Comment

Ergodic failure: The key vulnerability in derivatives modelling

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Editorial Board Member

Abstract *Ergodicity* is a principal property intrinsic to the mathematical modelling of capital markets. It is a structural symmetry underpinning the way in which information is extrapolated from today's state of the world to new, stochastic, future states. The failure of ergodicity — manifested in the myriad of dysfunctionality in capital markets during the financial crisis — represents a primordial breakdown in the way financial engineering has mathematically parameterised its underlying markets. This paper assesses the consequent implications of such failure with respect to the pricing of derivatives.

Keywords: *ergodic, volatility, derivatives, models, static-hedging*

Fundamental to the risk practices underpinning the derivatives industry is an appropriate process for mathematically parameterising uncertainty. In fact, a key factor facilitating the rapid evolution in the complexity of the derivatives markets has been the application of accessible mathematical methods to finance.

This facilitation began with the seminal work of Nobel laureates Scholes and Merton which demonstrated that, notwithstanding the intrinsic unknowns associated with a contingent liability, those unknowns are capable of being *completely* ameliorated through a carefully designed dynamic hedging strategy.

This cleverly solved the hitherto problem of pricing an uncertain, market-contingent cash flow (such as a call option on a stock): the price of the unknown is the cost of the hedging strategy which neutralises the unknown. Black, Scholes and Merton have explained how to design such a strategy.[1,2] Encoded in this line of reasoning, of course, is an assumed awareness of all relevant unknowns.

While derivative products, and their associated pricing models, have undergone a remarkably complex evolution over the last decade, the underlying premise for pricing construction has remained the same, that is, *the value of a derivative is its cost of replication through dynamic hedging in the underlying markets*. This axiom of applied quantitative finance implicitly assumes that the hedge markets preserve the structure and stability of their behavioural modes across different temporal periods — ie that the world is ergodic.

However, the world (and its capital markets) is not ergodic. This is not to say that markets are not capable of slyly exhibiting apparent ergodicity for, possibly prolonged, bouts. Aided and abetted by overly accommodative central bank policy and a decade of consumption/savings imbalances, 2003–07 was a period characterised by a cyclical decline in volatility, a contraction in risk premium, and the ostensible sustainability of market liquidity.

The abrupt migration of that state to a more disorderly one left derivatives portfolios, predicated on the ergodic belief in the ongoing persistence of prior market stability, hopelessly exposed.

Financial models founded on the ergodic principle necessarily fail to reflect the risk of state migration. Instead, derivatives models work as follows: the ergodic pricing premise dictates that future hedging will be conducted in a market state consistent with today's state (where 'state' represents the market's volatility characteristics implied by the breadth and depth of its prevailing liquidity, its impact cost and transactional friction, etc). As today's state evolves into that of tomorrow's, the parameters of the derivatives models get recalibrated *post ante*, to the new state — which is, of course, too late.

This is a critical flaw for an exotics portfolio because, when the state undergoes a disorderly transformation, the Black-Scholes hedging prescription pertaining to the newly attained state destabilises. This results in erratic, volatile and high-frequency fluctuations in the required hedging strategy.

The economic consequences of model failure to anticipate *ex ante* the risk of state migration have proved catastrophic: the equity exotics industry's net losses for 2008 are estimated to have been \$10bn.[3]

Non-ergodicity is the propensity for breakdown in a system's symmetry. While a suitably augmented derivative pricing theory can accommodate random fluctuations in volatility, non-ergodicity — a corollary of which is the asymmetric disappearance of hedge market liquidity — fundamentally undermines the 'pricing-by-replication' axiom intrinsic to derivatives markets and their underlying models. In short, non-ergodicity is a more primary instability than mere regime shifts in volatility (Figure 1).

Derivatives models are predicated on a number of other, more technical assumptions. These include the idea that markets have no memory (the Markov property), that markets never jump (the continuity hypothesis), and that markets' statistical attributes are stable (stationarity). To a greater or lesser extent, these assumptions are belied by market behaviour every day. Why then are models still developed under such flawed premises?

Part of the reason is the seduction of intellectual elegance. Derivatives modelling is based on a meretricious theory, more pseudoscience than the rigorous discipline of theoretical physics

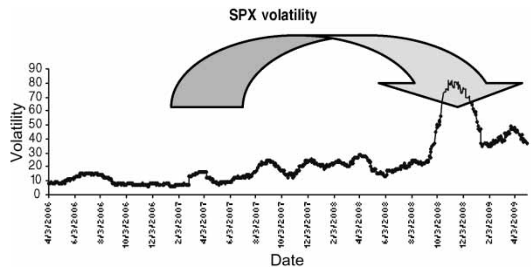

Figure 1: Historical volatility of SPX = S&P500: A failure of ergodicity is more fundamental than a regime shift in volatility

or applied mathematics. It has the deceptive appearance of distilling all the unknowns involved in pricing an uncertain cash flow into a single parameter: volatility.

Both traders and marketers are enamoured by this specious simplicity: it obviates the need for thinking at a deeper level, allowing sales people to get on with the business of originating more risk.

The reality is that the market is a highly-complex, nonlinear system which lurches from one meta-stable equilibrium to another. The risk dependencies and inter-dependencies of the system are manifold. Compactifying the dimension of these dependencies into a single parameter induces inherent instability in the hedge process. Derivatives models consummate risk on this misconstrued assumption that today's meta-stable equilibrium is a thing of permanence, that today's state will retain its structural characteristics for the tenor of the derivatives contract — which in some cases, is 30 years. This results in an under-pricing of the ensuing hedge instability and an under-allocation of capital against this risk.

Practical expedience also plays a part in this tolerance for modelling imprecision: were we to relax, for example, the Markov assumption that markets evolve independent of prior volatility patterns, then the computational burden involved in pricing even relatively simple exotics would prove prohibitively expensive. Distilling quantitative modelling to its simplest, tractable form has been an important factor in industrialising the scale of the structured derivatives market against a backdrop of increasing complexity. In so doing, there has been a misappropriation of the mathematics — we continue to count the cost.

Albert Einstein once recommended making everything as simple as possible, but not simpler. In pursuit of intellectual elegance, of a scaleable theory adapted to the reality of commercial expedience, the derivatives market has evolved out of sophistic modelling assumptions of devastating economic consequences.

Derivative models work on the principle of dynamic replication which claims to mitigate all future risk. The principle needs to say something about the future state of the world in which hedging will be conducted. It says nothing: it is *de facto* assumed ergodic. The brave new world of globalisation in market access and the improved efficiencies of information dissemination continue to interfere with the classic heterogeneity of risk thinking, inducing cyclical phases of game theoretical disequilibria as capital crowds into and then exits markets. Against the backdrop of these instabilities, ergodicity is a critical vulnerability in the derivatives markets' pricing of risk. To avoid repeating the consequent mistakes experienced in 2008, a thorough rethink of risk modelling is required.

An alternative approach to exotics pricing is to define the currently available set of fixed hedges that minimise the variance of future volatility of the derivative's value. In contrast to derivative pricing theory, this approach seeks merely to minimise such variance *statically* with no recourse to dynamic hedging during the tenor of the derivative — thus circumventing any assumption about the nature of future world states.

Unlike derivatives models, static hedging recognises its imperfections

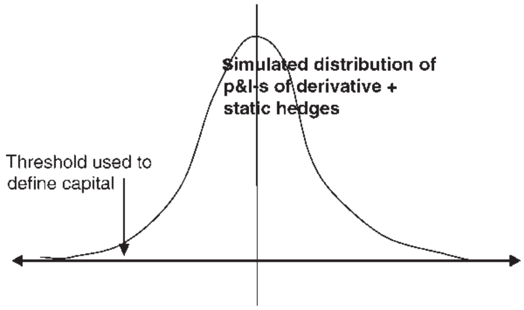

Figure 2: A Monte Carlo integration of a derivative and its portfolio of predefined static hedges

upfront. Capital allocation thus provides a more realistic reflection of the amount of risk retained by static hedge imperfection.

As shown in Figure 2, a Monte-Carlo integration of a derivative and its portfolio of predefined static hedges yields a distribution of losses.

The derivative price is that which yields the required return on equity from this simulated distribution (see above: the threshold cut-off of the percentile loss which defines equity capital is used to price the option based on a prerequisite return on this equity).

References

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