Minimising operational risk in portfolio allocation decisions

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Abstract Models play an important role in strategic asset allocation (SAA). Overdependence on the stability of models, however, can reduce the utility of the results for practitioners. Professionals face the problem of choosing an SAA model that matches their goals, and must bear the operational risk of making the wrong choice (the model risk). The present paper sets out to assist in this challenge by evaluating several methodologies for estimating efficient portfolios, according to the perspective of a global long-term investor. It also presents the resampling adjusted technique (RATE) approach to incorporate estimation risk into mean-variance portfolio selection, based on the portfolio resampling technique. The results support the use of the Michaud resampling methodology followed by the RATE, as they offer better results in terms of financial efficiency, allocation stability and diversification. The evaluation of the different models uses several international asset classes for the period between June 1998 and July 2006. The findings are very useful for practitioners who can benefit from a fairly simple and robust asset allocation methodology.

Keywords: *model risk, estimation risk, portfolio optimisation*
JEL Classification: *G11, G15, G23*

INTRODUCTION

Models play an important role in strategic asset allocation (SAA). Overdependence on the stability of models, however, can reduce the utility of the results for practitioners. A major issue in asset allocation problems, using Markowitz optimisation, is that the output is strongly driven by the risk-return estimation, which usually generates very unstable portfolios. The most well-known problem with this technique is the substitution problem, where two assets have the same risk but slightly different expected returns. An optimiser would give all the weight to the asset with the higher expected return, leading to very unstable asset allocation. This is particularly problematic for an investor with a long-term investment horizon (eg pension funds and central banks).

Professionals face the problem of choosing an SAA model that matches their goals, and must bear the operational risk of making the wrong choice (the model risk). Demonstrating the importance of model risk, Fritz and Erlebach[1] posit that the current subprime crisis is partly due to a systematic and widespread misuse of models with too little emphasis on validation and stress-testing of model assumptions. Model risk is possibly one of the most harmful sources of operational risk and practitioners should be aware of models' strengths and weaknesses, so as to avoid model misoperation. Too much trust on the risk-return estimation is critical in several SAA models and this is the main issue addressed in this paper.

Recent literature has tried to address the problems leading to unfeasible portfolios and some of the proposed models are discussed in the following section. These models aim to identify how to create realistic portfolios that recognise the values used for risk and returns are not deterministic but merely estimates (ie they are stochastic). It should be noted that the misspecification of returns is much more critical than that of variances.[2]

Despite the vast literature that has proposed methods to address the asset allocation problem, the consideration of a long investment horizon from a global markets perspective has been neglected. This is probably due to the lack of reliable international financial data and to the greater level of interest in the short-term horizon. However, the present study considers that several market participants, characterised by a more conservative long-term preference, would be willing to adjust their portfolios or benchmarks in a more efficient way, and in this sense, the available techniques are less suitable.

This paper evaluates several methodologies for estimating efficient portfolios and also presents the resampling adjusted technique (RATE) approach to incorporate estimation risk into mean-variance portfolio selection, based on the portfolio resampling technique. Thus, the paper addresses the operational risk of model misuse due to errors in the mean-variance estimation. The results support the use of the Michaud[3] resampling methodology followed by the RATE, as they offer better results in terms of financial efficiency, allocation stability and diversification. The evaluation of the different models uses several international asset classes for the period from June 1998 to July 2006. The findings are very useful for practitioners who can benefit from a fairly simple, easy to implement and robust asset allocation methodology.

The remainder of this paper is as follows. The next section offers a brief literature review, describing the other methodologies of asset allocation considered in this study as well as the RATE. This is followed by a description of the empirical study, including the data and the implementation, as well as the results. The subsequent section explains why the results of the present study are relevant for asset managers and the final section concludes the paper, reviewing the main achievements.

DESCRIPTION OF THE OPTIMISATION MODELS USED

Operational risk is defined as the risk of financial losses, reputational damage or inability to reach business objectives as a result of inadequate or failed internal processes, people, systems or external events.[4] Under the Basel II operational risk taxonomy, model risk is classified as an 'Execution, Delivery & Process Management' item, which is defined as losses from failed transactions processing or process management.

The sources of model risk are typically wrong model, model implementation and model usage. This paper focuses on the wrong model problem, which appears from inapplicability of the model or incorrect model specification. In particular, the paper is interested in evaluating the impact of estimation error on several SAA models, in an attempt to mitigate the model risk due to uncertainty of the risk-return estimates.

Under the same framework, this section reviews the existing optimisation approaches and then describes the RATE. The models addressed in this paper are given as follows:

- *Mark:* The Markowitz portfolio selection model;[5]
- *Jorion:* Jorion's Bayesian informative prior model;[6]
- *Bayes:*The Bayesian diffuse prior model;[6]
- *Kempf:* Kempf *et al.*'s Bayesian informative prior model;[7]
- *RiskAdj:* Black and Litterman's risk-adjusted model;[8]
- *Equil:* Black and Litterman's equilibrium model;[8]
- *Horst:* Horst *et al.*'s adjusted risk aversion model;[9]
- *Mich:* Michaud's portfolio resampling model;[3]
- *RATE:* The resampling adjusted technique (proposed by the authors).

The following notational conventions are used: constants and variables are typed in italics, operators and functions in regular roman text, and vectors and matrices in bold.

Markowitz portfolio selection model

The portfolio selection model developed by Markowitz[5] is considered the classical approach for portfolio optimisation. It is based on the trade-off between the mean and variance of a portfolio's returns. Assume a market with N risky securities and one risk-free security. Let R_i, $i = 1, \ldots, N$, be the random returns of risky securities and R_F the constant return of a risk-free security. This model does not specify the probability distributions of the returns but one can assume means and variances that are real finite numbers. Denote the variance-covariance matrix of the risky securities by $\mathbf{V}$, a positive definite matrix, which implies that there are no redundant securities.

Consider a portfolio composed of N risky assets plus the risk-free asset. The weights for each risky asset are given by $N \times 1$ vector of real numbers called $\mathbf{a}$. In this case, the return of the portfolio is given by:

$$R_a = (1 - \mathbf{a}^{\mathrm{T}}\mathbf{1})R_F + \mathbf{a}^{\mathrm{T}}\mathbf{R}$$

where $\mathbf{1}$ is an $N \times 1$ vector of ones, $\mathbf{R}$ is an $N \times 1$ vector of the risky securities' returns, and $^{\mathrm{T}}$ denotes the transpose operator.

Observe that weights of this portfolio add up to one, and short positions are permitted. The expected return and the variance of the return of this portfolio are:

$$\mathrm{E}(R_a) = R_F + \mathbf{a}^{\mathrm{T}}\mathbf{X}$$
$$\mathrm{VAR}(R_a) = \mathbf{a}^{\mathrm{T}}\mathbf{V}\mathbf{a}$$

where $\mathbf{X}$ is the $N \times 1$ vector of excess risky security expected returns over R_F.

Following Markowitz,[5] the study uses the term *portfolio efficient frontier* for the set of portfolios that have the lowest standard deviation for any given expected return, which is the same as to solve the following minimisation problem:

$$\underset{a}{\mathrm{Min}} \frac{1}{2}\mathbf{a}^{\mathrm{T}}\mathbf{V}\mathbf{a}$$

subject to

$$\mathrm{E}(R_a) = \mathbf{a}^{\mathrm{T}}\mathbf{X}$$

In the presence of a risk-free asset, the efficient frontier becomes a straight line — the so-called capital market line (CML). This paper is interested in defining the tangency portfolio — the portfolio that is both on the CML and on the portfolio frontier of risky assets hyperbola. As shown by Feldman and Reisman,[10] the weights for the tangency portfolio are given by:

$$\mathbf{a} = ((\mathbf{V}^{-1}\mathbf{X})^{\mathbf{T}}\mathbf{1})^{-1}\mathbf{V}^{-1}\mathbf{X}$$

Another key portfolio in Markowitz's framework is the minimum variance portfolio, ie the portfolio of the efficient frontier with lowest variance. The weights of the minimum variance portfolio are given by:

$$\mathbf{a_0} = ((\mathbf{V}^{-1}\mathbf{1})^{\mathrm{T}}\mathbf{1})^{-1}\mathbf{V}^{-1}\mathbf{1}$$

Jorion's Bayesian informative prior model and Bayesian diffuse prior model

This model offers a simple empirical Bayes estimator that should outperform the sample mean in the context of a portfolio. The main idea is to select an estimator with average minimising properties relative to a loss function, which depends on estimation risk. Instead of using sample mean, an estimator obtained by 'shrinking' the mean values towards a common value is proposed to reduce estimation error. The common value chosen is the expected return for the minimum variance portfolio.

The model of Jorion[6] has two main assumptions. First, the prior mean is identical across all N risky assets. Secondly, the estimation risk is proportional to the innovation risk. Thus, the optimal portfolio choice should be based on the predictive function of the vector of future returns, and the estimated expected returns and expected covariance matrix are given by:

$$\mathrm{E}(R_a) = (1 - w)\mathbf{X} + w\mathbf{1}\mathbf{a_0}\mathbf{X}$$

and

$$\mathrm{VAR}(R_a) = \mathbf{V}\left(1 + \frac{1}{Y + \lambda}\right) + \frac{\lambda}{Y(Y + 1 + \lambda)} \frac{N}{\mathbf{1}^{\mathbf{T}}\mathbf{V}\mathbf{1}}$$

where Y is the number of return observations used to estimate the mean and

$$w = \frac{\lambda}{Y + \lambda}$$

Observe that λ is the precision of the prior belief that the prior mean is the same for all N risky assets (the mean return for the minimum variance portfolio). Therefore, this approach will outperform the classical sample mean because it relies on a richer model and includes the sample mean as a special case when $\lambda = w = 0$. This special case is known as the Bayesian diffuse prior model.

In the general case ($\lambda \neq 0$) the shrinkage estimator is given by:

$$w = \frac{N+2}{(N+2) + (\mathbf{X} - \mathbf{a}_0\mathbf{X1})^{\mathbf{T}} Y \mathbf{V}^{-1} (\mathbf{X} - \mathbf{a_0X1})}$$

From the previous formulation, one can observe that, when Y is small, ie there is a low number of return observations, the shrinkage estimator seems to offer better results (w is higher), intuitively due to the lower confidence on the sample mean. As Y increases and w gets lower, the relevance of the prior belief is reduced and the model tends towards the classical approach of Markowitz.[5] Jorion[6] shows that the model's improvement is really relevant when $Y < 100$.

Kempf *et al.*'s Bayesian informative prior model

As with the model of Jorion,[6] this model assumes that the prior mean is identical across all N risky assets. However, Kempf *et al.*'s model[7] considers estimation risk as a second source of risk, determined by the heterogeneity of the market, which is represented by the standard deviation of the expected returns across risky assets τ^2. The estimated expected returns and expected covariance matrix are given by:

$$\begin{aligned} \mathrm{E}(R_a) &= [\mathbf{V} + \mathbf{Y}\tau^2\mathbf{I}]^{-\mathbf{1}} Y\tau^2\mathbf{X} \\ &\quad + [\mathbf{V} + \mathbf{Y}\tau^2\mathbf{I}]^{-\mathbf{1}}\mathbf{V1a_0X} \\ \mathrm{VAR}(R_a) &= \mathbf{V}[\mathbf{1} + \tau^2[\mathbf{V} + \mathbf{Y}\tau^2\mathbf{I}]^{-\mathbf{1}}] \end{aligned}$$

where $\mathbf{I}$ denotes the identity matrix. In this case, as in Jorion,[6] the conditional expected return is a weighted sum of the prior and the historical average. However, the weighting factors are no longer scalars but matrices. Kempf *et al.*[7] show that this model generates more efficient portfolios when compared with Jorion[6] or the classical Markowitz.[5] Results are robust with respect to different estimation periods.

Black and Litterman's models

Black and Litterman[8] propose a framework for portfolio optimisation that mixes different types of *ex ante* return estimates, some based on historical data as in Markowitz,[5] and others based on equilibrium conditions. For instance, the use of a global capital asset pricing model (CAPM) equilibrium model can significantly improve the usefulness of optimisation models, as it can provide a neutral starting point for estimating the set of expected returns. The paper will consider two of their approaches, as described below.

Risk-adjusted equal means

This approach assumes that different risky assets have the same Sharpe Ratio, ie the same expected return per unit of risk, where the risk measure is the standard deviation of returns. Thus, expected excess returns can be estimated by a linear regression where the independent variable is the standard deviation of each asset's returns:

$$\mathbf{X} = \alpha\boldsymbol{\sigma} + \varepsilon, \varepsilon \approx \mathrm{N}(0, 1)$$

where $\boldsymbol{\sigma}$ is a vector of the risky assets' standard deviations. This approach has a significant problem: it does not consider the correlation between risky assets when defining the expected excess returns.

Equilibrium approach

This approach uses the CAPM equilibrium model to derive expected returns. In this case, the expected returns are those that keep the market in equilibrium, if all investors have identical views. An asset is rewarded by the additional systematic risk it adds to the market portfolio. The formulation is given in the following way:

$$\mathbf{X} = \beta M + \varepsilon$$

and

$$\beta_i = \frac{\mathrm{Cov}(X_i, M)}{\mathrm{Var}(M)}$$

where M is the excess return of the market portfolio. This model overcomes the previous model's problem of not considering the correlation of risky assets. The main inconvenience of this approach is related to the selection of a market portfolio.

Horst *et al.*'s adjusted risk aversion model

Horst *et al.*[9] propose a model that takes into account the estimation risk. They use pseudo-risk aversion rather than actual risk aversion, which depends on the sample size, number of assets in the portfolio and curvature of the mean-variance frontier. Pseudo-risk aversion is always higher than actual risk aversion and this difference increases with the uncertainty about the expected return estimations. The pseudo-risk aversion is given by:

$$\alpha = (\mathbf{V^{-1}X})^{\mathbf{T}}\mathbf{1} \times \left(\mathbf{1} + \frac{\mathbf{1^T V^{-1} 1}}{(\mathbf{1^T V^{-1} 1})(\mathbf{X^T V}^{-1}\mathbf{X}) - ((\mathbf{V^{-1}X})^{\mathbf{T}}\mathbf{1})^2} \frac{N-1}{Y}\right)$$

and so the weights for the tangency portfolio are given by:

$$\mathbf{a} = \boldsymbol{\alpha}^{-1}\mathbf{V}^{-1}\mathbf{X}$$

Michaud's portfolio resampling model

Portfolio sampling allows an analyst to visualise the estimation error in traditional portfolio optimisation methods. Suppose one estimated both the variance-covariance matrix $\mathbf{V}$ and the excess return vector $\mathbf{X}$ by using Y observations. It is important to recall that the point estimates are random variables and so another sample from the same distribution would result in different estimates.

Sherer[11] suggests that sampling from a multivariate normal distribution (a parametric method termed *Monte Carlo simulation*) is a way to capture the estimation error. In this sense, $\mathbf{X}$ and $\mathbf{V}$ would be just the expected values for a multivariate normal distribution. By repeating the sampling procedure n times, one obtains n new sets of optimisation inputs, and then a different efficient frontier. The resampled weight for a portfolio would then be given simply by the average of portfolio weights of the n samples:

$$\mathbf{a}^{\text{Resamp}} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{a_i}$$

where $\mathbf{a_i}$ is the vector of portfolio weights for the i^{th} sample.

Resampled portfolios have two characteristics that are desirable for practitioners. First, resampled portfolios usually have greater diversification as more assets enter in the solution than the classical mean-variance efficient portfolios. Secondly, the model exhibits smoother transitions and less sudden shifts in allocations as return expectations change, meaning that the transaction costs of rebalancing the portfolio are lower. Recent literature has supported resampled portfolios when performing out-of-sample analysis.[12–14]

The resampling adjusted technique (RATE)

The paper has now discussed several models to deal with asset allocation problems and, as the intention is to consider a long-term investor, it is important to recall the main goal: to achieve an efficient stable asset allocation. As has been discussed, the classical approach outweighs the importance of estimated excess return, which may result in an inefficient portfolio.

All Bayesian methods approach this issue by incorporating prior beliefs in the estimation, thereby reaching smoother portfolio weights. However, this prior belief can always be questioned. The usual resampling technique explicitly considers estimation errors obtaining a more diversified portfolio. However, when they consider $\mathbf{X}$ and $\mathbf{V}$ to be the expected values for a multivariate normal distribution of excess returns, they double-count the effect of the variance-covariance matrix in the optimisation problem.

An alternative resampling technique might be used to overcome the previous problems. The main idea is to dissociate the estimation risk associated with the expected excess return estimation from its variance. First, one assumes that there is no estimation risk in the variance-covariance estimation. This is a usual assumption when dealing with portfolio optimisation techniques as the impact of this estimation error is much less pronounced than the errors associated with expected returns.

So, assume one has N risky assets, each associated to a time series of observed returns $\mathbf{r_i}$ (vector of observed returns for asset i) of length L. Then suppose an estimation period $p \ll L$. Here, the estimated excess return and variance-covariance matrix calculated over p is the classical $\mathbf{X}$ and $\mathbf{V}$. $\mathbf{V}$ is assumed to have no estimation error, while $\mathbf{X}$ is assumed to be the expected value of a multivariate normal distribution, with the variance-covariance matrix $\mathbf{W}$ calculated over the different values of $\mathbf{X}$, considering a rolling window from p to L. Thus:

$$\mathbf{W} = Cov(\mathbf{X}_p, \ldots, \mathbf{X}_L)$$

Then, similar to the usual resampling model, by repeating the sampling procedure n times, one obtains n new sets of optimisation inputs, and then a different efficient frontier. The resampled weight for a portfolio would then be given by:

$$\mathbf{a}^{\mathrm{Resamp}} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{a_i}$$

The main advantage of the proposed technique is that it dissociates market risk ($\mathbf{V}$) from the estimation risk ($\mathbf{W}$), thus helping to reach a stable efficient portfolio. The drawback of the procedure is that one must assume that $\mathbf{W}$ is stable over time, which tends to be true especially if one considers a long-term investment horizon, as is the purpose of this paper (eg the variance-covariance matrix of several assets' five-year average excess return tends to be constant over time).

EMPIRICAL STUDY

Data and implementation

The tests are based on monthly data of 15 Lehman Brothers and JPMorgan international total return indices and also two stock indices, from the period from June 1998 to July 2006. All total return time series are calculated on a US-dollar basis while excess returns are calculated using a three-month US treasury bill rate. (It is recognised that this assumption favours US assets as they are then free from currency risk; however, the US dollar is the typical numeraire considered by global investors, such as central banks, pension funds and multinational institutions.) Table 1 presents the description of each asset considered. The study uses a diversified group of assets in terms of currency, market and instrument type.

Figure 1 presents the risk-return information for the considered assets. Observe the upward slope of the tendency indicating the risk-return trade-off.

The performance of the nine optimisation strategies described previously is analysed. The estimation period (p) is varied in and out of sample analysis. The parameters are estimated using monthly return observations of the past p months. The efficient risky portfolio is defined and held for the next (e) months. The parameters are the re-estimated and the portfolio weights adjusted. The models' performance is evaluated with and without a short-selling constraint.

To judge the financial performance of the strategies, their empirical Sharpe Ratios are computed, and to evaluate the stability of the allocation weights generated by the model, the study uses the mean of the standard deviations of each asset's allocation weights for the period. The mean Herfindahl index, given by the sum of the squared asset allocation weights, is used to infer the diversification.

Results

The Sharpe Ratios of the different strategies are presented in Table 2 for the total period from 1998 to 2006, considering $e = 1$ or 6; and p varying from 60 to 72 months.

As expected, the Bayesian models (Jorion, Kempf, Horst and Bayes) offer better performance for lower estimation periods and, as the estimation period increases, their results approach the traditional Markowitz allocation. The resampling methods (Mich and RATE) seem to slightly improve the Markowitz results, especially if one considers a larger evaluation period. However, when compared with the other models, the results are mixed. Such an unfavourable result of resampling has already been highlighted by Sherer,[11] who pointed out that the Michaud methodology does not improve the Markowitz if short-selling is not constrained. When one takes into account the dispersion in the optimal allocations, one obtains the results presented in Table 3.

Jorion and Horst offer the low dispersions. RiskAdj and Equil obtain the worst results (higher dispersion). The poor results obtained by the Black-Litterman model (Equil) are probably due to the difficulty in defining an appropriate market portfolio with the

Table 1: Descriptive information of each asset considered in the analysis

Name	Mnemonic	Market	Currency	Instrument type	E[R](%)	Std. (%)
Asset Backed Securities	ABS	USA	USD	Bonds	0.470	0.778
DAX30	DAX	Germany	EUR	Stocks	0.391	5.721
EMBI plus	EMBI	Emerging	USD	Bonds	0.908	3.384
Treasuries EUR 1–5 years	EUR1–5	Euro Zone	EUR	Bonds	0.531	2.865
Corporates	Corp	USA	USD	Bonds	0.490	1.090
FTSE100	FTSE	UK	GBP	Stocks	0.442	3.492
Global Index	Global	Global	USD	Global	0.462	1.630
Gold	Gold	USA	USD	Currency	1.098	7.908
High Yield Bonds	HY	USA	USD	Bonds	0.463	2.272
Bank Deposits EUR 1M	DepEUR	Euro Zone	EUR	Cash	0.427	2.402
Bank Deposits JPY1M	DepJPY	Japan	JPY	Cash	0.188	2.663
Bank Deposits USD 1M	DepUSD	USA	USD	Cash	0.308	0.163
Mortgage Backed Securities	MBS	USA	USD	Bonds	0.467	0.765
S&P500	S&P	USA	USD	Stocks	0.572	3.707
Treasury Bill 1–3 month	Tbill	USA	USD	Bonds	0.278	0.151
Treasuries US 1–5 years	US1–5	USA	USD	Bonds	0.387	0.671
Guilts	GLT	UK	GBP	Bonds	0.656	2.619

EMBI, Emerging Markets Bond Index; ER, expected return or mean monthly return calculated over the sample period; Std, monthly standard deviation calculated over the sample period.

correct level of currency hedging. In addition, the model is being used without including the investors' views of the market, as the study is evaluating long-term passive strategies. One can also observe that Mich and RATE do not present any improvement in terms of lowering the dispersion in the asset allocations.

However, by adding a short-selling constraint (which is the usual situation with the kind of investor being considered here, such as pension funds and central banks) in the optimisation problem, one obtains the results presented in Tables 4 and 5.

From Table 4, the improvement of the resampling methods (Mich and RATE) is clear when compared with the others, and this result is robust for different estimation and evaluation periods. Additionally, Mich outperforms RATE. From the remaining models, Kempf seems to offer the best results.

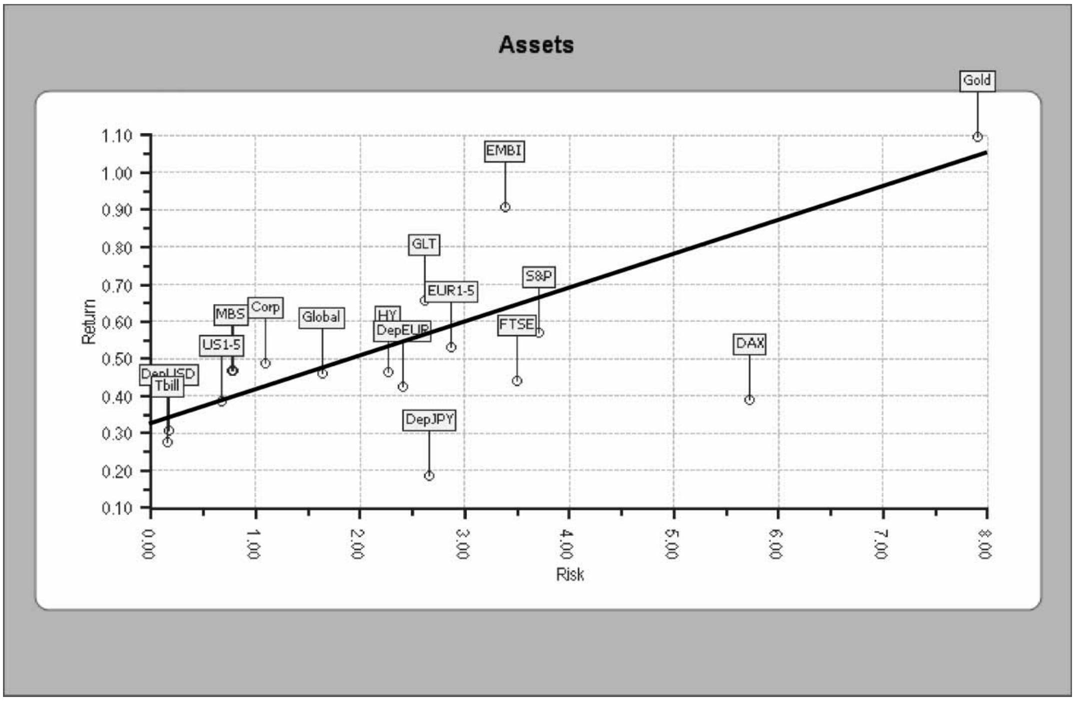

Figure 1: Risk-return estimates of asset classes

In Table 5 the dispersion is addressed and again Mich offers the best results, generating more stable optimum portfolios. RATE generally produces stable portfolios when compared with the remaining models. Table 6 evaluates the diversification of the optimum portfolios generated by each model and Mich offers the most diversified portfolio with a Herfindahl index of about 0.21, while its value for Markowitz is over 0.42. RATE also produces more diversified portfolios versus the Markowitz model.

In general, one can state that Mich and RATE offer the better results for a long-term passive short-selling constrained investor when considering financial efficiency, stability and diversification; the former outperforms the latter. However, RATE can still be considered as a viable alternative for generating more feasible portfolios than the other methodologies. The previous results are robust for different estimation and evaluation periods.

Table 2: Sharpe Ratios (unconstrained solution) of the efficient portfolio generated by each estimation model

	Mark	Jorion	Kempf	Horst	Bayes	RiskAdj	Equil	Mich	RATE
60-1	0.693	0.734	0.767	0.731	0.694	0.704	0.419	0.689	0.649
66-1	0.788	0.779	0.805	0.780	0.789	0.840	0.765	0.784	0.805
72-1	0.731	0.726	0.725	0.726	0.732	0.693	0.731	0.740	0.736
60-6	0.674	0.715	0.737	0.712	0.676	0.657	0.629	0.625	0.695
66-6	0.751	0.762	0.775	0.761	0.753	0.739	0.751	0.738	0.756
72-6	0.628	0.645	0.650	0.643	0.629	0.541	0.628	0.614	0.634

The Sharpe Ratio is calculated by dividing the excess return observed divided by the standard deviation.

Table 3: Dispersion (unconstrained solution) in the asset allocation weights given by the mean standard deviation calculated over the whole out-of-sample period

	Mark	Jorion	Kempf	Horst	Bayes	RiskAdj	Equil	Mich	RATE
60-1	0.53%	0.45%	0.54%	0.45%	0.53%	0.64%	0.75%	0.57%	0.78%
66-1	0.52%	0.41%	0.52%	0.43%	0.51%	0.88%	0.85%	0.60%	0.55%
72-1	0.47%	0.38%	0.47%	0.39%	0.46%	0.75%	0.47%	0.53%	0.50%
60-6	0.60%	0.51%	0.60%	0.52%	0.60%	0.65%	0.75%	0.60%	0.63%
66-6	0.58%	0.46%	0.58%	0.48%	0.57%	0.70%	0.58%	0.65%	0.61%
72-6	0.51%	0.41%	0.50%	0.43%	0.50%	1.03%	0.51%	0.57%	0.53%

Table 4: Sharpe Ratio (constrained solution) of the efficient portfolio generated by each estimation model

	Mark	Jorion	Kempf	Horst	Bayes	RiskAdj	Equil	Mich	RATE
60-1	0.219	0.221	0.217	0.221	0.214	0.189	0.206	0.251	0.240
66-1	0.216	0.215	0.245	0.185	0.207	0.244	0.161	0.250	0.233
72-1	0.324	0.327	0.329	0.276	0.316	0.319	0.324	0.406	0.354
60-6	0.194	0.197	0.221	0.183	0.185	0.180	0.106	0.222	0.213
66-6	0.221	0.219	0.257	0.209	0.214	0.175	0.113	0.270	0.234
72-6	0.270	0.273	0.307	0.241	0.265	0.200	0.270	0.359	0.304

Table 5: Dispersion (constrained solution) in the asset allocation weights given by the mean standard deviation calculated over the whole out-of-sample period

	Mark	Jorion	Kempf	Horst	Bayes	RiskAdj	Equil	Mich	RATE
60-1	6.08%	6.12%	6.17%	4.73%	6.17%	8.28%	7.96%	4.05%	4.84%
66-1	5.75%	5.77%	6.11%	4.53%	5.82%	8.13%	7.78%	3.21%	4.55%
72-1	4.65%	4.69%	5.09%	3.65%	4.67%	7.30%	4.65%	2.49%	3.79%
60-6	6.47%	6.51%	6.83%	5.06%	6.61%	8.64%	8.33%	4.58%	5.13%
66-6	6.43%	6.51%	6.63%	4.98%	6.43%	8.57%	8.59%	3.72%	4.92%
72-6	4.17%	4.21%	5.57%	3.38%	4.18%	7.30%	4.17%	2.75%	3.81%

Table 6: Mean Herfindahl index calculated over the optimum portfolio allocation weights considering the whole out-of-sample period

	Mark	Jorion	Kempf	Horst	Bayes	RiskAdj	Equil	Mich	RATE
60-1	0.473	0.468	0.537	0.332	0.492	0.502	0.512	0.211	0.350
66-1	0.495	0.492	0.577	0.349	0.517	0.492	0.520	0.217	0.385
72-1	0.456	0.458	0.566	0.321	0.468	0.456	0.456	0.221	0.384
60-6	0.480	0.472	0.550	0.338	0.507	0.516	0.501	0.222	0.356
66-6	0.520	0.520	0.550	0.347	0.526	0.523	0.550	0.209	0.388
72-6	0.426	0.430	0.547	0.298	0.432	0.452	0.426	0.221	0.390

HOW MIGHT ASSET MANAGERS BENEFIT FROM THESE RESULTS?

The findings of this paper could benefit practitioners in a number of ways. First, the paper has stressed the importance of a main source of operational risk: the model risk. As using several methodologies obtained very different results for the same asset allocation problem, it is clear that the choice of model is crucial. In addition, many of the results are not suitable for a typical long-term investor, as portfolio weights have extreme values and fluctuate dramatically over time. The best-known model, Markowitz, typically under-diversifies and generates unstable portfolios, even if short-selling is constrained, and this is very harmful for the performance of the portfolio, especially under crises where uncertainty is exacerbated.

The findings in Table 4 can be used by asset allocation firms to compare the financial performance of the models and decide upon the most appropriate. Additionally, the findings of Table 5 can be used by long-term investors to compare transaction costs of the various models and rebalancing schemes. It is reasonable to conclude that more stable portfolios, such as those given by Michaud or RATE, will imply lower transaction costs compared with the other models. As such, comparison of financial efficiency would favour those two optimisation techniques even more.

Both RATE and Michaud outperform the Markowitz model in terms of financial efficiency, level of diversification and allocation stability. They are also fairly simple, easy to implement and robust asset allocation methodologies. As such they provide good alternatives for practitioners to run their SAA activities.

However, to prevent the operational risk of over-reliance on model stability, even RATE and Michaud have their limitations; for example, they are not classified as equilibrium models. They can be considered heuristic in terms of the characteristics of data imprecision, and they propose statistical techniques to overcome the estimation risk problem.

CONCLUSIONS

This paper addresses the issue of model risk in strategic asset allocation. The estimation of model inputs is still a challenge for practitioners. The study evaluated several techniques to calculate the efficient portfolio, using data from a broad range of asset classes (bonds, cash and stocks), from the perspective of a long-term passive investor. Nine methodologies were considered from the traditional Markowitz model, Bayesian approaches, general equilibrium techniques and finally resampling methodologies.

The results support the use of resampling techniques with the one proposed by Michaud[3] offering the best results for a passive short-selling constrained global investor, as measured by financial efficiency (Sharpe Ratio), portfolio stability and diversification. The findings are very useful for practitioners who can benefit from a fairly simple and robust asset allocation methodology.

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