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On Pricing and Hedging the No-Negative-Equity Guarantee in Equity Release Mechanisms

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Abstract

In a roll-up mortgage, the borrower receives a loan in the form of a lump sum. The loan is rolled up with interest until the borrower dies, sells the house, or moves into long-term care permanently. The house is sold at that time, and the proceeds are used to repay the loan and interest. Most roll-up mortgages are sold with a no-negative-equity guarantee (NNEG), which caps the redemption amount at the lesser of the face amount of the loan and the sale proceeds. The core of this study is to develop a framework for pricing and managing the risks of the NNEG.

Introduction

For many people, a shortfall in retirement income can be met by participating in home equity release mechanisms, which enable a householder to draw down part of the equity in the house. Such mechanisms are particularly helpful to "asset-rich, cash-poor" pensioners, who may be on relatively low incomes but who own homes of fairly substantial value. Equity release products are available in several developed countries including Canada, France, the United Kingdom, and the United States. This article focuses on the UK equity release market, since it is well developed and has been expanding rapidly since the early 1990s. The total value of new reverse equity loans written in the United Kingdom for full year 2007 reached £1,210.4 million, which is five times what it was in 1999 (Safe Home Income Plans, 2008; Institute of Actuaries, 2005). Given the aging of the baby boomers and the fact that UK pensioners are quite rich in home equity,[1] we expect equity release to become a core financial product, and an integral part of retirement provision for many.

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[1] Using data from the English Longitudinal Study of Ageing 2002–2003, Sodha (2005) estimated that in Britain, of those who were retired, 10.2 percent had an income below the "modest but

In 2005, roll-up mortgages, or fixed interest lifetime mortgages, account for over 95 percent of the equity release sales in Britain (Safe Home Income Plans, 2006). In a typical roll-up mortgage, the borrower is advanced a lump sum of money at the outset, and interest on the amount advanced is compounded at a fixed rate. The principal and interest are repaid from the property sale proceeds when the borrower dies, sells the house, or moves into long-term care permanently. Of course, given that the value of the property when the loan is repaid is uncertain, a shortfall in the home sales proceeds relative to the mortgage repayable is possible. However, most roll-up mortgages are sold with a no-negative-equity guarantee (NNEG), which protects the borrower by capping the redemption amount of the mortgage at the lesser of the face amount of the loan and the sale proceeds of the home.

From the lender's viewpoint, providing an NNEG is equivalent to writing the borrower a European put option on the mortgaged property. To illustrate, let us suppose that the amount of cash advanced is X and that the borrower will have to repay the loan at an unknown future time T. The amount repayable (Y) will be the sum of the principal and the interest accrued at a fixed roll-up rate u; that is, $Y = Xe^{uT}$. At time T, if the property value (S_T) exceeds the amount repayable, then the lender will receive Y without further financial obligations; otherwise, the lender will obtain only S_T. Equivalently speaking, when the loan is repaid, the provider will receive an amount of Y plus the NNEG payoff:

$$-\max[Y - S_T, 0],$$

which is precisely the payoff from a European put option with strike price Y and written on an underlying asset with time-zero value S_0.

Although the NNEG is similar to a standard European put option, there are some very significant differences. First, the property market lacks many of the attributes we associate with a perfect frictionless market. In particular, the underlying asset, that is, the mortgaged property, is unique and infrequently traded. Second, the time to maturity is random, since it depends on the timing when the borrower leaves the house permanently. Third, as opposed to typical equity returns, house price returns are highly autocorrelated, with significant effects of leverage and heteroskedasticity. The classical Black–Scholes option pricing formula, which assumes returns on the underlying asset follow a geometric Brownian motion, is therefore inappropriate for the NNEG.

Besides the NNEG, there are other options that are related to real estate performance. For example, Booth and Walsh (2001a, b) consider the upward only rent review option embedded in property lease contracts. In an upward only rent review lease structure, at each review, the rent can be set at the higher of the rent receivable before the review and the level of market rents at the review. This embedded option can therefore be regarded as a call option on the level of market rents. However, the valuation techniques for this option are significantly different from those for the NNEG, partly because this option has a fixed (rather than random) time to maturity, and partly because this option depends on rents, which may not behave like property

adequate" standard (£157 per week before housing costs), but owned more than £100,000 of housing wealth.

prices by which the payoff of a NNEG is determined (see the "Econometrics of House Prices" section).

The core of this study is to develop a framework for pricing and managing the risks of the NNEG. We first model the randomness of the time to maturity by using a stochastic mortality model that takes account of the uncertainty of future death rates. Then, we search for a stochastic volatility process that adequately models the features of the house price returns. However, the stochastic volatility process we have chosen implies market incompleteness, which means there exists more than one equivalent risk-neutral probability measure. A critical issue in this article is therefore to identify an equivalent martingale measure that gives a justifiable price for the guarantee. Given the stochastic models and the pricing measure, we derive a pricing formula and investigate how we may manage the risks associated with the guarantee.

The remainder of this article is organized as follows: the "Econometrics of House Prices" section provides an in-depth analysis of the econometrics of house prices, the "Pricing the NNEG" section identifies an equivalent martingale measure based on the dynamics of house price returns and derives a formula for pricing the NNEG, the "Hedging the NNEG" section discusses how the house price inflation risk can be hedged, finally, the "Concluding Remarks" section concludes the article.

ECONOMETRICS OF HOUSE PRICES

The House Price Index

Unlike options on listed stocks, the underlying asset of the NNEG is unique and infrequently traded, leaving us no historical prices to base our analysis on. To deal with this problem, we may consider house price indexes that reflect the changes in residential property prices within a particular geographical region. Booth and Marcato (2004) provide a comprehensive review of the pros and cons of various UK house price indexes. According to Geltner et al. (2007), property indexes can be divided into two categories: valuation based and price based.

Valuation-based indexes are based on appraised values (or appraisals), that is, surveyors' infrequent and subjective valuations of the properties in the index. The Investment Property Databank (IPD) residential real estate index falls into this category. The basic problem of valuation-based indexes, as identified by Hager and Lord (1985), is that there is a tendency for valuers to use historical comparables when forming an opinion of the value of a property. This problem, which is often called valuation smoothing, will result in a valuation that is too "tied" to previous sales prices in a rapidly moving market. Also, as valuation smoothing dampens the effect of random noise, the volatility of a valuation-based index tends to be smaller than that of the actual property prices. The problem of valuation smoothing can be solved by desmoothing, which is carried out by removing the excess serial correlation arising from the valuation process on the basis of some assumptions about the statistical characteristics of the underlying transaction price process. We refer interested readers to Brown and Matysiak (1998), Chaplin (1997), Fisher et al. (1994), and Quan and Quigley (1991) for descriptions of different desmoothing techniques.

Price-based indexes rely on using the transaction data from properties that are actually sold. Examples in this category include the Halifax Bank House Price Index, the

Nationwide House Price Index, and the HM Land Registry data. Unlike securities, which are normally sold several times within each index calculation period, real estate properties are seldom transacted twice between two index calculation dates. Therefore, any sample of properties used to compile a price-based index directly would represent a very small proportion of properties and would be a biased sample (e.g., more frequently traded properties would probably be prime properties). A common way to use real estate data to produce an index from transaction prices is to use hedonic modeling in which characteristics (e.g., location) of all houses are recorded and a regression equation is fitted for the relationship between the house prices and these characteristics. For example, we can write the price (S_i) of house i as

$$S_i = b_0 + b_1 X_{1i} + b_1 X_{2i} + \cdots + b_j X_{ji} + e_i,$$

where $b_1, \ldots, b_j$ are the regression coefficients; $X_{1i}, \ldots, X_{ji}$ are values representing different characteristics of house i; and e_i is an error term. An important property of price-based indexes is that they do not rely on valuations and therefore should not have the problem of valuation smoothing.

In this study, we use the Nationwide House Price Index (Nationwide Building Society, 2008), partly because it is long enough for a meaningful long-term statistical projection and partly because it is priced-based and thus requires no ad hoc desmoothing process. The index, available on a quarterly basis from 1953Q4 to 2008Q1 (218 observations), is derived from the average purchase price of the properties in Nationwide's own mortgage portfolio and is not seasonally adjusted. The mortgage portfolio consists of residential properties all over Britain but with a bias toward the south of England. To prevent this geographical bias from affecting the index, regional weightings in the index are based on the figures from the Department of the Environment, Transport, and the Region instead of the actual composition of Nationwide's portfolio.

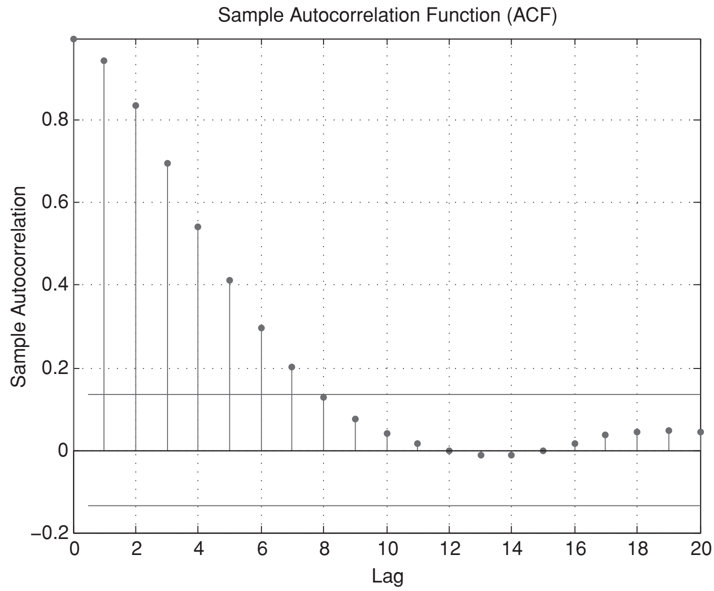
Since the Nationwide House Price Index is purely based on purchase prices with no consideration of rental income, returns on the index should be interpreted as capital value returns (i.e., excluding rents). As such, house price returns in this article refer to capital value returns unless otherwise specified.

No matter how representative the index is, returns on the index are unlikely to be identical to those on the mortgaged property. As a result, the use of a property index as a proxy will inevitably expose the lender to some basis risk. We will discuss the issue of basis risk in the "Hedging the NNEG" section.

Pre-Estimation Analysis

We perform a pre-estimation analysis to obtain an idea of what type of models may be suitable for modeling house price returns. First, we check for serial correlation in the return series. This correlation can be quantified by the sample autocorrelation function (ACF) of the log-returns. In Figure 1, we show the sample ACF of the log-returns, along with the upper and lower confidence bounds, based on the assumption that all autocorrelations are zero beyond lag zero. The sample ACF, which does not

FIGURE 1
Sample ACF of the Log-Return on the Nationwide House Price Index



cut off until the eighth lag, indicates a strong autocorrelation effect that lasts for approximately 2 years.

Second, we check for serial correlation in the conditional variance process. To perform this check, we filter out the serial correlation in the log-return series by fitting an autoregressive moving average (ARMA) model to capture the autocorrelation in the log-returns. In its general form, an ARMA(R, M) model can be expressed as

$$Y_t = c + \sum_{i=1}^{R} \phi_i Y_{t-i} - \sum_{j=1}^{M} \theta_j a_{t-j} + a_t,$$

where Y_t is the log house price return at time t, c is the drift term, and ϕ_i and θ_j quantify the dependence of Y_t on the ith previous log-return, Y_{t-i}, and the jth previous innovation, a_{t-j}, respectively. The innovations, a_{t-j}, $j = 1, \dots, M$, are assumed to be normally distributed with zero mean and constant variance. Using the Box and Jenkins (1976) approach, we found that an ARMA(1, 3) model gives the best fit. In Figure 2 we show the sample ACF of the residuals of the fitted ARMA(1, 3) model. We observe that the residuals are largely uncorrelated, indicating that the best fit ARMA model effectively captures most of the autocorrelation effect in the log-return series. Having filtered out the serial correlation in the return series, we can check for serial correlation in the conditional variance process by examining the sample ACF of the squared residuals, which is depicted in Figure 3. From Figure 3 we observe that the squared residuals are significantly correlated with one another. This correlation, which is sometimes referred to as conditional heteroskedasticity, can be further confirmed by

FIGURE 2
Sample ACF of the Residuals of the Best Fit ARMA Model

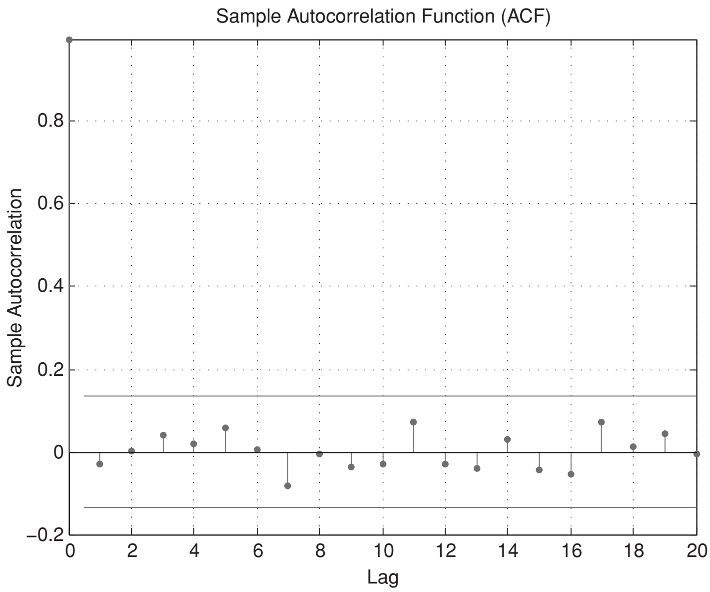


FIGURE 3
Sample ACF of the Squared Residuals of the Best Fit ARMA Model

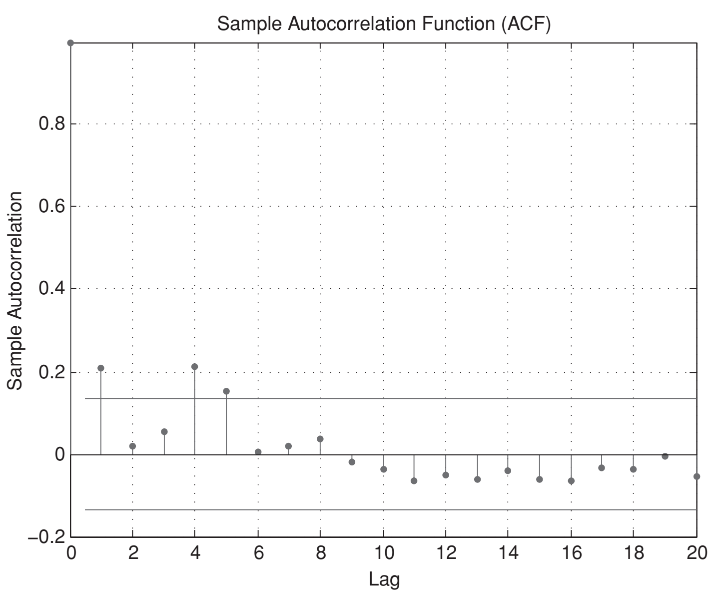
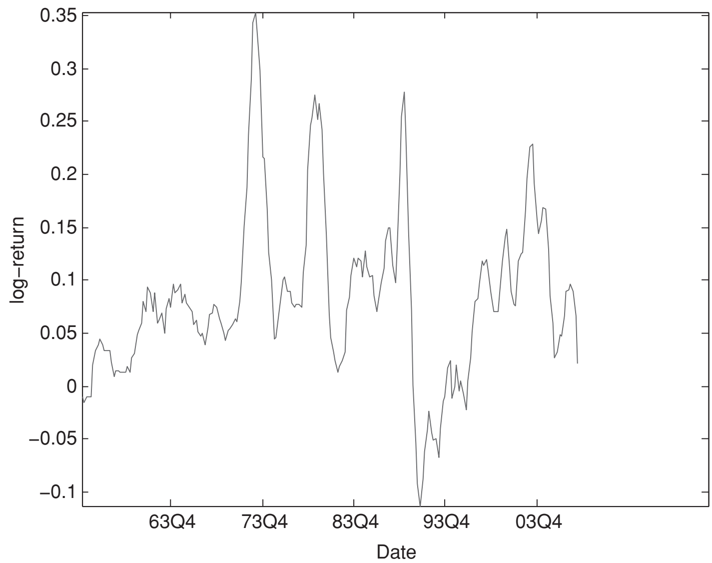


FIGURE 4
Log-Return on Nationwide House Price Index, 1953Q4-2008Q1



performing the Ljung–Box Portmanteau test (Ljung and Box, 1978)[2] on the squared residuals. The Ljung–Box Portmanteau test statistic gives a p-value of less than 10^{-4}, providing additional evidence for conditional heteroskedasticity.

Third, we check for asymmetric effects between positive and negative house price returns. From Figure 4, which shows the plot of the log-return series, we observe that volatility tends to rise in response to higher than expected returns and to fall in response to lower than expected returns. The phenomenon that volatility responds asymmetrically to negative and positive shocks is referred to as leverage effect.

In sum, the Nationwide House Price Index has the following statistical properties:

(1) there is a strong positive autocorrelation effect among the log-returns;

(2) the volatility of the log-returns varies with time;

(3) a leverage effect is present in the log-return series.

The Model

The traditional approach to modeling asset returns in the financial economics literature, including the original Black–Scholes paper, is to assume that in continuous time

[2] The Ljung–Box Portmanteau test statistic tests for the null hypothesis H_0: $\rho_1 = \rho_2 = \cdots = \rho_m = 0$ against the alternative hypothesis H_1: $\rho_i \neq 0$ for some $i \in \{1, 2, \ldots, m\}$, where ρ_i is lag i autocorrelation in the raw return series. The selection of m may affect the performance of the test statistic. Tsay (2002) recommended the use of $m \approx \ln(T)$, where T is the total number of observations. Following Tsay's recommendation, we use $m = 5 \approx \ln(218)$.

asset returns follow a geometric Brownian motion (GBM). This assumption has the following implications:

(1) For all $t > 0$, the single-period log-return, Y_t, is lognormally distributed with time-invariant parameters μ and σ; that is, there is no conditional heteroskedasticity and hence leverage effect.

(2) For all $t \neq u$, Y_t and Y_u are independent; that is, there is no autocorrelation in the log-return series.

The properties above mean that the GBM model does not suit the Nationwide House Price Index well.

Stochastic modeling of the UK real estate performance has been considered by several researchers including Wilkie (1995), Booth and Marcato (2004), and Booth and Walsh (2001b). Wilkie (1995) models the rental yield, $P(t)$, on a real estate portfolio by the following model:

$$\ln P(t) = PI \cdot I(t) \cdot \ln PMU + PA \cdot (\ln P(t-1) - \ln PMU) + PSD \cdot \mathrm{N}(0,1),$$

where PMU is the mean value, $I(t)$ is the rate of inflation at time t, PI is the elasticity of $P(t)$ with respect to $I(t)$, PA is the first-order autoregressive parameter, and $PSD \cdot \mathrm{N}(0,1)$ is a random term. Booth and Marcato (2004) extend the Wilkie (1995) model by introducing the following additional parameters:

- $D(t)$: dividend yield from equities;
- $C(t)$: redemption/running yield from consols; and
- $T(t)$: return from Treasury bills (as a proxy for short-term interest rates).

The generalized model therefore becomes:

$$\begin{aligned}\ln P(t) = {} & PI \cdot I(t) + PD \cdot \ln D(t) + PC \cdot \ln C(t) + \ln PMU \\ & + PA.(\ln P(t-1) - \ln PMU) + PT \cdot T(t) + PSD.\mathrm{N}(0,1),\end{aligned}$$

where PD, PC, and PT are the elasticities of $P(t)$ with respect to $D(t), C(t)$, and $T(t)$, respectively. In these models, parameter PA captures some of the serial correlations in the rental yield. Booth and Walsh (2001b) consider two continuous-time processes for modeling the IPD annual index of office rental values. They are the GBM, which we have shown to be inappropriate for the Nationwide House Price Index, and the Ornstein–Uhlenbeck process, which is equivalent to a first-order autoregressive process if observations are made in unit intervals.

Unfortunately, for a number of reasons, the models above are not suitable for the house price index we consider in this study. First, these models do not provide for the conditional heteroskedasticity and leverage effect that we observe in the index. Second, since there is at most one autoregressive term in these models, they are not likely to capture the higher order autocorrelation we found in the log-returns. Third, these models, originally designed for modeling rental yields, may give a good fit to indexes of property rental values but may not to indexes of house price appreciation. An alternative model for the house price index is therefore necessary.

Given the statistical properties of the house price index, we consider a conditional heteroskedastic time-series model, which consists of one submodel for modeling conditional mean and another for modeling conditional variance. The first submodel is an ARMA(R, M) process that models the single-period log-return, Y_t, at time t by the following equation:

$$Y_t = c + \sum_{i=1}^{R} \phi_i Y_{t-i} - \sum_{j=1}^{M} \theta_j a_{t-j} + a_t,$$

where c is the drift term, and ϕ_i and θ_j are the weight on the previous ith log-return, Y_{t-i}, and the previous jth innovation, a_{t-j}, respectively. This process provides for the autocorrelation effect by allowing the log-return at time t to be dependent on the previous R log-returns as well as the previous M innovations. We require both $1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_R B^R$ and $1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_M B^M$ to have no common factors and have zeros lying outside the unit circle[3] to ensure that the ARMA process is weakly stationary.[4] This process, in contrast to the ARMA model we considered in the pre-estimation analysis (the "Pre-estimation Analysis" section), allows the variance of a_t to vary with time. Specifically, we assume in this model that a_t follows a normal distribution with mean zero and variance h_t, which is time-varying. Given the information up to and including time $t - 1$, the variance of Y_t is simply h_t. The serial dependence of the conditional variance h_t is modeled by the second submodel model.

Possible candidates for the second submodel include, but are not limited to, the autoregressive conditional heteroskedastic (ARCH) model (Engle, 1982) and its variants such as the generalized ARCH (GARCH) model (Bollerslev, 1986) and the exponential GARCH (EGARCH) model (Nelson, 1991). Although all these models provide for heteroskedasticity, we use an EGARCH model due to its ability to quantify the leverage effect that we found in the Nationwide House Price Index. An EGARCH(P, Q) process models the logarithm of the conditional variance, $\ln(h_t)$, at time $t > 0$ by the following equation:

$$\ln(h_t) = k + \sum_{i=1}^{P} \alpha_i \ln(h_{t-i}) + \sum_{j=1}^{Q} \beta_j [|\tilde{a}_{t-j}| - \mathrm{E}(|\tilde{a}_{t-j}|)] + \sum_{j=1}^{Q} \gamma_j \tilde{a}_{t-j}, \tag{1}$$

where $\tilde{a}_t = a_t/\sqrt{h_t}$ is the standardized innovation at time t, and α_i and β_j are the weight on the previous ith log conditional variance, $\ln(h_{t-i})$, and the jth previous standardized innovation, $\tilde{a}_{t-j}$, respectively. An EGARCH process provides for the

[3] By outside the unit circle, we mean that absolute values of the zeros are greater than 1.

[4] A time-series $\{y_t\}$ is said to be weakly stationary if both $\mathrm{E}(y_t)$ and $\mathrm{cov}(y_t, y_{t-l})$, where l is an arbitrary integer, are time invariant. We refer readers to Tsay (2002) and Wei (2006) for further information regarding the concept of stationarity.

leverage effect through the leverage parameters, γ_j, $j = 1, \ldots, Q$, which allow conditional variances to respond asymmetrically to positive and negative innovations. For instance, if $\gamma_j > 0$, then a positive lag j standardized innovation ($\tilde{a}_{t-j}$) will bring higher conditional variance than a negative $\tilde{a}_{t-j}$ of the same magnitude will do. Since an EGARCH process models the logarithm of the conditional variance, we require no parameter constraints to ensure positive conditional variances. We only require $1 - \alpha_1 B - \alpha_2 B^2 - \cdots - \alpha_P B^P$ and $1 - \beta_1 B - \beta_2 B^2 - \cdots - \beta_Q B^Q$ to have no common factors and have zeros lying outside the unit circle to ensure that the EGARCH process is weakly stationary.

There is no simple way to specify the order of an EGARCH model, but lower order EGARCH models are sufficient in most applications. Following Box and Jenkins (1976), we use an iterative procedure to jointly estimate the optimal ARMA order, (R, M), and EGARCH order, (P, Q). Starting with $(R, M) = (0, 0)$ and $(P, Q) = (0, 1)$, we gradually increase the values of R, M, P, and Q until we reach an ARMA-EGARCH model that can adequately capture the serial correlation in both log-returns and conditional variances. We determine whether or not a specification is adequate by considering the sample ACF and the Ljung–Box portmanteau test statistic for both (1) standardized innovations and (2) squared standardized innovations. From this iterative procedure, we found that the most parsimonious specification that gives an adequate fit is ARMA(1, 3)-EGARCH(1, 1). It is interesting to note that although the EGARCH conditional variance process may capture some of the autocorrelation effect in the log-return series, an ARMA order of (1, 3) is still necessary for an adequate fit in this application.

All parameters of the jointly estimated ARMA(1, 3)-EGARCH(1, 1) model are highly significant (see Table 1). We can verify the adequacy of the fitted model by examining the standardized innovations, $\tilde{a}_t = a_t/\sqrt{h_t}$. Figures 5 and 6 show the sample ACFs for $\{\tilde{a}_t\}$ and $\{\tilde{a}_t^2\}$, respectively. These ACFs indicate that the fitted ARMA-EGARCH model has effectively captured the correlations in the house price returns and that in the conditional variances. The Ljung–Box Portmanteau test statistic for $\{\tilde{a}_t\}$ gives a p-value of 0.2977 and that for $\{\tilde{a}_t^2\}$ gives a p-value of 0.7444, providing no evidence that $\{\tilde{a}_t\}$ and $\{\tilde{a}_t^2\}$ are serially correlated. The results of the Ljung–Box Portmanteau test further confirm the explanatory power of the fitted ARMA-EGARCH model.

TABLE 1
Parameter Estimates of the Fitted ARMA(1,3)-EGARCH(1,1) Model

Parameter	Value	Standard Error	t-Statistic
c	0.0188	0.0055	3.4182
ϕ_1	0.7085	0.0586	12.0904
θ_1	0.8117	0.0608	13.3503
θ_2	0.7773	0.0680	11.4309
θ_3	0.6906	0.0454	15.2115
k	−1.3621	0.5475	−2.4879
α_1	0.8321	0.0675	12.3274
β_1^*	0.3556	0.1053	3.3770
γ_1	0.1034	0.0413	2.5036

FIGURE 5
Sample ACF of the Standardized Innovations of the Fitted ARMA(1,3)-EGARCH(1,1) Model

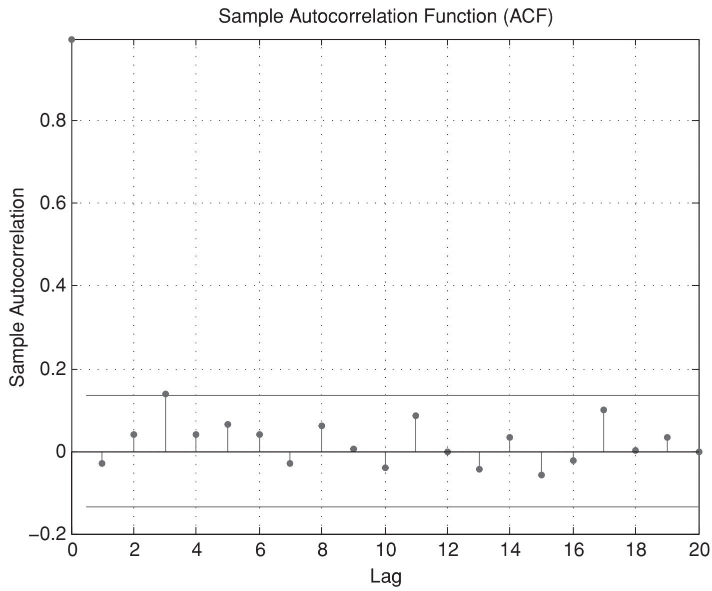


FIGURE 6
Sample ACF of the Squared Standardized Innovations of the Fitted ARMA(1,3)-EGARCH(1,1) Model

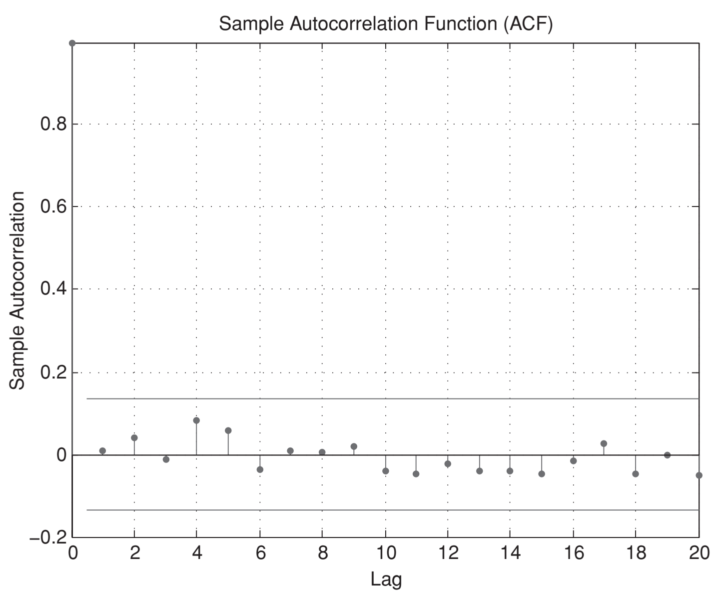
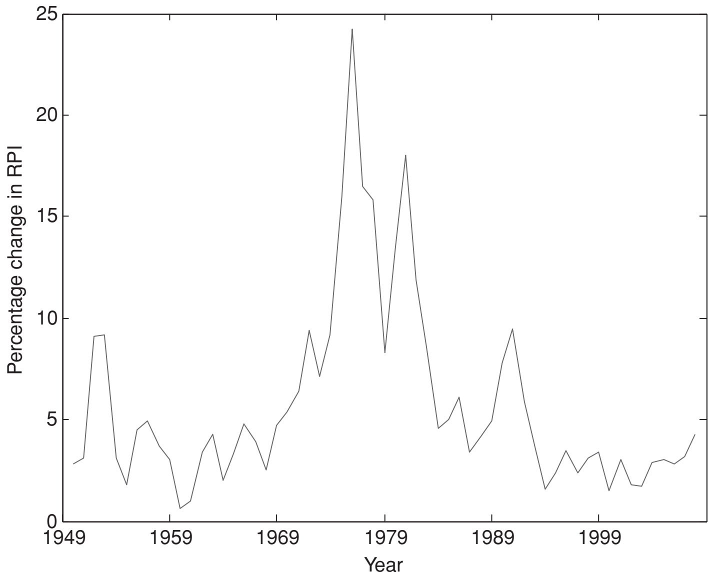


FIGURE 7
Annualized Percentage Change in the UK Retail Prices Index, 1949–2007



Source: Office for National Statistics.

Stability of the Model

From the diagnostic check in Section "The model" we conclude that the jointly estimated ARMA(1, 3)-EGARCH(1, 1) model provides a sufficient fit to the Nationwide House Price Index from 1954-Q1 to 2008-Q1. However, the diagnostic check provides no information regarding the following issues:

1. whether or not the model form is stable over time;
2. whether or not the model form is robust to extra data;
3. whether or not the model form will change in different inflation environments.

From Figure 7, which depicts the historical changes in the UK Retail Prices Index (RPI), we observe that the UK economy has experienced episodes of high (1970–1990) and low (1949–1969, 1991–2007) regimes of inflation. To address the three issues aforementioned, particularly the third one, we perform a subsample analysis in which we fit time-series processes to the Nationwide House Price Index for subperiods 1970-Q1 to 1990-Q4 (when inflation was high) and 1993-Q3 to 2008-Q1 (when inflation was low). We found that the ARMA(1, 3)-EGARCH(1, 1) specification is adequate for both subperiods. This conclusion can be drawn from the p-values of the relevant Ljung–Box Portmanteau tests (see Table 2) and the patterns of the relevant sample ACFs (not shown).

Although the form of ARMA(1, 3)-EGARCH(1, 1) is fairly stable across a range of circumstances, the model has some limitations that are common to all GARCH-type models. One problem is that the model may fail to capture highly irregular

TABLE 2
The p-Values of the Ljung-Box Portmanteau Tests Applied to $\{\tilde{a}_t\}$ and $\{\tilde{a}_t^2\}$ Resulting from the Best-Fit ARMA(1,3)-EGARCH(1,1) for Subperiods 1970-Q1 to 1990-Q4 and 1993-Q3 to 2008-Q1

	p-Value	
	1970-Q1 to 1990-Q4	1993-Q3 to 2008-Q1
Standardized residuals, $\{\tilde{a}_t\}$	0.8760	0.2165
Squared standardized residuals, $\{\tilde{a}_t^2\}$	0.8016	0.9822

phenomena, including wild market fluctuations (e.g., crashes and later rebounds) and other unanticipated events that can lead to significant structural changes. Another problem is that estimation may sometimes be difficult due to the constraints that ensure economically feasible parameter values. An ARMA-EGARCH specification guarantees positive conditional variances, but model parameters are still required to satisfy four constraints that ensure the processes for conditional mean and variance are weakly stationary (see the "The Model" section).

PRICING THE NNEG

Identification of an Equivalent Martingale Measure

Given that the NNEG is very similar to a standard European put option, a natural starting point for placing a value on the NNEG is to consider the classical Black–Scholes option pricing methodology. A key assumption in the Black–Scholes formula is that the returns on the underlying asset follow a GBM. The GBM assumption, as we mentioned in the "The Model" section, is not appropriate for the house price returns in which serial correlation is strong and volatility is clearly time varying. An ARMA-EGARCH model is more suitable for modeling the house price returns, but the use of such a model implies market incompleteness.

Market completeness refers to the situation that the payoff of an option can be obtained as the terminal value of a self-financing portfolio. Hence, by the principle of no arbitrage, the price of the option at any time before the maturity date must be the value of the replicating portfolio at that time. Equivalently speaking, under a complete market, there exists only one equivalent martingale (risk-neutral) probability measure, and the price of any option is simply its expected discounted payoff at the maturity date under the martingale measure. On the other hand, if a market is incomplete, there exists more than one equivalent martingale measure, leading to a range of no-arbitrage prices for an option. Therefore, one critical issue in this study is to identify an equivalent martingale measure that gives an economically consistent and justifiable price for the NNEG.

The identification of such a martingale measure can be performed in various ways; for example, Rubinstein (1976) adopted the equilibrium or utility-maximization-based approach; Föllmer and Sodermann (1986), Föllmer and Schweizer (1986), and Schweizer (1996) identified a unique equivalent measure by minimizing the variance of the hedging loss; Gerber and Shiu (1994) suggested a feasible way to choose an equivalent martingale measure using the Esscher transform, introduced by Esscher

(1932). In option pricing under the GARCH framework, Duan (1995) used the equilibrium approach and the concept of locally risk-neutral relationship (LRNVR) in which the conditional variances under the physical and risk-neutral measures are equal; Siu et al. (2004) utilized the conditional Esscher transform, which was first introduced by Bühlmann et al. (1996). The advantage of using the conditional Esscher transform is that different distributions, such as translated gamma, can be assumed for the GARCH innovations. Under the conditional normality assumption for the GARCH innovations, the results of Duan and Siu et al. are consistent.

However, in previous work, the conditional mean is assumed to follow a GARCH-in-mean specification, instead of an ARMA process that we found to be highly suitable for describing the serial correlation in the house price returns. To overcome this problem, we extend the work of Siu et al. (2004) to obtain an equivalent martingale measure when the house price returns in the physical measure follow an ARMA-EGARCH process.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space, where $\mathbb{P}$ is the data-generating probability measure. We assume that all economic activities take place at time t, where $t \in \mathcal{T}$ and $\mathcal{T}$ is the time index set $\{0, 1, \ldots, T\}$. Also, let $\Phi = \{\Phi_t\}_{t \in \mathcal{T}}$ be the natural filtration such that, for each $t \in \mathcal{T}$, Φ_t contains all market information up to and including time t, and that $\Phi_T = \mathcal{F}$.

Let us assume that, under $\mathbb{P}$, the log house price returns, $\{Y_t\}_{t \in \mathcal{T}}$, follow an ARMA($R$, M)-EGARCH(P, Q) process with Gaussian innovations. It follows that under $\mathbb{P}$, $Y_t \mid \Phi_{t-1} \sim \mathrm{N}(\mu_t, h_t)$, where $\mu_t = c + \sum_{i=1}^{R} \phi_i Y_{t-i} - \sum_{j=1}^{M} \theta_j a_{t-j} + a_t$, and h_t is specified by Equation (1). Note that μ_t and h_t are nonrandom given the information Φ_{t-1}.

Following Bühlmann et al. (1996), we define a sequence $\{Z_t\}_{t \in \mathcal{T}}$ with $Z_0 = 1$, and for $t \geq 1$,

$$Z_t = \prod_{k=1}^{t} \frac{e^{\lambda_k Y_k}}{\mathrm{E}(e^{\lambda_k Y_k} \mid \Phi_{k-1})},$$

for some constants $\lambda_1, \lambda_2, \ldots, \lambda_t$. It can be shown easily that $\{Z_t\}_{t \in \mathcal{T}}$ is a martingale.

Let $\mathbb{P}_t$ be the restriction of the measure $\mathbb{P}$ on the information Φ_t, for each $t \in \mathcal{T} \backslash \{0\}$, where $\mathbb{P}_T = \mathbb{P}$. The fact that $\{Z_t\}_{t \in \mathcal{T}}$ is a martingale allows us to construct a family of measures $\{\tilde{\mathbb{P}}_t\}_{t \in \mathcal{T}}$ such that $d\tilde{\mathbb{P}}_t = Z_t \; d\mathbb{P}_t$ and $\tilde{\mathbb{P}}_t = \tilde{\mathbb{P}}_{t+1} \mid \Phi_t$, and a probability measure $\tilde{\mathbb{P}} = \tilde{\mathbb{P}}_T$ on the sample space $(\Omega, \mathcal{F})$. Let A be an open interval on the real line, and $I_{\{Y_t \in A\}}$ be the indicator function for the event $\{Y_t \in A\}$. The conditional distribution

$$\tilde{\mathbb{P}}_t(Y_t \in A \mid \Phi_{t-1}) = \mathrm{E}_{\mathbb{P}_t}\left[I_{\{Y_t \in A\}} \frac{e^{\lambda_t Y_t}}{\mathrm{E}_{\mathbb{P}_t}(e^{\lambda_t Y_t})} \,\middle|\, \Phi_{t-1} \right] \tag{2}$$

is called the conditional Esscher transform. Substituting $(-\infty, y]$, where y is a real number, for A in Equation (2), we obtain the following distribution function of Y_t

given Φ_{t-1} under the measure $\tilde{\mathbb{P}}_t$:

$$F_{\tilde{\mathbb{P}}_t}(y;\lambda_t \mid \Phi_{t-1}) = \frac{\displaystyle\int_{-\infty}^{y} e^{\lambda_t x}\, dF_{\mathbb{P}_t}(x \mid \Phi_{t-1})}{\mathrm{E}_{\mathbb{P}_t}(e^{\lambda Y_t} \mid \Phi_{t-1})}.$$

It immediately follows that the moment generating function of Y_t given Φ_{t-1} under the measure $\tilde{\mathbb{P}}_t$ is given by

$$\mathrm{E}_{\tilde{\mathbb{P}}_t}[e^{Y_t z};\lambda_t \mid \Phi_{t-1}] = \frac{\mathrm{E}_{\mathbb{P}_t}[e^{Y_t(z+\lambda_t)} \mid \Phi_{t-1}]}{\mathrm{E}_{\mathbb{P}_t}[e^{Y_t(\lambda_t)} \mid \Phi_{t-1}]}. \tag{3}$$

Using Equation (3) and the fact that $Y_t \mid \Phi_{t-1} \sim \mathrm{N}(\mu_t, h_t)$, we have

$$\mathrm{E}_{\tilde{\mathbb{P}}_t}[e^{Y_t z};\lambda_t \mid \Phi_{t-1}] = e^{(\mu_t + h_t\lambda_t)z + \frac{1}{2}h_t z^2}. \tag{4}$$

To construct a risk-neutral probability measure $\mathbb{Q}$, which is equivalent to the data-generating measure $\mathbb{P}$ on the sample space $(\Omega, \mathcal{F})$, we choose a sequence of conditional Esscher parameters $\{\lambda_t^q\}_{t\in\mathcal{T}\setminus\{0\}}$ such that the following set of equations are satisfied:

$$\mathrm{E}_{\tilde{\mathbb{P}}_t}\left[e^{Y_t};\lambda_t^q \,\middle|\, \Phi_{t-1}\right] = e^{r-g}, \quad t \in \mathcal{T}\setminus\{0\},$$

where r is the risk-free interest rate and g is the rental yield on the portfolio of properties from which the house price index is constructed.[5] We refer interested readers to Bühlmann et al. (1996) for a detailed proof. Substituting λ_t^q into Equation (4), we obtain

$$\mathrm{E}_{\mathbb{Q}_t}[e^{Y_t z} \mid \Phi_{t-1}] = e^{(r-g-\frac{1}{2}h_t)z + \frac{1}{2}h_t^2},$$

where $\mathbb{Q}_t$ is the restriction of $\mathbb{Q}$ on the information Φ_t, and $\mathbb{Q}_T = \mathbb{Q}$. Hence, under $\mathbb{Q}$, $Y_t \mid \Phi_{t-1} \sim \mathrm{N}(r - g - h_t/2, h_t)$, or equivalently, $Y_t = \mu_t + a_t$ and $a_t \sim \mathrm{N}(-\mu_t + r - g - h_t/2, h_t)$. In other words, the dynamics of Y_t under the equivalent risk-neutral measure are the same as that under the physical measure, except that the distribution of a_t is translated by an amount of $-\mu_t + r - g - h_t/2$.

Finally, under the equivalent martingale measure $\mathbb{Q}$, the time $t, t \in \mathcal{T}$, the price of an option that gives a payoff V_T at maturity is given by

$$V_t = \mathrm{E}_{\mathbb{Q}}[e^{-r(T-t)}V_T \mid \Phi_t].$$

[5] In the risk-neutral world, the expected total return from any asset is the risk-free rate, r. Since the total return on the house price index is the sum of the capital return and rental income (net of insurance and maintenance costs) from the portfolio of properties from which the index is constructed, the expected return on the house price index (i.e., the capital value return) in the risk-neutral world is r less the rental yield g.

The Pricing Formula

Let us define the following notation:

- X: the amount of loan advanced at time zero;
- S_t: the value of the mortgaged property at time t;
- r: the risk-free interest rate;
- u: the roll-up interest rate;
- g: the rental yield;
- $P(t, S_0, X, u, r, g)$: the time-zero value of a put option on an asset with a dividend yield g; the option matures at time t and has a strike price Xe^{ut}; and
- δ: the average delay in time from the point of home exit until the actual sale of the property.

Using discrete annual time steps, the value of a NNEG in a portfolio of roll-up mortgages sold to persons who are aged x at inception can be expressed as

$$V_{NNEG} = \sum_{k=0}^{\omega-x-1} {}_k p_x \, q_{x+k} P\left(k + \frac{1}{2} + \delta, S_0, X, u, r, g\right), \tag{5}$$

where ω is the highest attained age, ${}_k p_x$ is the probability that a borrower who is aged x at inception survives to age $x + k$, and q_{x+k} is the probability that a borrower who is aged x at inception dies in the time interval k to $k + 1$ given that he or she has survived to age $x + k$. Assuming that all deaths occur at midyear, the time to maturity for each put option is $k + 1/2 + \delta$.

Furthermore, if we assume that the house price returns follow an ARMA-GARCH process, then $P(k + \frac{1}{2} + \delta, S_0, X, u, r, g)$ is given by

$$e^{-r(k+\frac{1}{2}+\delta)} E_{\mathbb{Q}}\left[\left(Xe^{u(k+\frac{1}{2}+\delta)} - S_{k+\frac{1}{2}+\delta}\right)^+\right], \tag{6}$$

where $\mathbb{Q}$ is the equivalent martingale measure identified in the "Identification of an Equivalent Martingale Measure" section. The expectation in Expression (6) can be evaluated numerically using Monte Carlo simulations.

We use the following baseline assumptions to illustrate the use of Equation (6) to value the guarantee:

- The average delay in time from the point of home exit until the actual sale of the property is 6 months.
- The risk-free interest rate is 4.56 percent per annum, compounded continuously. This rate is the average yield from 20-year nominal zero-coupon British government securities in year 2007.[6]
- The roll-up rate is 6.39 percent per annum, compounded continuously. This is the average roll-up rate for the top 10 UK equity release providers in May 2007.[7]

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[6] Source: Bank of England.
[7] Source: Safe Home Income Plans.

TABLE 3
Minimum Initial House Values

Age at Inception	60	70	80	90
Initial house value	£176,500	£111,000	£81,000	£60,000

TABLE 4
NNEG Costs as a Percentage of the Cash Advanced

	Age at Inception							
	Period Life Tables (2007)				Projected Cohort Life Tables			
Model	60	70	80	90	60	70	80	90
Males								
EGARCH	3.47%	2.85%	1.60%	1.09%	4.51%	3.38%	1.75%	1.12%
GBM	2.58%	2.03%	0.93%	0.53%	3.65%	2.58%	1.09%	0.56%
Females								
EGARCH	5.50%	4.57%	2.49%	1.54%	7.41%	5.70%	2.90%	1.67%
GBM	4.51%	3.57%	1.64%	0.83%	6.41%	4.57%	1.92%	0.89%

- The rental yield is 2 percent per annum, compounded continuously.
- The amount of cash advanced is £30,000.
- Death is the only mode of decrement.
- The following two sets of life tables are used:
  (1) Period life tables for the English and Welsh population in 2007.
  (2) Cohort life tables derived from a Lee–Carter mortality projection (Lee and Carter, 1992) for the English and Welsh population.

  Both sets of life tables are extended to age 100 and above by the threshold life table technique proposed by Li et al. (2008). The required mortality data are provided by the Human Mortality Database (2008).
- The initial house values are equal to the minimum permitted values shown in Table 3. These values are broadly in line with products currently available in the UK equity release market.

The resulting NNEG values, expressed as a percentage of the cash advanced, are displayed in Table 4. In Table 4 we also compare the NNEG values based on the ARMA-EGARCH process with those based on the GBM assumption. The NNEG values based on the GBM assumption are computed by replacing $P(k+\frac{1}{2}+\delta, S_0, X, u, r, g)$ in Equation (5) with the Black–Scholes formula:

$$Xe^{((u-r)(k+\frac{1}{2}+\delta))}\mathrm{N}(-d_2) - S_0\,e^{(-g(k+\frac{1}{2}+\delta))}\mathrm{N}(-d_1),$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - g - u + \frac{\sigma^2}{2}\right)\left(k + \frac{1}{2} + \delta\right)}{\sigma\sqrt{k + \frac{1}{2} + \delta}},$$

and $d_2 = d_1 - \sigma\sqrt{k + \frac{1}{2} + \delta}$. Here the values of X, S_0, δ, r, u, and g are the same as those we use in the ARMA-EGARCH framework, and we estimate σ from the historical returns on the house price index.

On the basis of the ARMA-EGARCH assumption, the cost of the guarantee ranges from 1.1 percent to 7.4 percent of the cash advanced, depending on the gender and age of the borrower. For all genders and ages, the GBM assumption yields lower NNEG prices, indicating that we may risk underpricing the guarantee if the classical Black–Scholes approach is used. From Table 4 we also observe that the NNEG values based on projected mortality are higher than those based on current mortality. The impact of mortality improvement is very significant on guarantees issued to persons at lower ages; for example, the change for 60-year-old males is more than 50 percent. Providers of NNEG should therefore be careful not to underestimate future reduction in mortality rates.

Alternatively, we can express the NNEG values as a yield cost per annum. Let y be the annual yield cost, compounded continuously. If the loan is repaid at a future time t, then the present value of the NNEG margin is given by

$$M(t, X, u, r, y) = e^{-rt} X(e^{ut} - e^{(u-y)t}).$$

The annual yield cost y is derived so as to produce the expected present value of NNEG margin equal to the monetary cost of the NNEG. In other words, we solve the following equation for y:

$$\sum_{k=0}^{\omega-x-1} {}_k p_x \, q_{x+k} \left[P\left(k + \frac{1}{2} + \delta, S_0, X, u, r, g\right) - M\left(k + \frac{1}{2} + \delta, X, u, r, y\right)\right] = 0.$$

Table 5 shows the NNEG values, expressed as an annual yield cost, for different genders and ages. Note that the spread $u - r$, which equals 1.83 percent under our assumptions, is the lender's loading for profit, expenses and contingencies. The results we show in Table 5 therefore indicate that, in terms of an annual yield cost, the NNEG accounts for approximately 5–16 percent of the assumed loading. Also, note that the higher the age at inception is, the shorter the expected time during which the yield cost is paid. As a result, as an annual yield cost, the NNEG values do not necessarily decrease with the age at inception.

Readers are reminded that the option prices in this article are based on index returns, but the options are sold with their prices depending on individual property prices. Guarantee prices in practice depend on individual house price volatility, which can be substantially different than the index volatility (the volatility of the average of

TABLE 5
NNEG Costs as an Annual Yield Cost

	Age at Inception							
	Period Life Tables (2007)				Projected Cohort Life Tables			
Model	60	70	80	90	60	70	80	90
Males								
EGARCH	0.10%	0.15%	0.16%	0.22%	0.12%	0.17%	0.17%	0.22%
GBM	0.07%	0.10%	0.09%	0.11%	0.10%	0.13%	0.11%	0.11%
Females								
EGARCH	0.13%	0.19%	0.21%	0.27%	0.17%	0.23%	0.23%	0.29%
GBM	0.11%	0.15%	0.13%	0.14%	0.14%	0.18%	0.15%	0.15%

TABLE 6
NNEG Costs (as a Percentage of Cash Advanced) Under Different Scenarios, Males

	Age at Inception			
Scenario	60	70	80	90
Baseline				
$u = 6.39\%, g = 2\%, \delta = 0.5$	4.51%	3.38%	1.75%	1.12%
Roll-up rate				
$u = 7.39\%$ ($\uparrow$ 1%)	14.92%	8.91%	3.80%	1.99%
$u = 5.39\%$ ($\downarrow$ 1%)	1.32%	1.26%	0.81%	0.63%
Rental yield				
$g = 3\%$ ($\uparrow$ 1%)	10.59%	6.93%	3.18%	1.77%
$g = 4\%$ ($\uparrow$ 2%)	20.64%	12.17%	5.22%	2.62%
Delay in sale of property				
$\delta = 1.5$ ($\uparrow$ 1 year)	7.03%	5.59%	3.19%	2.38%
$\delta = 2.5$ ($\uparrow$ 2 years)	11.12%	9.19%	5.72%	4.99%
Mortality				
20% lighter	6.53%	5.14%	2.84%	1.89%
20% heavier	3.33%	2.44%	1.19%	0.71%

house prices). Lenders may want to charge difference prices for houses with different price volatility.

Sensitivity Testing

The results in the "The Pricing Formula" section have not taken any account of the uncertainty in the assumptions used. Here we perform a sensitivity analysis to examine how much the guarantee would cost under different scenarios of (1) roll-up rate (u), (2) rental yield (g), (3) delay in property sale (δ), and (4) mortality. Tables 6 and 7 show the NNEG costs based upon various scenarios.

The cost of the guarantee is highly sensitive to the assumption on the roll-up interest rate; for example, an increase in u by 1 percent would raise the value of the NNEG for

TABLE 7
NNEG Costs (as a Percentage of Cash Advanced) Under Different Scenarios, Females

	Age at Inception			
Scenario	60	70	80	90
Baseline				
$u = 6.39\%, g = 2\%, \delta = 0.5$	7.41%	5.70%	2.90%	1.67%
Roll-up rate				
$u = 7.39\%$ (↑ 1%)	24.41%	14.55%	6.08%	2.87%
$u = 5.39\%$ (↓ 1%)	2.06%	2.02%	1.28%	0.91%
Rental yield				
$g = 3\%$ (↑ 1%)	16.46%	10.86%	4.85%	2.43%
$g = 4\%$ (↑ 2%)	31.82%	19.13%	8.10%	3.67%
Delay in sale of property				
$\delta = 1.5$ (↑ 1 year)	11.00%	8.71%	4.75%	3.17%
$\delta = 2.5$ (↑ 2 years)	17.15%	14.05%	8.31%	6.34%
Mortality				
20% lighter	9.97%	7.87%	4.22%	2.55%
20% heavier	5.92%	4.35%	2.04%	1.08%

a 60-year-old male borrower by more than 200 percent. Increasing the roll-up rate will widen the spread $(u - r)$ for profit and expenses but will dramaticially increase the burden from the NNEG. Lenders should be aware of this relationship when pricing a roll-up mortgage contract.

Since the borrower of a reverse mortgage is both the owner and occupant of the mortgaged property, the borrower is effectively renting the property to himself or herself. In an arbitrage-free pricing model, the rental income from the property (even if notional) is relevant to the price of the guarantee. From Tables 6 and 7 we observe that the rental yield assumption is quite important; for example, the NNEG cost for a 60-year-old male borrower would raise by about 130 percent if g is increased by 1 percent, and by about 350 percent if g is increased by 2 percent.

Marketability and liquidity of the mortgaged property would also affect the price of the NNEG. A longer delay between home exit and sale of property will raise the cost of the guarantee. An additional 1-year delay would increase, for example, the NNEG cost for a 60-year-old male borrower by approximately 50 percent.

Mortality deduction will result in a more valuable NNEG due to the increase in both the time to maturity[8] and the strike price of the option. In the baseline scenario, we estimate future mortality reduction by the Lee–Carter model. However, there is substantial uncertainty involved in making the projection. Should the actual mortality

[8] The value of an European put option does not necessarily increase with the time to maturity if the strike price is fixed. However, for a straight Black–Scholes framework, the value of a put option will be an increasing function of the time to maturity if the strike price increases at a rate greater than the risk-free interest rate.

be 20 percent lighter than the projected mortality (i.e., values $q_x, x = 60, 61, \ldots$, are reduced by 20 percent), then the NNEG values would increase by 30–70 percent.

On the other hand, morbidity and early redemption would lower the NNEG values, since these additional modes of decrement would reduce the likelihood of survival to advanced ages. In the baseline scenario, we assume that death is the only mode of decrement. Here we consider how the NNEG values would be affected if mortality is 20 percent heavier than the baseline assumption due to morbidity and early redemption. From Tables 6 and 7 we observe that the NNEG costs would increase by 20–30 percent if decrement is 20 percent heavier than expected.

HEDGING THE NNEG

Although the underlying asset for the NNEG is unique and infrequently traded, we can dynamically hedge the guarantee by forming a hedge portfolio that has a price sensitivity profile similar to that of the NNEG liability. More specifically, the hedge portfolio and the NNEG liability should have similar deltas and possibly gammas. Possible assets for forming a hedge portfolio include residential property index funds and real estate income trusts (REITs). These assets are traded, and the dynamics of their returns should be somewhat similar to that of the Nationwide House Price Index (the proxy for the actual underlying asset).

However, even if we can find suitable assets for constructing the portfolio, the hedge will not be perfect due to the following factors:

- *Hedging error*
  In the ARMA-EGARCH framework, the hedge portfolio is not self-financing. Hence, when the hedge portfolio is rebalanced, there may be a shortfall or excess of funds, leading to a hedging error.
- *Transaction cost*
  A transaction cost is incurred when the hedge portfolio is rebalanced.
- *Longevity risk*
  The lender is exposed to longevity risk, that is, the uncertainty of future mortality reduction. This risk cannot be diversified by writing a large number of contracts, since any change in the general level of mortality will affect all borrowers of the same cohort. One way to deal with this risk is to write roll-up mortgages in a portfolio of business that includes life insurance. This method, which is sometimes referred to as natural hedging (Cox and Lin, 2007), stabilizes aggregate cash outflows by the fact that life insurance and reverse mortgages react to mortality changes in an opposite manner. Another way to hedge this risk is to rely on mortality-linked securities such as longevity bonds (Blake et al., 2006), which pay coupons that are linked to a survivor index, and survivor swaps (Dowd et al., 2006), which allow reverse mortgage providers to swap mortality contingent cash flows with fixed cash flows.
- *Basis risk*
  As the Nationwide House Price Index is not a traded security, in practice we have to resort to using a portfolio of risky assets that behaves like the index. Any difference between the returns on the index and the risky assets in the hedge portfolio will lead to basis risk. Another source of basis risk is the discrepancy between the returns

on the mortgaged property and the house price index. This basis risk is difficult to measure in the absence of individual house prices data. If sufficient information on individual house prices is available, we may be able to adapt Calhoun's (1996) model, which relates a property price index to individual house prices, to quantify the basis risk entailed.

The factors above lead to additional costs, on top of the hedge portfolio, for dynamic hedging. These costs, which if often called unhedged liability, can be estimated by using Monte Carlo simulations. The procedure for such simulations are detailed in Hardy (2003).

CONCLUDING REMARKS

In a booming property market, one may feel that the NNEG is not valuable at all. However, it is entirely possible that the recent house price trends in Britain will cease. On the basis of historical house price returns, we found that the NNEG can be a significant financial burden to reverse equity product providers. Risk management for the NNEG is therefore highly important. In this article, we suggest a dynamic hedging strategy that can hedge the house price inflation risk associated with the guarantee. However, this strategy is not perfect due to various factors such as hedging error. The cost arising from these factors can be estimated by Monte Carlo simulations. Given an estimate of this cost, the lender can set aside an appropriate amount of capital to cushion against the unhedged liability.

Decrement assumptions play a crucial role in pricing the NNEG. By means of the Lee–Carter methodology, we have estimated the impact of potential reduction in future mortality on the cost of the guarantee. Through a sensitivity analysis, we have also examined how the cost of the guarantee would be affected if there exists other modes of decrement, such as long-term care entry and early redemption, that would lower the home exit rates. A more accurate way to study the effect of various modes of decrement is to use a multiple-state model (see, e.g., Waters, 1984), but the use of such a model requires relevant morbidity and lapse data that are not readily available.

When the NNEG becomes "in the money," the borrower would have little incentive to maintain his or her home. The lack of maintenance may lead to a depreciation of property value, resulting in a reduction of the lender's profit. We call this phenomenon moral hazard. Shiller and Weiss (2000) argued that we might be able to prevent moral hazard by penalizing the borrower deviations of home selling price from the value predicted by a house price index and the property value at inception of the contract. It would be interesting to investigate how the existing design of roll-up mortgages may be modified to avoid the risk of moral hazard without reducing the attractiveness of the product.

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