An alternative methodology for estimating credit quality transition matrices

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Abstract This study presents an alternative method of estimating credit quality transition matrices using a hazard function model. The model is useful both for testing the validity of the Markovian assumption, frequently made in credit rating applications, and also for estimating transition matrices conditioning on firm-specific and macroeconomic covariates that influence the migration process. The model presented in the paper is likely to be useful in other applications, although extrapolating numerical values of the coefficients outside of the present context is not recommended. Transition matrices estimated this way may be an important tool for a credit risk administration system, in the sense that a practitioner can use them to forecast the future behaviour of clients' ratings, and their possible change of state.

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INTRODUCTION

Financial institutions use credit ratings to express their risk perception about their clients. Credit ratings feed institutions' internal credit scoring models, allowing them to evaluate the current state of the quality of their balances and to calculate the reserves required to provision their loan portfolios. The information they provide therefore constitutes a useful tool for evaluating credit demands and for assigning the corresponding interest rates to approved credits.

Moreover, within a credit risk administration system, it is crucial to be able to forecast the future behaviour of clients' ratings and any possible changes of state. From this perspective, transition matrices constitute a fundamental tool for financial institutions as they measure migration probabilities among states. Transition probabilities are at the core of modern credit risk models and represent a standard point for risk dynamics; therefore they must be estimated with rigorous precision using the most appropriate techniques available.

In many important economic applications (eg JP Morgan's Credit Metrics), transition matrices are estimated under the Markovian assumption in a discrete-time setting using a cohort method. In a discrete and finite space setting, the probability of migrating from state i to state j is estimated by dividing the number of observed migrations from i to j in a given time period by the total number of firms in state i at the beginning of the period. One implication of this cohort method is that, if no firm migrates directly from state i to j during the observation period, the estimate of the corresponding probability is zero. This is not a desirable feature, especially when dealing with the estimation of rare event probabilities, which, should they occur, may have a deep impact.

Various studies have proposed using continuous time methodologies as an alternative to the cohort approach (eg Lando,[1] Gómez-González[2] and Gómez-González and Kiefer[3]), which not only overcomes the problem of the zero estimates for rare event probabilities, but also offers additional advantages such as allowing simple tests for non-Markovian behaviour. Most empirical applications assume the Markov property holds and proceed to estimate transition matrices under that assumption. Thus, the veracity of results depends on whether or not the Markov property holds in a particular setting. (With an absorbing state of default, the Markovian assumption essentially implies that all individuals will eventually migrate to the absorbing state, meaning that the relevant question is not whether the ratings are actually Markovian, but rather whether the Markovian specification, which provides simplicity, is adequate for the short term.)

There are several reasons why the Markov assumption is likely to fail in practice. First, ratings tend to exhibit momentum or inertia; in other words, the longer an individual has been rated in a particular rating, the less probable they are to move to another rating within a period (as documented, for example, in Lando and Skodeberg[4]). Secondly, the business cycle influences rating dynamics (as shown by Nickel *et al.*[5]). Thirdly, if individuals are heterogeneous, migration probabilities may depend on their individual characteristics (as documented by Gómez-González and Kiefer[3]).

The present study contributes to the literature on rating transition dynamics in two ways. First, it provides evidence of non-Markovian behaviour in the process of rating transitions of commercial loans of financial institutions in Colombia (rigorously speaking, the study tests the Markov property of first degree, ie it tests the null hypothesis that $Pr(X_{t+1} \mid X_t, \ldots, X_{t-n}) = Pr(X_{t+1} \mid X_t, \ldots, X_{t-k})$, where in general $n > k$ and in particular $k = 0$; here $\{X_t\}$ represents a draw from the random process of states; the number of states is finite). Secondly, it constructs a duration model capable of estimating more precise transition matrices for this process. The methodology presented in this study can be used to calculate migration probabilities in many other applications. While the qualitative results are likely to be applicable to modern banking systems generally, the extrapolation of numerical values of coefficients outside of this application is not recommended.

The dataset used in this paper is unique. It is the result of the merger of a dataset that includes all individual commercial loans from the universe of financial institutions in Colombia, including their main characteristics, and a dataset containing the financial statements of the debtor firms. Given this level of disaggregation and the richness of the information, it is possible to test whether individual characteristics of the debtors influence the dynamics of rating migrations.

In the following two sections, the paper first describes the data and then presents the techniques used to construct the model to test the validity of the Markovian assumption and to estimate the transition matrices. The subsequent section presents the results of the estimation as well as empirical tests to check the validity of the model. The final section presents conclusions.

DESCRIPTION OF THE DATA

The econometric exercises presented in the results section of the paper use a dataset resulting from the merger of two datasets. The first dataset contains information on individual commercial loans, as reported by Colombian financial institutions to the Superfinanciera (the regulator of Colombia's financial system). This information, reported quarterly by the institutions in Format 341 of the Superfinanciera, contains detailed data on credit characteristics, including their ratings, from December 1998 to December 2006. The level of disaggregation allows analysis of credit risk that considers the heterogeneity among debtors and credit contracts.

Although credit ratings (and other information) are reported periodically (monthly) by the credit institutions to the Superfinanciera, institutions monitor these ratings continuously. This fact will justify the use of continuous-time survival function models in the results section of the paper.

The second dataset, provided by the Supersociedades (the Superintendence of Corporations), contains individual balance sheets reported on an annual basis by an important proportion of firms. Given the richness of the dataset, in this analysis the individuals are credits (not firms). Each credit has a corresponding credit rating by year, and an associated financial statement. Table 1 shows the number of individuals with financial statement information in the dataset resulting from the merger, by year.

Table 1: Description of the dataset

Year	Number of matches
1999	4,101
2000	4,307
2001	3,851
2002	5,236
2003	5,457
2004	11,215
2005	14,879
2006	14,660

Covariates were constructed using both firms' balance sheets (firm-specific variables) and macroeconomic variables. A brief description of each of the chosen covariates is presented below:

- *liquidity (LIQ):* ratio of the sum of current assets, long-term investments and long-term debtors to the sum of current liabilities, long-term financial and labour obligations, long-term unpaid accounts, and long-term bonds — this indicator measures the firm's long-term liquidity position;
- *hedging (HED):* ratio of liabilities to equity — among the accounts considered as equity, those corresponding to second-tier capital are weighted 50 per cent;
- *size (SIZE):* assets of the institution divided by a common number to scale the variable appropriately;
- *efficiency (EFF):* ratio of operating expenses to assets;
- *debt composition (COMP):* ratio of current liabilities to the sum of current and long-term liabilities;
- *number of relations (NUM):* number of financial institutions with which the firm has a credit relation;
- *age (AGE):* number of periods in which the firm has at least one credit contract with a financial institution;
- *profitability of assets (PROF):* ratio of profits to assets;
- *GDP growth (GDP):* annualised quarterly growth rate of the economy;
- *real lending interest rate (RATE):* quarterly average lending interest rate.

These financial indicators are proxies of the variables traditionally considered in the literature (eg see Audretsch and Mahmood[6]).

Most correlations between the variables were small and in no case did one exceed 0.4 in absolute value.

CONTINUOUS-TIME SURVIVAL ANALYSIS METHODS FOR ESTIMATING TRANSITION MATRICES

Transition matrices are widely used to estimate migration probabilities within states. Preliminary analyses showed the presence of time heterogeneity in the transition matrices estimated using both discrete time and continuous time methodologies. Unsurprisingly, the results are in line with the findings of other studies, including Lando and Skodeberg,[4] who document evidence of rating momentum; Kavvathas,[7] who finds dependence of rating migrations on macroeconomic variables; Jonker,[8] who using a dataset of ratings of European, US and Japanese banks finds that the country of origin matters in the downgrading process; Bangia *et al.*,[9] who find that the state of the business cycle matters; and Gómez-González and Kiefer[3] who find that both macroeconomic and bank-specific variables influence the process of bank rating dynamics.

To study the origin of the time heterogeneities further, the study constructs a duration or hazard function model to evaluate the impact of several variables on credit quality transition dynamics. This approach generalises the more common binary response (logit or probit) approach by modelling not only the occurrence of the transition but the time to migration, allowing finer measurement of the effect of different variables on migrations.

Survivor functions and hazard functions

In duration models, the dependent variable is duration — the time that it takes a system to change from one state to another. In the case of credit quality migrations, duration is the time that it takes from a loan to change of state. For this paper, the (finite) state space is constituted by the five loan quality categories existing in Colombian banks' balances, specifically A, B, C, D and E.

In theory, duration T is a non-negative, continuous random variable. However, in practice, duration is usually represented by an integer number of months, for example. When T can take a large number of integer values, it is conventional to model duration as being continuous.[10]

Duration can be represented by its density function $f(t)$ or its cumulative distribution function $F(t)$, where $F(t) = Pr(T \leq t)$, for a given t. The survival function, which is an alternative way of representing duration, is given by $S(t) = 1 - F(t) = Pr(T > t)$. In other words, the survival function represents the probability that the duration of an event is larger than a given t. Now, the probability that a state ends between period t and $t + \Delta t$, given that it has lasted up to time t, is given by:

$$Pr(t < T \leq t + \Delta t | T > t) = \frac{F(t + \Delta t) - F(t)}{S(t)} \quad (1)$$

This is the conditional probability that the state ends a short time after t, provided it has reached time t. For example, in the case of loan quality dynamics, it is the probability that a loan state changes from quality i to quality j in a short time after time t, conditional on the fact that the loan was rated i at time t.

The hazard function $\lambda(t)$, which is another way of characterising the distribution of T, results from considering the limit when $\Delta t \to 0$ in Equation (1). This function gives the instantaneous probability rate that a change of state occurs, given that it has not happened up to moment t. The cumulative hazard function $\Lambda(t)$ is the integral of the hazard function. The relation between the hazard function, the cumulative hazard function and the survival function is given by Equation (2):

$$\Lambda(t) = \int_{u=0}^{t} \lambda(u) du = -\log[S(t)] \quad (2)$$

Some empirical studies use parametric models for duration. Commonly used distributions are the exponential, the Weibull and the Gompertz. The exponential implies a constant hazard while the Weibull admits decreasing or increasing hazards. The Gompertz distribution allows non-monotonic hazard rates, but is not particularly flexible. Further, the baseline hazard in this formulation reflects changes in conditions of the environment (eg regulatory changes) common to all credits. There is no reason to think these will correspond to a monotonic hazard, and indeed there is evidence it does not.

The study begins by estimating the unconditional (raw — no covariates) survivor function, using the Kaplan-Meier non-parametric estimator, which takes censored data into account. Suppose that changes of rating from quality i to quality j are observed at different moments in time, $t_1, t_2, \ldots, t_m$, and that d_k loans change state at time t_k.

For $t \geq t_k$

$$\hat{S}_{ij}(t) = Y_i(t_k)\prod_{t_k \leq t}\left[1 - \frac{d_k}{N_k}\right] \qquad (3)$$

where $Yi(t_k)$ is an indicator function that takes the value 1 whenever the loan is rated i at time t_k, and N_k represents the total number of loans rated i at time t_k. Tests of equality of the survivor function for credits of tradable firms versus credits of non-tradable firms were performed for each of the transitions. In most cases the null hypothesis was not rejected at standard confidence levels. Therefore, all credits were treated as belonging to the same group.

To estimate the hazard function, it is first necessary to obtain an estimation of the cumulative hazard function. The Nelson-Aalen non-parametric estimator is natural for this purpose. Equation (4) shows how to compute this estimator. For $t \geq t_k$:

$$\hat{\Lambda}_{ij}(t) = Y_i(t_k)\sum_{t_i \leq t}\frac{d_k}{N_k} \qquad (4)$$

The hazard function can be estimated as a kernel-smoothed representation of the estimated hazard contributions $\Delta\hat{\Lambda}(t_k) = \hat{\Lambda}(t_k) - \hat{\Lambda}(t_{k-1})$. (The kernel-smoothed estimator of $\lambda(t)$ is a weighted average of these 'crude' estimates over event times close to t. The proximity of the events is determined by the bandwidth, b, so that events lying in the interval $[t - b, t + b]$ are included in the weighted average. The kernel function determines the weights given to points at a distance from t. In the present case, the Epanechnikov kernel is used.)

Hence:

$$\hat{\lambda}_{ij}(t) = \frac{1}{b}\sum_{k=1}^{D} K\left(\frac{t - t_k}{b}\right)\Delta\hat{\Lambda}(t_k) \qquad (5)$$

where $K(\cdot)$ represents the kernel function, b is the bandwidth, and the summation is over the total number of failures D observed.[11]

The form of the estimated hazard functions, which in all cases exhibited non-monotonicities, shows that the most commonly used parametric models for the distribution of duration do not seem to be appropriate for modelling the baseline hazard of credit quality dynamics in Colombia.

Proportional hazards

The present objective is to understand how macroeconomic and bank-specific variables affect the conditional probability of migration between states i and j. In ordinary regression models, explanatory variables affect the dependent variable by moving its mean around. However, in duration models it is not straightforward to see how explanatory variables affect duration and the interpretation of the coefficients in these types of models depends on the particular specification of the model. However, there are two widely used special cases in which the coefficients can be given a partial derivative interpretation: the proportional hazards model and the accelerated lifetime model.[12]

Building on the above analysis indicating that conventional candidates for parametric models are inappropriate, this paper estimates a proportional hazards model in which no parametric form is assumed for the baseline hazard function. According to a specification test (Schoenfeld's residual test), this assumption seems to be appropriate for the problem of interest. The hazard rate can be written as:

$$\lambda_{ij}^{n}(t) = Y_i^n(t)\alpha_{ij}^{n}(\beta_{ij}, t, X^n(t)) \qquad (6)$$

where $\lambda_{ij}^{n}(t)$ denotes the transition intensity from category i to category j of credit n; $Y_i^n(t)$ is an indicator function which takes the value 1 if the loan is rated in category i at time t and 0 otherwise; $\alpha_{ij}^{n}(\beta_{ij}, t, X^n(t))$ is a function both of time and of a vector of covariates of loan n at time t, denoted $X^n(t)$. This study uses time-varying covariates; however, if time-varying covariates are not available or if the covariates to be included do not vary during the observation period, a vector of fixed covariates can be used. It is assumed that the function $\alpha_{ij}^{n}(\beta_{ij}, t, X^n(t))$ has the multiplicative (proportional hazards) form, as per Cox:[13]

$$\alpha_{ij}^{n}(\beta_{ij}, t, X^n(t)) = \alpha_{ij}^{0}(t)\phi(\beta_{ij}, X^n(t)) \qquad (7)$$

where $\alpha_{ij}^{0}(t)$ represents the baseline intensity, common to all loans, which captures the direct effect of time on the transition intensity. (The duration model used in this paper allows for a non-constant baseline hazard, common to all observations, which captures changes in environmental conditions that are not being controlled by the inclusion of microeconomic and macroeconomic covariates.) For estimation purposes, a functional form is specified for $\Phi(\beta_{ij}, X^n(t))$, while the baseline intensity is left unspecified (the only restriction is that it is non-negative). A functional form which is frequently chosen for $\Phi(.)$, the transformation function, is the exponential form, $\Phi(\beta_{ij}, X^n(t)) = \exp(X^n(t)'\ \beta_{ij})$, which has the advantage of guaranteeing non-negativity without imposing any restrictions on the values of the parameters of interest ($\beta_{ij}'s$). The model is estimated by the method of partial likelihood estimation, developed by Cox.[13]

Estimation technique

In the case of specifications which explicitly model the baseline hazard by making use of a particular parametric model, estimation can be done by the method of maximum likelihood. When the baseline hazard is not explicitly modelled, the conventional estimation method is partial likelihood estimation, as developed by Cox.[13] The key point of the method is the observation that the ratio of the hazards (Equation 6) for any two individuals, i and j, depends on the covariates, but does not depend on duration:

$$\frac{\lambda_{ij}^{n}(t)}{\lambda_{ij}^{m}(t)} = \frac{exp(X^n(t)'\beta_{ij})}{exp(X^m(t)'\beta_{ij})} \qquad (8)$$

Suppose there are N observations and there is no censoring. If there are no ties, durations can be ordered from the shortest to the longest, $t_1 < t_2 < \ldots < t_N$. Note that the index denotes both the observation and the moment of time in which the duration for that particular observation ends. The contribution to the partial likelihood function of any observation j is given by:

$$\frac{exp(X^m(t)'\beta_{ij})}{\sum_{n=j}^{N} exp(X^n(t)'\beta_{ij})} \qquad (9)$$

This provides the ratio of the hazard of the individual whose spell ended at duration t_j to the sum of the hazards of the individual whose spells were still in progress at the instant before t_j. The likelihood can then be written as:

$$L(\beta) = \prod_{j=1}^{n} \frac{exp(X^m(t)'\beta_{ij})}{\sum_{n=j}^{N} exp(X^n(t)'\beta_{ij})} \quad (10)$$

Thus, the log-likelihood function is:

$$\ell(\beta) = \sum_{j=1}^{n} \left[X^m(t)'^{\beta_{ij}} - log \sum_{n=j}^{N} exp(X^n(t)'\beta_{ij}) \right] \quad (11)$$

By maximising Equation (11) with respect to β, estimators of the unknown parameter values are obtained. The intuition behind partial likelihood estimation is that, without knowing the baseline hazard, only the order of durations provides information about the unknown coefficients.

When there is censoring, the censored spells will contribute to the log-likelihood function by entering only in the denominator of the uncensored observations.

Censored observations will not enter the numerator of the log-likelihood function at all.

Ties in durations can be handled by several different methods. In this paper, ties are handled by applying the Breslow method. In continuous time, ties are not expected. Nevertheless, given that the moment of failure in practical applications is aggregated into groups (here months), ties are possible, and indeed do occur. Suppose there are four individuals in the risk pool (a_1, a_2, a_3, a_4), and, at a certain moment, a_1 and a_2 change state. According to the Breslow method, it is unknown which of the changes preceded the other, and so the largest risk pool will be used for both changes. In other words, this method assumes that a_1 changed state from the risk pool a_1, a_2, a_3, a_4, and a_2 also changed state from the risk pool a_1, a_2, a_3, a_4. The Breslow method is an approximation of the exact marginal likelihood, and is used when there are not many ties at a given point in time.

ESTIMATION RESULTS

The model was estimated using the partial likelihood method. Results for each transition are presented in Tables 2–6, which show the values of the estimated coefficients and their standard errors.[14] An important preliminary conclusion from these tables is the clear rejection of the null hypothesis that none of the indicators included in the model is important in explaining the behaviour of duration. This provides evidence to support the idea that credit rating migrations can be explained by differences in financial health and prudence existing across institutions. Additionally, migrations vary along the business cycle, supported by the fact that the macroeconomic variables included in the regressions are jointly significant. Thus, transition matrices estimated without conditioning on firm-specific and macroeconomic variables can be misleading.

Regarding the role played by individual indicators, it can be seen that the different indicators explain the inter-credit variability in the hazard rate for the different transitions. Covariates such as LIQ, SIZE, EFF and COMP are statistically significant at conventional levels for most of the cases. However, for the purpose of this study, the central

Table 2: Transition intensities out of A

	AB		AC		AD		AE	
Covariate	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.
LIQ	−0.0002	0.0000	0.0007	0.0002	−0.0106	0.0012	−0.0094	0.0013
HED	0.0004	0.0043	0.0005	0.0012	−0.0008	0.0013	−0.0008	0.0001
SIZE	−0.0007	0.0001	−0.0077	0.0011	−0.0120	0.0030	−0.0121	0.0034
EFF	−0.1162	0.0270	−0.3025	0.0891	−0.1465	0.1601	−1.4767	0.3359
COMP	−0.0099	0.0004	−0.0127	0.0010	−0.0189	0.0021	−0.0157	0.0023
NUM	0.0322	0.0046	0.0908	0.0126	0.0126	0.0295	−0.0752	0.0360
AGE	−0.0108	0.0013	−0.0281	0.0032	−0.0053	0.0066	0.0083	0.0075
GDP	−0.0709	0.0054	−0.1591	0.0145	−0.1624	0.0306	−0.1100	0.0345
RATE	0.0103	0.6841	0.0477	0.0175	0.0289	0.0355	0.0534	0.0402
PROF	0.0000	0.0000	−0.0001	0.0001	0.0000	0.0002	−0.0006	0.0003
Log-like	−97,287.6		−13,591.7		−3,161.6		−2,517.8	
LR χ^2 (10)	1,319.1		1,053.8		546.4		410.2	
Prob > χ^2	0.0000		0.0000		0.0000		0.0000	

Table 3: Transition intensities out of B

	BA		BC		BD		BE	
Covariate	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.
LIQ	0.0000	0.0002	−0.0008	0.0002	−0.0053	0.0009	−0.0041	0.0015
HED	0.0008	0.0003	0.0000	0.0002	−0.0001	0.0005	0.0002	0.0001
SIZE	0.0003	0.0001	−0.0073	0.0007	−0.0090	0.0020	−0.0193	0.0059
EFF	0.0820	0.0206	−0.1130	0.0666	−0.2086	0.1725	−0.1528	0.2766
COMP	0.0085	0.0006	−0.0016	0.0008	−0.0015	0.0019	−0.0078	0.0031
NUM	−0.0212	0.0062	0.0873	0.0106	0.0704	0.0257	0.0569	0.0442
AGE	−0.0085	0.0016	−0.0274	0.0026	−0.0334	0.0057	−0.0241	0.0098
GDP	0.0788	0.0068	−0.0429	0.0116	−0.1968	0.0274	−0.1404	0.0451
RATE	0.0261	0.0090	−0.0165	0.0145	−0.0201	0.0321	−0.0019	0.0533
PROF	0.5459	0.0416	−0.0386	0.0168	−0.0523	0.0434	−0.0433	0.0690
Log-like	−52,009.9		−16,145.2		−3,000.5		−1,077.6	
LR χ^2 (10)	1,016		369.6		348.8		104.53	
Prob > χ^2	0.0000		0.0000		0.0000		0.0000	

issue is not to identify individually significant indicators; the central point is to show that together the included covariates are significant to explain credit migrations.

The duration model presented above can be used to construct credit migration matrices conditioning on relevant covariates. The method followed here, which to the authors' knowledge has not been proposed in the related literature, is a two-step method. In the first step, the baseline hazard function is recovered at every analytic time at which a failure occurs, t_i, following Kalbfleisch and Prentice,[15] and with this, one obtains transition intensities for each possible transition and each possible failure time. In the second step, the time-scaled intensity matrix is exponentiated to obtain the transition matrix for the desired period of time. Following are the resulting one-year transition matrices for 1999 ($year_{1999}$ a year of financial crisis and recession), for 2006 ($year_{2006}$ the last year of data) and the annual average for

Table 4: Transition intensities out of C

	CA		CB		CD		CE	
Covariate	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.
LIQ	0.0000	0.0000	0.0006	0.0007	−0.0006	0.0002	−0.0035	0.0012
HED	0.0002	0.0001	0.0000	0.0001	0.0000	0.0000	0.0000	0.0003
SIZE	0.0013	0.0007	0.0011	0.0005	−0.0083	0.0013	−0.0063	0.0025
EFF	0.5081	0.1148	−0.1419	0.1502	−0.0434	0.0698	0.0160	0.2058
COMP	0.0106	0.0017	0.0042	0.0016	0.0027	0.0009	0.0036	0.0025
NUM	−0.1032	0.0245	−0.0573	0.0215	0.0539	0.0132	0.0922	0.0332
AGE	−0.0074	0.0050	0.0171	0.0052	−0.0241	0.0030	−0.0221	0.0078
GDP	0.0246	0.0226	0.0229	0.0225	−0.0077	0.0134	−0.0785	0.0353
RATE	0.0882	0.0277	0.0595	0.0273	−0.0018	0.0168	−0.0087	0.0424
PROF	1.4237	0.2177	−0.0475	0.0526	−0.0899	0.0265	0.0327	0.1577
Log-like	−4,072.7		−3,995.4		−10,836.6		−1,581.9	
LR χ^2 (10)	173.7		33.8		219.4		49.79	
Prob $> \chi^2$	0.0000		0.0002		0.0000		0.0000	

Table 5: Transition intensities out of D

	DA		DB		DC		DE	
Covariate	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.
LIQ	0.0001	0.0001	0.0000	0.0002	0.0002	0.0001	0.0000	0.0001
HED	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0006	0.0005
SIZE	0.0007	0.0014	0.0037	0.0008	0.0007	0.0012	−0.0039	0.0012
EFF	0.0681	0.0783	−0.0677	0.0601	0.1293	0.1321	0.0159	0.0329
COMP	0.0079	0.0024	0.0031	0.0030	0.0017	0.0024	0.0049	0.0010
NUM	−0.1245	0.0402	−0.0367	0.0448	−0.0973	0.0395	0.0364	0.0154
AGE	0.0061	0.0076	0.0070	0.0097	0.0303	0.0086	−0.0243	0.0034
GDP	0.1041	0.0364	−0.0784	0.0438	−0.0650	0.0359	−0.0264	0.0152
RATE	0.1839	0.0452	−0.6405	0.0526	−0.0811	0.0428	−0.0386	0.0193
PROF	0.0801	0.0596	−0.0542	0.0241	0.1511	0.0705	0.0104	0.0311
Log-like	−1,630.6		−1,057.5		−1,492.3		−7,969.1	
LR χ^2 (10)	48.45		26.58		25.2		131.3	
Prob $> \chi^2$	0.0000		0.0030		0.0050		0.0000	

the period 1999 to 2006 ($year_{average}$). It is important to note that these matrices are all calculated using the average values of each of the covariates included in the duration model. An interesting exercise, that is not presented here but that can be easily done after estimating the coefficients of the model, is to stress the values of some (or all) of the covariates and get transition matrices for the corresponding simulated scenarios.

$$year_{1999} = \begin{pmatrix} 0.952 & 0.028 & 0.118 & 0.006 & 0.002 \\ 0.039 & 0.786 & 0.082 & 0.054 & 0.039 \\ 0.032 & 0.043 & 0.762 & 0.109 & 0.054 \\ 0.013 & 0.054 & 0.014 & 0.893 & 0.026 \\ 0.022 & 0.015 & 0.006 & 0.010 & 0.947 \end{pmatrix}$$

Table 6: Transition intensities out of E

	EA		EB		EC		ED	
Covariate	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.
LIQ	0.0000	0.0001	0.0000	0.0004	0.0000	0.0004	0.0000	0.0001
HED	0.0000	0.0001	0.0000	0.0002	0.0000	0.0002	0.0002	0.0000
SIZE	0.0007	0.0006	−0.0004	0.0029	0.0007	0.0010	0.0005	0.0007
EFF	0.0403	0.0675	−0.1878	0.4656	0.4067	0.6584	0.0204	0.1587
COMP	0.0103	0.0027	0.0018	0.0045	0.0058	0.0053	−0.0034	0.0024
NUM	−0.0591	0.0490	0.0181	0.0763	0.0103	0.0813	0.0367	0.0383
AGE	0.0001	0.0090	0.0048	0.0154	0.0408	0.0196	0.0108	0.0083
GDP	0.0139	0.0413	−0.1254	0.0674	0.0355	0.0770	−0.0386	0.0361
RATE	0.1209	0.0484	−0.0862	0.0762	0.0331	0.0955	0.0046	0.0424
PROF	0.0421	0.0706	0.0134	0.1693	0.1734	0.2268	0.0539	0.0943
Log-like	−1,262.5		−445		−325		−1,545.6	
LR χ^2 (10)	41.77		4.4		8.37		16.31	
Prob $> \chi^2$	0.0000		0.9278		0.5925		0.0910	

$year_{2006}$

$$= \begin{pmatrix} 0.986 & 0.011 & 0.002 & 0.001 & 0.001 \\ 0.097 & 0.816 & 0.066 & 0.013 & 0.008 \\ 0.037 & 0.033 & 0.789 & 0.111 & 0.031 \\ 0.014 & 0.034 & 0.011 & 0.916 & 0.024 \\ 0.011 & 0.007 & 0.006 & 0.008 & 0.968 \end{pmatrix}$$

$year_{average}$

$$= \begin{pmatrix} 0.978 & 0.016 & 0.004 & 0.002 & 0.001 \\ 0.067 & 0.831 & 0.067 & 0.019 & 0.015 \\ 0.029 & 0.037 & 0.799 & 0.099 & 0.036 \\ 0.011 & 0.037 & 0.015 & 0.913 & 0.023 \\ 0.013 & 0.011 & 0.007 & 0.009 & 0.961 \end{pmatrix}$$

From the above matrices, it is clear that transitions to worse categories were more common during the crisis than during normal times. For example, a credit in the best rating in 1999 was 2.6 percentage points more likely to migrate out of that category in a one-year horizon than in 2006. Similarly, upgradings were less probable during the crisis. These results reinforce the idea that unconditional transition matrices are likely to be misleading because they do not take into account variables related to the business cycle.

An interesting exercise would be to compare these matrices with those estimated unconditionally under the assumption that the stochastic process underlying rating migrations is Markovian. Although this is not explored here, the exercise is straightforward.

CONCLUSIONS

This study presents an alternative way of estimating credit transition matrices using a hazard function model. The model is useful both for testing the validity of the Markovian assumption, frequently made in credit rating applications, and also for estimating transition matrices conditioning on firm-specific and macroeconomic covariates that influence the migration process. The model presented in the paper is likely to be useful in other applications, although extrapolating the numerical values of coefficients outside of the present application is not recommended.

Transition matrices estimated in this way may be an important tool for a credit risk administration system, in the sense that a practitioner can use them to forecast the future behaviour of clients' ratings and possible changes of state.

Disclaimer

The views herein are those of the authors and do not necessarily represent the views of the Banco de la República or the Office of the Comptroller of the Currency.

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