Managing structured bonds: An analysis using RAROC and EVA

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Rosa Cocozza*
is Professor of Financial Risk Management at the University of Naples. Her research interests are in managerial quantitative modelling.

Albina Orlando
is Senior Researcher at the Italian National Research Council. Her research interests are in numerical modelling for finance and insurance.

*Università degli Studi di Napoli Federico II, Facoltà di Economia, Dipartimento di Economia Aziendale, Via Cinthia Monte S. Angelo, 80126, Napoli, Italy
Tel: +39 (0)81675083; Fax: +39 (0)81675058; E-mail: rosa.cocozza@unina.it

Abstract The paper investigates a decision criterion for structured bonds portfolio choices. The main issue is the application of risk-adjusted indicators as tools to select either the asset portfolio given a structured bond, or the bond structure given an existing coverage asset portfolio. Such an indicator is suitable for the appraisal of both portfolio management and the potential profits of the structured issue. The selection tool is put into an asset and liability management decision-making context, where the relationship between the expected profit and the capital-at-risk are compared in order to evaluate the issue of the bond and the expected rate of return of the whole portfolio. The case is referred to an equity-linked bond and treated by means of Monte Carlo simulations to identify the best portfolio according to the issuer targets and constraints.

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INTRODUCTION

Modern financial markets are characterised by a growing number of structured securities. A structured security can be defined as a contract where the payoff can be replicated by a combination of elementary financial instruments and derivatives. As the complexity of the products grows, that is, as the number of implied cash flows increases, so does the number of constituents required to replicate the desired payoff. In other words, as the involvement of the product grows, so does the complexity of the replicating portfolio, of the pricing process and of the managing practice.

As is known, the constituents of the replicating portfolio will be easier to price, understand and analyse than the original element. All else being equal, a perfect replicating portfolio must have the same price as the original instrument.[1] Thus, by adding the values of the constituent assets, one obtains the cost to form a replicating portfolio.

Nevertheless, there is a practical problem which always arises when dealing with financial products — and especially with structured securities — namely the gap between the fair (theoretical) and the real (actual) price.

In a rigorous systematic perspective, the gap should be void and the price should be fair, in the sense that it should respect the 'law of one price'. Within this context, the optimal estimate should be the arbitrage-free price as equilibrium value. In a wider business perspective, the actual price of a debt instrument issued by any financial intermediary has to exhibit a value that, on the one hand, can be appraised by subscribers and that, on the other hand, proves adequately profitable to the issuer. Therefore, the cost of forming a replicating portfolio will give the actual price of the original instrument once the market practitioner adds a proper margin. This is the case for any bid-ask differential that one might find on the market for any financial intermediation process; this is also the case for loadings applied to pure premiums, within the insurance context.

This paper focuses on the adequacy of the issue's profitability, given a fair valuation of the product. This adequacy has to be appreciated on the basis of the risk-return trade-off. In a sense, one could reformulate the question, directing attention towards the optimality of the issue, as the pricing is set by the usual standard.

This optimality has to be pragmatically evaluated: the risk-return trade-off has to be compared with the return on capital-at-risk set by top management and/or by shareholders and is, by definition, based on a portfolio approach. At the same time, the main answer is conditional on two other sub-questions:

- the identification of the factor relevant to the *optimality* of the issue;
- the measure (or the family of measures) that can be used for the purposes of the present study.

If one can identify bond or security design variables influencing future outcomes, it is possible to obtain an answer to the first question. In this respect, the solution is related not only to the technical features of the bond and to the discretionary parameters but also to the selected investment strategy. As far as the features are concerned, the offer of higher guarantees and/or participation rate should reduce the profitability of the issue. However, the cross-section of this information with different investment strategies could give rise to different results if the adopted strategy is opportunely elected. Such is the case with a beneficial mismatch between asset and liability, as the decision on the (investing) replicating portfolio could make a difference. The selection of a perfect replicating portfolio (a perfect matching strategy) reduces both risk and return, but does not necessarily prove competitive on the market. Conversely, the selection of a favourable imperfect matching will increase both risk and return and could give rise to a portfolio with acceptability that must be carefully evaluated. This leads directly to the question regarding the sustainability of the risks arising from a mismatched position. If the increase in the profitability is connected to an increase in the risk of the portfolio, the evaluation must consider the classical trade-off between risk and reward. In a sense, this is the bridge to the second sub-question.

The measures (or family of measures) must be able to depict the two aspects coherently.[2] As a consequence, the main object of the present investigation is the distribution of the future net worth of the portfolio on the set of states of nature at bond maturity, that is to say, at the end of the portfolio life cycle. A crucial measurement step is whether the prospective value of the position belongs or does not belong to the subset of acceptable risks that represent positions with acceptable future net worth. The acceptability of the future net worth can be measured by an economic indicator such as the economic value added.

This paper is organised in six main sections, including this introductory one, plus appendices. The section on 'The bond' illustrates the technical features of the contract and two possible investment alternatives (a perfect and imperfect matching portfolio) to address the question of the profitability of the issue. The section on 'The acceptability set' defines risk and reward indicators, from a managerial perspective, and seeks to find a general answer to the question of the family of measures that can be useful to the scope of the present study. The section on 'The risk framework' pioneers the applicative section, setting the modelling context and the theoretical reference structure. The section on 'The computational procedure' provides all the computational remarks and the estimated results. A summary of estimations and conclusion are in the section on 'Findings and future research prospects'. The appendices provide the semantics of the symbols used in the paper (Appendix A), and detailed mathematical derivation of some of the product pricing formulas (Appendix B) and the profit formulas (Appendix C).

THE BOND

The analysis is applied to a specific equity-linked bond, with the aim of illustrating the potential application, but it can be applied to any structured security, given the opportune refinements. According to the agreement, by paying a defined amount (U), the buyer will earn — at each settlement time t ($t \leq m$) — a benefit (B_t) that is calculated at the maximum rate between an annual guaranteed one (g) and a percentage (η) of the annual equivalent total return of the reference asset (s) in the same period. In other words, the benefit is set by incrementing each year the present value of the fixed payment R_g (the guaranteed payment) by a fraction η (participation rate) of the return on the investment of the corresponding quota in the reference asset.

At each settlement time t the buyer will therefore receive:

- a fixed amount (R_g) calculated as the instalment of an annuity whose present value is the invested amount (U), ie $R_g = U / \sum_{t=1}^{m} e^{-\delta_g t}$, where δ_g is the force of interest corresponding to the annual guaranteed rate (g);
- an eventual additional benefit calculated as a percentage (η) of the total return of the reference asset, conditional on an actual return higher than the guaranteed rate.

At each maturity, the buyer will earn the following cash flow:

$$B_t = R_g \left[1 + \frac{\eta}{s_0 e^{\delta_g t}} \max(0; S_t - s_0 e^{\delta_g t}) \right] \quad (1)$$

The cash-flow mapping of the single bond is given by Table 1.

Table 1: Buyer flows

Time	0 (issue time)	1	t	m
Flow	$-U$	$+B_1$	$+B_t$	$+B_m$

Cash-flow mapping and perfect replicating portfolio

As the present analysis is aimed at evaluating the profitability of the whole issue, the question is approached on a portfolio basis. The portfolio cash flows are simply obtained by multiplying the individual benefits (Equation 1) by the number of issued securities (n) as shown by Table 2.

By applying a Black and Scholes valuation methodology[3] to the plain-vanilla option portfolios, the value at issue of the single contract can be set as follows:

$$B_0 = \left\{ R_g \sum_{t-1}^{m} \frac{1}{e^{\delta_{r(0;t)}t}} + \eta \left[\frac{N(d_{1(t)})}{e^{\delta_g t}} - \frac{N(d_{2(t)})}{e^{\delta_{r(0;t)}t}} \right] \right\} \tag{2}$$

where $N(\cdot)$ is a standard normal distribution and parameters $d_{1(t)}$ and $d_{2(t)}$ are defined respectively as:

$$d_{1(t)} = \frac{\left(\delta_{r(0;t)} - \delta_g + \frac{\sigma_s^2}{2} \right) t}{\sigma_s \sqrt{t}}$$

and

$$d_{2(t)} = \frac{\left(\delta_{r(0;t)} - \delta_g + \frac{\sigma_s^2}{2} \right) t}{\sigma_s \sqrt{t}}$$

Table 2: Writer flows

Time	0 (issue time)	1	t	m
Flow	$+n^*U$	$-n^*B_1$	$-n^*B_t$	$-n^*B_m$

and where $\delta_{r(0;t)}$ is the force of interest corresponding to the spot rate $r(0;\, t)$ for the period $0 - t$ (see Appendix B).

Notice that the present value of the bond portfolio (OF_0^{P}) is equivalent to the sum of:

- a portfolio of m zero coupon bonds with maturity going from 1 to m whose nominal value is exactly the compulsory t-outflow; and
- m portfolios of plain-vanilla call options with varying maturity from 1 to m, increasing strike prices from $s_0 e^{\delta_g}$ to $s_0 e^{\delta_g m}$ and a correspondingly decreasing number of options according to the growing discount factor.

Formally:

$$OF_0^{\mathrm{P}} = nB_0 = nR_g \sum_{t=1}^{m} \left\{ \frac{1}{e^{\delta_{r(0;t)}t}} + \eta \left[\frac{N(d_{1(t)})}{e^{\delta_g t}} - \frac{N(d_{2(t)})}{e^{\delta_{r(0;t)}t}} \right] \right\} \tag{3}$$

By virtue of the put-call parity property,[3] any equity-linked bond with a minimum guaranteed rate of return is equivalent either to a plan providing a fixed benefit plus a call option or to a plan providing a benefit of the value of the reference portfolio plus a put option. From a theoretical point of view, the two strategies are perfectly equivalent. Nevertheless, from a practical perspective, if the reference asset is not directly traded on the market, the put replication can give rise to higher replication cost, for the need to build up on a synthetic. For this reason, the call option strategy is selected. (Naturally, if the reference portfolio is directly traded, the put replication strategy could equally be pursued.) Hence, the earned sum is considered to be invested into a portfolio made up by bonds and plain-vanilla call options, giving rise to the following balance sheet structure, the related cash flows of which are reported in Table 3:

- assets:
  - plain-vanilla call options;
  - zero coupon bonds;
- liabilities:
  - equity-linked bonds;
  - capital.

By adding the writer's capital, the investment out-flow will also comprise this quota. If the writer decides to invest the initial capital together with the shareholder capital C_0 in the bond market, they will end up with a profit that is:

- definitively certain if they buy only zero coupon bonds with duration equal to m;
- almost certain (moderately risky) if they go for a roll-over bond strategy;
- risky if they select some other risky assets (stocks, derivatives and so on).

Hence, as long as the selected investing instruments are available — and no dynamic hedging is required — the profit will exhibit the earlier risk ranking. The investment strategy selected, being a perfect replicating portfolio (denoted by $^{\mathrm{P}}$), provides the writer with a final profit (Φ_m^{P}) substantially conditional on the magnitude of the initial net inflow ($NF_0^{\mathrm{P}} = nU - OF_0^{\mathrm{P}}$) given different levels of the parameters g and η as it can be shown to be equal to (see Appendix C):

$$\Phi_m^{\mathrm{P}} = C_0\left[\prod_{t=0}^{m-1} e^{\delta_{r(t;1)}} - 1\right] + NF_0^{\mathrm{P}} \prod_{t=0}^{m-1} e^{\delta_{r(t;1)}} \tag{4}$$

Consequently, from a security design perspective, relevant variables influencing the outcome are the guaranteed and the participation rate, being $\Phi_m^{\mathrm{P}} = f(g, \eta)$. Therefore, the acceptability of the outcome can be evaluated with reference to these parameters (as discussed previously).

An imperfect replicating portfolio

As an alternative to the previous investing hypothesis, a different strategy is also considered, which can provide the issuer with a higher level of initial surplus, with some beneficial mismatching between inflows and outflows, in order to evaluate the levels of profitability offered by different values of parameters g and η and to perform a comparative analysis with the perfect matching strategy.

A possibility could be an asset portfolio made up by:

Table 3: The asset and liability cash flows (perfect matching)

Time	Investment cash flows	Bond cash flows	Net flows
0	$-OF_0^{\mathrm{P}} = -nR_g\left[\sum_{t=1}^{m}\frac{1}{e^{\delta_{r(0;t)}t}} + \eta\sum_{t=1}^{m}\left(\frac{N(d_{1(t)})}{e^{\delta_g t}} - \frac{N(d_{1(t)})}{e^{\delta_{r(0;t)}t}}\right)\right]$	$+IF_0 = nU$	$NF_0^{\mathrm{P}} = IF_0 - OF_0^{\mathrm{P}}$
1	$+IF_1^{\mathrm{P}} = +R_g n\left[1 + \frac{\eta}{s_0 e^{\delta_g}}\max(0; S_1 - s_0 e^{\delta_g})\right]$	$-OF_1 = -R_g n\left[1 + \frac{\eta}{s_0 e^{\delta_g}}\max(0; S_1 - s_0 e^{\delta_g})\right]$	0
t	$+IF_t^{\mathrm{P}} = +R_g n\left[1 + \frac{\eta}{s_0 e^{\delta_g t}}\max(0; S_t - s_0 e^{\delta_g t})\right]$	$-OF_t = -R_g n\left[1 + \frac{\eta}{s_0 e^{\delta_g t}}\max(0; S_t - s_0 e^{\delta_g t})\right]$	0
m	$+IF_m^{\mathrm{P}} = +R_g n\left[1 + \frac{\eta}{s_0 e^{\delta_g m}}\max(0; S_m - s_0 e^{\delta_g m})\right]$	$-OF_1 = -R_g n\left[1 + \frac{\eta}{s_0 e^{\delta_g m}}\max(0; S_m - s_0 e^{\delta_g m})\right]$	0

- the zero coupon bonds as showed in the perfect matching strategy;
- a portfolio of plain-vanilla call options with maturity 1 and strike price equal to $s_0 e^{\delta_g}$;
- $m-1$ portfolios of path-dependent call options with varying maturity from 2 to m, variable strike prices defined as $\hat{s}_{t-1}e^{\delta_g}$ (with $t = 2, 3, \ldots, m$) and a correspondingly variable number of options defined on the basis of the different strikes where the circumflex accent accounts for a forecasted/simulated value of the reference asset.

In this case, the final profit, denoted by $^{\mathrm{I}}(\Phi^{\mathrm{I}}_m)$ is equal to (see Appendix C):

$$\Phi^{\mathrm{I}}_m = C_0\left[\prod_{t=0}^{m-1} e^{\delta_{r(t;1)}} - 1\right] + \sum_{t=0}^{m}\left[NF^{\mathrm{I}}_t \prod_{t'=t}^{m-1} e^{\delta_{r(t';1)}} - 1\right] + NF^{I}_m \tag{5}$$

The bond portfolio cash-flow breakdown is given by Table 4.

THE ACCEPTABILITY SET

The final decision on the issue has to take into account both the risk-return trade-off and the return on capital-at-risk set by top management as a threshold value to set the 'acceptability' of the issue and the backing asset. Within this context, a primary insight can be gained by analysing the expected profit $E[\Phi_m]$ and the expected return on equity (RoE), defined as the ratio of the expected profit to the initial issuer's capital C_0. These two indicators provide management with an immediate picture of future outcomes and differentials of portfolio alternatives, although they do not weight the information with the risk associated to each strategic choice. As a consequence, the return has to be compared with the risk undertaken; to this end, value-at-risk (VaR) measures are considered.

Recalling value-at-risk and conditional value-at-risk

If one denotes W_m the final (random) value of the net value of the asset and liability portfolio (intermediation portfolio), w_0 the corresponding (known) value at present and $G_m(x)$, the distribution function of W_m whose density function is $g_m(x)$, the definition of $VaR_\alpha(m)$ is given by:

$$\Pr[W_m - w_0 \leq -VaR_\alpha(m)] = \alpha \tag{6}$$

from which it is easy to infer that the VaR at the confidence level α is realisation of the loss amount $w_0 - W_m$,

Table 4: The asset and liability cash flows (imperfect matching)

Time	Investment cash flows	Bond cash flows	Net flows
0	$-OF^{\mathrm{I}}_0 = -(zero\ coupon + path\ dependent)$	$+IF_0 = nU$	$NF^{\mathrm{I}}_0 = IF^{\mathrm{I}}_0 - OF_0$
1	$+IF^{\mathrm{I}}_1 = +R_g n\left[1 + \frac{\eta}{s_0 e^{\delta_g}}\max(0; S_1 - s_0 e^{\delta_g})\right]$	$-OF_1$	0
t	$+IF^{\mathrm{I}}_t = +R_g n\left[1 + \frac{\eta}{\hat{s}_{t-1} e^{\delta_g}}\max(0; S_t - \hat{s}_{t-1} e^{\delta_g})\right]$	$-OF_t$	$NF^{\mathrm{I}}_t = IF^{\mathrm{I}}_t - OF_t$
m	$+IF^{\mathrm{I}}_m = +R_g n\left[1 + \frac{\eta}{\hat{s}_{m-1} e^{\delta_g}}\max(0; S_m - \hat{s}_{m-1} e^{\delta_g})\right]$	$-OF_m$	$NF^{\mathrm{I}}_m = IF^{\mathrm{I}}_m - OF_m$

which will not be exceeded with probability $1-\alpha$. By denoting the α-quantile distribution of W_m as $W_\alpha(m) = G_m^{-1}(\alpha)$, one obtains:

$$\Pr[W_m \leq -W_\alpha(m)] = \alpha \qquad (7)$$

and, consequently:

$$VaR_\alpha(m) = w_0 - W_\alpha(m) \qquad (8)$$

The VaR of the entire net value can be interpreted as the VaR of a portfolio exposed to those risk factors that are the changing parameters of the net value itself. This can be measured by modelling the risk factors and by using risk filters to proportionate the effect on the net value. As known, the classical VaR measure suffers from many serious limitations,[2] which can be partially overcome by conditional VaR (CVaR) (also known as conditional tail expectation), defined as the average of losses exceeding VaR,[4] that is:

$$CVaR_\alpha(m) = E[(w_0 - W_\alpha(m)) | (w_0 - W_\alpha(m)) > VaR_\alpha(m)] \qquad (9)$$

Risk-adjusted performance indicators and the value-added measure

Due to the need for efficient risk management and to compare different business units, risk-adjusted performance measures (RAPMs) have become popular in the finance industry. As known, business evolution can be described by means of both the intermediation portfolio (economic value approach) and the income flows (current earning approach). The first approach accounts for the difference between asset and liability at any time of the portfolio cycle, while the second explains the difference between the profit components periodically accrued. Both are used to compute RAPMs, concentrating on an average yearly measure as it is the global profitability of the issue that is of concern.

Many acronyms and definitions for RAPMs can be found in the literature. The following are used:

- *the expected return on equity* ($E[RoE]$), defined as the ratio of the expected profit to initial capital, the average annual rate of which is set as:

$$Annualised\ E[RoE] = \left[1 + \frac{E[\Phi_m]}{C_0}\right]^{\frac{1}{m}} - 1 \qquad (10)$$

- *the risk-adjusted return on capital (RAROC)*, defined as the ratio of the expected profit to the VaR of the portfolio.[5] There are many different methods for calculating the ratio of economic result to capital, and it is common for near-identical measures to assume different names in different contexts; as such, the present paper uses the term 'RAROC' to avoid counterintuitive results arising from the numerous names used in practice as well as in the literature. The corresponding average annual value of RAROC is given by:

$$Annualised\ RARoC = \left[1 + \frac{E[\Phi_m]}{VaR}\right]^{\frac{1}{m}} - 1 \qquad (11)$$

- *the coherent RAROC (CRAROC)*, defined as the ratio of the expected profit to the CVaR of the portfolio, the corresponding average annual value of which is given by:

$$Annualised\ CRARoC = \left[1 + \frac{E[\Phi_m]}{CVaR}\right]^{\frac{1}{m}} - 1 \qquad (12)$$

As known, the last measure can be considered the most useful because it respects the majority of coherence axioms. As far as value creation is concerned, a general indicator is also selected. A very broad definition of value added is given by:

$$\text{value added} = \text{profit} - (TR \times \text{allocated capital})$$

where TR is a threshold rate setting the acceptability of the future result and the capital-at-risk is set equal to the CRAROC. The value added should be calibrated to a specific measure of profit which is the net operating profit and the hurdle rate should be the return on capital-at-risk set by top management. Here the value-added measure (EVA), on an average yearly basis, is defined as:

$$Economic\ Value\ Added = \frac{\hat{E}[\Phi_m]}{m} - \left(TR \cdot \frac{CVaR}{\sqrt{m}}\right) \tag{13}$$

As can be easily seen, the EVA increases according to the increase in the expected profit and reduction in risk exposure.

THE RISK FRAMEWORK

Interest rate

The single local source of uncertainty is the spot rate, which is a diffusion process described by Cox-Ingersoll-Ross (CIR)[6] by means of the well-known stochastic differential equation:

$$dr_t = -\alpha(r_t - \mu_r)dt + \sigma_r\sqrt{r_t}dZ_r(t) \tag{14}$$

where $Zr(t)$ is a standard Brownian motion. As known, this specification assumes a mean-reverting drift, with long-term rate μ_r speed of adjustment α and a square-root diffusion, with volatility parameter σ_r. The CIR process implies non-central chi-square transition distribution for r_t. The risk-adjusted parameters of the CIR component of the valuation model were estimated by a two-step procedure developed by Barone and Cesari.[7]

Stock index

A single source of uncertainty is also assumed for the stock market, expressed by the stock index S_t; the diffusion process for the stock index is given by the stochastic differential equation:

$$dS_t = \mu S_t d_t + \sigma_s dZ_s(t) \tag{15}$$

where $Z_s(t)$ is a standard Brownian motion with the property:

$$cov_t[dZ_r(t)dZ_s(t)] = kdt$$

This presents a geometric Brownian motion, with instantaneous expected return μ_s and volatility σ_s which implies a lognormal transition density for S_t as follows:

$$S_t = s_0 e^{\mu_S - (\sigma_S^2/2) + \sigma_S dZ_S(t)} \tag{16}$$

As proposed by De Felice and Moriconi,[8] the volatility of the stock index and the covariance parameter k can be exogenously specified. With reference to this, it is worth remarking that, as the covariance between interest rates and stock prices reveals a slightly negative correlation, a null value of the κ parameter is selected. By such adjustment, one actually considers a higher value of the variance of the portfolio and hence a sort of 'prudential variance (risk) margin' is used to counterbalance the model risk.

THE COMPUTATIONAL PROCEDURE

The study goal is the computation of the expected profit at the end of the business, together with the connected capital-at-risk in the two prospected strategies. To this end, the formal flow definition from Tables 3 and 4 is applied. Computation of the final result has been derived by numerical methods, using Monte Carlo simulations for the identified process using 10,000 paths. The risk sources are the interest rate (modelled on the basis of the CIR process) and the price of the underlying index (modelled on the basis of a log-normal distribution). For any simulation, the intermediate net flows are reinvested at the corresponding (expected) rates up to the maturity (m) according to the roll-over strategy defined in the section on 'The bond'. Computation provides the expected profit $\hat{E}[\Phi_m]$ at maturity both for the plain-vanilla replication ($\hat{E}[\Phi_m^{\mathrm{P}}]$) and the path-dependent replication ($\hat{E}[\Phi_m^{\mathrm{I}}]$) as well as VaR and CVaR for the whole portfolio at a confidence level of 99 per cent. The two error term sets for the simulation procedures have been kept constant across different alternatives in order to maximise the comparison results. Due to having worked on a portfolio basis, the individual benefit has been multiplied by the number of issued bonds (1,000).

As far as the initial value of the investment portfolio is concerned, it is important to distinguish between the two cases. In the case of the plain-vanilla replication, the initial outflow connected to the investment process has been set by means of a market-consistent pricing procedure as far as both the bonds and the options are concerned (see *supra*). Therefore the corresponding values derive from closed formulas: traditional bond valuation and Black-Scholes prices. In the case of the imperfect replication (path-dependent), the initial outflow has been set by a market pricing procedure for the bonds and a numerical market-consistent pricing procedure for the options.

To perform the simulation procedure it has been necessary to consider the discrete time equation for the chosen stochastic differential equation describing the time evolution of the interest rates. On the basis of the first-order Euler scheme, the discrete version of Equation (15), considering the relevant time interval is:

$$r_{t_i+1} = r_{t_i}\alpha(\mu_r - r_{t_i})\Delta + \sigma_r\sqrt{r_{t_i}\Delta\varepsilon_{r_i}} \quad (17)$$

with $i = 1, 2, \ldots, n-1$, where $\Delta = T/n$ is the sampling interval, with ε_{r_i} being the increment ΔZ_{r_i} of the Brownian motion between $t_i = i\Delta$ and $t_{i+1} = (i+1)\Delta$. The random variable ε_{r_i} follows a normal standard distribution. All estimates have been based on the Eonia and Euribor data and calibration has been set on 11th April, 2007. To perform the simulation procedure, it has been necessary to consider the discrete time equation for the chosen stochastic differential equation describing the time evolution of the stock index price. On the basis of the first-order Euler scheme, the discrete version of Equation (16) is:

$$S_{t_{i+1}} = s_{t_i}e^{\mu_S-(\sigma_S^2/2)\Delta+\sigma_S\sqrt{\Delta}\varepsilon_{S_i}} \quad (18)$$

with $i = 1, 2, \ldots, n-1$.

All estimates have been based on the SPMIB data (Bloomberg dataset; fixing values) and calibration has been set on 11th April, 2007. Other computational details remain unchanged across the two hypotheses. Table 5 reports the whole set of input data.

The results

Tables 6–19 provide the results of the computational procedure, based on previously-defined values and indicators. The EVA analysis is based on Equation (13) with the threshold rate based on the average rate of return of the Italian financial sector over the last three years. This rate, equal to 17 per cent, was inferred from the performance of listed companies in the Italian financial sector and was regarded as a proxy for the rate of return on investment required by shareholders. This rate is a surrogate of the actual rate of return because it suffers from many shortcomings, the most important being that it is a gross rate short of any fiscal adjustment. In this respect, an appropriate EVA appraisal should reflect not only the proper net rate of return but also a net profit measure, the so-called net operating profit after taxes. The present analysis uses the gross measure for both the expected profit and the rate of return. As one would expect, in a practical implementation, the financial intermediary can adapt all the measures to net values by considering proper fiscal loading and other expenses adjustments.

FINDINGS AND FUTURE RESEARCH PROSPECTS

Tables 6–11 provide insight into the acceptability of the two different investment alternatives. As expected, plain-vanilla replication gives rise to less risky and less profitable portfolios. This would appear to imply that the actual selection of the investment portfolio depends on the risk aversion of the writer. Although this general conclusion is correct, many other remarks also arise from the risk-return framework.
Figures 1 and 2 show that a reduction of the parameters g and η improves the expected return on equity, with a different degree of convexity across the two strategies. As expected, the imperfect replication offers the opportunity to sustain a higher number of combinations with respect to the perfect replication strategy. This indicates that an 'opportune' imperfect replication strategy provides for higher returns for bondholders due to a

Table 5: The input dataset

Description	Symbol	Figures
Single minimum investment	U	€2,500.00
Guaranteed annual rate	g	[0.00; 0.005; 0.01; 0.015; 0.02]
Guaranteed payment based on different g rates	R_g	[833.33; 841.68; 850.06; 858.46; 866.89]
Reference asset (SPMIB)	s	SPMIB points multiplied by €2.50
Participation rate	η	[0.45; 0.50; 0.55; 0.60; 0.65]
Maturity	m	3 years
Number of issued bonds	n_b	1,000
Total value of the issuance		€2,500,000.00
Issuer's initial capital	C_0	€250,000.00
Number of simulation paths		10,000
Threshold rate	TR	17%

Table 6: $\hat{E}[\Phi_m^P] = f(g, \eta)$ — Plain-vanilla replication (perfect matching)

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	132,102.80	114,164.00	95,860.61	77,181.13	58,115.61
0.50	117,440.90	100,422.20	83,008.43	65,186.20	46,944.51
0.55	102,778.90	86,680.76	70,156.28	53,191.26	35,773.37
0.60	88,116.95	72,939.15	57,304.13	41,196.34	24,602.25
0.65	73,454.99	59,197.26	44,451.98	29,201.43	13,431.13

Table 7: $\hat{E}[\Phi_m^I] = f(g, \eta)$— Path-dependent investment (imperfect matching)

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	409,866.10	395,046.60	379,934.80	364,370.20	348,103.10
0.50	398,289.80	384,737.20	370,869.50	356,508.10	341,375.80
0.55	386,713.40	374,427.80	361,804.10	348,646.10	334,648.50
0.60	375,137.00	364,118.40	352,738.80	340,784.10	327,921.20
0.65	363,560.60	353,809.00	343,673.40	332,922.00	321,193.90

Table 8: VaR 99% — Plain-vanilla replication (perfect matching)

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	177,635.30	157,558.90	137,074.40	116,169.00	94,831.53
0.50	161,226.20	142,179.80	122,690.60	102,744.70	82,329.27
0.55	144,817.10	126,800.60	108,307.00	89,320.38	69,826.92
0.60	128,408.00	111,421.50	93,923.36	75,896.22	57,383.16
0.65	111,998.80	96,042.07	79,539.76	62,471.86	44,822.38

Table 9: VaR 99% — Path-dependent investment (imperfect matching)

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	481,725.20	464,085.30	445,442.40	425,403.90	405,405.00
0.50	473,956.40	457,411.40	439,760.10	420,806.30	400,691.10
0.55	466,168.50	450,473.90	433,893.80	415,735.50	396,955.20
0.60	458,032.40	443,694.80	428,159.50	411,322.90	393,183.80
0.65	449,706.00	436,717.60	422,358.40	406,866.30	389,262.50

Table 10: CVaR 99% — Plain-vanilla replication (perfect matching)

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	187,758.20	167,206.50	146,237.10	124,836.90	102,994.30
0.50	170,960.70	151,463.30	131,512.90	111,094.80	90,196.12
0.55	154,163.10	135,720.20	116,788.80	97,352.74	77,397.84
0.60	137,365.50	119,977.00	102,064.70	83,610.70	64,599.58
0.65	120,568.00	104,233.50	87,340.54	69,868.66	51,801.30

convexity gain (Figures 1 and 2). This would appear to imply that the actual selection of the investment portfolio depends on the risk aversion of the writer. Although this general conclusion is correct, again, there are many other remarks arising from the risk-return framework.

Table 11: CVaR 99% — Path-dependent investment (imperfect matching)

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	494,228.20	475,801.20	456,563.20	436,528.40	415,533.00
0.50	486,123.60	468,728.40	450,601.50	431,587.10	411,452.00
0.55	478,412.60	462,069.80	444,901.00	426,913.40	407,665.80
0.60	471,119.30	455,882.80	439,566.20	422,491.20	404,045.00
0.65	464,091.80	449,925.40	434,464.50	418,232.10	400,595.50

Table 12: Average annual E[RoE] — Plain-vanilla replication

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	0.1519	0.1336	0.1143	0.0938	0.0722
0.50	0.1370	0.1191	0.1003	0.0803	0.0590
0.55	0.1216	0.1043	0.0859	0.0664	0.0456
0.60	0.1059	0.0891	0.0712	0.0522	0.0318
0.65	0.0897	0.0734	0.0561	0.0375	0.0176

Table 13: Average annual E[RoE] — Path-dependent investment

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	0.3820	0.3716	0.3608	0.3495	0.3375
0.50	0.3739	0.3642	0.3542	0.3437	0.3324
0.55	0.3656	0.3568	0.3476	0.3379	0.3273
0.60	0.3573	0.3493	0.3409	0.3320	0.3222
0.65	0.3489	0.3417	0.3341	0.3260	0.3171

Table 14: Average annual RAROC — Plain-vanilla replication

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	0.4294	0.4183	0.4038	0.3836	0.3538
0.50	0.4206	0.4078	0.3906	0.3663	0.3292
0.55	0.4098	0.3947	0.3740	0.3438	0.2958
0.60	0.3962	0.3779	0.3523	0.3134	0.2475
0.65	0.3787	0.3559	0.3227	0.2699	0.1730

Table 15: Average annual RAROC — Path-dependent investment

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	0.4912	0.4915	0.4924	0.4945	0.4957
0.50	0.4852	0.4856	0.4869	0.4891	0.4919
0.55	0.4789	0.4799	0.4814	0.4842	0.4867
0.60	0.4729	0.4738	0.4756	0.4783	0.4815
0.65	0.4668	0.4677	0.4698	0.4724	0.4764

A clearer insight into the phenomenon can be gained by CRAROC analysis. As shown in Figures 3 and 4, the two indicators show a markedly different evolution as regards the guaranteed rate.

Selection of the proposed imperfect replication strategy presents the

Table 16: Average annual CRAROC — Plain-vanilla replication

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	0.4062	0.3942	0.3785	0.3570	0.3258
0.50	0.3966	0.3828	0.3644	0.3388	0.3005
0.55	0.3849	0.3687	0.3468	0.3155	0.2669
0.60	0.3704	0.3510	0.3242	0.2845	0.2199
0.65	0.3517	0.3279	0.2938	0.2413	0.1497

Table 17: Average annual CRAROC — Path-dependent investment

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	0.4788	0.4794	0.4804	0.4819	0.4837
0.50	0.4730	0.4739	0.4752	0.4769	0.4790
0.55	0.4667	0.4678	0.4695	0.4715	0.4739
0.60	0.4597	0.4611	0.4633	0.4657	0.4686
0.65	0.4523	0.4540	0.4567	0.4596	0.4629

Table 18: Average annual EVA — Plain-vanilla replication

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	25,605.88	21,643.42	17,600.43	13,474.36	9,263.03
0.50	22,367.25	18,608.01	14,761.55	10,824.83	6,795.46
0.55	19,128.60	15,572.71	11,922.66	8,175.29	4,327.89
0.60	15,889.96	12,537.36	9,083.77	5,525.76	1,860.33
0.65	12,651.30	9,501.95	6,244.89	2,876.23	−607.23

Table 19: Average annual EVA — Path-dependent investment

	$g = 0.00$	0.005	0.01	0.015	0.02
$\eta = 0.45$	88,113.76	84,982.53	81,833.46	78,611.67	75,249.99
0.50	85,050.46	82,240.25	79,396.83	76,475.96	73,408.11
0.55	81,948.49	79,457.33	76,934.54	74,314.01	71,537.29
0.60	78,805.52	76,628.11	74,436.38	72,127.38	69,650.23
0.65	75,636.47	73,776.36	71,915.31	69,924.71	67,746.37

opportunity to benefit from a convexity gain and therefore offer the bondholder a higher guaranteed rate. The sustainability of these two strategies is synthesised by the EVA measure (Figures 5 and 6) showing that both strategies are sustainable and therefore acceptable, but that imperfect replication creates levels of value far higher than perfect replication, given the different convexities of the two positions. Clearly, the most profitable solutions are those that exhibit riskier profiles, as can be easily appreciated.

The computation of risk-adjusted performance indicators is useful in appreciating the optimality of the issue with reference to both relevant product design variables and investment decisions. Within this respect, one can observe that both portfolio and product design decision can be usefully evaluated by the

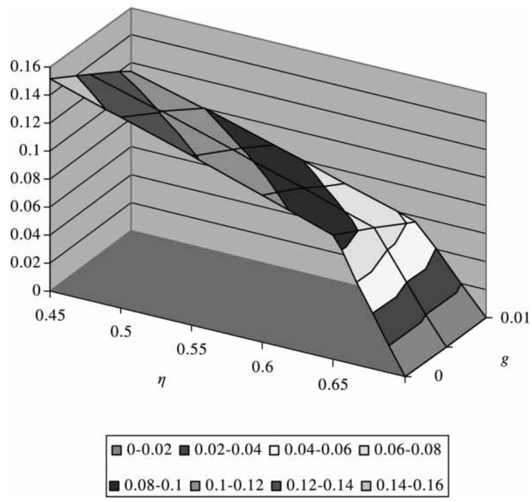

Figure 1: ROE plain-vanilla replication

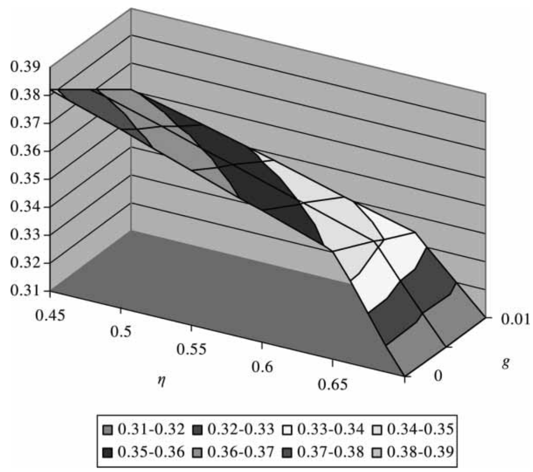

Figure 2: ROE path-dependent investment

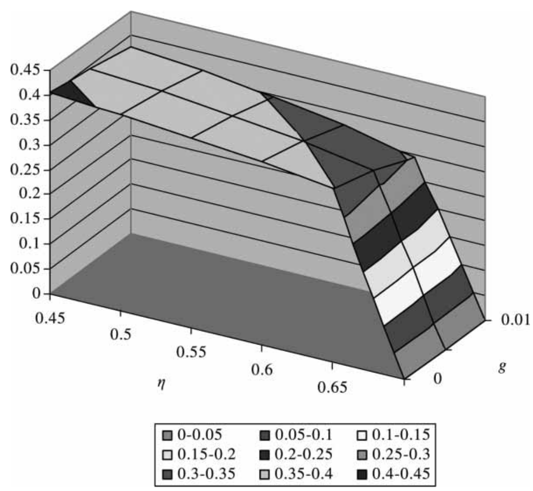

Figure 3: CRAROC plain-vanilla replication

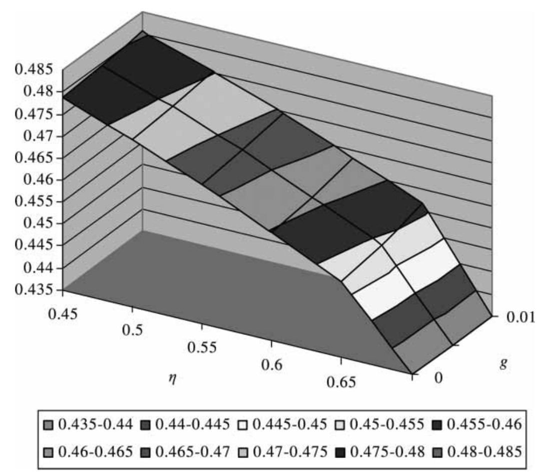

Figure 4: CRAROC path-dependent investment

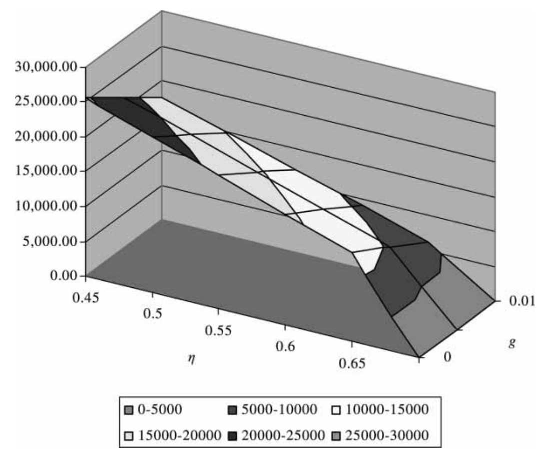

Figure 5: EVA plain-vanilla replication

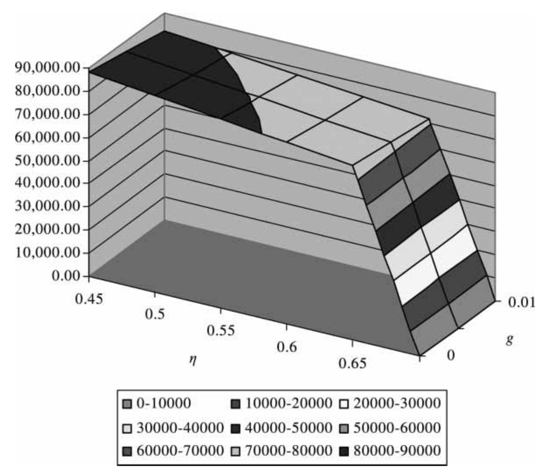

Figure 6: EVA path-dependent investment

set of indicators adopted. Nevertheless, the development of a simulation procedure is prone to model risk.

In this regard, future research should consider modelling the financial risk factors with reference not only to the possibility of using multi-factor models but also with reference to a complete breakdown of covariance analysis and corresponding simulation techniques. This aspect is particularly important when there are multiple risk factors. This is the case for any multi-factor model for the interest rate dynamic as well as for any more complex replicating investment portfolio. As regards the specific features of the instruments used, special attention should be devoted to using the whole informative set concerning the volatility surface in order to consider the inner smile effect.

Acknowledgment

Although the paper is the result of a joint study, the first three sections are the work of R. Cocozza, while the last three sections are by R. Cocozza and A. Orlando.

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APPENDIX A: THE SEMANTICS OF THE RELEVANT SYMBOLS

Semantics of section on ‘The bond’:

U	Single initial minimum investment
m	Bond (or issue) maturity
t	Relevant payment dates with t varying from 1 to m
B_t	Periodical benefit maturing at each time t
g	Guaranteed annual rate
η	Participation rate
s	Reference asset
R_g	Guaranteed annual payment based on the g rate
δ_g	Force of interest corresponding to the g rate
S_t	Value at time t of the reference asset
s_0	Value at issue time of the reference asset
n_b	Number of sold bonds
$r(0;\ t)$	Risk-free spot rate for the period $0-t$

$\delta_{r(0;t)}$ Force of interest corresponding to the $r(0;\, t)$ rate
σ_s Volatility of the reference asset
C_0 Issuer's initial capital
Φ_m^{P} Final profit of the perfect replicating strategy
NF_0^{P} Initial surplus of the perfect replicating strategy
Φ_m^{I} Final profit of the imperfect replicating strategy
NF_0^{I} Initial surplus of the imperfect replicating strategy
NF_t^{I} Periodical surplus of the imperfect replicating strategy maturing at each time t
$\hat{s}_{t-1}e^{\delta_g}$ Strike prices of the path-dependent call options
$\hat{E}[\Phi_m]$ Simulated expected final profit

Semantics of section on 'The acceptability set':

W_m Final value of the asset and liability portfolio
W_0 Present value of the asset and liability portfolio
$W_\alpha(m)$ α — quantile distribution of W_m
$VaR_\alpha(m)$ Value-at-risk of W_m at a confidence level α
$CVaR_\alpha(m)$ Conditional value-at-risk of W_m at confidence level α

Semantics of section on 'The risk framework':

r_t Instantaneous spot interest rate at time t
μ_r Long-term mean of the rate of interest
σ_r Volatility of the spot rate of interest
$Z_r(t)$ Standard Brownian motion
S_t Price at time t of the reference asset
μ_s Expected return of the reference asset
σ_s Volatility of the reference asset
$Z_s(t)$ Standard Brownian motion

APPENDIX B: THE FORMAL DEVELOPMENT OF THE CONTRACT

If $S_t \leq s_0 e^{\delta_g t}$ then the payoff equals R_g; if $S_t > s_0 e^{\delta_g t}$ then the payoff equals:

$$R_g\left[1+\eta\left(\frac{S_t}{s_0 e^{\delta_g t}}-1\right)\right] = R_g\left[1+\frac{\eta}{s_0 e^{\delta_g t}}(S_t - s_0 e^{\delta_g t})\right]$$

Therefore, at each settlement date t the buyer will receive a benefit defined as:

$$B_t = R_g\left[1+\frac{\eta}{s_0 e^{\delta_g t}}\max(0;\, S_t - s_0 e^{\delta_g t})\right]$$

Being m the final maturity, the whole contract is featured by the summation of m flows defined as stated. At issue, the value of the contract is given by the present value of the forthcoming flows and is therefore given by the present value of the annuity whose instalment is set equal to R_g and the present value of m portfolios of a number $\eta/s_0 e^{\delta_g t}$ of plain-vanilla options with strikes equal to $s_0 e^{\delta_g t}$ and maturity equal to t ($t \leq m$). The annuity and the options have to be evaluated at the current interest rates and therefore for each flow maturing at time

t the present value is given by:

$$\frac{B_t}{e^{\delta_{r(0;t)}t}} = R_g\left\{\frac{1}{e^{\delta_{r(0;t)}t}} + \left(\frac{\eta}{s_0 e^{\delta_g t}}\right) \times \left[s_0 N(d_{1(t)}) - s_0 \frac{e^{\delta_g t}}{e^{\delta_{r(0;t)}t}} N(d_{2(t)})\right]\right\}$$

$$= R_g\left\{\frac{1}{e^{\delta_{r(0;t)}t}} + \left(\frac{\eta}{e^{\delta_g t}}\right) \times \left[N(d_{1(t)}) - \frac{e^{\delta_g t}}{e^{\delta_{r(0;t)}t}} N(d_{2(t)})\right]\right\}$$

$$= R_g\left\{\frac{1}{e^{\delta_{r(0;t)}t}} + \eta\left[\frac{N(d_{1(t)})}{e^{\delta_g t}} - \frac{N(d_{2(t)})}{e^{\delta_{r(0;t)}t}}\right]\right\}$$

The overall value of the contract at issue is therefore given by the summation for each time t of the present value of the single flow as follows:

$$B_0 = \sum_{t=1}^{m} \frac{B_t}{e^{\delta_{r(0;t)}t}} = R_g \sum_{t=1}^{m} \frac{1}{e^{\delta_{r(0;t)}t}} + \frac{\eta}{s_0 e^{\delta_g t}}\left[s_0 N(d_{1(t)}) - s_0 \frac{e^{\delta_g t}}{e^{\delta_{r(0;t)}t}} N(d_{2(t)})\right]$$

$$= R_g \sum_{t=1}^{m} \frac{1}{e^{\delta_{r(0;t)}t}} + \eta\left[\frac{N(d_{1(t)})}{e^{\delta_g t}} - \frac{N(d_{2(t)})}{e^{\delta_{r(0;t)}t}}\right]$$

For each option contract d_1 and d_2 parameters are defined respectively as:

$$d_{1(t)} = \frac{\ln \frac{s_0}{s_0 e^{\delta_g t}} + \left(\delta_{r(0;t)} + \frac{\sigma_s^2}{2}\right)t}{\sigma_s\sqrt{t}} = \frac{-\delta_g t + \left(\delta_{r(0;t)} + \frac{\sigma_s^2}{2}\right)t}{\sigma_s\sqrt{t}} = \frac{t}{\sigma_s\sqrt{t}}\left(\delta_{r(0;t)} - \delta_g + \frac{\sigma_s^2}{2}\right)$$

$$d_{2(t)} = d_{1(t)} - \sigma_s\sqrt{t} = \frac{t}{\sigma_s\sqrt{t}}\left(\delta_{r(0;t)} - \delta_g + \frac{\sigma_s^2}{2}\right) - \sigma_s\sqrt{t} = \frac{t}{\sigma_s\sqrt{t}}\left(\delta_{r(0;t)} - \delta_g - \frac{\sigma_s^2}{2}\right)$$

APPENDIX C: THE PROFIT EQUATIONS

The final profit is given by:

$$\Phi_m = ((((C_0 + NF_0)e^{\delta_{r(0;1)}} + NF_1) \times e^{\delta_{r(1;1)}} + NF_2)e^{\delta_{r(2;1)}} + \cdots + NF_m) - C_0$$

$$= C_0 e^{\delta_{r(0;1)}} e^{\delta_{r(1;1)}} e^{\delta_{r(2;1)}} \cdots e^{\delta_{r(m-1;1)}} + NF_0 e^{\delta_{r(0;1)}} e^{\delta_{r(1;1)}} e^{\delta_{r(2;1)}} \cdots e^{\delta_{r(m-1;1)}} + NF_1 e^{\delta_{r(1;1)}} e^{\delta_{r(2;1)}} \cdots e^{\delta_{r(m-1;1)}} + NF_2 e^{\delta_{r(2;1)}} \cdots e^{\delta_{r(m-1;1)}} + \cdots + NF_m - C_0$$

$$= C_0 \prod_{t=0}^{m-1} e^{\delta_{r(t;1)}} + \sum_{t=0}^{m}\left[NF_t \prod_{t'=t}^{m-1} e^{\delta_{r(t';1)}}\right] + NF_m - C_0$$

$$= C_0\left[\prod_{t=0}^{m-1} e^{\delta_{r(t;1)}} - 1\right] + \sum_{t=0}^{m}\left[NF_t \prod_{t'=t}^{m-1} e^{\delta_{r(t';1)}}\right] + NF_m$$

By definition, in the case of perfect replication $NF_t^{\mathrm{P}} = 0\,\forall t \neq 0$ and therefore the final profit reduces to:

$$\Phi_m^{\mathrm{P}} = C_0 \left[\prod_{t=0}^{m-1} e^{\delta_{r(t;1)}} - 1 \right] + NF_0^{\mathrm{P}} \prod_{t=0}^{m-1} e^{\delta_{r(t;1)}}$$

In the case of imperfect replication, intermediate net flows are all not null and therefore the final profit is:

$$\Phi_m^{\mathrm{I}} = C_0 \left[\prod_{t=0}^{m-1} e^{\delta_{r(t;1)}} - 1 \right] + \sum_{t=0}^{m} \left[NF_t^{\mathrm{I}} \prod_{t'=t}^{m-1} e^{\delta_{r(t';1)}} \right] + NF_m^{\mathrm{I}}$$