Is China's bond market really inefficient?

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Abstract China controls its exchange and interest rates, and does not offer RMB interest rate derivatives in China. As China is an emerging market, people expect a certain level of inefficiencies in its bond market. This can be illustrated in the way that some bonds are transacted at prices that appear to bear little relation to their fair values. However, closer inspection reveals that this is not due to market inefficiency *per se*, but rather the fact that bond traders in China may buy and sell bonds in bundled form. As an example, a trader may seek to hide his losses on one bond by selling it at an inflated price at the same time as buying another bond, also at an inflated price; thus, while the bonds may, individually, deviate from their fair price, bundled together, the aggregate price is fair. This is only possible for over-the-counter bond transactions, due to the opaque nature of the trade process; all bonds traded on the exchanges are transacted at their fair values.

Keywords: *market efficiency, yield curve, cubic spline function, China's bond market*

INTRODUCTION

As the largest emerging market, China attracts global investors. Foreign investors have been allowed to invest directly into the Chinese domestic securities market since July 2003 through the Qualified Foreign Institutional Investors Scheme. Many domestic and foreign institutions are especially interested in China's bond market because the insurance and banking sectors are growing exponentially and need to buy and sell large numbers of bonds in order to manage their balance sheet.

This paper aims to demonstrate that China's bond market is efficient, despite the fact that China is an emerging market. Proving this requires a function that can provide a good estimate of the Chinese treasury yield curve. To the best of the author's knowledge, this paper is the first that proves that China's bond market is efficient and shows that the cubic spline function[1] without knot points provides a good fit to China's treasury yield curve.

Two popular approaches in term structure modelling are arbitrage-free models and equilibrium models. However, it is futile to forecast the Chinese interest rate as the People's Bank of China controls the RMB interest rate and does not offer RMB interest rate derivatives in China. Nevertheless, the cubic spline function remains a good model to estimate the treasury yield curve. The findings of this study are as follows:

- The cubic spline function without knot points provides a better fit to China's treasury yield curve than that with knot points.
- Including illiquid treasury bonds in the treasury yield curve estimation impairs the fitness of the curve.
- Agency bonds should not be included in the estimation.
- Corporate bonds should not be included in the estimation.
- The Chinese treasury bonds being traded on the stock exchanges are transacted at their fair values while some Chinese treasury bonds being traded over the counter are transacted at prices that deviate considerably from their fair values. This implies that traders may deliberately buy or sell treasury bonds over the counter using prices that are significantly different from their fair values, as it is unreasonable to believe that treasury bonds are priced correctly on the stock exchanges while they are tremendously mispriced over the counter.

This paper is organised as follows. The next section introduces China's bond market. The subsequent section reviews related literature. Data collection and the proposed model are then described. This is followed by the preliminary statistics and a discussion of the results. The penultimate section demonstrates the fitness of the cubic spline function by means of the butterfly strategy. The final section concludes the paper, showing that China's bond market is indeed efficient.

CHINA'S BOND MARKET

There are three bond markets in China: the interbank market, the Shanghai Stock Exchange and the Shenzhen Stock Exchange (the 'Three Markets'). The interbank market is an over-the-counter market. From the turnover perspective, it is also the biggest among the Three Markets, while the Shenzhen Stock Exchange is the smallest.

There are treasury, agency and corporate bonds in China. Agency bonds are issued by the China Export and Import Bank, the China Development Bank and the China Agriculture Development Bank. These bonds may be coupon bonds, coupon-payment-in-arrears bonds, zero-coupon bonds and floating-rate bonds. Coupon-payment-in-arrears bonds are those bonds where the coupon payment is made at the same time as the principal, ie when the bond matures (for example, on a ten-year 2 per cent coupon-payment-in-arrears bond with a face-value of RMB 100, there will only be one payment of RMB 120). The floating rate in China is defined as the one-year retail fixed deposit rate with commercial banks with a spread.

As of 29th March, 2005, the total amount of issued and outstanding treasury bonds was RMB 3.20trn with

124 treasury bonds outstanding; the total issued and outstanding amount of the whole bond market was RMB 4.63trn with 282 bonds outstanding.

LITERATURE REVIEW

Arbitrage-free models and equilibrium models are two popular approaches in term structure modelling. The arbitrage-free tradition focuses on fitting the term structure perfectly at a point in time to ensure that arbitrage opportunities do not exist; this is important in pricing derivatives. The equilibrium tradition focuses on modelling the dynamics of the instantaneous rate, typically using affine models.[2,3] Yields at other maturities can be derived thereafter under various assumptions regarding the risk premium.

The earliest single-factor short-rate models include Rendleman and Bartter (1980).[4] Prominent contributions in the arbitrage-free vein include Hull and White (1990)[5] and Heath *et al.* (1992).[6] The latter authors extended the Ho and Lee (1986) model,[7] which pioneered an arbitrage-free lattice approach for interest rate models, in four directions: (1) using forward rates as basic building blocks; (2) incorporating continuous trading; (3) allowing for multiple factors; and (4) allowing any volatility structure. Hull and White (1990)[5] extended the Vasicek and Fond (1982) model[8] to fit both the current term structure and volatilities of interest rates. In their model, the short rate follows a normal, mean-reverting process with time-dependent parameters. The model is popular in practice because it produces closed-form solutions for bond prices.

Black *et al.* (1990)[9] created the first model to combine the mean-reverting behaviour of the short rate with lognormal distribution. The major contribution of the model is the transparent calibration procedure to the yield and volatility curves. Unfortunately, the trade-off for this is the mutually-dependent mean-reversion and volatility terms. Black and Karasinski (1991)[10] rectified this trade-off by allowing for independent parameters. Sandmann and Sondermann (1993)[11] studied a general arbitrage-free model, dynamically incorporating properties of both normal and lognormal models. Brace *et al.* (1997)[12] and Jamshidian (1997)[13] developed a unifying framework based on Heath *et al.*[6] for forward Libor rates, known as the Market Model.

Prominent contributions in the affine equilibrium tradition include Vasicek (1977),[14] Cox *et al.* (1985),[15] and Duffie and Kan (1996).[16] Unfortunately, the aforementioned models are not applicable in China, as China controls the RMB interest rate and there are no RMB interest rate derivatives in China.

DATA AND MODEL DESCRIPTIONS

Although it is not possible to use arbitrage-free models or equilibrium models, as the RMB interest rate is manipulated by the People's Bank of China and there are no interest rate derivatives in China yet, it is still possible to estimate the Chinese treasury yield curve. The study examines all bond data in the Three Markets from 30th September, 2000 to 30th March, 2005 (Table 1), and uses the following cubic spline function (please refer to Appendix B for how to construct the cubic spline curve):

$$P(t) = 1 + at + bt^2 + ct^3 + a_1 \max[(t-t_1)^3, 0] + a_2 \max[(t-t_2)^3, 0] + \ldots + a_n \max[(t-t_n)^3, 0] \qquad (1)$$

Table 1: The number of outstanding issues of each type of coupon-payment-in-arrears and zero-coupon bond from 30th September, 2000 to 30th March, 2005

Bond types	Outstanding issue numbers
Treasury bonds — coupon-payment-in-arrears	2
Agency bonds — coupon-payment-in-arrears	17
Corporate bonds — coupon-payment-in-arrears	10
Treasury zero-coupon bonds	13
Agency zero-coupon bonds	25
Corporate zero-coupon bonds	0

where $P(t)$ is the discount function for time t. When all knot points are equal to zero, the equation above will reduce to:

$$P(t) = 1 + at + bt^2 + ct^3 \tag{2}$$

To estimate the treasury yield curve, the next stage is to minimise the root mean square errors ($RMSE_P$) on clean closing prices (ie prices without the accrued interest) using the following equation:

$$RMSE \equiv \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(P_i - \hat{P}_i\right)^2} \tag{3}$$

where N is the number of bond closing prices being considered, P_i is the daily clean closing price on the ith closing price, and $\hat{P}_i$ is the predicted clean closing price of the ith closing price using the cubic spline function (a clean price is a bond price that excludes the accrued interest; a dirty price includes the accrued interest).

A nonlinear optimal search algorithm is used to determine a set of coefficients, a, b and c, such that the RMSE is minimised; in other words, the search is for the set of discount functions that fits the observed prices best. The same weight is assigned to each bond in the fitting process. This optimal set of coefficients determines the set of theoretical discount functions.

It is possible to define RMSE in terms of the yield-to-maturity rather than in terms of the price error. The problem with minimising price errors is that long-term bond prices have greater sensitivity to yield errors than do short-term bond prices. As a result, weighting price errors equally actually influences the yield error of short-term bonds much less than that of long-term bonds. Therefore, the following $RMSE_{ye}$ should be used instead:

$$RMSE_{ye} \equiv \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(y_i - \hat{y}_i\right)^2} \tag{4}$$

where y_i is the yield-to-maturity on the ith closing price and $\hat{y}_i$ is the estimated yield-to-maturity on the ith closing price. In the case of Chinese treasury bonds, however, calculating RMSE using prices tends to obtain a better result. The $RMSE_P$ is therefore used.

The cubic spline function is estimated using only liquid treasury bonds (eg coupon, zero-coupon and coupon-payment-in-arrears[17] treasury bonds) and repurchase agreements (repos) on all liquid and illiquid treasury bonds. A bond with a turnover greater than RMB 50m in one day is defined as a liquid bond on that day. Excluded are floating-rate treasury bonds being traded on the Shanghai Stock Exchange, on the Shenzhen Stock Exchange and in the interbank market.

Floating-rate treasury bonds are excluded from estimates of the treasury yield curve because the floating rate in China is the one-year retail fixed deposit rate with commercial banks with a spread. The People's Bank of China controls domestic interest rates and, hence, this one-year short rate rarely changes. The repo market, however, is volatile and data from the Three Markets are included (although the Shenzhen Stock Exchange repo turnover is very small). The repo market in China only trades treasury bonds with a repo term not longer than nine months. Also included in estimating the treasury yield curve are over-the-counter liquid treasury bonds.

STATISTICS AND FINDINGS

The base case is constructed as follows: all knot points are set to zero. The set of bond prices consists of liquid treasury bonds, except those with a floating rate, as described previously. Table 2 shows the statistical results of the base case. The total number of clean treasury closing prices used in the estimation is 18,887. The deviation is defined as follows:

$$\text{Deviation} = \frac{(\text{Predicted clean price} - \text{Clean price})}{\text{Clean price}} \tag{5}$$

The base case results show that the cubic spline function without knot points provides a good estimate of the chosen set of bond prices. The mean deviation is –0.21 per cent. The median deviation is 0.05 per cent. Within the 80 per cent confidence interval, the deviation ranges from –1.82 per cent to 1.40 per cent. Within the 97 per cent confidence interval, the deviation ranges from –6.79 per cent to 3.16 per cent. Within the 99 per cent confidence interval, the deviation ranges from –10.24 per cent to 5.23 per cent. Figure 1 shows the distribution.

As expected, those bonds traded with large deviations were traded over the counter. This suggests either that the traders mispriced those bonds tremendously, or they intentionally bought or sold those bonds at prices that deviated from their fair values for reasons not apparent in the data set.

To test whether the base case is desirable, nine cases are constructed to compare versus the base case (Table 3). Statistical summaries of each case are presented in Tables 4–12. The results are summarised below.

Case 1: Liquid agency bonds without floating-rate bonds

The mean deviation is 0.42 per cent and the median deviation is 0.14 per cent. The deviations are in general slightly larger in this case than in the base case. This suggests that the market does not deem agency bonds as risk-free. As expected, bonds being traded with large deviations are traded over the counter. This implies that the traders have either mispriced those bonds tremendously, or

Table 2: Base case: Zero knot points; liquid treasury bonds (including coupon, zero-coupon and coupon-payment-in-arrears treasury bonds), excluding those with a floating rate

Total	18,887	Range (99%): −10.24 to 5.23
Minimum deviation (%)	−43.32	Range (97%): −6.79 to 3.16
Maximum deviation (%)	27.30	Range (95%): −5.66 to 2.64
Mean deviation (%)	−0.21	Range (90%): −3.56 to 1.88
Median deviation (%)	0.05	Range (80%): −1.82 to 1.40

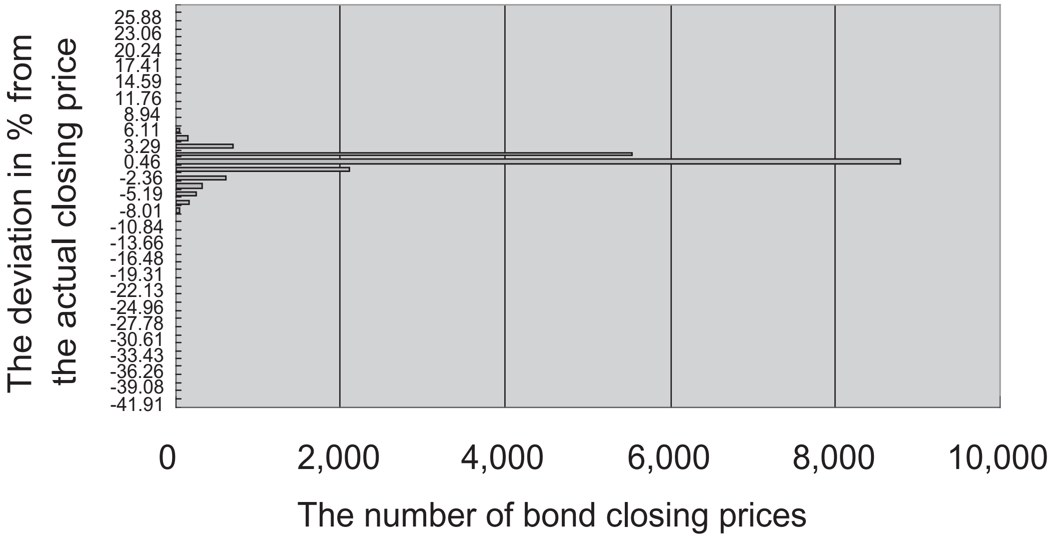

Figure 1: The deviation of predicted prices from the actual prices in the base case

Table 3: Cases used to compare with the base case

Case	Table	Knot points	Bonds being estimated
1	4	0	Liquid agency bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate
2	5	0	Liquid corporate bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate
3	6	0	Illiquid and liquid treasury bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate
4	7	0	Illiquid and liquid agency bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate
5	8	0	Illiquid and liquid corporate bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate
6	9	0	All bonds excluding those with a floating rate
7	10	0	All bonds including those with a floating rate
8	11	{0.7, 1.5, 2.25, 5.18, 6.25 and 25}	Liquid treasury bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate
9	12	{0.7, 1.5, 2.25, 5.18, 6.25 and 25}	All bonds including those with a floating rate

Table 4: Case 1: Zero knot points; liquid agency bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate

Total	5,024	Range (99%):	–21.40 to 13.81
Minimum deviation (%)	–31.61	Range (97%):	–11.68 to 10.15
Maximum deviation (%)	36.69	Range (95%):	–9.38 to 8.16
Mean deviation (%)	0.42	Range (90%):	–6.18 to 6.93
Median deviation (%)	0.14	Range (80%):	–3.35 to 4.76

they intentionally bought or sold those bonds at prices different from their fair prices — even after taking into consideration a possible credit spread.

Case 2: Liquid corporate bonds without floating-rate bonds

The mean deviation is 8.63 per cent and the median deviation is 9.56 per cent.

Table 5: Case 2: Zero knot points; liquid corporate bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate

Total	1,714	Range (99%):	−0.78 to 21.90
Minimum deviation (%)	−10.91	Range (97%):	−0.25 to 19.52
Maximum deviation (%)	25.29	Range (95%):	−0.07 to 18.78
Mean deviation (%)	8.63	Range (90%):	0.12 to 17.44
Median deviation (%)	9.56	Range (80%):	0.50 to 15.90

Table 6: Case 3: Zero knot points; illiquid and liquid treasury bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate

Total	32,717	Range (99%):	−9.78 to 5.02
Minimum deviation (%)	−43.32	Range (97%):	−6.82 to 3.13
Maximum deviation (%)	27.30	Range (95%):	−5.73 to 2.55
Mean deviation (%)	−0.32	Range (90%):	−3.93 to 1.88
Median deviation (%)	−0.04	Range (80%):	−2.28 to 1.39

Table 7: Case 4: Zero knot points; illiquid and liquid agency bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate

Total	6,572	Range (99%):	−21.00% to 13.87%
Minimum deviation (%)	−31.61	Range (97%):	−12.08% to 10.18%
Maximum deviation (%)	36.69	Range (95%):	−9.54% to 8.35%
Mean deviation (%)	0.56	Range (90%):	−6.43% to 7.03%
Median deviation (%)	0.23	Range (80%):	−3.20% to 5.10%

Table 8: Case 5: Zero knot points; illiquid and liquid corporate bonds (including coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate

Total	15,547	Range (99%):	−0.83 to 19.13
Minimum deviation (%)	−10.91	Range (97%):	−0.44 to 16.70
Maximum deviation (%)	35.62	Range (95%):	−0.28 to 15.55
Mean deviation (%)	5.20	Range (90%):	−0.01 to 13.87
Median deviation (%)	3.85	Range (80%):	0.25 to 12.07

Table 9: Case 6: Zero knot points; all bonds excluding those with a floating rate

Total	54,836	Range (99%):	−10.38 to 16.60
Minimum deviation (%)	−43.32	Range (97%):	−6.72 to 13.89
Maximum deviation (%)	36.69	Range (95%):	−5.38 to 12.59
Mean deviation (%)	1.35	Range (90%):	−3.28 to 10.09
Median deviation (%)	0.44	Range (80%):	−1.68 to 6.99

Table 10: Case 7: Zero knot points; all bonds including those with a floating rate

Total	63,211	Range (99%):	−9.92 to 16.33
Minimum deviation (%)	−43.32	Range (97%):	−6.36 to 13.66
Maximum deviation (%)	36.69	Range (95%):	−5.01 to 12.31
Mean deviation (%)	1.23	Range (90%):	−3.38 to 9.66
Median deviation (%)	0.36	Range (80%):	−1.86 to 6.55

Table 11: Case 8: Knot points = {0.7, 1.5, 2.25, 5.18, 6.25 and 25}; liquid treasury bonds (coupon, zero-coupon and coupon-payment-in-arrears), excluding those with a floating rate

Total	18,887	Range (99%):	−13.45 to 20.63
Minimum deviation (%)	−81.08	Range (97%):	−6.32 to 15.74
Maximum deviation (%)	165.26	Range (95%):	−4.99 to 12.26
Mean deviation (%)	0.41	Range (90%):	−2.85 to 4.19
Median deviation (%)	0.04	Range (80%):	−1.22 to 1.78

Table 12: Case 9: Knot points = {0.7, 1.5, 2.25, 5.18, 6.25 and 25}; all bonds including those with a floating rate

Total	63,211	Range (99%):	−24.72 to 22.81
Minimum deviation (%)	−118.43	Range (97%):	−9.79 to 17.65
Maximum deviation (%)	183.19	Range (95%):	−6.41 to 15.26
Mean deviation (%)	1.47	Range (90%):	−3.75 to 12.22
Median deviation (%)	0.35	Range (80%):	−1.83 to 7.86

The deviations are substantially larger in this case than in the base case. This suggests that the market deems corporate bonds to have substantial credit risks that differ from one corporate bond to the next. However, there are only 1,714 data points in this case; this means that few of the corporate bonds being traded in this period are liquid. As a result, the deviations should be due to both the credit risk and the liquidity risk, in addition to the mispricing factor.

As expected, those bonds being traded with large deviations were traded over the counter. Such deviations may be due to the mispricing factor, the credit spread, the liquidity premium (ie when a trader sells a corporate bond in an illiquid market and they are obliged to sell it at a cheaper price), or all of the above. Alternatively, traders intentionally bought or sold those bonds at prices that were different from their fair prices for reasons not discernible from the data set.

Case 3: Illiquid and liquid treasury bonds without floating-rate bonds

The mean deviation is –0.32 per cent and the median deviation is –0.04 per cent. The deviations are slightly larger in this case than in the base case. Such larger deviations are likely to be due to the illiquidity issue, as the only difference between this case and the base case is the liquidity.

Case 4: Illiquid and liquid agency bonds without floating-rate bonds

The deviations are in general slightly larger in this case than in case 1. The larger deviations are likely to be due to the illiquidity issue.

Case 5: Illiquid and liquid corporate bonds without floating-rate bonds

The deviations are in general better in this case than in case 2. However, this should not come as too great a surprise given that there is a significantly larger volume of data here compared with case 2.

The following is a summary of the results when Equation (1) is applied to the set of bond prices. The knot points are {T1, T2, T3, T4, T5, T6} = {0.7, 1.5, 2.25, 5.18, 6.25 and 25}; T1 means the first knot point is knotted at 0.7 years; T2 means the second knot point is knotted at 1.5 years, and so on..

Case 8: Liquid treasury bonds without floating-rate bonds

Surprisingly, the deviations are generally worse in this case than in the base case. This is likely to be due to the fact that there are very few long bonds in China — as shown in Tables 13 and 14 and Figures 2 and 3. There are only 11 treasury bonds longer than ten years, as shown in Table 14, and thus the error is increased.

Case 9: All bonds including floating-rate bonds

The deviations in this case are in general worse than in case 7. As with case 8, this case shows that using the cubic spline function without knot points is more suitable in China.

USING THE BUTTERFLY STRATEGY TO TEST THE ROBUSTNESS OF THE CUBIC SPLINE FUNCTION

The butterfly strategy is used to test if the cubic spline function works reliably. A butterfly strategy is a three-security trade. In this instance, it involves going long on a cheap (ie underpriced) treasury bond (time to maturity of five years — the body) at the same time as shorting half a unit of a rich (ie overpriced) treasury bond (time to maturity of 4–5 years — the wing) and shorting another half unit of a rich treasury bond (with time to maturity of 5–6 years — the wing). The next stage is to wait for them to converge to their fair prices. If they do, the cubic spline function represents a good method of estimating the fair value of treasury bonds.

Table 13: Bond distribution across terms and credit ratings, 9th February, 2004

Years	0–1	>1–5	>5–10	>10–15	>15–20	>20–25	>25–30	>30	Total
Treasury	60	1,088	1,619	82	116		296		3,260
Financial	60	228	684	5	40		5		1,022
Corp-AAA		13	51	28	14		3		108
Corp-AA+		4							4
Corp-AA		8							8
Corp-Other		0							0
Total	120	1,340	2,354	114	170	—	304	—	4,402

Source: www.wind.com.cn. Units: RMB billion.

Table 14: Number of bonds, 29th March, 2005

Remaining life/ bond type	Treasury bonds	Corporate bonds	Agency bonds	Subtotal
$\leq$ 5 years	110	19	61	190
>5 & $\leq$ 7 years	32	6	20	58
>7 & $\leq$ 10 years	10	28	30	68
>10 & $\leq$ 15 years	4	9	5	18
>15 years	7	4	6	17

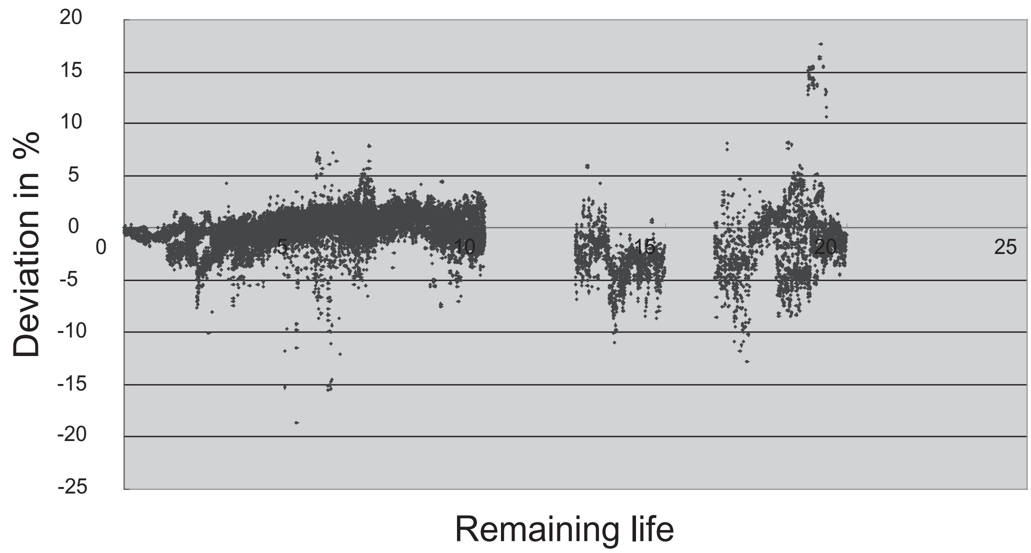

Figure 2: The distribution of deviations versus the remaining life of bonds for 30th September, 2000 to 30th March, 2005

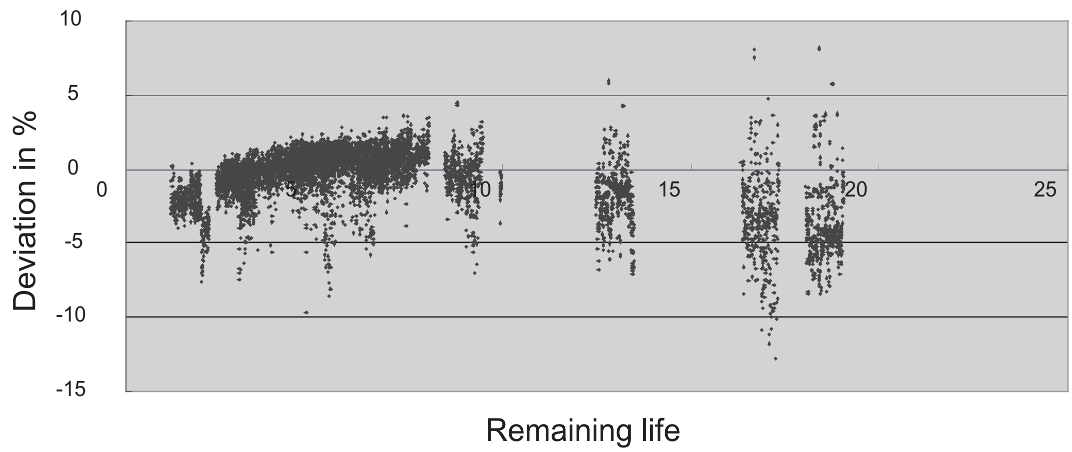

Figure 3: The distribution of deviations versus the remaining life of bonds for 30th March, 2004 to 30th March, 2005

Butterfly opportunities are only attempted on stock exchanges. Delta is defined as follows:

$$\delta = \frac{\text{Predicted clean price} - \text{Clean closing price}}{\text{Clean closing price}} \quad (6)$$

To identify a butterfly opportunity, the delta of the body is set to be larger than 3 per cent and the delta of each wing to be larger than 2 per cent. The remaining life of the corresponding wings must be within ± 1 year of the remaining life of the body.

As shown in Appendix A, six butterfly opportunities can be identified: 28th December, 2001, 15th January, 2002, 30th March, 2004, 20th April, 2004, 29th April, 2004 and 12th May,

2004. For instance, if the first butterfly positions are closed on 18th January, 2002, this will make RMB 0.17 on half a unit of the treasury 995, RMB 6.73 on a unit of treasury 973, and RMB 0.06 on half a unit of the treasury 903. The total profit on this butterfly trade will be RMB 6.96. The total profit of each of the other five butterfly trades — even after the typical transaction costs of a few basis points — is also impressive: RMB 6.64, 5.58, 5.89, 4.38 and 6.58, respectively. However, the corresponding turnovers are not sufficiently large for profitable arbitrages.

The above profitable butterfly opportunities without significant turnovers on the stock exchanges show that the cubic spline function without knot points is a good function to estimate the fair value of treasury bonds in China, as arbitrage opportunities are almost nonexistent. For instance, Appendix A shows that in the third butterfly opportunity, on 30th March, 2004, although the wings present a decent turnover — RMB 662.45m and RMB 56.25m, respectively — the body shows only a turnover of RMB 70,000. Although traders may find other wings, the above results are sufficient to demonstrate the efficiency of the treasury bond market on the stock exchanges.

CONCLUSION

The findings in this paper, albeit slightly surprising, are consistent with modern finance theory. First, the cubic spline function (Equation (2)) without knot points provides a better fit to the Chinese treasury yield curve than the cubic spline function with knot points (Equation (3)). This can be explained by the lack of treasury bonds with maturities of more than ten years.

Secondly, including illiquid treasury bonds into the treasury yield curve estimation impairs the quality of the fit of the curve. Consistent with modern finance theory, the fair price of an illiquid bond includes a liquidity premium.

Thirdly, agency bonds should not be included in the estimation. Consistent with finance theory, the market does not deem agency bonds as risk-free.

Fourthly, corporate bonds should not be included in the estimation. Again, consistent with finance theory, corporate bonds have credit risks.

Fifthly, the Chinese treasury bonds traded on the stock exchanges are correctly priced while some traded over the counter deviate substantially from their fair values. This implies that traders have specific reasons for deliberately buying or selling treasury bonds using prices that deviate significantly from their fair values, as it is unrealistic to believe that treasury bonds are priced correctly on the stock exchanges while being tremendously mispriced over the counter. The misperception of inefficiency is due to the fact that bond traders in China may buy and sell bonds as a bundle.

The results in this paper are encouraging and suggest that, by using the parsimonious cubic spline function without knot points, traders can identify overpriced and underpriced Chinese bonds, while risk managers will have an objective method to price illiquid Chinese treasury bonds.

NOTE

The conclusions herein do not represent the views of Goldman Sachs (Asia) LLC.

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17 Confusingly, two types of bond are known as zero-coupon treasury bonds in China. The first type is the genuine zero-coupon treasury bond; ie a treasury bond issued at a discount without coupons. The second type is described in the present paper as the coupon-payment-in-arrears treasury bond; ie a treasury bond issued at face value which also has a coupon rate, where the coupon payments are paid along with the principal when the bond matures. For instance, a ten-year 2 per cent coupon-payment-in-arrears treasury bond will have only one payment — RMB 120 — on the maturity day (such payment should therefore be discounted).

APPENDIX A: BUTTERFLY STRATEGY RESULTS

Market	Price (clean)	Value (clean)	v - p	(v - p) over p	Date	Turnover (in 10,000)	Bond name		Price (clean)	Value (clean)	v - p	(v - p) over p	Date	Turnover (in 10,000)	Profit or loss	Net profit
SZ	101.01	103.06	2.05	0.02	28/12/2001	253	Treasury995		101.35	103.39	2.04	0.02	18/01/2002	109	0.17	
SZ	146.92	137.01	-9.91	-0.07	28/12/2001	755	Treasury973		140.19	137.18	-3.00	-0.02	18/01/2002	11	6.73	
SZ	100.77	103.10	2.33	0.02	28/12/2001	44	Treasury903		100.89	103.25	2.36	0.02	18/01/2002	25	0.06	6.96
SZ	101.16	103.34	2.18	0.02	15/1/2002	10	Treasury995		101.60	102.41	0.81	0.01	22/01/2002	230	0.22	
SZ	143.51	136.74	-6.76	-0.05	15/1/2002	159	Treasury973		137.25	135.81	-1.44	-0.01	22/01/2002	250	6.26	
SZ	100.61	103.30	2.69	0.03	15/1/2002	21	Treasury903		100.94	102.49	1.56	0.02	22/01/2002	134	0.16	6.64
SSE	92.66	94.80	2.14	0.02	30/3/2004	57,100	02Treasury(10)									
SZ	92.75	94.80	2.05	0.02	30/3/2004	199	Treasury0210									
SSE	97.03	99.16	2.13	0.02	30/3/2004	66,245	99Treasury(8)		96.96	98.73	1.77	0.02	06/04/2004	81,668	-0.03	
SZ	100.72	97.16	-3.56	-0.04	30/3/2004	7	Treasury0215		95.02	96.78	1.76	0.02	06/04/2004	2	5.70	
SSE	93.08	95.00	1.92	0.02	30/3/2004	5,625	03Treasury(7)		92.90	94.76	1.86	0.02	06/04/2004	548	-0.09	5.58
SSE	88.90	91.08	2.18	0.02	20/4/2004	108,922	03Treasury(1)									
SSE	87.87	90.16	2.29	0.03	20/4/2004	22,460	03Treasury(7)		90.01	90.33	0.32	0.00	23/06/2004	34,697	1.07	
SZ	97.53	94.42	-3.11	-0.03	20/4/2004	10	Treasury0311		93.42	94.46	1.04	0.01	23/06/2004	21	4.11	
SSE	88.01	89.91	1.90	0.02	20/4/2004	4,087	21Treasury(10)		89.43	89.98	0.55	0.01	23/06/2004	76,009	0.71	5.89
SSE	87.96	89.75	1.79	0.02	29/4/2004	454,781	03Treasury(1)		89.71	90.11	0.40	0.00	20/05/2004	341,699	0.88	
SZ	97.53	92.59	-4.94	-0.05	29/4/2004	10	Treasury0311		97.48	92.98	-4.50	-0.05	20/05/2004	79	0.05	
SZ	85.33	87.95	2.62	0.03	29/4/2004	28	Treasury912		92.25	88.44	-3.81	-0.04	20/05/2004	14	3.46	4.38
SZ	88.45	90.48	2.03	0.02	12/5/2004	4	Treasury0301		90.51	91.26	0.75	0.01	23/06/2004	5	1.03	
SZ	97.53	93.37	-4.16	-0.04	12/5/2004	8	Treasury0311		93.42	94.46	1.04	0.01	23/06/2004	21	4.11	
SSE	86.56	88.38	1.82	0.02	12/5/2004	4,474	21Treasury(10)		89.43	89.98	0.55	0.01	23/06/2004	76,009	1.44	6.58
SZ	85.25	88.79	3.54	0.04	12/5/2004	5	Treasury912									
SSE	86.95	88.79	1.84	0.02	12/5/2004	21,789	21Treasury(12)									

SZ, Shenzhen Stock Exchange; SSE, Shanghai Stock Exchange; V, predicted clean bond price; P, clean closing bond price. The body is assumed to be bought (sold) with one unit. Both wings are assumed to be sold (bought) with half a unit each.

APPENDIX B: HOW TO CONSTRUCT THE CUBIC SPLINE CURVE

Yield curve estimation:

$$\chi^2(\mathrm{a}) = \sum_{i=1}^{N}\left[\frac{y_i - y(x_i;\mathrm{a})}{\sigma_i}\right]$$

where

a are the coefficients of the yield curve function, such that **a** = [a_0, a_1, a_2, . . . , a_{n-1}]

discount-factor(t) = $a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 \max[(t - t_1)^3, 0] + a_{n-1} \max[(t - t_{n-4})^3]$

t_i is a knot point; when $t_i = 0$, the function is a cubic spline function without knot points

y_i is the market price of bond i

$y(x_i; \mathbf{a})$ is the model price of ith bond with coefficients **a**

σ_i is the weight of bond i

σ_i = {20, if remaining life of bond <0.4 year; 1 if remaining life of bond ≥0.4 year

$$\beta_k \equiv \frac{1}{2}\frac{\partial\chi^2}{\partial a_k}\alpha_{kl} \equiv -\frac{1}{2}\frac{\partial\chi^2}{\partial a_k \partial a_l}$$

Defining a new matrix α' as:

$$\alpha'_{jj} \equiv \alpha_{jj}(1+\lambda)$$

$$\alpha'_{jk} \equiv \alpha_{jk} \qquad (j \neq k)$$

$$\sum_{l=1}^{M} \alpha'_{kl}\delta a_l = \beta_k$$

Equation Δ(alpha)

Given an initial guess for the set of fitted parameters **a**

Computer $\chi^2(\mathbf{a})$

Pick a modest value for λ (0.001)

(loop) solve the linear equation Δ(alpha) for $\delta\mathbf{a}$ and evaluate $\chi^2(a + \delta a)$

If χ^2 (**a** + δ**a**) ≥ = χ^2 (**a**), increase λ by a factor of 10 and go back to (loop)

If χ^2 (**a** + δ**a**) < χ^2 (**a**), decrease λ by a factor of 10, update the trial solution

a ← **a** + δ**a** and go back to loop

Pseudo code:

Function cal_modelvalue (xi, &model_price, &dyda)

Where xi is bong i, return model price and the vector of derivatives dyda $= \partial y / \partial a_k$

Function mrqmin()

Performing one iteration of Marquardt's method.

Function mrqcof()

Computing the matrix [α] and vector [β]

```
mrqmin()
{
  Loop for (λ) until fitted
   mrqcof(&α, &β);
  return a
}
mrqcof(&α, &β)
{
  Loop from 1 to N trading bonds
  cal_modelvalue(xi, &model_price,
  &dyad)
  calculate_deltay()
  calculate_sigma()
  build_alpha_matrix()
  build_beta_vector()
}
```

