Retail credit capital charge optimisation and the new Basel Accord

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Abstract The Basel Committee for Banking Supervision's new Basel Accord and accompanying credit risk capital equations are designed to encourage the improvement of risk management practices. However, over a range of loan quality for some loan types, improvement of risk management practices by enhanced borrower discrimination leads to *increased* regulatory capital charges. The effect — entirely due to underlying mathematics — could discourage banks from improving risk management for such loans, thereby contravening the Committee's aims. This paper locates and investigates the source of the problem and illustrates its effect on regulatory capital.

Keywords: *Basel II, probability of default, credit risk, capital requirements*

INTRODUCTION

The Basel Accord of 1988 was the first attempt by the Basel Committee on Banking Supervision (BCBS) to improve risk management practices in banks.[1] With much of the mathematics of risk management then still in its infancy, the accord did not address all risks faced by banks. Those risks that *were* covered required several amendments and improvements to the 1988 accord as practitioners' skills were honed and theory became best practice.[2,3] Basel I addressed only credit and later market risk, but it soon became apparent that the former was too punitive in some aspects and too lenient in others. Basel I also completely ignored operational risk. These shortcomings and omissions have now been addressed in the new Capital Accord (or Basel II, as it has come to be known) with a much-improved treatment of credit risk as well as the incorporation of a set of methodologies to assist in the estimation of operational risk.[4] Market risk has been left largely unchanged in Basel II, and weaknesses that do remain in the treatment of operational risk will no doubt be addressed in future versions of the accord.

The impact of the BCBS's new credit risk methodologies on banks' regulatory capital calculations are yet to be fully ascertained. The BCBS rolled out five versions of the Quantitative Impact Study (QIS) questionnaire to determine precisely this impact, but the results were mixed and not well attended.[5] Basel II has now been implemented (January 2008) in many developed and emerging countries; and those not yet fully Basel II compliant are expected to follow suit by 2011. Thus, the theoretical results gleaned from the BCBS's QIS surveys have begun to be replaced by empirical data from banks' risk and compliance departments. The BCBS relies on practitioner feedback to adjust shortcomings in the accords: it is important to remember that the Basel Accords themselves are not legislation, but rather 'global best practice' guidelines established and implemented by representatives from major central banks.

The Basel Accords, however, seek to be all things to all banks in order to level the global playing field and provide useful benchmarks. As a result, a necessarily limited set of models have been developed to cover all risky possibilities facing banking institutions worldwide. While these efforts are to be applauded, it is perhaps inevitable that some flaws and inconsistencies will appear. One of these potential inconsistencies will be explored in this paper — a feature of Basel II's treatment of credit risk measurement that is incompatible with its stated goal of providing capital incentives for better risk management practices.

The remainder of this paper is arranged as follows. The next section establishes the relevance of Basel II in banks by providing a brief background to its development and implementation. The paper then goes on to provide a survey of current, popular credit risk models including the single and multi-factor models.

The credit risk model adopted by Basel II is the asymptotic single risk factor (ASRF) model. After introducing this model, the paper explains the components of the model and discusses the problems introduced by convex variation of minimum capital with probability of default. The subsequent section explores analytical results from hypothetical portfolios using different numbers of pooled probabilities of default. Analysis is then extended to real portfolios and the results discussed. The final section concludes the paper.

THE BASEL ACCORDS

The implementation of the new Basel Accord's non-advanced approaches in January 2008 heralded the inauguration of an ambitious, global banking project to encourage improved risk management in banks. The timetable established by the Banking Committee of the Bank for International Settlements originally intended this implementation to commence one year earlier, in 2007. The hiatus was blamed on implementation difficulties and a largely unprepared banking community. The situation has only marginally improved: many institutions — in both developed and emerging markets — remain woefully ill-prepared and local regulators sometimes lack the requisite sophistication.

Many of these problems stem from the complexities of credit risk measurement. The enormous popularity of value at risk (as the metric of choice for measuring market risk) is largely due to its endorsement by the 1996 amendment to the 1988 Basel Accord, but its relative simplicity, adaptability and applicability to a wide range of products have also played a role. Credit risk, by contrast, is more complex: the distribution of institutional credit losses is highly skewed; it relies on complicated mathematics (eg copulas) to explain links between the economic milieu and loan values; it involves an understanding and differentiation of equity, asset and default correlations; and it requires several more parameters than market risk (eg loss given default, probability of default, exposure at default) — each related to the other in complex ways.

The new accord has undergone (planned) multiple revisions, augmentations and excisions, and, given the experience garnered from the 1996 amendment to Basel I, it is likely that Basel II will be further revised at some or several points in the future. These alterations are generally welcomed as they represent an egalitarian, best-practice approach to global banking. For the moment, the credit risk environment is highly fluid, constantly evolving and improving as techniques are honed and perfected. Enthusiastic debate and concentrated research are in progress and it is to be expected that the fruits of these labours will show segments of the new accord to be inadequate, inaccurate or completely wrong. Given the intensity of the research that has already been incorporated into Basel II, complete rewrites seem unlikely, but it is inevitable that models will improve as more information becomes available.[6]

Banks may choose between two approaches to calculate the capital requirement for credit risk: the standardised approach (a slightly modified version of the current Basel I accord) and the internal ratings based (IRB) approach (in which banks are permitted to use their own internal estimates of prescribed key risk drivers as inputs to the capital calculation). In the IRB approach, regulatory minimum capital for a loan portfolio is calculated in a bottom-up manner, by estimating and then summing capital requirements at the individual loan level. Loan capital requirements are derived via the ASRF model. Although the BCBS neither cites nor documents this model, it is widely believed that Gordy's[7] work (itself largely derived from an adaptation of the single asset model of Merton,[8] later extended to an entire portfolio by Vasicek[9]) was the precursor to the regulatory equations. Pykhtin and Dev[10] have further extended Gordy's work.

In this model, portfolio credit risk is separated into two categories: systematic and idiosyncratic risk. Systematic risk represents the effect of unexpected changes in macroeconomic and financial market conditions on borrower performance. Idiosyncratic risk represents the effects of risk connected to individual companies. One key assumption of the ASRF approach is that the credit portfolio comprises a large number of relatively small exposures. As the portfolio becomes more and more fine-grained (ie the largest individual exposures account for smaller and smaller portfolio exposures), *portfolio* idiosyncratic risk is diversified away and all systematic risk — such as industry or regional risk — is modelled with only a single, common systematic risk factor which drives all dependence across credit losses in the portfolio. The model thus assumes that banks are well diversified across all geographic and industrial sectors in large economies.

The ASRF model also assumes that the capital charge for a lending exposure is based solely on loan-specific information. Capital charges are thus calculated on a decentralised loan-by-loan basis first, and then aggregated up to portfolio-wide value at risk (VaR) afterwards.

Principal inputs supplied by the bank include the exposure at default (EAD), the probability of default (PD), the loss given default (LGD) and the effective remaining loan maturity (M). Given these inputs, the IRB capital charge is computed by calculating capital charges on a decentralised loan-by-loan basis and then aggregating these up to a portfolio-wide capital charge.

REGULATORY CAPITAL USING THE IRB APPROACH

The standardised approach essentially mimics the Basel I approach to credit risk (with some minor adjustments) in which all loan types are assigned a risk weighting determined by the BCBS. The IRB approach — which comprises the Foundation (FIRB) and the Advanced (AIRB) approach — allows banks to assign their own internal ratings to loans. In the FIRB approach, banks may only determine and use their own internal ratings (and associated PD values), while in the AIRB approach, banks may measure and use other inputs (over and above their own internal ratings) in the specified regulatory capital equations.

Banks are expected to forecast the average level of credit losses they can reasonably expect to experience over a one-year horizon, known as expected losses. Losses above this expected level — known as unexpected losses — occur occasionally, but their timing and severity are both unknown. Banks cover their expected losses continuously by provisions, write-offs and the incorporation of these expected losses into instrument pricing. Basel II only requires banks to hold capital against unexpected losses: the required capital per unit of currency exposure. A number of approaches exist to determine a bank's requisite capital. Basel II's IRB approach estimates the annual loss which will be exceeded with a small, pre-selected probability, $1 - \alpha$ (see Equation 1). This is considered to be the probability of bank insolvency (meant in a broad sense, including, for example, the case of a bank failing to meet senior loan obligations) and is shown graphically in Figure 1.

It is instructive to compare the distribution of credit losses and relevant loss regions with the more familiar equivalent graph for market risk losses, shown in Figure 2. For regulatory capital purposes, the expected loss here is assumed to be zero, the confidence interval is set at a much lower value (99 per cent) and the period over which the losses are expected to occur is ten days rather than one year (where the confidence interval is 99.9 per cent and the period over which losses are expected to occur is one year for credit risk).

BCBS specifies Basel II's regulatory

capital charge for the IRB approach — per unit of currency exposure — as:

$$K = LGD \times \left(N\left[\sqrt{\frac{1}{1-\rho}} \cdot N^{-1}(PD) + \sqrt{\frac{\rho}{1-\rho}} \cdot N^{-1}(\alpha) \right] - PD \right) \cdot M^{*} \quad (1)$$

where: LGD is the loss given default; $N[x]$ denotes the cumulative distribution function for a standard normal random variable (ie the probability that a normal random variable with mean zero and variance of one is less than or equal to x); $N^{-1}(z)$ denotes the inverse cumulative distribution function for a standard normal random variable (ie the value of x such that $N(x) = z$); PD is the probability of default; ρ is the asset correlation which indicates the dependence of the asset value of a borrower on the general state of the economy (all borrowers are linked to each other by this single risk factor); α is the confidence level, set at 99.9 per cent (ie an institution is expected to suffer losses that exceed its economic capital only once in 1,000 years on average);

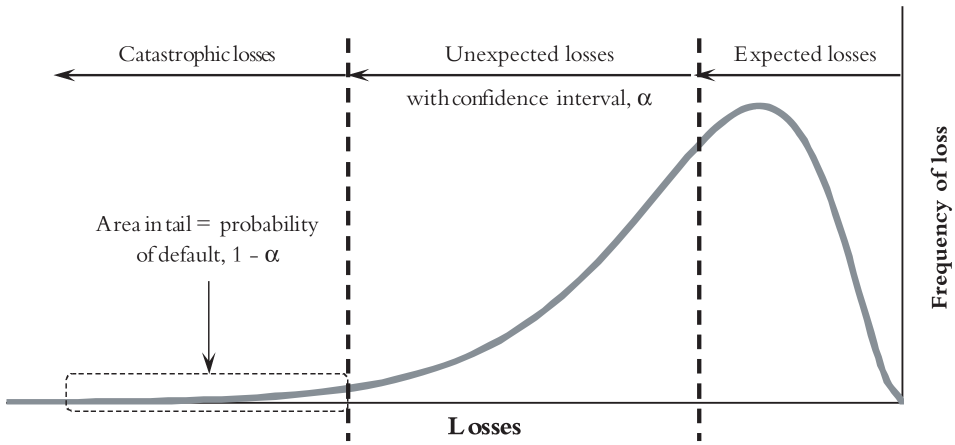

Figure 1: Characteristic shape and relevant parameters of credit loss distributions

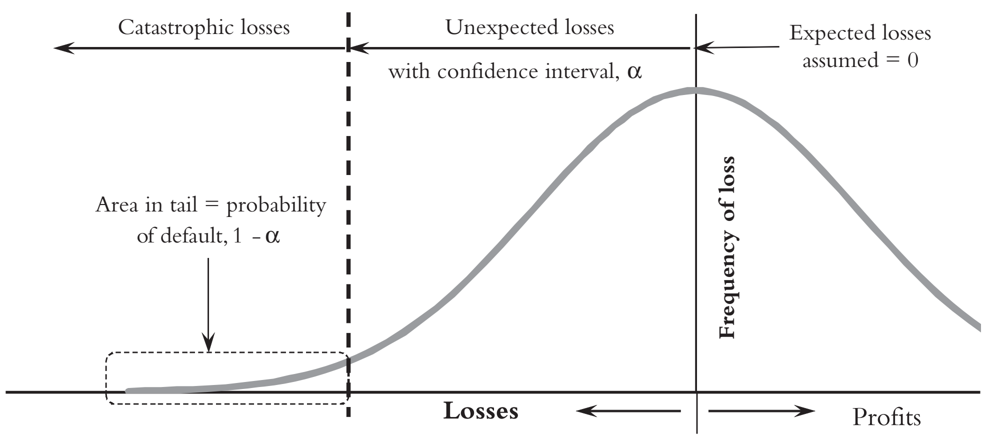

Figure 2: Characteristic shape and relevant parameters of market loss distributions

and $M^\star$ is a maturity adjustment which embraces the empirical evidence that long-term loans are riskier than short-term loans and the fact that downgrades from a higher rating category to a lower one are more likely for long-term loans. Maturity effects are also more pronounced for obligors with low probabilities of default. This maturity adjustment, $M^\star$, is given by:

$$M^* = \frac{1 + (M - 2.5) \cdot b}{1 - 1.5 \cdot b} \qquad (2)$$

where: M is the maturity of the (wholesale) loan and b is a scaling factor dependent only on PD, and is given by: $b = (0.11852 - 0.05478 \cdot \ln(PD))^2$.

While Equation 1 differs by loan type, these differences are relatively minor, eg linear adjustment scaling for turnover in loans to small to medium-sized firms.

A remarkable characteristic of Equation 1 (apart from its simplicity) is its property of asymptotic capital additivity: the total capital for a large portfolio of loans is the weighted sum of the marginal capital for individual loans. That is, the capital required to add a loan to a large, diversified portfolio depends only on the properties of that loan and not on the portfolio to which it is added. The ASRF credit model is thus said to be portfolio invariant, a property that depends strongly on the asymptotic assumption, and especially on the assumption of a single systematic risk-factor. In addition, portfolios that are not asymptotically fine-grained (ie any single obligor represents a negligible share of the portfolio's total exposure) contain undiversified idiosyncratic risk. In this case, the marginal contributions to the economic capital depend on the rest of the portfolio. For a loan portfolio of unit of currency exposure EAD, the capital charge is (using Equation 1) simply $K \cdot EAD$.

A brief description of each of the principal input parameters follows.

Probability of default

Under the IRB approach, the PDs are obtained from the internal rating system of the bank. According to the BCBS, these should be average quantities, reflecting expected default rates under normal business conditions.

Two kinds of models for determining probabilities of default are commonly addressed in the literature: accounting-based models and market-based models. Discriminant analysis and logistic regression models belong to the first class. The popular Z-score[11] is based on linear discriminant analysis, while Ohlson's O-Score[12] is based on generalised linear models with the logit link function. Newer accounting based models are founded on neural networks[13] and generalised additive models.[14]

Market models are based on the firm's asset value as determined by the market, such as Moody's KMV model. Stock prices are used as proxies for the asset value of the firm — implying that these models require stock-exchange registered (publicly-listed) firms, a criterion not fulfilled for many small and medium-sized borrowers.

Loss given default

In the capital formula (Equation 1) it is assumed that the LGD rate (which is equal to one minus the recovery rate) is known and non-stochastic.[5] During an economic downturn, losses on defaulted loans are likely to be higher than under normal business conditions, because, for example, collateral values may decline. Average loss severity figures over long periods of time can understate LGD rates during an economic downturn, and may therefore need to be adjusted upward to reflect adverse economic conditions appropriately.

Conservative values should thus be chosen so as not to underestimate portfolio risk. Equation 1 therefore requires that a 'downturn' LGD is estimated for each client/risk segment. Due to the evolving nature of bank practices in the area of LGD quantification, the BCBS has not proposed a specific rule for estimating the LGDs. Instead, banks are required to provide their own estimates, but they may use supervisory estimates if they have adopted the foundation IRB approach for wholesale exposures.

Exposure at default

Under the advanced IRB approach, banks are allowed to use their own estimates of expected exposure at default for each facility. EAD comprises two parts: the amount currently drawn and an estimate of future draw-downs of available, but untapped, credit. Estimates of potential future draw-downs (ie how the client may decide to draw unused commitments) are known as credit conversion factors (CCFs). As the CCF is the only random or unknown proportion of EAD, estimating EAD amounts to estimating this CCF. CCFs depend on both the type of loan and the type of borrower.

Default correlations

The single systematic risk factor required in the ASRF model may be interpreted as reflecting the state of the global economy. The degree of an obligor's exposure to this systematic risk factor is expressed by the asset correlation. Asset correlations link the movements of how asset values of one borrower depend on the asset values of another. In a similar way, the correlation can be described as the dependence of the asset value of a borrower on the general state of the economy — all borrowers are linked to each other by this single risk factor.

Asset correlations influence the structure of the risk weight formulas. They are asset-class dependent as different borrowers and/or asset classes show different degrees of dependency on the overall economy. It is important to note that *asset* correlation and *default* correlation are not the same; their relationship is explained elsewhere.[15]

In the IRB approach, asset correlations are not estimated by banks. Instead they are calculated according to equations provided by the BCBS. These are based on two empirical observations,[16] namely:

- asset correlations decrease with increasing probability of default; and
- asset correlations increase with firm size.

This means that the higher the probability of default, the higher the idiosyncratic risk components of an obligor. Moreover, conditional on a certain probability of default, the assets of small and medium-sized enterprises are less correlated. Hence, if two companies of different size have the same PD, it follows that the larger one is assumed to have a higher exposure to the systematic risk factor. In other words, larger firms are more closely related to the general conditions in the economy, while smaller firms are more likely to default for idiosyncratic reasons. The BCBS has provided different formulas for computing the asset correlations for different business segments. These are discussed in detail by the BCBS[4] and are summarised in Table 1.

Note the differences — for asset correlations — between upper and lower bounds as well as the exponent coefficients for 'other retail' and corporate loans.

CAPITAL CHARGE OPTIMISATION

For the remainder of this paper, 'capital charges' refers specifically to charges incurred due to credit risk. Market risk capital charges are usually much smaller than credit risk capital charges[17] and remain outside the scope of this investigation. Operational risk charges — where applicable — are referred to explicitly.

Many banks have reported significant regulatory capital relief due largely to more accurate capital allocation in, for example, large mortgage portfolios. This is due to the significant reduction in risk weights applied to loans in this asset class by the shift from Basel I (50 per cent) to Basel II (variable). Although risk weights vary (Equation 1), for a loan portfolio comprising high-quality mortgage loans (ie PDs not exceeding 1 per cent), it is not unusual to observe significant capital charge reductions through the use of Basel II.

Despite this — and other — capital charge reductions, most banks in the transition from Basel I to Basel II have remained approximately 'capital neutral' with capital benefits (derived from more accurate capital allocation for credit risk) being reabsorbed by capital charges for operational risk. Many banks continue to emphasise that the reduction of capital charges by ever more refined methods will become a principal focus in the future.[18–20]

In the light of these efforts, this study was instituted to investigate the possibility of minimising capital charges by optimising the allocation of risk grades. More obvious methods of capital charge reduction involve improving the loan portfolio quality (eg lower PDs and LGDs). As such considerations are the chief business of banks, they remain outside the orbit of scrutiny and influence of academic studies and will thus not be considered here. Optimising the capital charge an institution faces involves *minimising* these charges within both the Basel II framework and actual loan parameters unique to each bank. It should be noted, however, that

Table 1: Loan input parameters under Basel II's advanced internal ratings based approach

	Loan type	LGD (%)	Correlation (%)	
Retail				
	Mortgage	25	Fixed	15
	Qualifying revolving	85	Fixed	4
	Other retail	85	Varies with PD	$0.03 \cdot \left(\frac{1-e^{-35\,PD}}{1-e^{-35}}\right) + 0.16 \cdot \left[1 - \left(\frac{1-e^{-35\,PD}}{1-e^{-35}}\right)\right]$
Corporate, sovereign, banks		45	Varies with PD	$0.12 \cdot \left(\frac{1-e^{-50\,PD}}{1-e^{-50}}\right) + 0.24 \cdot \left[1 - \left(\frac{1-e^{-50\,PD}}{1-e^{-50}}\right)\right]$

LGD, loss given default; PD, probability of default 'optimisation' in this context refers to capital charge optimisation through PD grade allocation and not to capital charge optimisation through 'improved risk management'. The BCBS endeavours to encourage banks to embrace the latter aim, not the former.

The fifth and final Quantitative Impact Study was instituted and reported by the BCBS in 2005.[5] The principal focus of the study was to ascertain the impact of Basel II on banks before actual implementation of the FIRB approach in 2007 and the AIRB approach in 2008. The results of the study indicated — *inter alia* — that all six categories of banks held retail portfolios with significantly higher average PDs than other loan types. Notably, as shown in Figure 3, the average retail loan portfolio PD of 'other non-G10' countries exceeds 10 per cent.

DATA ANALYSIS

Loan parameters and PD buckets

The BCBS requires banks using the IRB approach to develop a rating system that estimates the PD of each obligor based on data such as historic and projected financial performance, industry risk and non-financial contingencies such as labour problems. The risk rating system

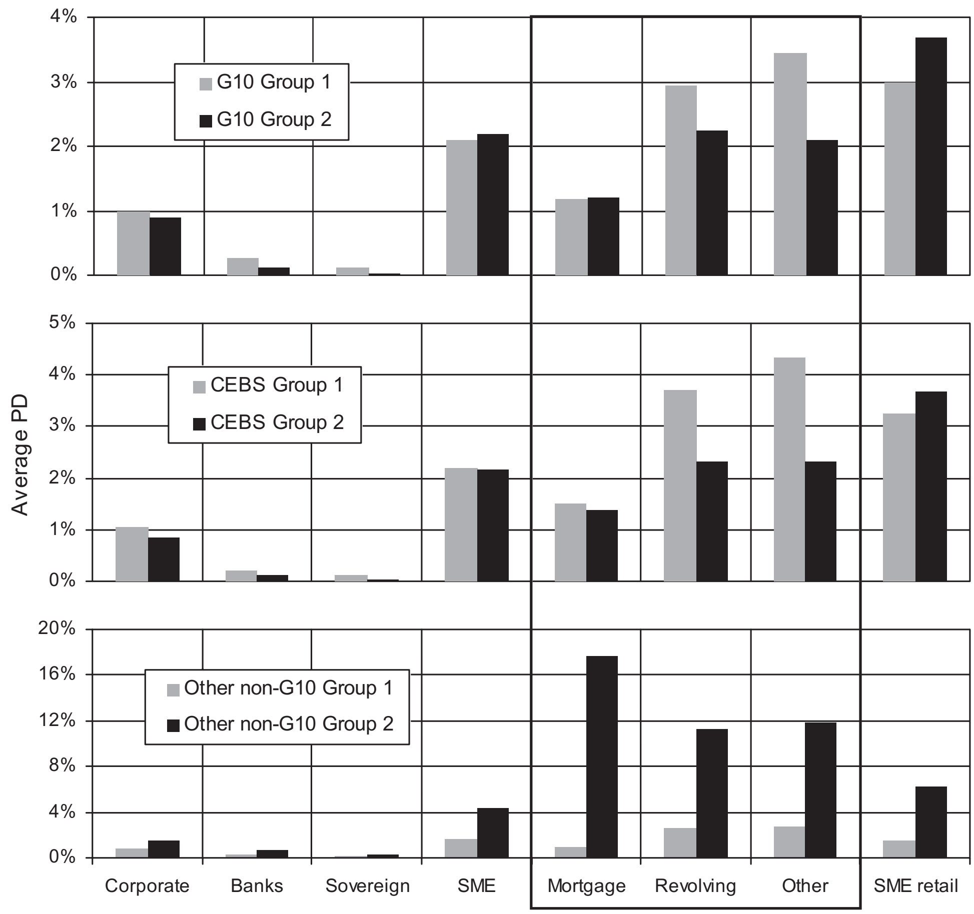

Figure 3: Average PD results from the BCBS Fifth Quantitative Impact Study

must have a sufficient number of PD grades (usually taken to mean at least eight, including default[21]) to allow for a meaningful differentiation of credit risk.[4]

Further discrimination is introduced within the range (ie from best to worst) of obligor PDs. As the PD grades are not (necessarily) of the same width, this added discrimination — known as PD buckets — serves only to facilitate the optimal allocation of PD grades.

As retail loan portfolios suffer the highest PDs (and hence, via Equation 1, the highest capital charges) the goal of optimisation focused on loan asset type. A loan portfolio was constructed with characteristics as described in Table 2.

The PD buckets in Table 2 are constructed such that each embraces an equal fraction of the total loan exposure, in this case one-fifth per bucket, or 20 per cent. This retail loan portfolio structure was chosen as it is fairly typical of non-G10 banks' portfolio composition.[19] These buckets, and their corresponding loan quality, are defined in Table 3.

Optimisation constraints

The optimisation of loan portfolios involves allocating the bank's internally-chosen PD *grades* (this study used PD grades of 8, 12, 16, 20 and 24) into five PD *buckets* in such a way that the capital charge for each portfolio is minimised.

Table 3: PD buckets and corresponding loan quality

PD bucket range	Corresponding loan quality
$0.001\% < PD \leq 0.1\%$	Excellent — highest grade
$0.1\% < PD \leq 1\%$	Very good
$1\% < PD \leq 5\%$	Good to fair
$5\% < PD \leq 10\%$	Fair to poor
$10\% < PD \leq 15\%$	Poor — lowest grade

This process is shown graphically in Figure 4.

This optimisation procedure is subject to the following constraints:

- each PD bucket must contain *at least one* PD grade (recall that for the simulated loan portfolio, PDs range from 0.001 per cent to 15 per cent, see Table 2);
- all PD grades *must* be utilised (ie allocated to one of the PD buckets);
- the width of each PD *bucket* may differ;
- within each PD bucket, the width of each PD *grade* is identical — so, for example, if there were two PD grades in PD Bucket 1 (Figure 4), the width of each PD grade would be 0.05 per cent ([0.1% – 0.001%] / 2); likewise, if there were six PD grades in PD Bucket 2, the width of each would be 0.015 per cent ([(1% – 0.1%)] / 6), and so on;
- portfolio exposure is equally distributed among *buckets*;
- portfolio exposure is equally distributed among *grades* within a bucket.

Table 2: Loan portfolio parameters

Parameter	Value
EAD	1
Minimum PD (%)	0.001
Maximum PD (%)	15
Number of PD regions (buckets)	5
LGD	Dependent on loan type: specified by Foundation IRB approach
Correlation	As there is no Foundation IRB approach for retail (October 2008), these are determined using the Advanced IRB approach

EAD, exposure at default; IRB, internal ratings based; LGD, loss given default; PD, probability of default

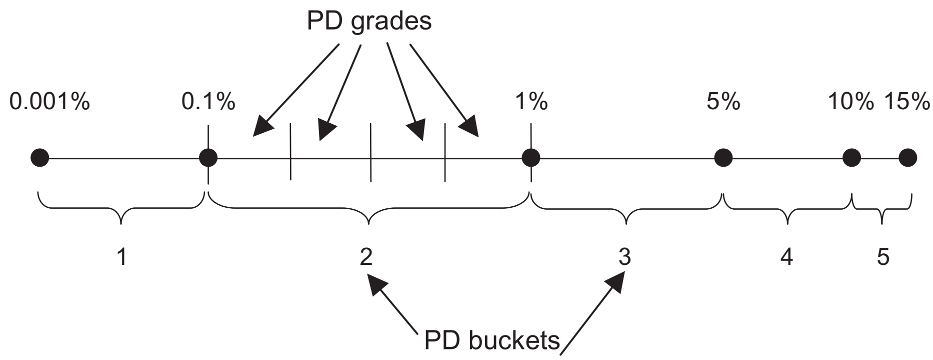

Figure 4: PD grade allocation into PD buckets, shown on a logarithmic scale

Optimisation procedure

A variety of methods exist for solving constrained, nonlinear programming problems. Widely used are the generalised reduced gradient[22] and sequential quadratic programming[23] methods. Both of these methods were employed to optimise PD grade allocation within PD buckets (and determine the corresponding minimum capital charge) subject to the requisite constraints.

The arrangement of PD grades within PD buckets that yielded the lowest and highest capital charge (the latter for comparison purposes only) was then selected for each portfolio listed in Table 2 (ie for the portfolio comprising eight PD grades, 12 PD grades etc, up to 24 PD grades). The results are shown in Figure 5.

For each loan type, it is possible to reduce capital charges by efficient allocation of PD grades. 'Other retail' portfolios benefit the most from this allocation. Figure 6 shows the optimal allocation of PD grades — within the five PD buckets — for each loan type.

Optimisation results

The results for mortgage and qualifying revolving portfolios are quite similar. Minimum capital charges arise from those portfolios with few PD grades within the 0.001–0.1 per cent range, more with PD grades in the 0.1–1 per cent range, more still in the 1–5 per cent and 5–10 per cent ranges, and then a slightly declining number of PD grades in the 10–15 per cent range. This trend holds for all numbers of PD grades.

The allocation of PD grades to the five buckets in 'other retail' portfolios, however, shows significant differences. While the trend remains broadly the same for PDs below 5 per cent, for PDs above 5 per cent — and in all cases — the minimum capital charge portfolio has only a single PD grade allocated to both the $5\% < PD \leq 10\%$ and $10\% < PD \leq 15\%$ buckets. These results seem to indicate that discrimination for PDs in the 5–15 per cent range in 'other retail' portfolios is detrimental to the goal of capital charge minimisation. All loans with PDs in this range should be grouped into a single grade if the minimum capital charge is sought. It is not immediately obvious why this should be so.

Local convexity in 'other retail' capital charges

The relationship between capital charges and PD for mortgage and qualifying revolving portfolios is convex (using

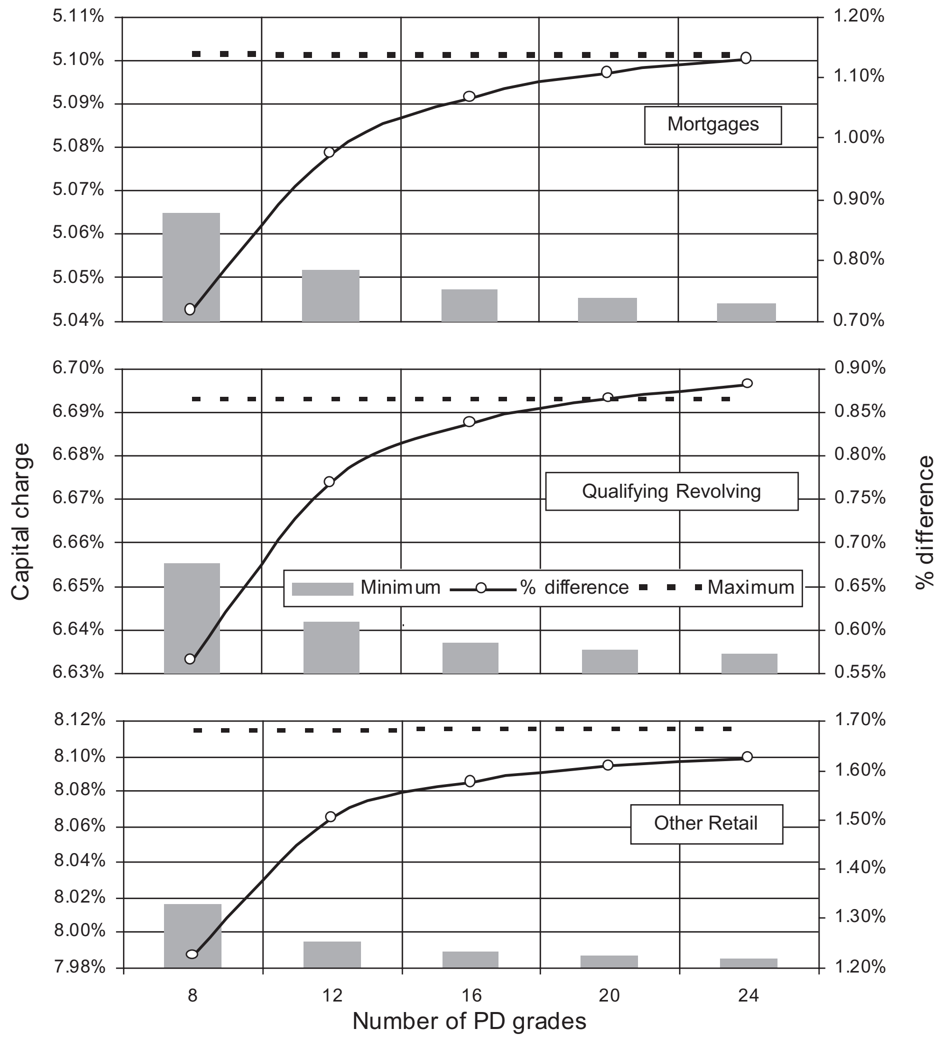

Figure 5: Minimum/maximum capital charges and percentage differences for retail exposures

Equation 1 and the relevant parameters for these portfolios): capital charges increase steeply for low PDs with an ever-decreasing gradient which reaches 0 in the $30\% \leq \text{PD} \leq 40\%$ range and then becomes negative for PDs greater than this range (Figure 7). While on a broad scale this relationship also holds true for 'other retail' portfolios, closer examination reveals a region of local negative convexity in the $5\% < \text{PD} < 15\%$ range, as shown in Figure 8(a). A similar effect has been noted in the market,[24] but the problem was approached from a stressed PD point of view with the Gini coefficient employed to determine the effect of discrimination on minimum capital requirements. This range was determined by differentiating Equation 1 (using the relevant asset correlation equation from Table 1). The results indicate that the capital charge curve is concave over (0%, 4.903%), convex over (4.903%, 15.184%) and concave over (15.184%, 100%).[25]

The explanations of this local negative convexity must be embedded within the mathematics of Equation 1. In fact, it

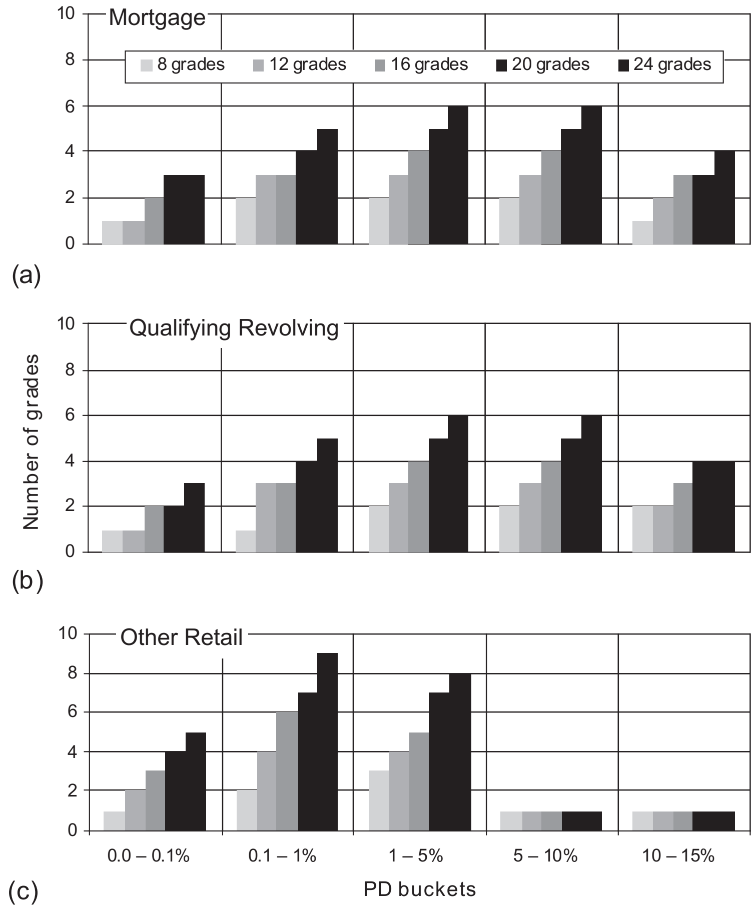

Figure 6: Retail loan portfolio capital charge optimisation

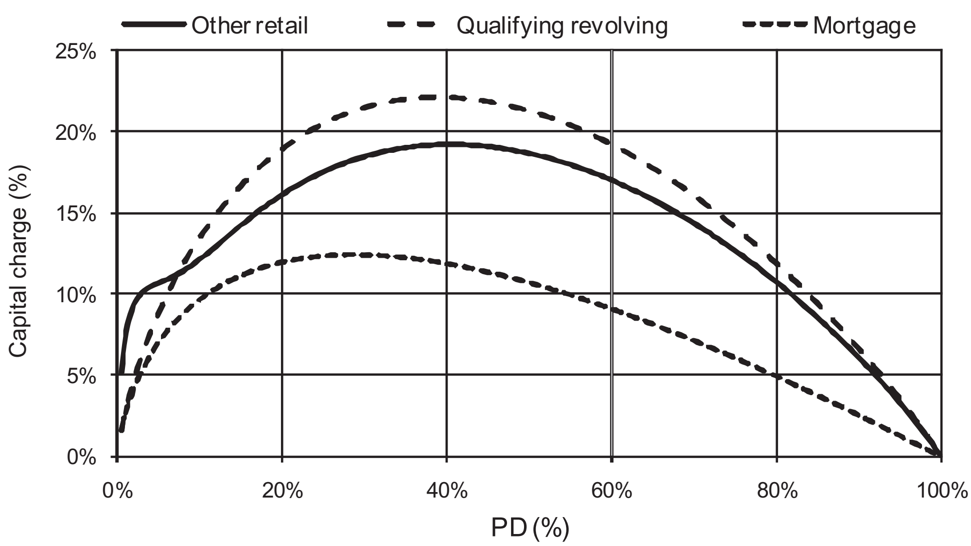

Figure 7: Variation of retail capital charge versus PD for retail exposures

must lie within the formulation of the asset correlation as this is the only real difference between the treatment of mortgage + qualifying revolving portfolios and that of 'other retail' portfolios. The obvious difference between these loan types is the fixed versus variable asset correlation formulation. This observation provides a necessary — but insufficient — explanation for the negative convexity. It is insufficient because corporate loans also employ variable asset correlation and yet do not exhibit this local negative convexity. Note from Table 1, however, that corporate loan asset correlations are bounded by 0.12 and 0.24 and the exponential coefficient is 50 while 'other retail' loan asset correlations are bounded by 0.03 and 0.16 with exponential coefficient 35. Figure 8(b) shows the shape of the capital charge versus PD curve using a fixed asset correlation range between 0.03 and 0.16 with exponential coefficients of 35 and 50 (ie the asset correlation curve for 'other retail' portfolios using both exponential coefficients). Figure 8(c) shows the capital charge versus PD curve using a fixed asset correlation range between 0.12 and 0.24 with exponential coefficients of 35 and 50 (ie the asset correlation curve for *corporate* portfolios using both exponential coefficients). No negative convexity is observed in Figure 8(c). The parameter responsible for the local negative convexity, then, must be the asset correlation. Figure 8(d) shows

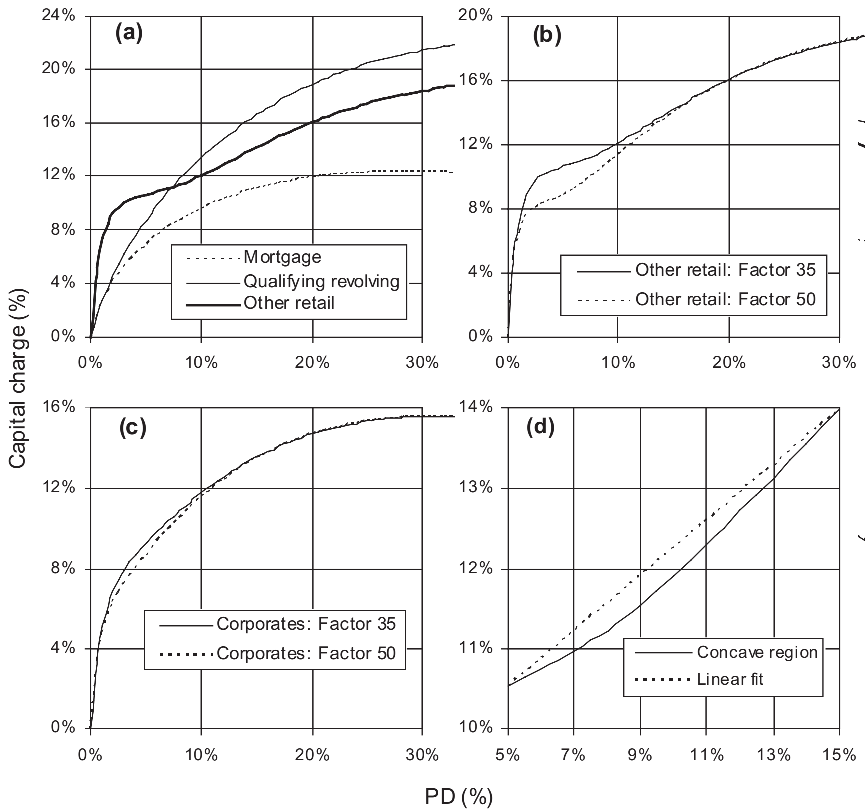

Figure 8: (a) Retail portfolio capital charges; (b) effect of changes in exponent coefficient on capital charge for 'other retail' loan capital charges; (c) effect of changes in exponent coefficient on capital charge for corporate loan capital charges; and (d) detail of the negative convexity capital charge region for 'other retail' portfolios

an expanded portion of the capital charge versus PD curve from the 4% $<$ PD $\leq$ 15% range, clearly showing the negative convexity. As this region of negative convexity is the sole cause of the difference between optimal allocation of PD grades for mortgage, qualifying revolving and 'other retail' portfolios, an explanation must be sought as to why these different-shaped capital charge curves yield such disparate PD grade allocations for capital charge minimisation.

Worked examples

Consider two mortgage loans each with an EAD of 0.5, but one with a PD of 1 per cent and the other with PD of 3 per cent. If the financial institution which issued these loans did not discriminate between these PDs, but rather grouped them into the same PD grade (say between 0.001 per cent and 4 per cent) then these loans would both be assigned a PD of 2 per cent — the average PD of the grade into which they fall (ie [4% + 0.001%] / 2 = 2%). Using Equation 1, the capital charge for these loans is 8.763 per cent (2 × 0.5 × 8.763%), where the factor of 2 arises from the two loans and the 0.5 is the EAD for each loan. The value of 8.763 per cent is calculated using Equation 1.

Now consider the case in which a bank had employed a more discriminating rating system, ie one which allocated, say, *two* grades in the 0.001% $<$ PD $\leq$ 4% range. Assume also that these grades were of equal width — ie 2 per cent in each case: one grade from 0.001% $<$ PD $\leq$ 2% and another from 2% $<$ PD $\leq$ 4%. The average PDs for these grades are 1 per cent and 3 per cent respectively. In this case, the first loan (with PD = 1 per cent) will fall into the first grade and the second loan (with PD = 3 per cent) into the second. The capital charges for these two loans (each with EAD = 0.5) are 3.458 per cent (0.5 × 6.917%) and 4.744 per cent (0.5 × 9.489%). Due to the additive property of the Basel II capital charge equations, the capital charge for this latter portfolio of loans is 8.203 per cent — or some 6.4 per cent *less* the capital charge obtained from the portfolio of identical loans but with less discrimination between them. This disparity is clearly shown in Figure 9(a): a single PD grade embracing both loans results in a capital charge read from the actual capital charge versus PD curve (thick black line), while a more discriminating PD grade allocation results in the *linear average* of the two individual capital charges. This capital charge is read from the straight line joining the two individual capital charges. Adding more loans to the portfolio and increasing the discrimination between these loans results in lower and lower capital charges. It is clear, in this example, how the BCBS's stated goal of rewarding improved risk management practices is accomplished.

Figure 9(b) shows a region of the capital charge versus PD curve for the 'other retail' loan category. This region, from the 4% $<$ PD $<$ 16% range, was deliberately chosen because it is over this range that the capital charge curve exhibits negative convexity.

Consider again two loans, one with EAD = 0.5 and PD = 7 per cent and another with EAD = 0.5 and PD = 13 per cent. Again assume that both loans are grouped — in the first case — into a single PD grade spanning say 4% $<$ PD $\leq$ 16%. This PD range is chosen for ease

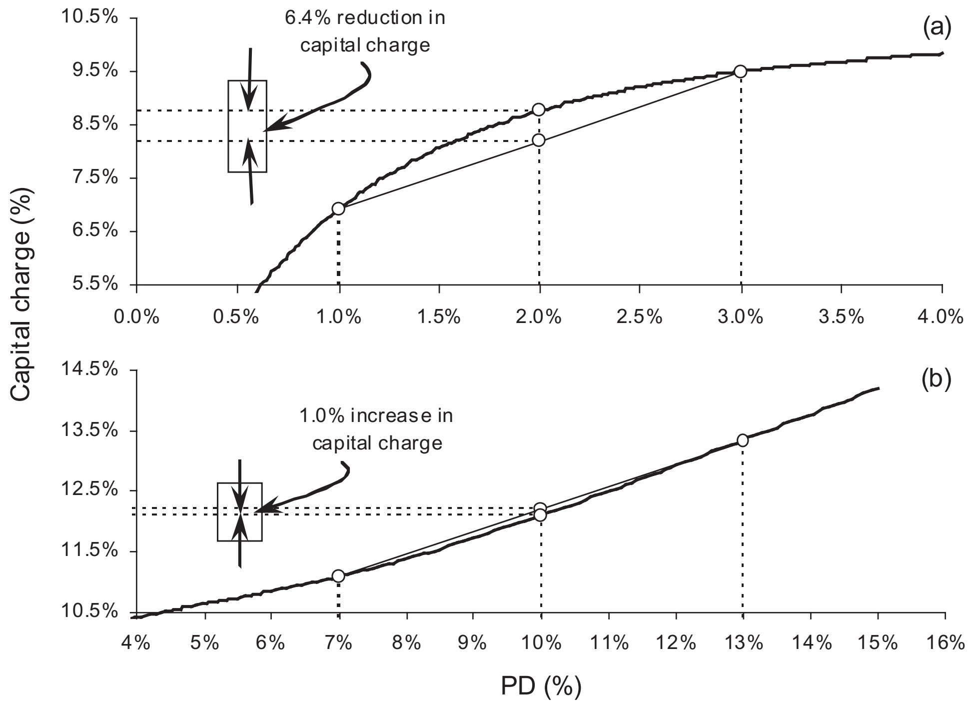

Figure 9: PD discrimination effect on 'other retail' portfolio for (a) PD $\leq$ 4% and (b) 5% $\leq$ PD $\leq$ 16%

of calculation. Although not specifically encompassing the full PD range over which negative convexity occurs, this range falls well within it. The average PD of this grade is 10 per cent ([4% + 16%] / 2). Using Equation 1, the capital charge for these two loans is 12.100 per cent ($2 \times 0.5 \times 12.100$), where, as before, the factor of 2 arises from the fact that there are two loans, the factor of 0.5 is the EAD for each loan and the value of 12.100 per cent is derived from Equation 1.

Assume now that the bank adopts a more discriminatory PD grade allocation: in this case assume the bank splits the PD region into two grades spanning 4% < PD $\leq$ 10% and 10% < PD $\leq$ 16%. In this case, the first loan will fall into the first bucket (of average PD 7 per cent and capital charge 5.547 per cent) and the second loan into the second bucket (of average PD 13 per cent and capital charge 6.674 per cent). The capital charge for this portfolio of loans is 12.212 per cent or 1.0 per cent *more* than the charge paid for a *less* discriminating PD allocation. Adding more loans — and more grades — only exacerbates the problem: capital charges *increase* the more grades that are applied.

Influence on PD discrimination

The outcome of this investigation may now be applied to the results obtained in Figure 6. When the capital charge versus PD curve is convex, greater PD discrimination leads to a *reduction* in capital charges. Thus, no matter how many grades a financial institution employs with which to attribute loans of different quality (and hence PD), PD grades are more or less evenly spaced over the PD range for mortgages and qualifying revolving loans. The convexity lends itself to greater discrimination: the more grades, the lower the capital charge and the allocation of those grades within the PD region is more or less equal.

On the other hand, in the region of the 'other retail' capital charge versus PD curve that exhibits negative convexity, *greater* PD discrimination leads to an *increasing* capital charge. At best, assuming a financial institution actually has loans of low PD quality (ie $5\% < \text{PD} \leq 15\%$), only a single PD grade should be allocated to this region as further discrimination yields comparatively higher capital charges.

CONCLUSION

Basel II proposes novel and improved risk-sensitive methodologies for the allocation of adequate capital requirements to cover credit risk. Derived from single-factor credit models which are known not to provide the perfect framework for capital charge allocation, these methodologies introduce convexity into the otherwise concave capital charge function for 'other retail' portfolios. The stated goal of Basel II is to provide capital incentives for better risk management practices. The results from this study, however, indicate that, in poor-quality loan portfolios (ie precisely the type of portfolio most in need of stringent PD discrimination), greater loan quality discrimination leads to *higher* capital charges — thereby illuminating a potential weakness of the Basel II calculations for credit capital charges under the IRB approach. It has been shown that, even for simple examples of 'other retail' loan portfolios presented in this study, capital charge reductions are easily affected with non-complex segmentation of the PD grades.

This anomaly is unsatisfactory from a regulatory point of view. Any incentives to allocate time and resources to better PD discrimination are effectively reduced as capital requirements can be reduced simply by decomposing the relevant portfolio into segments (the most favourable segmentation determined via optimisation). In addition, there are incentives to segment portfolios purely for capital allocation purposes rather than improved risk management. Future studies will concentrate on real, large, heterogeneous retail loan portfolios to ascertain the degree of regulatory arbitrage that can be attained through optimised PD segmentation.

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