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Individual Annuity Demand Under Aggregate Mortality Risk

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Abstract

Aggregate mortality risk—the risk that the mortality trend in a population changes in a nondeterministic way—and its implications for corporate decisions has recently been the subject of lively scientific discussion. We show that aggregate mortality risk is also a key determinant for individual annuitization decisions. Aggregate mortality risk appears to be a risk very difficult to transfer for individuals. Whether its existence leads to a higher or lower annuity demand depends on objective factors (e.g., insurers' vulnerability to aggregate mortality changes). Subjective factors (i.e., individuals' preferences) determine only the intensity of the annuity demand reaction to aggregate mortality risk. Our results are of significant importance not only for financial planning approaches of individual annuity buyers but also for strategic decisions in insurance companies and for solvency regulators. Furthermore, consideration of aggregate mortality risk may alleviate, but also intensify, the annuity puzzle.

Introduction

Initiated by Yaari's seminal work, the study of optimal individual annuitization decisions became of increasing interest in the economics literature. Yaari (1965) showed that, in a perfect market setting, expected utility maximizers with no utility of bequest would manage the uncertainty regarding their lifetime by annuitizing their entire wealth. Under more general assumptions on utility functions, this result was recently confirmed by Davidoff, Brown, and Diamond (2005). Empirical studies find, however, that only a small portion of private wealth is used to purchase annuities. Explanations for this "annuity puzzle" are as follows:

- unfair pricing due to adverse selection and transaction costs (Friedman and Warshawsky, 1988; Mitchell et al., 1999);
- constant annuity payouts in combination with borrowing and short-selling constraints may induce a suboptimal consumption profile (Brown, 2001; Gupta and Li, 2007);

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- the existence of bequest motives (Yaari, 1965);
- the crowding-out effect of government pensions (Mitchell et al., 1999; Brown and Poterba, 2000);
- intrafamily risk sharing (Kotlikoff and Spivak, 1981; Post, Gründl, and Schmeiser, 2006);
- default-risk of the annuity providing insurer (Babbel and Merrill, 2006; Lopes and Michaelides, 2007; Huang, Tsai, and Tzeng, 2008);
- substitution of annuities by long-term care insurance (Davidoff, 2008); and
- behavioral biases, like framing (Agnew et al., 2008; Brown et al., 2008).

This article provides a normative analysis of the impact of *aggregate mortality risk*—which is also called stochastic mortality (e.g., Cairns, Blake, and Dowd, 2006) or demographic risk (e.g., Gründl, Post, and Schulze, 2006)—on individual annuity demand. Aggregate mortality risk refers to the fact that population mortality rates may change in a nondeterministic way (see Olivieri, 2001; Cairns, Blake, and Dowd, 2006; Brown and Orszag, 2006). Such unexpected changes in mortality may result from, e.g., medical innovations (leading to lower mortality rates; see, e.g., Hayflick, 2000; Held, 2002; Olshansky, Hayflick, and Carnes, 2002) or increased occurrence of very hot summers—comparable to the 2003 and 2006 summers in Europe—as a consequence of a global climate change (on hot summers and higher mortality rates, see Valleron and Boumendil, 2004; Conti et al., 2005; Deschenes and Moretti, 2007). The importance of aggregate mortality risk for insurance company risk management decisions has been highlighted in recent publications (see, e.g., Lin and Cox, 2005; Cowley and Cummins, 2005; Cairns, Blake, and Dowd, 2006; Gründl, Post, and Schulze, 2006; Brown and Orszag, 2006). We show that aggregate mortality risk is also a key determinant in individual risk management, especially for annuitization decisions.

Our investigation utilizes an intertemporal expected utility framework with borrowing and short-selling constraints and uninsurable government pension income risk where we, as, e.g., in Davidoff, Brown, and Diamond (2005), or Huang, Tsai, and Tzeng (2008) use a two-points-in-time approach. The risk-averse individual must optimize decisions regarding consumption, saving, and how to allocate savings between a risky asset, a risk-free asset, and annuities. In accordance with Olivieri (2001) and Gründl, Post, and Schulze (2006), aggregate mortality risk is modeled by setting alternative possible levels for the probability of surviving at the end of the period. In particular, possible deviations (or shocks) in respect of a best-estimate assumption will be considered.[1]

Within this model environment, we derive the partial equilibrium and show that aggregate mortality risk does not influence optimal decisions if it is stochastically independent of other sources of the individual's income risk. We make clear that this is a direct consequence of the preference assumptions under which optimal decisions are derived. This result is in accordance with Rothschild and Stiglitz (1976) in the context of informational asymmetry.

[1] For a general overview on modeling options for aggregate mortality risk, see Cairns, Blake, and Dowd (2006).

However, for most individuals, aggregate mortality risk will not be independent from other sources of income risk. For example, Gründl, Post, and Schulze (2006) made clear that the solvency situation of an annuity-providing life insurer is strongly influenced by aggregate mortality risk. Depending on the structure of the life insurer's contract portfolio and asset allocation, mortality shocks can lead to an at least partial insurer default, thus linking the distribution of effective annuity payments to aggregate mortality risk.[2] Furthermore, government pension payments—especially pay-as-you-go systems—may depend on aggregate mortality developments.[3] Also, the insurer's investment in the risky asset influences its solvency situation. Thus, the performance of the risky asset influences the distribution of effective annuity payments. Finally, the performance of the individual's investment in the risky asset might negatively depend on the development of future survival rates ("asset meltdown").[4]

After integrating these dependences into our framework, the individual's optimal decisions become highly dependent on aggregate mortality risk. We show that whether or not aggregate mortality risk leads to an increase in individuals' annuity demand is determined by objective factors (such as the exposure of government pensions and/or the insurance industry to aggregate mortality shocks). Subjective factors (such as individuals' risk aversion) determine only the intensity of the annuity demand reaction with respect to aggregate mortality risk. We find that—depending on the model parameters—consideration of aggregate mortality risk may alleviate, but also intensify, the annuity puzzle.

This article is organized as follows. In the next section, we formalize our model. As the model will be analytically tractable only under very restrictive assumptions, we calibrate the model parameters in order to perform further analyses numerically. Then, our results are presented and policy implications are derived. The last section summarizes.

THE MODEL

Formalization

We consider a two-period framework. Let w_1 denote the individual's known initial wealth in period 1, and $c_i \geq 0$, $i = 1, 2$, the individual's nominal consumption in period i. Denoting by u the individual's one-period utility function with respect to a

[2] For a general analysis of the influence of an insurer's default risk on annuity demand, see Babbel and Merrill (2006).

[3] For example, a positive shock on survival rates means that—*ceteris paribus*—a greater number of retirees will have to be paid out of a given amount of social security taxes. Thus, the average pension must shrink. For further analyses, see, e.g., Aaron (1966) or Kotlikoff (1979). Ongoing reforms of the German government pension system provide a recent example for such a demographically driven reduction of pension payments (see, e.g., Börsch-Supan and Wilke, 2004).

[4] The asset meltdown hypothesis is motivated by the idea that an increase in survival rates results in a greater number of retirees. Besides the general argument that an "older" society is said to be less innovative and productive, increased pension obligations of business firms may have an adverse effect on their financial performances (Bakshi and Chen, 1994). That demographic variables may have an effect on asset prices, and the direction of this effect, is not unequivocally accepted in the literature (see Bakshi and Chen, 1994; Abel, 2001, 2003; Poterba, 2001; Brooks, 2002; Krueger and Ludwig, 2007).

real monetary argument (period consumption), we assume that from the perspective of period 1, the individual evaluates total two-period utility additively separably by

$$u(c_1) + \beta \cdot I_{Ind} \cdot u(c_2 \cdot \delta), \tag{1}$$

where I_{Ind} is a Bernoulli-distributed random indicator variable with success probability p taking value 1 if the individual survives (so that p is the individual's survival probability), and value 0 if the individual dies before period 2 (implying that there are no bequest motives). We assume that p is representative for the whole population and subject to aggregate mortality risk. Similarly, as done by Olivieri (2001) and Gründl, Post, and Schulze (2006), we model aggregate mortality risk as a symmetrically distributed additive deviation Δp from individuals' best estimate survival probability p_0. Thus, the survival probability p is random and given by $p = p_0 + \Delta p$. By $\beta \in (0, 1)$ we denote the individual's subjective discount factor (expressing time preference) and $\delta > 0$ is the one-period monetary deflator.

The consumption in period 2, c_2, depends on the individual's savings, the investment return on these savings, annuity payments and payments from the government pension system. Next, each component of c_2 will be defined in detail. Note that as all of these components will be modeled to be stochastic, from the perspective of period 1, c_2 also is stochastic.

Let s_R and s_f be the amounts of money the individual saves and invests in a risky and a risk-free asset in period 1, providing him with a risky return R and a risk-free return r_f in period 2, respectively. The value of savings in period 2 thus is given by $s_R \cdot (1 + R) + s_f \cdot (1 + r_f)$. We assume both s_R and s_f to be nonnegative, implying that the individual is not allowed to either short sell the risky asset or go into debt. We further assume that the individual can decide to put some money $\pi \geq 0$ into a life annuity, providing him with an annuity payment in period 2 given he survives until period 2, i.e., if I_{Ind} realizes as 1. Total savings, i.e., $s_R + s_f + \pi$, will henceforth be denoted as S.

Let us assume that the annuity payment in period 2 is subject to the default risk of the insurance company having sold the contract and that in the case of ruin, the individual receives only a fraction $\psi \in [0, 1]$ of every monetary unit of the agreed-upon survival benefit (depending, of course, on the premium π). We can understand ψ as the fraction of the claims against the insurer that can be recovered by some guarantee fund. Let us denote by $d(r, \varphi) \in [0, 1]$ the insurer's ruin probability *given* realizations r and φ of R and Δp. With respect to the influences of the actual outcomes of R and Δp, we require that the first derivative of d with respect to its first argument is negative, while the sign of the first derivative of d with respect to its second argument is positive. The first assumption implies that the insurer's ruin probability is the higher the poorer is the performance of the risky asset. This is plausible since risky assets, e.g., stocks, usually are part of an insurer's asset portfolio.[5] The second assumption implies that the insurer cannot completely utilize natural hedging effects between annuity and

[5] In our model economy, there is only one risky asset available, in which both the individual and the insurer can invest. Our results, however, will be valid also for the in practice probably more applicable weaker requirement of a positive correlation between the value of risky assets in which the individual invests and the value of the insurer's assets.

term-life business or that natural hedging effects are not possible at all.[6] In case the derivative of d with respect to its second argument is equal to zero, the insurer's solvency situation appears to be immune to aggregate mortality shocks.

Note that while the ruin probability d is an indicator of the insurer's idiosyncratic risk of payment default, the default payoff fraction ψ may be interpreted as an indication of the unprotected exposure of the whole insurance industry to systematic risks such as asset return risk and, especially, aggregate mortality risk. If ψ is very low, exposure to systematic risks is very high (which could be the case if the guarantee fund is insufficiently funded to provide adequate cover against systematic risks that hit many life insurers at the same time). On the contrary, a high ψ may mean that there is a reliable guarantor to support the insurance industry in case of a systematic default.

Let us further denote by $A \geq 0$ the actuarially fair survival payment of the annuity. Accounting for the insurer's possible ruin and assuming symmetric beliefs between the individual and the insurer regarding survival probability distributions and, as in Doherty and Schlesinger (1990), the insurer's ruin probability,[7] for a given ruin probability d and a given survival probability p, the actuarially fair (i.e., zero expected profit) survival payment A is determined via the valuation formula[8,9]

[6] As discussed by Gründl, Post, and Schulze (2006), the actual dependence of the insurer's ruin probability on aggregate mortality shocks depends on the insurer's underwriting policy. For example, if the insurer holds a liability portfolio that mostly consists of annuity contracts, the relationship between d and Δp should be positive, whereas for an insurer that mostly sold term-life insurance contracts, natural hedging effects can—in case both the mortality for old adults and young adults are similarly affected by a shock—lead to a negative relationship (for a discussion of natural hedging opportunities, see Blake and Burrows, 2001; Cox and Lin, 2007). Furthermore, an insurer may hold mortality derivatives (see, e.g., Cowley and Cummins, 2005; Blake et al., 2006). In that case, the insurer's vulnerability to aggregate mortality risk depends on the degree of coverage and the basis risk of these instruments (see Blake, Cairns, and Dowd, 2006).

[7] For an analysis of asymmetric information about the ruin probability, see Cummins and Mahul (2003).

[8] We discount using the risk-free return in order to allow for meaningful sensitivity analyses in the Results section. In general, an insurer, having access to a perfect capital market, would take into account the dependence of the survival payment on capital market and aggregate mortality events. Thus, he would consider some sort of risk adjustment for bearing systematic risk in his pricing. In parts of this article, we assume stochastical independence between mortality and capital market risks which makes an adjustment of the discount rate unnecessary. In the case where dependence between capital market and mortality risks is assumed, such an adjustment would be appropriate. From the perspective of an annuity buyer (here facing an incomplete market), however, changing the discount rate is just equivalent to charging a loading factor on top of the expected value of payments. In the Results section we will analyze the impact of different assumptions on aggregate mortality risk and loading factors on annuity demand. For this, a framework where only one of these factors is varied at once is helpful. Consequently, while analyzing the impact of different assumptions on aggregate mortality risk we keep the annuity price constant and equal to the expected value of payments by discounting with the risk-free return; i.e., we abstract from possible changes in the risk premium charged. The risk premium itself should be thought of being part of the loading factor introduced and analyzed later.

[9] Note, that within this framework the payout structure of an annuity is equivalent to a pure endowment with a maturity of one period.

$$\pi = (1 + r_f)^{-1} \cdot p \times [(1 - d) \cdot A + d \cdot y \cdot A]. \tag{2}$$

Thus, the higher the annuity value is, the lower is the insurer's ruin probability and the higher is the default payoff fraction; i.e., the annuity value increases as product quality increases (Doherty and Schlesinger, 1990). Rearranging and now accounting for the stochastic character of the arguments of d, and p itself, we achieve for the actuarially fair survival payment

$$A(\pi) = (1 + r_f) \cdot \pi \bigg/ \left(\iint (p_0 + \varphi)[1 - (1 - \psi) \cdot d(r, \varphi)] \cdot \mathrm{d}F(r, \varphi) \right), \tag{3}$$

where F denotes the joint distribution function of R and Δp. The fair survival payment A thus depends—beyond the individual's decision on π—on the default payoff fraction ψ and on the joint distribution of the risky asset return R and the survival probability shock Δp. To implement premium loadings, let us further introduce the parameter $\lambda \in [0, 1]$. Instead of the fair amounts $A(\pi)$ and $\psi \cdot A(\pi)$ that are paid out depending on the insurer's solvency situation, only $\lambda \cdot A(\pi)$ and $\lambda \cdot \psi \cdot A(\pi)$ are paid out. A certain value for λ thus stands for a percentage premium loading of $(1 - \lambda)/\lambda$ 100 percent.

If I_{Ins} is a Bernoulli-distributed indicator variable with success probability $1 - d(r,)$ for given realizations r and φ of R and Δp (i.e., which is 0 if the insurer defaults and is 1 if it does not), we can write the effective annuity payment–let us call it $L(\pi)$–the individual receives given he survives until period 2 as

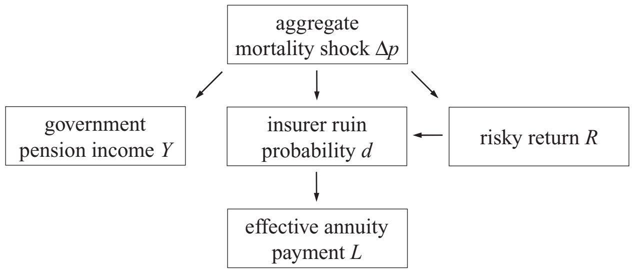
$$L(\pi) = \lambda \cdot A(\pi) \cdot [I_{Ins} + (1 - I_{Ins}) \cdot \psi], \tag{4}$$

which follows a (on the realization of d) conditional binary distribution and is purely random as soon as $\psi > 0$ and $d(r, \varphi) > 0$. *Unconditionally*, also the success probability of I_{Ins} is random, which transfers to L, too, so that $L(\pi)$ in Equation (4) actually should be written as $L(\pi | R = r,\ \Delta p = \varphi)$.

Additionally, for a given realization φ of Δp, let $Y(\varphi)$ denote the individual's income from the pay-as-you-go government pension system receivable in period 2, given the individual is alive then. Since pay-as-you-go pension systems strongly depend on the development of demographic variables (see, e.g., Aaron, 1966; Kotlikoff, 1979), we assume that $Y'(\varphi) \leq 0$, which means that an increase (decrease) in the general and individual survival probability will lead to an immediate drop (rise) or constancy in the pension payment.[10]

[10] We hereby think of a pension system, where an adjustment of pensions that are already in the payout phase is still possible. This is the case, for example, in Germany, where pension levels experienced zero nominal growth during recent years as opposed to nominal or even real increases in the past (see, e.g., Börsch-Supan and Wilke, 2004). See also Gomes, Kotlikoff, and Viceira (2007) for a model that analyzes the welfare effects of government indecision, i.e., a case where the government tries to procrastinate the announcement of bad news on retirement benefits.

FIGURE 1
Summary of Model Dependences



The individual's (from the perspective of period 1) random consumption in period 2 is thus given by

$$c_2 = s_R \cdot (1 + R) + s_f \cdot (1 + r_f) + L(\pi) + Y(\Delta p), \tag{5}$$

where s_R, s_f, and π together with first period consumption c_1, are linked by the budget constraint $w_1 = c_1 + s_R + s_f + \pi$, and each of these four decision variables is subject to its nonnegativity constraint. The individual's evaluation of total utility is thus given by

$$u(c_1) + \beta \cdot I_{Ind} \cdot u((s_R \cdot (1 + R) + s_f \cdot (1 + r_f) + L(\pi) + Y(\Delta p)) \cdot \delta), \tag{6}$$

which, of course, is also random. To determine optimal decisions on consumption, investment, and annuity purchase, we assume that the individual maximizes the expected value of total utility, which is given by

$$\begin{aligned} &u(c_1) + \beta \cdot \iint (p_0 + \varphi)\cdot \\ &u([s_R \cdot (1 + r) + s_f \cdot (1 + r_f) + L(\pi \mid R = r, \Delta p = \varphi) + Y(\varphi)] \cdot \delta)\mathrm{d}F(r, \varphi), \end{aligned} \tag{7}$$

with respect to c_1, s_R, s_f, and π, subject to $w_1 = c_1 + s_R + s_f + \pi$ and $c_1, s_R, s_f, \pi \geq 0$.

Figure 1 summarizes the endogenous dependences in the model.

Most of the calculations necessary to perform our analyses are not analytically tractable. We therefore determine optimal solutions numerically, making it necessary to first calibrate our model.

Calibration and Solving Technique

In this section, we define the base values assigned to our model parameters. The Results section contains a thorough analysis of the consequences different values would have.

To express the individual's *intra*temporal risk preferences, we use a utility function with constant relative risk aversion (CRRA), given by parameter $\gamma > 0$. Thus, for $c > 0$,

TABLE 1
Considered Probability Distributions of the Survival Probability Shock Δp

	Probability That Δp Realizes as		
Probability Distribution	−0.01 (Decrease in Survival Probability)	0 (No Aggregate Mortality Shock)	+0.01 (Increase in Survival Probability)
1	0.000	1.000	0.000
2	0.025	0.950	0.025
3	0.050	0.900	0.050
4	0.100	0.800	0.100
5	0.200	0.600	0.200

we have $u(c) = c^{(1-\gamma)}/(1-\gamma)$. The parameter of constant relative risk aversion γ is set to 3; the *inter*temporal subjective discount factor β is set to 0.95 (see, e.g., Laibson, Repetto, and Tobacman, 1998). For the individual's initial wealth, we set $w_1 = 100$.

To model aggregate mortality risk, we use three-point distributions as done in Gründl, Post, and Schulze (2006). For the state space of Δp, we set (−0.01, 0, +0.01), which means that the individual's survival probability—starting from p_0—will either increase by 1 percentage point, stay constant, or drop by 1 percentage point. Onto the set of measurable subsets of this state space, we set five alternative probability measures, which we choose as done in Gründl, Post, and Schulze. Table 1 contains the probability distributions used.

In all distributions, the main probability mass—depending on the probability model, between 60 percent and 100 percent—is assigned to the state of nature where Δp realizes as 0, i.e., if $E(p) = p_0$ turns out to be the individual's actual survival probability. The remaining mass is then assigned equally to the "deviating" states of nature, where the individual's survival probability realizes as $p_0 - 0.01$ or $p_0 + 0.01$, respectively. Such assumption about changes of survival probabilities can, to some extent, be looked at as a mean-preserving shock. For the mean survival probability we set $p_0 = 0.85$.[11]

The one-period return of the risk-free asset is set to $r_f = 0.05$, which is the sample mean of the annualized return time series of the German money market from 1950 to 2003 (see Gründl, Post, and Schulze, 2006). The one-period deflator δ is set to 0.9744,which is the inverse of the mean inflation rate given by the German consumer price index (CPI) between 1950 and 2003, which is about 1.0263.

For the return R of the risky asset, we initially assume that its conditional distribution given any realization of the aggregate mortality shock is normal with mean 0.1567 and standard deviation 0.3192, implying stochastical independence between R and Δp.[12]

[11] In general, a value of 0.85 for 1-year survival probabilities would refer to very old persons (say aged more that 80–85), which is not a usual age for the annuitization decision. However, this value was chosen here in order to induce the individual to buy annuities in most of the scenarios considered. This gives a basis to make results of the sensitivity analyses in the Results section more apparent.

[12] These values were also taken from Gründl, Post, and Schulze (2006). They were estimated from a time series (1950–2003) of the German stock market index DAX.

TABLE 2
Probability Distributions of Risky Return R Conditional on Realization φ of the Survival Probability Shock Δp

	Conditional Distribution of R in the Case of	
Realization φ of Survival Probability Shock Δp	Independence of R and Δp	A Negative Dependence of R and Δp (Asset Meltdown)
-0.01	N(0.1567, 0.3192)	N(0.1767, 0.3192)
0	N(0.1567, 0.3192)	N(0.1567, 0.3192)
$+0.01$	N(0.1567, 0.3192)	N(0.1367, 0.3192)

Introducing the asset meltdown hypothesis into our model later on, we then drop this independence assumption and instead assume the distribution of the risky return to be negatively correlated with the aggregate mortality shock (see footnote 4). We model this assuming, again, that conditional distributions of R given a realization of Δp are normal. Keeping conditional standard deviations constant at the level used before, i.e., at 0.3192, we now, however, model conditional expected returns to depend on the actual realization of Δp. We do this by assuming that the conditional expectation of R decreases by 2 percentage points to 0.1367 if Δp realizes as $+0.01$ (i.e., improvement of the survival probability leads to a drop in asset returns) and increases to 0.1767 if Δp realizes as -0.01. Table 2 summarizes the conditional probability distributions of R.[13]

To model the insurer's ruin probability $d(r, \varphi)$ for every pair of realizations (r, φ) of $(R, \Delta p)$, we choose the log-linear functional form

$$d(r, \varphi) = \exp(\alpha_{\Delta p} \cdot \varphi + \alpha_R \cdot (1 + r) + const), \tag{8}$$

where $\alpha_{\Delta p}$ and α_R determine the degree of reaction of the insurer's ruin probability to the aggregate mortality scenario and the performance of the risky asset. From the partial derivative conditions above, we require $\alpha_{\Delta p}$ and α_R to be nonnegative and nonpositive, respectively.[14] In cases where the insurer's ruin probability is not dependent on aggregate mortality risk or the performance of the risky asset, the respective reaction parameters are set to 0. Besides the natural requirement $0 \leq d(r, \varphi) \leq 1$, for every pair of realizations (r, φ) of $(R, \Delta p)$, we additionally impose the restrictions that, first, if R realizes below the 0.1 percent percentile of the risky asset return distribution and at the same time Δp realizes as 0 (no survival probability

[13] Dependence between capital market return and survival probability actually would require a risk adjustment in the discount factor when calculating the fair annuity premium in Equation (2). For the sake of facilitating comparative statics we refrain from that.

[14] Modeling ruin probability to be log-linear is in accordance with the findings of Darooneh (2005); see also Klugman, Panjer, and Willmot (2004, chap. 6). It can be supported by the following economic reasoning: If—as in our calibration—the maximum ruin probability already lies in the concave part of an unimodal equity capital distribution function, then the ruin probability is a decreasing and convex function of the equity capital available (see, e.g., Dowd, 2002, chap. 2.2).

shift), the insurer's ruin probability is equal to 3 percent; and that, second, every ruin probability induced by a realization of R increases (decreases) tenfold if Δp realized as $+0.01$ (-0.01), i.e., if there is an increase (decrease) in the survival probability. In the—with respect to the insurer's solvency situation—worst case of an asset return below about -0.874 and an increase in the survival probability, the insurer will fail to meet the individual's annuity payment claims with a probability of 30 percent. In cases where R realizes at the other end of its support, and aggregate mortality risk realizes as a reduction in survival probability, the insurer's ruin probability is approximately 0.[15] For the unconditional (expected) ruin probability $E[d(R,\Delta p)]$ (i.e., the *a priori* ruin probability of the insurer from the perspective of period 1), we assume that it coincides with some regulator-fixed maximum ruin probability (this is why we need the fitting variable *const* in Equation (8)). As done in Gründl, Post, and Schulze (2006), we set the insurer's maximum unconditional ruin probability to 0.01, i.e., 1 percent.

We set the default payoff fraction ψ alternatively to 0 and 0.6. The loading factor λ is set to 0.9, leading to a loading of $1/9 \cdot \pi$. For the payment from the government pay-as-you-go pension system in period 2, we assume as true the functional relationship

$$Y(\varphi) = w_1 \cdot \vartheta \cdot (1 - \alpha_Y \cdot \varphi), \tag{9}$$

where the parameter $\alpha_Y \geq 0$ determines the degree of reaction of the government pension payment on a given realization φ of the aggregate mortality shock Δp. The parameter $\vartheta \geq 0$ is some pension factor determining the fraction of initial wealth the individual expects to receive as government pension in period 2.[16] A positive aggregate mortality shock on the individual's survival probability will thus lead to a proportional drop in the individual's pension payment in period 2 (starting with $E[Y(\Delta p)] = w_1 \cdot \vartheta$), while a negative aggregate mortality shock will lead to a proportional pension increase (see footnote 3). We set ϑ to 0.15. The severity of the change in the pension payment for a given aggregate mortality shock φ is determined by α_Y. Depending on whether or not Y reacts to the aggregate mortality shock, we set α_Y to 1 or 0, respectively.

The optimization of objective (7) was performed by using the Mathematica® 5.1 implemented nonlinear optimizer NMaximize. To keep the optimization problem tractable, we discretized the risky return density function using Gaussian quadrature method as done, e.g., by Cocco, Gomes, and Maenhout (2005).

The variables and parameters used and their base calibration are summarized in Table 3.

RESULTS

As the Introduction and Model sections of this article made clear, our model incorporates complex dependences between aggregate mortality risk, government pension

[15] We checked the consequences of changes in these two assumptions (3 percent and tenfold increase/decrease) and found robustness of the tendencies of our results.

[16] By doing this, the CRRA feature of the one-period utility function makes our results arbitrarily scalable in the period 1 wealth-to-pension factor ratio (see Pratt, 1964; Carroll, 2004).

TABLE 3
Summary of Variables and Parameters, and Their Calibration

Symbol	Variable/Parameter	Value/Specification
A	Actuarially fair survival payment	See Equation (3)
α_Y	Reaction parameter of government pension w.r.t. aggregate mortality shock	$\{0, 1\}$
β	Subjective discount factor	0.95
c_i	Consumption in period i	Decision variable
d	Insurer's ruin probability	$\mathrm{E}[d(R, \Delta p)] = 0.01$
δ	One-period deflator	1/1.0263
γ	Parameter of relative risk aversion	3
L	Effective annuity payment	See Equation (4)
λ	Annuity loading factor	0.9
Δp	Aggregate mortality shock	See Table 1
p	Survival probability	$p_0 + \Delta p$
p_0	Mean survival probability	0.85
π	Annuity premium	Decision variable
ψ	Default payoff fraction	$\{0, 0.6\}$
R	Risky return	See Table 2
r_f	Risk-free return	0.05
S	Total savings	$s_R + s_f + \pi$
s_f	Risk-free investment	Decision variable
s_R	Risky investment	Decision variable
ϑ	Government pension factor	0.15
u	One-period utility function	CRRA
w_1	Initial wealth	100
Y	Government pension payment	See Equation (9)

payments, the insurer's ruin probability, effective annuity payouts, and the performance of the risky asset. To gain insight into how aggregate mortality risk, per se, is perceived by an expected utility maximizer, in the first subsection we analyze a situation where government pension payments, the risky return, and the insurer's ruin probability do not depend on aggregate mortality risk. After this, we investigate the situation including all dependences illustrated by Figure 1, but neglecting for the moment the possibility of a dependence between aggregate mortality risk and the performance of the risky asset (asset meltdown). Since this kind of dependence has generated some controversy in the scientific literature (see, e.g., Abel, 2001, 2003; Poterba, 2001; Brooks, 2002), we will analyze that case separately in another subsection. Then, the parameters used in our calibration are varied to generalize our results. Finally, we derive policy implications for potential insurance buyers as well as for the insurance industry and regulators.

Aggregate Mortality Risk as an Independent Multiplicative Background Risk

From the expected utility maximization objective (7) it becomes apparent that a mean-preserving aggregate mortality risk does not influence the individual's decisions at all if

(i) government pension payments do not react to aggregate mortality developments (or the individual simply does not participate in such a pension scheme),

(ii) annuity providers—even if subject to default risk—appear to be immunized against aggregate mortality shocks with respect to their solvency situation, and

(iii) aggregate mortality shocks and the risky asset return are stochastically independent from each other.

If conditions (i) to (iii) hold, the survival probability in objective (7) can be simply evaluated separately via the expected value operator. In the case—and that is what we assumed—that $E(p) = p_0$, objective (7) has the same form and will lead to the same optimal consumption, saving, and asset allocation decisions as it would if aggregate mortality risk did not exist at all. The individual thus behaves *risk neutrally* toward this intertemporal risk. In fact, for the individual, the risk of having used the wrong total discount factor $\beta \cdot p$ when determining optimal decisions (and thus, e.g., to have consumed too much if his survival probability turns out to go up) does not matter at all. In other words, the mere existence of aggregate mortality risk does not produce any wish for risk protection and thus does not influence the individual's annuity demand. Although at first this result seems surprising, it is, in fact, a standard implication of the individual's assumed preference structure, as analyzed in other contexts by Rothschild and Stiglitz (1976) and Gollier (2001, chap. 19).[17]

Even if the shock was not mean preserving but only symmetric around some mean deviation (or trend), it would have the effect of a change in discounting tomorrow's consumption rather than the effect of an (additive) background risk: the presence of a background risk generally reduces risky investment (Kimball, 1993) and increases insurance demand (Eeckhoudt and Kimball, 1992). As, on the contrary, a change in discounting for additively separable CRRA utility evaluation does not influence asset allocation (see Gollier, 2001, chap. 19), the introduction of aggregate mortality risk in the manner considered so far does not influence investment and insurance purchasing decisions.

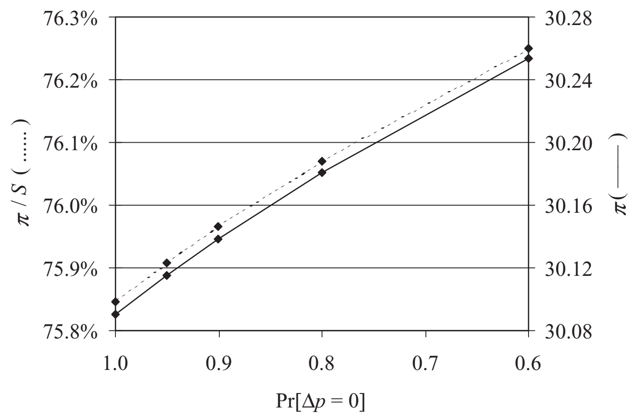
The individual's risk-neutral perception of aggregate mortality risk changes as soon as at least one of the three conditions (i) to (iii) above does not hold. In what follows, we will study the impacts of first dropping conditions (i) and (ii), and then look at the effect of dropping all three conditions.

A First Analysis of the Model With Dependences: The Insurer's Ruin Probability Depends on Aggregate Mortality Risk and the Risky Return; Government Pension Payments Depend on Aggregate Mortality Risk

We now assume that the insurer's ruin probability depends on aggregate mortality risk and the risky return. Furthermore, government pension payments also depend on aggregate mortality risk. Here, aggregate mortality risk becomes an important risk. From the perspective of the individual the market for aggregate mortality risk is incomplete and he cannot fully insure it. In states of nature where the survival

[17] Note that for these results, it is not necessary that the insurer's ruin probability is also stochastically independent from the risky asset return R.

FIGURE 2
Annuity Demand π and Annuity Demand in Proportion to Total Savings π / S Dependent on $\Pr[\Delta p = 0]$, i.e., the Probability That the Aggregate Mortality Shock Is 0 (*ADRF*); Default Payoff Fraction $\psi = 0.6$



probability increases due to an aggregate mortality shock, the government pension system performs poorly and the insurer's ruin probability increases; i.e., there is a higher probability that the annuity will also perform poorly. To complete the distress, exactly those cases are weighed more heavily within the expected utility evaluation (see objective (7)). On the one hand, this occurs because for those cases the survival probability, i.e., the weight in the expected utility evaluation, is high. On the other hand, due to the concavity of u, the insurer's worsening solvency situation when survival probability increases is perceived more badly than a reduction in the insurer's ruin probability (when the survival probability decreases) is appreciated. For high values of the default payoff fraction ψ (and low values of α_Y), these effects are somewhat alleviated, since the annuity payout (and government pension payout) reacts less to aggregate mortality risk. Thus, both parameters indicate the importance of aggregate mortality risk.

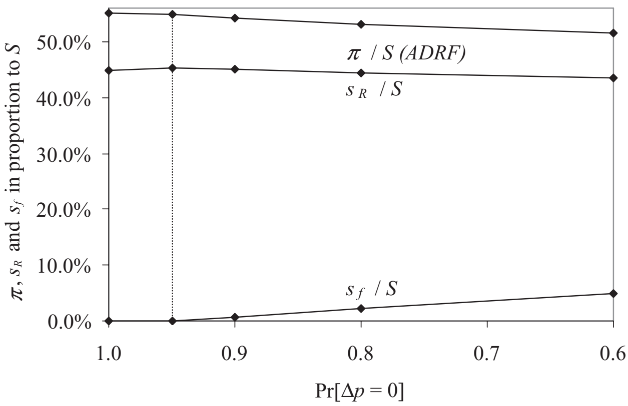
The annuity demand reaction function with respect to the strength of aggregate mortality risk—henceforth abbreviated by *ADRF*—is shown in Figure 2 for a default payoff fraction of $\psi = 0.6$.

Figure 2 illustrates that insurance demand increases in absolute terms and also in proportion to total savings with the strength of aggregate mortality risk, i.e., with decreasing probability that the aggregate mortality shock is 0.[18] This increase in insurance demand as aggregate mortality risk increases is the reaction one would intuitively expect: higher risk is accompanied by higher insurance demand. The

[18] Total savings decrease slightly with increasing aggregate mortality risk. This is the normal (prudent) reaction of a CRRA investor to an increase in background risk (see Deaton, 1991; Elmendorf and Kimball, 2000). The tendency of annuity demand reaction w.r.t aggregate mortality risk is in absolute terms and relative terms equal. In the remainder of this article, we show only relative values, i.e., portfolio shares.

FIGURE 3

Annuity, Risky, and Risk-Free Investment in Proportion to Total Savings S Dependent on $\Pr[\Delta p = 0]$, i.e., the Probability That the Aggregate Mortality Shock Is 0; Default Payoff Fraction $\psi = 0$



actual causal mechanisms leading to such positively sloped *ADRF*, however, are rather complex as will be demonstrated in the following.

As the independence case demonstrated, the individual has no incentive to protect himself against aggregate mortality risk per se, for example, by buying annuities. Furthermore, due to the additional dependence between the insurer's ruin probability, annuity default, and aggregate mortality risk, an annuity cannot be used to fully hedge against this risk. Things have to be different. Government pension payments negatively depend on the realization of the aggregate mortality shock. Aggregate mortality risk thus increases the background risk stemming from the government pension system and therefore increases the individual's wish to create a safer asset allocation (cf. Kimball, 1993; Elmendorf and Kimball, 2000). To do this, the individual shifts savings out of the risky asset (not shown). In deciding where the assets should be shifted to, the individual compares the specific payment characteristics of the risk-free asset and the annuity. If the annuity is risk free itself (and not loaded too heavily), it will clearly dominate the risk-free asset due to the inherent mortality premium on top of the risk-free return (see, e.g., Brown, 2001). However, if the annuity becomes risky, the choice now depends on a comparison between the situation-dependent certainty equivalent of the annuity and the risk-free investment. If the default payoff fraction ψ is high, this comparison will favor the annuity leading to a zero demand for the risk-free investment.

If the default payoff fraction ψ of the annuity shrinks, the risk-free investment becomes comparatively more attractive, and finally, it will be used for saving. This is the case in Figure 3, where ψ is 0; i.e., in the case of the insurer's ruin, the individual receives no payment at all from the annuity.[19]

[19] This extreme case is what Kahneman and Tversky (1979) and Wakker, Thaler, and Tversky (1997) called probabilistic insurance.

ADRF now *decreases* with increasing aggregate mortality risk. This reaction by the individual to aggregate mortality risk can be observed for $\psi < 0.2$. Inside the interval $0.2 \leq \psi \leq 0.3$, the slope of *ADRF* reverses by changing its shape from a decreasing function to a parabola, and from there ($0.3 < \psi$) to an increasing function.

Moving from $\Pr[\Delta p = 0] = 1$ to $\Pr[\Delta p = 0] = 0.95$, i.e., introducing aggregate mortality risk, the emerging (government pension) background risk induces the individual to reduce annuity demand and to shift savings into the risky asset, which now, for $\psi = 0$, appears to be the less risky investment opportunity (for $\psi = 0.6$, this shift did not take place; see Figure 2). In this aggregate mortality probability model, the risk-free investment is still not attractive enough (or, in other words, the background risk is not severe enough) to induce risk-free savings. In fact, the individual would even go into debt, if the borrowing constraint did not keep him from doing so.

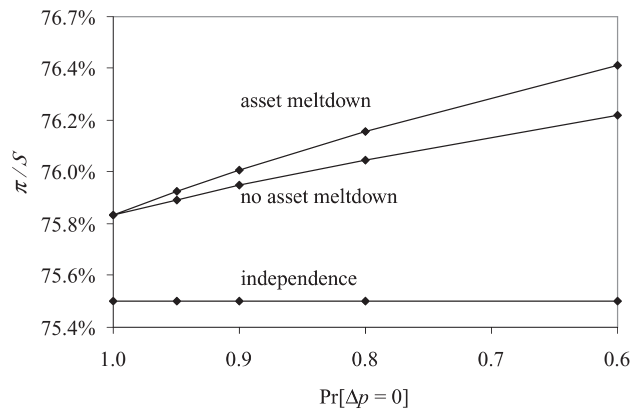
For lower values of $\Pr[\Delta p = 0]$, higher aggregate mortality risk or, to be more precise, the increased background risk, now creates a situation where any portfolio of annuity and the risky asset alone is too risky for the individual. He begins to shift savings away from annuities and the risky investment into the risk-free investment, which means that, due to the systematic character of aggregate mortality risk, *ADRF* now decreases. Note that this decrease is lower in the region where the borrowing constraint is binding, which can be explained as follows. The slope of the *ADRF* in the region where the borrowing constraint is nonbinding can be seen as the individual's "normal" reaction to aggregate mortality risk. If there was no borrowing constraint, for $\Pr[\Delta p = 0] \geq 0.95$ the individual would go into debt. However, as this is not possible, the individual seeks another way to get some risk into his portfolio. One way is to become more tolerant toward aggregate mortality risk, leading to a less strong reaction to aggregate mortality risk in the constrained regions.

Comparing the annuity demand shown in Figures 2 and 3 it is apparent that a reduction in the default payoff fraction ψ does not only reverse the portfolio decisions for different strengths of aggregate mortality risk. Also, given a certain value of $\Pr[\Delta p = 0]$, a decrease in the default payoff fraction ψ leads to a reduction in annuity demand, which is the normal reaction: a reduction of ψ neither changes the expected value of the return from the annuity nor its actuarial fairness (see Equations (2) and (3)). However, it makes the return more volatile, increases the interval of possible effective payments, and increases the annuity's exposure to aggregate mortality risk (thus also increasing the positive dependence on the government pension payment; see Equation (4)). Due to risk aversion and the prudence feature of u, this results in a lower annuity demand (cf. Kimball, 1993; Elmendorf and Kimball, 2000).

The Insurer's Ruin Probability Depends on Aggregate Mortality Risk and the Risky Return; Government Pension Payments Depend on Aggregate Mortality Risk; the Risky Return Depends on Aggregate Mortality Risk (Asset Meltdown)

We now introduce the asset meltdown hypothesis into the model. So far, the two systematic risks (aggregate mortality and asset return risk) have been assumed to be stochastically independent from each other. The asset meltdown hypothesis now links aggregate mortality risk and the return of the risky asset directly, in a way that a combination of both risks becomes more undesirable to the individual (cf. Table 2). Furthermore, if the individual decides to hold the risky asset, government pension

FIGURE 4
Annuity Demand in Proportion to Total Savings S Dependent on $\Pr[\Delta p = 0]$, i.e., the Probability That the Aggregate Mortality Shock Is 0 (*ADRF*); Default Payoff Fraction $\psi = 0.6$



payments also become riskier because the dependence between aggregate mortality risk and the return of the risky asset implicitly links government pension payments to the performance of the risky asset (see Figure 1). Finally, an asset meltdown increases the importance of aggregate mortality risk, since the states of nature that are weighed more heavily in the expected utility evaluation (increases in the survival probability) are accompanied by rather poor returns from the risky asset.

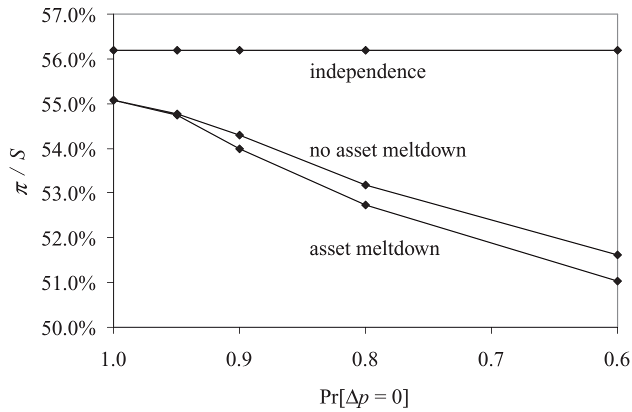
Consequently, the tendencies shown in Figures 2 and 3 become more strongly pronounced when the asset meltdown hypothesis is introduced. Figure 4 shows *ADRF* for the case of a default payoff fraction $\psi = 0.6$.

The additional dependence of both the risky asset and the government pension payments on the performance of the risky asset return leads—when ψ is high—to a higher demand for the comparatively safe risky asset: the annuity (cf. Viceira, 2001; Cocco, Gomes, and Maenhout, 2005). This occurs because the just introduced dependence in the first instance disturbs the payoff characteristics of the risky asset. As the annuity was already linked to both aggregate mortality and risky return risk, the new dependence affects the (for ψ high, in any case rather safe) annuity only slightly.

If the annuity becomes increasingly risky as ψ decreases, *ADRF* goes down (shown in Figure 5) and, as before (see Figure 3), the individual shifts some of his wealth into the risk-free asset (not shown here). As above, the effects, having led to a decreasing *ADRF* as shown in Figure 3, are intensified by introducing the asset meltdown.

Figures 4 and 5 also contain the independence case, where annuity demand did not react at all to aggregate mortality risk. The fact that—compared to this case—annuity demand increases (decreases) for higher (lower) values of the default payoff fraction ψ underlines the arguments made above. For higher values of ψ (Figure 4), the individual reacts to the increase in risk stemming from the introduction of dependences by shifting savings from the risky asset to the comparatively less risky annuity. For

FIGURE 5
Annuity Demand in Proportion to Total Savings S Dependent on $\Pr[\Delta p = 0]$, i.e., the Probability That the Aggregate Mortality Shock Is 0 (*ADRF*); Default Payoff Fraction $\psi = 0$



lower values of ψ, again, the reaction is the opposite—*ADRF* becomes negatively sloped (see Figure 5).

A Deeper Analysis of the Impacts of Model Parameters on *ADRF*

The numerical examples of the last subsections revealed that there are several influences that must be considered regarding an individual's annuity demand reaction w.r.t. aggregate mortality risk (*ADRF*). We observed that *ADRF* may have a negative or positive slope; i.e., in some situations the individual reacts to increasing aggregate mortality risk by increasing, in other cases by decreasing, the portfolio share of annuities. The sign of the slope of *ADRF* appears to be controlled by the default fraction payoff parameter ψ. In what follows we provide a deeper analysis of the mechanisms driving our model.

When we vary the parameters γ, β, λ, p_0, α_Y, the strength of the aggregate mortality shock, the strength of the mean return shift in the case of an asset meltdown, and ϑ, we observe fundamental differences in parameter influence: some parameters (γ and β) control only the *intensity* of the individual's reaction to aggregate mortality risk, i.e., the magnitude of the slope of *ADRF*, whereas other parameters (ψ, λ, p_0, α_Y, the strength of the aggregate mortality shock, the strength of the mean return shift in the case of an asset meltdown, and ϑ) also control the *direction* of the individual's reaction, i.e., the sign of the slope of *ADRF*. Moreover, emphasizing our previous results, it turns out that whether or not the borrowing constraint, $s_f \geq 0$, is binding is also a factor that influences the magnitude of the slope (but not the sign) of *ADRF*.

In the following, we discuss these influences in detail. We start with parameters that do not influence the sign of the slope of *ADRF*. Next, we examine other parameters that are able to change the sign of the slope of *ADRF*. Then we draw general conclusions and derive some policy implications.

Parameters Influencing Only the Magnitude of the Slope of *ADRF*

An increase in the relative risk aversion parameter γ generally leads to a safer asset allocation (see Gollier, 2001, chap. 4). From the discussion of Figures 2 and 3, we know that the most important factors influencing how the individual will achieve this increased safety are, first, whether the annuity is rather safe (ψ high) or not (ψ low) and, if ψ is low, second, whether the borrowing constraint is binding or not.

If ψ is rather high (increasing *ADRF*) and thus the annuity appears to be more attractive than the risk-free asset, increased desire for a safe asset allocation will lead to an even stronger shift out of the risky asset into the (safer) annuity than for a lower γ. The increasing *ADRF* becomes steeper. In the case where ψ is rather low (decreasing *ADRF*) and the borrowing constraint is binding, our discussion of Figure 3 showed that the individual develops some risk tolerance toward aggregate mortality risk. As risk aversion goes up, however, the individual's level of additional risk tolerance becomes much more sensitive to an increase in aggregate mortality risk. Thus, the individual will react more strongly to increasing aggregate mortality risk: *ADRF* becomes steeper.

Only when the borrowing constraint is nonbinding, and the individual is permitted to act freely in the face of increasing aggregate mortality risk, we find a decrease in reaction intensity. For a higher γ, the individual invests a greater amount in the risk-free asset and chooses a lower exposure to aggregate mortality risk (via purchasing annuities) than if γ is low. Due to his lower exposure, the individual's adjustments to the annuity share of the portfolio will be less pronounced than for lower γ. Thus, *ADRF* becomes flatter.

The influence of the subjective discount factor β on the optimal decisions depends on whether the individual receives government pension income. Generally, an increase in β leads to higher total savings since the individual puts a larger weight on future consumption. If government pension income is 0 ($\vartheta = 0$), the individual's asset allocation is independent of β for CRRA utility (see, e.g., Gollier, 2001, chap. 19). If the individual does receive pension income ($\vartheta > 0$), the pension income works to some extent as a substitute for risk-free savings (see, e.g., Cocco, Gomes, and Maenhout, 2005). Higher total savings due to an increase in β naturally lead to an increase in the relation of total savings to pension income. This relative reduction of a risk-free position induces the individual to choose a safer asset allocation for his savings, or in other words, the individual acts more risk averse. Thus, the effects of an increase in β on the slope of *ADRF* are the same as in the case of an increase in γ. Table 4 summarizes the results of this subsection.

Parameters Influencing the Sign of the Slope of *ADRF*

Whereas γ and β control *behavior* in a given risk situation, the parameters ψ, λ, p_0, α_Y, the strength of the aggregate mortality shock, the strength of the mean return shift in the case of an asset meltdown, and ϑ control the risk situation *itself*, thus leading to changes in the way the individual reacts to aggregate mortality risk, i.e., the sign of the slope of *ADRF*. At this, it turns out that only for variations in ϑ, the strength of the mean return shift in case of the asset meltdown, and ψ is it of importance whether the borrowing constraint is binding or not.

TABLE 4
Influence of Model Parameters on the Annuity Demand Reaction Function *ADRF*

	Magnitude of Absolute Slope of *ADRF* When Borrowing Constraint	
Influence of Increase in	Does Not Bind	Binds
Relative risk aversion γ	↓	↑
Subjective discount factor β (for $\vartheta > 0$)	↓	↑

A decrease in the loading factor λ, i.e., an increase in the premium loading, leads to a proportional drop in annuity payments for a given annuity premium, which makes the annuity—compared to alternative investment opportunities—less attractive in all states of nature. Annuity demand shrinks (see, e.g., Mitchell et al., 1999; Brown and Poterba, 2000). An increase in the volatility of effective annuity payouts with increasing aggregate mortality risk is now penalized more heavily. If the slope of *ADRF* already has a negative sign (compare Figure 2), a decrease in λ makes *ADRF* even steeper. The slope of a positive *ADRF* (compare Figure 3) becomes flatter; with increasing aggregate mortality risk, the individual more reluctantly shifts savings to the now less attractive annuity. If the annuity price increases, further making the spread on top of the actuarily fair premium too high, the slope of *ADRF* will even reverse and become negative. Thus, a decrease in λ turns *ADRF* clockwise.

Varying the mean survival probability p_0 creates two effects. On the one hand, from objective (7) it follows that an increase in p_0 is equivalent to an increase in the mean "overall" discount rate $\beta \cdot p_0$, which leads to the same tendencies as in the case of an increase in β. On the other hand, from Equation (3) it follows that an increase in p_0 leads to a drop in the mortality credit of the annuity. Thus, the expected return from the annuity shrinks in a way similar to the case of a decrease in λ. Again, in all states of nature the annuity becomes less attractive compared to alternative investment opportunities, and annuity demand decreases (see, e.g., Mitchell et al., 1999; Brown and Poterba, 2000). In our calibration, the second effect dominates the first, and thus an increase of p_0 leads to the same (although more moderate) tendencies as a decrease in λ.

An increase in the reaction parameter of government pensions w.r.t. aggregate mortality shocks, α_Y, makes government pension payments more volatile, i.e., background risk and the importance of aggregate mortality risk increase. Since effective annuity payouts are positively correlated with government pension payments, the annuity becomes less attractive relative to other (uncorrelated) assets, similar to what happens in the case of a decrease in λ. Consequently, the tendency of change in the slope of annuity demand curves is the same as in the case of a decrease of λ.

An increase in the strength of the aggregate mortality shock (in the base case it was set to ± 0.01) also leads to an increase in background risk and makes the impact of aggregate mortality risk more pronounced, since bad states of nature are weighed even more heavily in the expected utility evaluation. Thus, an increase in the strength of the aggregate mortality shock leads to the same tendencies as an increase in α_Y, although they will be more pronounced.

Increases in the strength of the mean return shift in the case of an asset meltdown make the link between the risky asset and aggregate mortality risk even more risky to the individual. Here, the effect on the *ADRF* depends on whether the borrowing constraint is binding. If the borrowing constraint binds, i.e., if the individual perceives the annuity as a rather favorable, safe investment, the individual reacts to stronger aggregate mortality risk by reducing the risky position in favor of the annuity; *ADRF* turns counterclockwise (compare Figure 4). If the borrowing constraint does not bind, the individual increases his risk-free position at the expense of annuity holdings. The stronger the aggregate mortality risk and mean return shift, the more likely he will be to transfer assets in this manner; *ADRF* turns clockwise (compare Figure 5).[20]

As previously discussed, also the default payoff fraction ψ is important for the sign of the slope of *ADRF*. This parameter's influence on *ADRF* depends on whether or not the borrowing constraint binds. For low ψ, the annuity is a rather risky asset. The risk-free asset is attractive enough so that the borrowing constraint does not bind and risk-free savings are induced. *ADRF* was shown to have a negative slope. Increasing ψ here leads to a steeper *ADRF*; it turns clockwise. Although the risk–return trade-off of the annuity improves, the individual is less willing to hold this asset as aggregate mortality risk increases because where ψ is low, aggregate mortality risk—when it occurs—has a stronger impact. When increasing ψ at a low level, the risk exposure of the annuity remains high. Nevertheless, especially for $\Pr[\Delta p = 0] = 1$, the individual increases the amount in the annuity with increasing ψ due to the better risk–return trade-off. However, since the individual is now more invested in the annuity in the first place,[21] he reacts more sensitively to the introduction of aggregate mortality risk. If aggregate mortality risk is introduced, i.e., $\Pr[\Delta p = 0] < 1$, the higher but nevertheless still low ψ is not enough to overcome the individual's fear of aggregate mortality risk. This changes as soon as ψ takes values high enough to dominate the risk-free asset, i.e., to make the borrowing restriction bind. With increasing ψ, *ADRF* turns counterclockwise until, eventually, the sign of its slope becomes positive (see Figure 2).

An increase in the government pension income factor ϑ (see Equation (9)) means, on the one hand, an increase in the individual's exposure to aggregate mortality risk. On the other hand, it also means an increase in the individual's risk-free income position. As a consequence, the effects of variations in the parameter ϑ on *ADRF* are less clear-cut. Here, the value of the default payoff fraction parameter ψ, i.e., the systematic exposure of the annuity to aggregate mortality risk, and the binding or nonbinding nature of the borrowing constraint are of great importance. For high ψ, we observed the following general tendency: an increase in government pension factor ϑ and the thus induced increase in the individual's exposure to aggregate mortality risk turns

[20] The *ADRF* turning tendencies observed here do also hold for "asset meltups," i.e., for the case of a positive dependence between Δp and R. Such dependence may result from changes in saving behavior due to aggregate mortality shocks. For example, if survival probability rises, younger investors will save more to cope with their now increased expected lifespan, whereas older investors will decumulate their assets more slowly. Both effects may lead to increases in asset prices (see, e.g. Poterba, 2001).

[21] An increase in annuity demand due to an increasing default payoff fraction is consistent with the results of Babbel and Merrill (2006).

TABLE 5
Influence of Model Parameters on the Annuity Demand Reaction Function *ADRF*

	Turning Direction of *ADRF* When Borrowing Constraint	
Influence of Increase in	Does Not Bind	Binds
Loading factor λ	Counterclockwise	Counterclockwise
Mean survival probability p_0	Clockwise	Clockwise
Reaction of government pensions w.r.t. aggregate mortality shocks α_Y	Clockwise	Clockwise
Strength of aggregate mortality shock	Clockwise	Clockwise
Strength of mean return shift in the case of an asset meltdown	Clockwise	Counterclockwise
Default payoff fraction ψ	Clockwise	Counterclockwise
Government pension income factor ϑ given a low ψ	Switch clockwise to counterclockwise	Switch from counterclockwise to clockwise
Government pension income factor ϑ given a high ψ	Clockwise	Clockwise

the positively sloped *ADRF* clockwise until it finally becomes negatively sloped. For low ψ, *ADRF* stays negatively sloped, but the complex interplay of the factors mentioned above leads to different turning directions depending on the specific value of ϑ.[22]

The influence on *ADRF* of variations in model input parameters as discussed in this section is summarized in Table 5.[23]

Conclusions From Section Results and Policy Implications

The analyses provided here showed that model parameters can be classified into two groups: those parameters influencing the individual's risk perception (γ and β) and those that influence the risk situation itself (ψ, λ, p_0, α_Y, the strength of the aggregate mortality shock, the strength of the mean return shift in the case of an asset meltdown, and ϑ). While the parameters from the first group, which are all *subjective* (preference) parameters, determine only the intensity of the individual's reaction to aggregate mortality risk, the parameters from the second group, which are all *objective* parameters, also influence the direction of the annuity demand reaction to aggregate mortality risk. In the numerical examples presented, we demonstrated this by varying ψ in such a way that in one case ($\psi = 0.6$), the annuity was perceived as being rather safe, while in the other case ($\psi = 0$), the annuity was perceived as rather risky due to its exposure to the systematic risks. As we have seen, the sign of the slopes of the individual's respective *ADRF* changed. From the later analyses, we now know that this result also would occur if we vary one (or more) of the other parameters

[22] A thorough analysis of this interplay is provided in an appendix that is available upon request from the authors.
[23] We tested the validity of our results by using several thousand numerical examples.

of the second group (i.e., parameters influencing the risk situation) such that the individual's exposure to systematic risks in general, and to aggregate mortality risk in particular, is increased. Any constellation of objective parameters thus creates a risk situation determining the sign of the slope of the individual's *ADRF*. The intensity of reaction within this risk setting is then only fine-tuned by the individual's subjective risk perception parameters. That means that even if ψ is rather high, a high value of the government pension reaction parameter α_Y can lead to a negatively sloped *ADRF*. Such a turn in *ADRF* cannot be achieved by increases in γ or β within their domains.

A first and important implication of our results derived in a partial equilibrium approach is that aggregate mortality risk, per se, does not increase the demand for annuities. Rather, it depends on the riskiness of the whole situation (or more exactly: the stochastic dependences with aggregate mortality shocks) in which potential insurance buyers decide how much of their wealth they are willing to annuitize. Parameters influencing this risk situation can be:

- individual specific (e.g., claims on the government pension system, or the expected survival probability),
- valid for all potential insurance buyers (e.g., the extent to which systematic insurance defaults can be absorbed, or in how far asset returns and/or the stability of the government pension system is exposed to aggregate mortality shocks), and
- controllable by the insurer (e.g., exposure to systematic risks or choice of premium loadings).

All the objective parameters together determine whether a potential insurance buyer will react with an increased or decreased demand for annuities when new information on the characteristics of aggregate mortality risk becomes available, e.g., by the publication of a new study in this field. Regarding this, it is not only the expected trend of future development in mortality but also the *precision* of the survival probability forecasts (with respect to both the extent and the probability of possible deviations) that are of importance.

People whose (future) government pension income is very high compared to present wealth already have a high exposure to aggregate mortality risk. For these people, annuities become less attractive with increasing aggregate mortality risk, even if the guarantee fund is working quite well and the insurer itself provides a robust solvency situation. For other potential insurance buyers who have a rather low or no government pension income, also at lower degrees of safety of the whole risk situation, higher uncertainty about future mortality can even result in an increase in annuity demand.

Since we have seen that implementing the consideration of aggregate mortality risk into individual decision making may result in both decreases and increases in annuity demand, our results may alleviate, but also intensify, the annuity puzzle. In any case, potential insurance buyers should check their aggregate mortality risk exposure when making an annuity purchasing decision. For such a decision, the direction of their demand reaction does not depend on their subjective risk perception characteristics.

When considering the specific market situation (e.g., the characteristics of potential insurance buyers) and additionally the costs that will accompany the actions taken, our model can help the insurance industry answer the following questions:

- How can strategic decisions with respect to premium policy, or measures that aim at decreasing idiosyncratic (via asset liability management) or systematic risk exposure (via financing guarantee funds), stimulate annuity demand in dependence of the severity of aggregate mortality risk?
- How will increased research activity with respect to the actual changes in mortality rates and public awareness of aggregate mortality risk lead to an increase or a decrease in annuity demand?
- What will be the consequences for the annuity business if the asset meltdown hypothesis turns out to be true?
- Can combinations of annuities with mortality derivatives aimed at reducing the annuities' vulnerability to aggregate mortality risk stimulate annuity demand?

For the field of insurance regulation, our model illustrates that the ruin probability alone is not sufficient to describe and control an insurer's solvency situation (see also Butsic, 1994; Wang, 1998). Additional measures, such as expected policyholder deficit or the tail value at risk, should be considered.[24]

SUMMARY AND DIRECTIONS FOR FURTHER RESEARCH

In this contribution, we studied the impact of uncertainty about future mortality developments (aggregate mortality risk) on individuals' annuity demand. Within a partial equilibrium, intertemporal expected utility maximization framework, we showed that aggregate mortality risk does not influence annuity purchasing decisions if it is stochastically independent from all other sources of income risk. However, for most potential insurance buyers, this condition does not hold. Rather, aggregate mortality risk appears to be an important risk that is difficult to transfer. From the perspective of the individual, the market for trading this risk is incomplete and thus he cannot insure it fully. We showed that whether the existence of aggregate mortality risk will lead to an increase or a decrease in individuals' annuity demand is determined by objective factors (such as the exposure of the government pension system and/or the insurance industry with respect to aggregate mortality shocks) and not by subjective factors (such as individual risk aversion). Subjective factors determine only the intensity of the annuity demand reaction to aggregate mortality risk. Our results show that consideration of aggregate mortality risk may alleviate, but also intensify, the annuity puzzle.

As a model extension, bequest motives together with the additional possibility to buy term life insurance contracts could be incorporated into the model leading to a certain hedging opportunity for the individual. Due to the systematic character of aggregate mortality risk, however, this hedging opportunity at the same time may increase the individual's (and his heirs') exposure to aggregate mortality risk as, depending on the vulnerability of the insurer to aggregate mortality risk, annuities and term life

[24] The Swiss solvency regulation already uses such measures (see, e.g., Keller, Luder, and Stober, 2005).

contracts may default at the same time, once more emphasizing the importance of aggregate mortality risk.

In a further step, the individual annuity demand model may be combined with a corporate model such as of Gründl, Post, and Schulze (2006). Within the arising game situation, the shareholder value maximizing insurer then would choose its optimal exposure to aggregate mortality and capital market risk. The resulting solvency situation of the insurer is then incorporated into the decision calculus of expected utility-maximizing individuals deciding on their demand for annuities and/or term insurances, which again is an input parameter for the calculus of the insurer. In a final step, a general equilibrium analysis considering several insurers and the impacts of aggregate mortality risk on other sectors of the economy could help estimate the total effect that this risk would have on the insurance market. The resulting solution could provide valuable input for policymakers concerned with insurance regulation and the consequences of the privatization of retirement systems.

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