviation changes relatively slowly with time, even during periods of abrupt loss data changes, because it is an *average* of

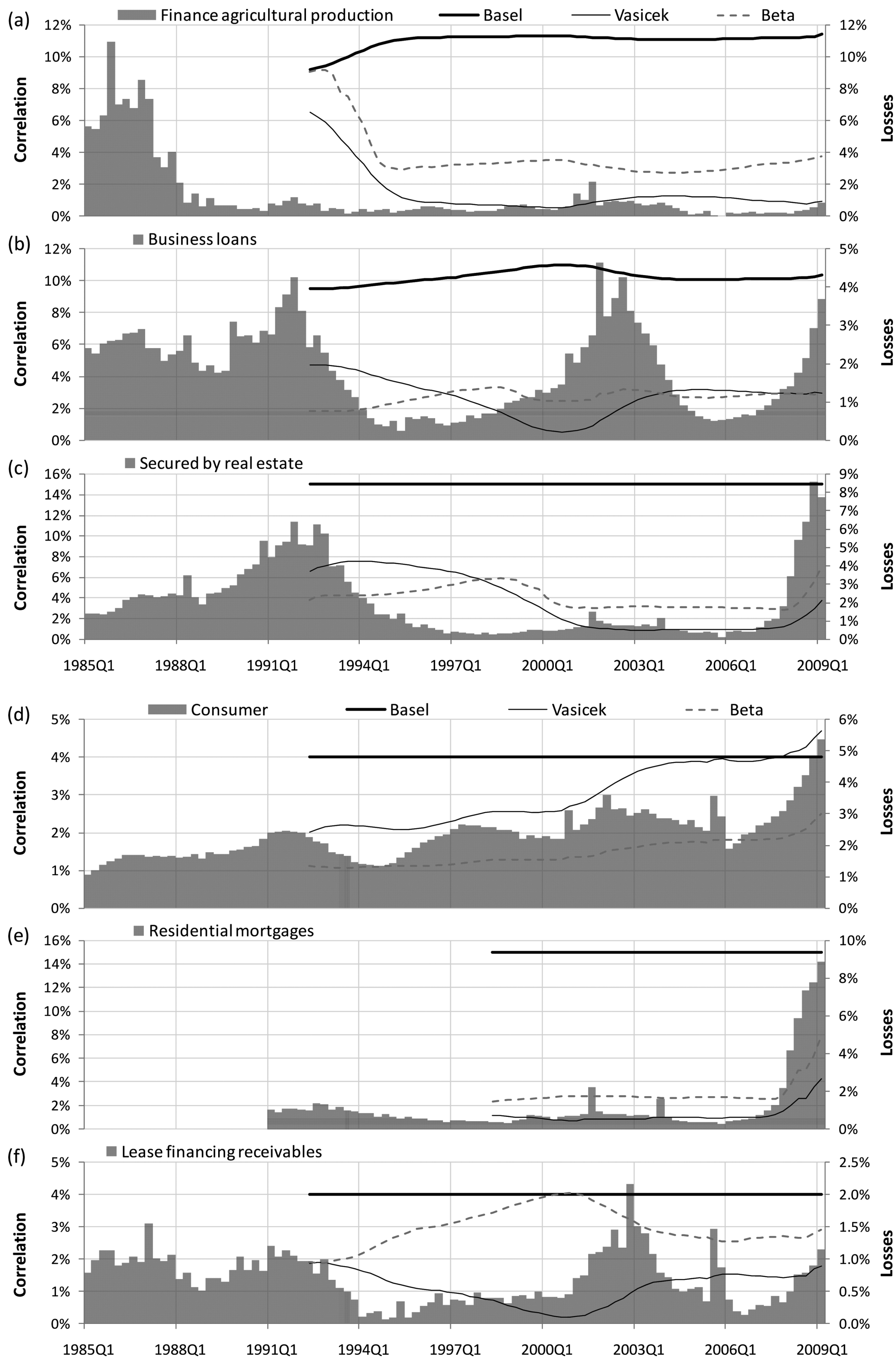

Figure 7: Comparison of rolling seven-year empirical correlations and Basel II specified correlations (right-hand axis). To assess how the underlying gross loss data affect the empirical correlations, the data are shown as grey histograms on the same scales

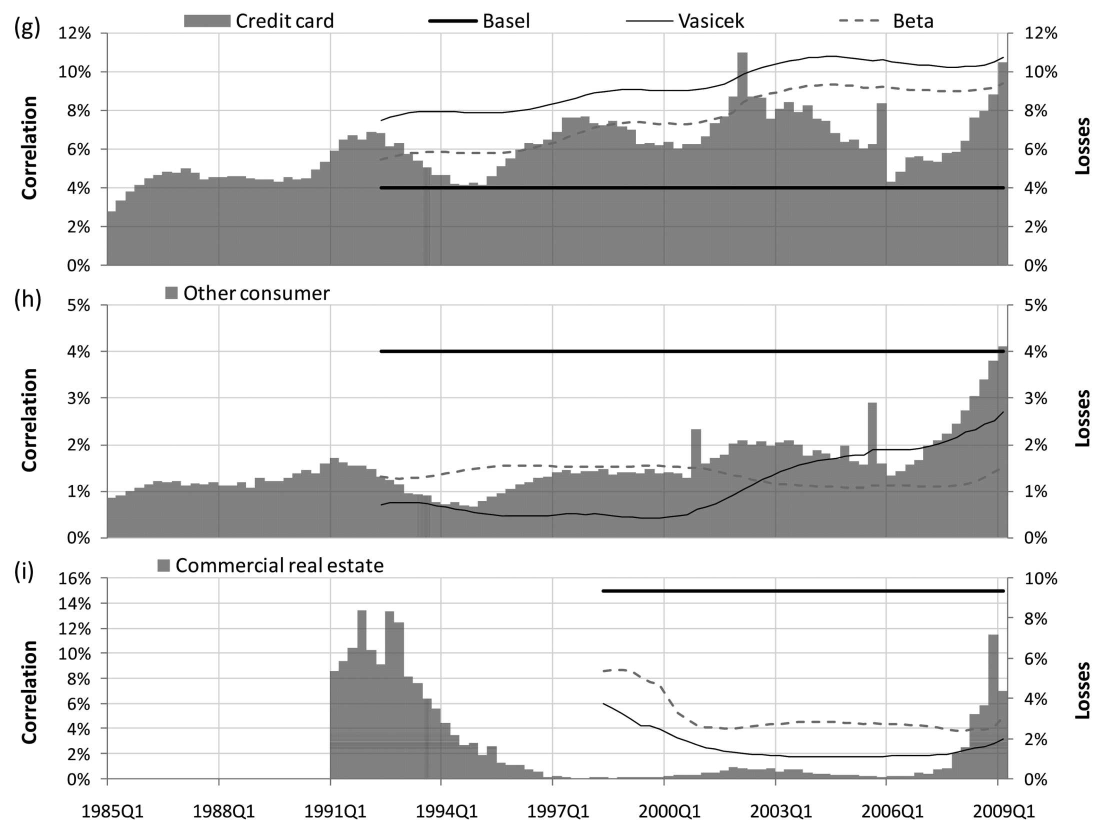

Figure 7: *Continued*

squared deviations from the mean. This averaging has the effect of smoothing out any large spikes that arise in underlying data. This smooth changing of σ feeds through into the shape of the beta distribution.

The mode, on the other hand, changes discretely — in varying step sizes — changing value only when (and if) the data under scrutiny yield a more populous gross loss value than the previous mode. If data are analysed during a period of diminished and then elevated losses, the mode will undergo a discrete 'jump' between a low value and a high value with no intermediate values in between. This will have the effect of amplifying changes in the correlations implied by the Vasicek distribution (as shown in Figure 7).

An interesting feature of the BCBS specified correlations — at least those which depend on the PD value — is that they appear to be counter-cyclical to the derived correlation values. In Figures 7(b) and (f) the BCBS specified asset correlations move counter to the way in which the derived correlations change. A possible explanation is that the BCBS specified asset correlations use a *current* PD and are therefore far more reactive to changing economic conditions than the empirical distributions which use gross loss *averages* measured (in these cases) over seven years to estimate asset correlations.

CONCLUSIONS

The decision by the BCBS to set prespecified correlations was designed to introduce a level of conservatism into the credit risk IRB framework and elevate capital charges to a satisfactory level. Analysis of empirically-derived asset correlations in retail portfolios demonstrates that this is indeed the case: empirical correlations embedded in gross loss data are lower than those set by the BCBS. The theoretical basis on which the IRB approach is built is non-trivial, but nevertheless accessible, and extracting empirical correlations from loss data need not necessarily be an onerous affair. Using two different distribution assumptions, it has been shown how these empirical correlations may be calculated from minimal input data (ie only gross losses over, at least, a seven-year period), how these differ from the BCBS specified correlations and how they change over changing economic conditions. The analysis should be of benefit to banks interested in establishing their own internal measure of correlation for both regulatory and economic capital purposes.

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