

TIME DEDUCTIBLES AS SCREENING DEVICES: COMPETITIVE MARKETS

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ABSTRACT

Seminal papers on asymmetric information in competitive insurance markets, analyzing the monetary deductible as a screening device, show that any existing equilibrium is of a separating type. High risks buy complete insurance, whereas low risks buy partial insurance—and this result holds for the Nash behavior as well as for the Wilson foresight. In this article, we analyze the strength of screening based on limitations to the period of coverage of the contract. We show that in this case (1) the Nash equilibrium may entail low risks not purchasing any insurance at all, and (2) under the Wilson foresight, a pooling equilibrium may exist.

INTRODUCTION

In insurance markets with adverse selection and two different risk types, insurers can implement a screening tool to separate low-risk and high-risk individuals. The most common device is the monetary deductible. Rothschild and Stiglitz (1976) have shown that if this instrument is applied in a competitive market with a Nash behavior, an equilibrium exists if the proportion of high risks in a population exceeds a certain threshold. This equilibrium, in the sequel called a *Cournot–Nash equilibrium*, is always of a separating type. High risks buy full coverage, and low risks buy partial coverage, both at actuarially fair terms. Miyazaki (1977) and Spence (1978) demonstrate that in a competitive market with a *Wilson foresight*,¹ an equilibrium always exists. Like the Cournot–Nash equilibrium derived in Rothschild and Stiglitz (1976), such an equilibrium is always of a separating type, except that high risks may be subsidized by low risks. Crocker and Snow (1985) show that according to the definition of efficiency developed in Harris and Townsend (1981), such a *Wilson–Miyazaki equilibrium* is second-best efficient.

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¹ The Wilson foresight (see Wilson, 1977) implies that the insurance companies take into account that existing contracts may be withdrawn as a result of a new contract under consideration.

An alternative screening device, which will be the topic of this article, a time deductible, may be considered. Such a device excludes the coverage for events that occur during a predefined period after the inception of the policy. The method, possibly aiming to rule out preexisting conditions, has found applications in some dental and medical policies. Besides, over recent years, it has gained popularity among Dutch group life companies as a consequence of a new legislation concerning the medical examination of employees. By the new law, which came into force at the beginning of 1998, insurance companies are strongly restricted in their possibilities to perform medical tests on individual members of a group life scheme. A time deductible may then be an appropriate instrument to identify individuals who are likely to make a claim soon after inception.

In the previous literature, the main focus has been on the probationary period, which is a special case of a time deductible, in which the coverage of the contract starts at a certain date after the inception of the policy. Several characteristics of the probationary period, such as, for example, its implication for the expected utility of consumers, have been investigated in Eeckhoudt et al. (1988). The authors' main conclusion is that most of the basic properties of the abovementioned monetary deductibles do not carry over to probationary periods. For example, the equilibrium in a competitive market (with symmetric information regarding risk class) is completely ambiguous when there is a positive loading factor—as opposed to the monetary deductible case in which the optimal solution involves incomplete yet positive coverage. The intuition behind this result is that with a probationary period, it is not possible to rank different degrees of coverage according to their riskiness (in the sense of Rothschild and Stiglitz, 1970). Hence, preferences will typically exhibit nonconvexities with regard to premium rates and the period of probation.

Fluet (1992) applies the concept of a probationary period in a competitive insurance market with asymmetric information and firms exhibiting a Nash behavior. He adopts the screening device of a time-dependent monetary deductible. Assuming that the proportion of high-risk agents is large enough, the high risks buy full coverage, whereas the low risks buy partial coverage in monetary terms. The monetary deductible may vary over time, but is always positive. Hence, the contract with a pure probationary period can never be an equilibrium, but a combination of an initial probationary period and subsequent monetary deductibles may well arise.

Fluet's (1992) finding suggests a certain kind of inferiority of the probationary period as a screening device when compared with the monetary deductible. This has been confirmed in Spreeuw (2005), who shows that using this instrument in a monopolistic insurance market may lead to a pooling equilibrium in which both classes of risk buy full coverage. This would never be possible with a monetary deductible, as shown in Stiglitz (1977).

This article deals with a competitive insurance market and focuses on the general properties of a time deductible, rather than on the monetary deductible, as an instrument to make individuals self-select. In this respect, the approach is less general than Fluet (1992) as contracts incorporating both time and monetary deductibles are not considered. It should be noted, however, that a monetary deductible always implies some nonlinear pricing that may be difficult to implement in practice. Moreover, Fluet has shown that the combination of both devices makes it difficult to draw conclusions,

unless restricting assumptions are being made. By concentrating on the time deductible, one can get an idea of possible equilibria resulting if, just as in Fluet, allowance is made for partial coverage in monetary terms. We will, however, deal with both the cases of the Nash behavior and the cases of the Wilson foresight. We will show that in the former case, a separating equilibrium may be degenerate if the risk of accident is high, and in the latter case, a pooling equilibrium may exist.

The model is described in "The Basic Assumptions and the Nature of a Time Deductible," in which the basic assumptions are also listed. In "The Nash Equilibrium," it is assumed that firms are myopic in the sense that they do not take subsequent withdrawals of contracts into account when designing policies. We show that if the low risks' probability of incurring a loss is high and the distinction between the high risks and low risks is strong, the separating equilibrium will be degenerate in the sense that the low-risk type gets no insurance coverage at all. Such an equilibrium exists, provided that the share of high risks in the population is sufficiently high.

"The Wilson Foresight" deals with the Wilson foresight. There we show that if the insurer's strategy is restricted to pooling contracts, full coverage may constitute an equilibrium. This is a result that contradicts Miyazaki's (1977) and Spence's (1978) findings. Finally, we drop the restriction to pooling contracts and allow the insurer to offer any pair of policies. It is shown that even then a pooling contract may constitute an equilibrium. Under the assumptions stated in this article, such a pooling contract would involve complete coverage.

The conclusions are given in "Conclusions and Final Remarks."

THE BASIC ASSUMPTIONS AND THE NATURE OF A TIME DEDUCTIBLE

The basic assumptions are mainly derived from Fluet (1992). For the ease of exposition, they are listed below:

- A population consists of two risk classes, namely, the high risks and the low risks. In the remainder of this article, all variables pertaining to high-risk and low-risk individuals will be accompanied by the subscripts H and L , respectively. All individuals have an initial wealth equal to W .
- All individuals within the population are identical, except with respect to the probability of having an accident in the period $[0, n]$, where 0 is the current time. In case an individual is faced with an accident, there is a monetary loss D . The probability of having an accident for an individual of risk class i is denoted by $\eta_i, i \in \{H, L\}$, with $\eta_L < \eta_H < 1$. It is assumed that an accident can occur to each individual at most once.
- All risks are insurable.
- The population consists of N individuals, of which N_H and N_L belong to the category of high risks and low risks, respectively. Hence $N = N_H + N_L$. The proportion of high risks among the entire population is denoted by ρ , so $N_H = \rho N$.
- The time at which any accident occurs is perfectly observable by both the individual concerned and the company and is verifiable.

- The probability for an individual of risk class $i, i \in \{H, L\}$, of facing an accident before time $t(0 \leq t \leq n)$ is denoted by $F_i(t)$ (hence, $F_i(n) = \eta_i$), and it is assumed to be differentiable in $[0, n]$, with derivative $f_i(t) > 0, \forall t \in [0, n]$. All individuals fully know these probabilities. These probabilities are exogenous so that the risk of moral hazard is nonexistent.
- The difference between the probability density functions $f_H(t)$ and $f_L(t)$ is determined by $b(t)$

$$f_H(t) = b(t)f_L(t), \quad \text{with } b(t) > 0, \quad 0 \leq t \leq n, \quad (1)$$

where $b(t)$ is a real-valued and differentiable function.²

- To each individual, the same utility function $U(\cdot)$ applies, which is assumed to be increasing, strictly concave, twice continuously differentiable, and independent of time.
- Insurance companies are risk-neutral profit maximizers and can offer any set of contracts that result in a nonnegative expected profit.
- There are no transaction costs involved in the supply of insurance and no administrative costs for the insurance business. Nor are there costs of obtaining classification information on a potential insured when it is possible to do so.
- Contracts are specified by (t_1, t_2, P) , with t_1 denoting the date at which the insurance coverage starts (i.e., the end of the probationary period), and t_2 is the end of the coverage period; hence, it is assumed that the screening mechanism is restricted to time deductibles concentrated at the beginning and/or at the end of the contract. P denotes the premium of a contract, payable irrespective of whether and when the accident occurs. Thus, for the given contract, no indemnity is paid if an accident occurs outside the period $(t_1, t_2]$, nor will the premium P , to be paid at time 0, be refunded to the insured. On the other hand, if an accident occurs during the period $(t_1, t_2]$, the insured will get a benefit equal to D (= loss).
- We denote the expected utility resulting from taking out the policy (t_1, t_2, P) by $E_i(t_1, t_2, P)$. This implies

$$\begin{aligned} E_i(t_1, t_2, P) &= (\eta_i - F_i(t_2) + F_i(t_1))U(W - P - D) \\ &\quad + (1 - \eta_i + F_i(t_2) - F_i(t_1))U(W - P), \quad i \in \{H, L\}. \end{aligned}$$

The special case of no insurance is denoted by E_i , so

$$E_i = E_i(t, t, 0) = \eta_i U(W - D) + (1 - \eta_i)U(W), \quad i \in \{H, L\}, t \in [0, n]. \quad (2)$$

² It should be noted that, apart from the assumption $\eta_L < \eta_H < 1$ made above, this is the only restriction we put on the loss distributions. Hence, we do not impose the additional condition $\frac{f_H}{1-F_H} > \frac{f_L}{1-F_L} \forall t$, which is necessary for some of Fluet's (1992) results.

THE NASH EQUILIBRIUM

In this section, we work with the assumption that insurance companies are myopic in the sense that they do not take potential withdrawals of competitors' contracts into account when offering policies. Rothschild and Stiglitz (1976) have shown that if all firms in the insurance market are myopic and the proportion of high risks exceeds a certain level, a Cournot–Nash separating equilibrium exists. High risks buy full insurance, whereas the policy for the low risks is subject to a monetary deductible.

We will show that a combination of time deductibles at the beginning and toward the end of the duration of the contract will only occur under some special conditions. We also argue that the conditions for a probationary period to be preferred over a limited term are fairly simple and intuitive. We then proceed to show that, with the time deductible as a screening device, a separating Cournot–Nash equilibrium may involve having low risks purchasing no insurance at all. Such an equilibrium would involve the contract $(0, n, \eta_H D)$ for the high risks (i.e., full coverage at an actuarially fair premium), in combination with a certain contract $(t_1^*, t_2^*, (F_L(t_2^*) - F_L(t_1^*))D)$ (partial coverage at an actuarially fair premium) for the low risks. It exists if the proportion of bad risks exceeds a certain level.

The Degenerate Equilibrium

First, we define our equilibrium concept.

Definition 1: A Nash equilibrium is a set of contracts (t_1^H, t_2^H, P_H) and (t_1^L, t_2^L, P_L) that maximizes expected welfare $E_i(t_1, t_2, P)$ of each risk type, subject to the following constraints:

$$E_H(t_1^H, t_2^H, P_H) - E_H(t_1^L, t_2^L, P_L) \geq 0, \quad (\text{ICH})$$

$$E_L(t_1^L, t_2^L, P_L) - E_L(t_1^H, t_2^H, P_H) \geq 0, \quad (\text{ICL})$$

$$(F_L(t_2^i) - F_L(t_1^i))D - P_i \leq 0, \quad i = L, H, \quad (\text{BE})$$

$$t_1^i \geq 0, \quad i = L, H, \quad (\text{TC1})$$

$$t_2^i \leq n, \quad i = L, H, \quad \text{and} \quad (\text{TC2})$$

$$t_1^i \leq t_2^i, \quad i = L, H. \quad (\text{TC3})$$

Hence, the equilibrium concept requires that expected utility be maximized for both types (since, otherwise, another contract could attract consumers and make non-negative profits), given the incentive compatibility constraints (ICH) and (ICL), the zero-profit constraint (BE), the requirement that the probationary period is nonnegative and less than n (TC1), the requirement that the limited term t_2 commences at

a positive date that is less than n (TC2), and the requirement that short selling of insurance coverage is not allowed (TC3). The following lemma suggests that some combinations of contracts can be ruled out immediately.

Lemma 1: *Among actuarially fair contracts according to (BE), the full coverage contract $(0, n, \eta_i D)$ provides the consumer with the highest expected utility.*

Proof: An individual with utility function $U(\cdot)$ is risk-averse if and only if for any $p \in [0, 1]$

$$pU(W - (F_i(t_2) - F_i(t_1))D - D) + (1 - p)U(W - (F_i(t_2) - F_i(t_1))D) < U(W - (F_i(t_2) - F_i(t_1))D - pD). \quad (3)$$

Replacing $p = \eta_i - F_i(t_2) + F_i(t_1)$ in Equation (3) leads to the left-hand side (LHS) being equivalent to expected utility from any actuarially fair contract, and the right-hand side (RHS) is the utility from the contract with full coverage, $(0, n, \eta_i D)$. Q.E.D.

An implication of Lemma 1 is that the low-risk type never gets full coverage in a separating equilibrium, since such a contract would be purchased by the high-risk type as well. Hence, the equilibrium contract for L will entail either a probationary period or a limited term or both. We now proceed to identify conditions under which these different types of contracts emerge in equilibrium.

Proposition 1: *If the low-risk type purchases insurance, the following conclusions can be drawn about equilibrium contracts:*

- (i) *A combination of probationary period and limited term can occur in equilibrium only if there exist at least two dates t and t' for which*

$$\frac{f_H(t)}{f_L(t)} = \frac{f_H(t')}{f_L(t')}. \quad (4)$$

- (ii) *If the ratio $\frac{f_H(t)}{f_L(t)}$ is strictly monotone decreasing for all $t \in [0, n]$, then the contract cannot entail a limited term.*
 (iii) *If the ratio $\frac{f_H(t)}{f_L(t)}$ is strictly monotone increasing for all $t \in [0, n]$, then the contract cannot entail a probationary period.*
 (iv) *If the ratio $\frac{f_H(t)}{f_L(t)}$ is constant for all $t \in [0, n]$, then the existence of an equilibrium with partial coverage for the low-risk type implies the existence of a continuum of equilibrium contracts.*

Proof: It follows from Lemma 1 above that the high-risk type will be provided with full coverage in equilibrium. Hence, we can derive the conditions for the equilibrium contract by maximizing $E_L(t_1^L, t_2^L, P_L)$ with respect to all the constraints from

Definition 1. Denoting the equilibrium contract offered to the low-risk type by (t_1^*, t_2^*, P_L^*) , and the Lagrange multipliers by λ_i (where the index i corresponds to the various constraints in the definition; i.e., λ_{ICH} is the Lagrange multiplier for type H 's incentive compatibility constraint), we have the following:

- (i) If there is a combination of probationary period and limited term (i.e., $0 < t_1^* < t_2^* < n$), then the following necessary conditions apply:

$$\begin{aligned} \left. \frac{\partial L}{\partial t_1} \right|_{t_1=t_1^*} &= f_L(t_1^*) \left\{ \lambda_{BE} D - \left(1 + \lambda_{ICL} - \lambda_{ICH} \frac{f_H(t_1^*)}{f_L(t_1^*)} \right) \right. \\ &\quad \left. \times (U(W - P_L) - U(W - P_L - D)) \right\} = 0, \\ \left. \frac{\partial L}{\partial t_2} \right|_{t_2=t_2^*} &= -f_L(t_2^*) \left\{ \lambda_{BE} D - \left(1 + \lambda_{ICL} - \lambda_{ICH} \frac{f_H(t_2^*)}{f_L(t_2^*)} \right) \right. \\ &\quad \left. \times (U(W - P_L) - U(W - P_L - D)) \right\} = 0. \end{aligned}$$

Since the two conditions are identical, $t_1^* \neq t_2^*$ requires that condition (4) be satisfied.

- (ii) Consider an equilibrium contract with limited term such that $0 \leq t_1^* < t_2^* < n$. Then, the following first-order conditions (FOC) apply:

$$\begin{aligned} \left. \frac{\partial L}{\partial t_1} \right|_{t_1=t_1^*} &= f_L(t_1^*) \left\{ \lambda_{BE} D - \left(1 + \lambda_{ICL} - \lambda_{ICH} \frac{f_H(t_1^*)}{f_L(t_1^*)} \right) \right. \\ &\quad \left. \times (U(W - P_L) - U(W - P_L - D)) \right\} + \lambda_{TC1} = 0, \\ \left. \frac{\partial L}{\partial t_2} \right|_{t_2=t_2^*} &= -f_L(t_2^*) \left\{ \lambda_{BE} D - \left(1 + \lambda_{ICL} - \lambda_{ICH} \frac{f_H(t_2^*)}{f_L(t_2^*)} \right) \right. \\ &\quad \left. \times (U(W - P_L) - U(W - P_L - D)) \right\} = 0. \end{aligned}$$

Solving the second FOC for λ_{BE} and rearranging, we get

$$\begin{aligned} \left. \frac{\partial L}{\partial t_1} \right|_{t_1=t_1^*} &= -f_L(t_1^*) \lambda_{ICH} \left(\frac{f_H(t_2^*)}{f_L(t_2^*)} - \frac{f_H(t_1^*)}{f_L(t_1^*)} \right) \\ &\quad \times (U(W - P_L) - U(W - P_L - D)) + \lambda_{TC1} = 0. \end{aligned}$$

Since $\lambda_{TC1} \geq 0$, the condition requires that $\frac{f_H(t_2^*)}{f_L(t_2^*)} - \frac{f_H(t_1^*)}{f_L(t_1^*)} \geq 0$, i.e., the ratio is not strictly monotone decreasing in t .

- (iii) Consider an equilibrium contract with probationary period such that $0 < t_1^* < t_2^* \leq n$. Then, the following FOCs apply

$$\begin{aligned} \frac{\partial L}{\partial t_1} \Big|_{t_1=t_1^*} &= f_L(t_1^*) \left\{ \lambda_{BE} D - \left(1 + \lambda_{ICL} - \lambda_{ICH} \frac{f_H(t_1^*)}{f_L(t_1^*)} \right) \right. \\ &\quad \left. \times (U(W - P_L) - U(W - P_L - D)) \right\} = 0, \\ \frac{\partial L}{\partial t_2} \Big|_{t_2=t_2^*} &= -f_L(t_2^*) \left\{ \lambda_{BE} D - \left(1 + \lambda_{ICL} - \lambda_{ICH} \frac{f_H(t_2^*)}{f_L(t_2^*)} \right) \right. \\ &\quad \left. \times (U(W - P_L) - U(W - P_L - D)) \right\} - \lambda_{TC2} = 0. \end{aligned}$$

Solving the first FOC for λ_{BE} and rearranging, we get

$$\begin{aligned} \frac{\partial L}{\partial t_2} \Big|_{t_2=t_2^*} &= f_L(t_2^*) \lambda_{ICH} \left(\frac{f_H(t_1^*)}{f_L(t_1^*)} - \frac{f_H(t_2^*)}{f_L(t_2^*)} \right) \\ &\quad \times (U(W - P_L) - U(W - P_L - D)) - \lambda_{TC2} = 0. \end{aligned}$$

Since $\lambda_{TC2} \geq 0$, this requires that $\frac{f_H(t_1^*)}{f_L(t_1^*)} - \frac{f_H(t_2^*)}{f_L(t_2^*)} \geq 0$, i.e., that the ratio is not strictly monotone increasing in t .

- (iv) Suppose we denote the constant ratio by β . Then, we can write the FOCs as

$$\begin{aligned} \frac{\partial L}{\partial t_1} \Big|_{t_1=t_1^*} &= f_L(t_1^*) \{ \lambda_{BE} D - (1 + \lambda_{ICL} - \lambda_{ICH} \beta) \\ &\quad \times (U(W - P_L) - U(W - P_L - D)) \} + \lambda_{TC1} = 0, \\ \frac{\partial L}{\partial t_2} \Big|_{t_2=t_2^*} &= -f_L(t_2^*) \{ \lambda_{BE} D - (1 + \lambda_{ICL} - \lambda_{ICH} \beta) \\ &\quad \times (U(W - P_L) - U(W - P_L - D)) \} - \lambda_{TC2} = 0. \end{aligned}$$

We know that the constraints $TC1$ and $TC2$ cannot both be binding as that would imply full coverage for the low-risk type. Similarly, since $f_L(\cdot)$ is strictly positive, the FOCs imply that $\lambda_{TC1} = \lambda_{TC2}$, thus ruling out a situation in which only one of them is binding. On the other hand, the break-even constraint needs to be binding in equilibrium. Otherwise, the above two conditions would imply that $(1 + \lambda_{ICL} - \lambda_{ICH} \beta) = 0$; however, this would contradict $\frac{\partial L}{\partial P_L} \Big|_{P_L=P_L^*} = 0$. Thus, we have $\lambda_{BE} > 0$, and it follows that $P_L^* = (F_L(t_2^*) - F_L(t_1^*)) D$. Hence, for a given solution P_L^* , it follows that if the contract (t_1^*, t_2^*) constitutes an equilibrium, then any other contract (t_1', t_2') such that $F_L(t_2') - F_L(t_1') = F_L(t_2^*) - F_L(t_1^*)$ will be an equilibrium as well. Observe further that the second derivatives of L are equal to zero at (t_1^*, t_2^*) , confirming the existence of a range of equilibria. Q.E.D.

These results all have straightforward interpretations. They are all related to situations in which the low-risk types have their probability of accident concentrated compared with the high-risk types. The first result, (i), states that the two types of screening mechanism can only occur together if the ratio between the two risk densities is not monotonically increasing or decreasing throughout. Parts (ii) and (iii) of the proposition state the interesting result that a pure probationary period will be chosen whenever the relative risk ratio between high and low is monotonically decreasing, and that a pure limited term will be used in the opposite case. This result is fairly intuitive. In most situations, the choice between probationary period and limited term depends entirely on the incentive compatibility problem of the high-risk type. When the high-risk types have most of their risk concentrated after the beginning of the contract, the low-risk types will have to give up relatively little insurance coverage to convince the insurer that they belong to the low-risk type, and the opposite holds for situations in which the ratio is monotonically increasing. Case (iv), finally, highlights the fact that the timing of the event carries no information of risk type in the case in which the ratio between the two probability density functions is constant—and hence time deductibles become less useful as screening devices in this case (cf. Milgrom, 1981).

Thus, Proposition 1 gives the general conditions under which a probationary period will be preferred over the limited term as a screening mechanism and the screening mechanism that minimizes the loss to the low-risk type will always be chosen. In what follows, we will argue that time deductibles might nevertheless be poor screening mechanisms by showing that the contract offered to the low-risk types might be so poor that they actually prefer to purchase no insurance at all in equilibrium. We restrict ourselves to pure probationary periods and pure limited terms; hence, it is assumed in the following proposition that the ratio $\frac{f_H(t)}{f_L(t)}$ is strictly monotonically increasing or decreasing throughout. Since we have $\eta_H > \eta_L$, it should be noted that this assumption means that (1) in the case of $\frac{f_H(t)}{f_L(t)}$ monotonically decreasing, $F_H(t) > F_L(t)$ for all $t \in [0, n]$, which means that, given the absence of a limited term, the incentive compatibility constraint for the low risks is always satisfied, and (2) in the case of $\frac{f_H(t)}{f_L(t)}$ monotonically increasing, we have $\eta_H - F_H(t) > \eta_L - F_L(t)$ for all $t \in [0, n]$, which means that, given the absence of a probationary period, again the incentive compatibility constraint for the low risks is always satisfied. So in both cases, we would have $\lambda_{ICL} = 0$.

The conditions for the low risks not purchasing insurance in equilibrium are provided in the next proposition.

Proposition 2: *Whenever the ratio $\frac{f_H(t)}{f_L(t)}$ is monotonically increasing or decreasing throughout $[0, n]$, the following result holds:*

- (i) *For sufficiently high η_L and η_H , the separating equilibrium will involve low-risk types purchasing no insurance at all, whereas high-risk types purchasing complete coverage.*

(ii) Such an equilibrium exists if the share of high-risk types is higher than $\hat{\rho}$, where

$$\hat{\rho} = \max_{t_1} \frac{(1 - F_L(t_1))(U'(W - P - D) - U'(W - P))}{\left(\frac{f_H(t_1)}{f_L(t_1)} - 1\right)(U'(W - P) + F_L(t_1)(U'(W - P - D) - U'(W - P)))}.$$

Proof: (i) It follows from Proposition 1 that any contract offered to the low-risk type will have either a pure probationary period or a pure limited term. Concerning the pure *probationary period*, we need to establish conditions for $t_1^* = n$. First, consider the expected utility of low-risk type with $t_2 = n$ and actuarially fair premiums:

$$E_L(t_1, n, P_L) = F_L(t_1)U(W - (\eta_L - F_L(t_1))D - D) \\ + (1 - F_L(t_1))U(W - (\eta_L - F_L(t_1))D).$$

Taking the derivative with respect to t_1 gives

$$\frac{dE_L(t_1, n, P_L)}{dt_1} = f_L(t_1)D \cdot \left\{ F_L(t_1)U'(W - P_L - D) + (1 - F_L(t_1))U'(W - P_L) \right. \\ \left. - \frac{U(W - P_L) - U(W - P_L - D)}{D} \right\}, \quad (5)$$

which, according to the mean value theorem, is equal to

$$d \frac{E_L(t_1, n, P_L)}{dt_1} = f_L(t_1)D \cdot \{F_L(t_1)U'(W - (\eta_L - F_L(t_1))D - D) + (1 - F_L(t_1))U'(W \\ - (\eta_L - F_L(t_1))D) - U'(W - (\eta_L - F_L(t_1))D - \beta(t_1))\}, \quad (6)$$

for some $\beta(t_1) \in [0, D]$. For $t_1 = n$, Equation (6) reduces to

$$\left(d \frac{E_L(t_1, n, P_L)}{dt_1} \right)_{(t_1=n)} = f_L(n)D(\eta_L U'(W - D) + (1 - \eta_L)U'(W) - U'(W - \beta(n))). \quad (7)$$

For large η_L , this derivative is positive, indicating that in such cases, the individual prefers no insurance to coverage with a long probationary period. Hence, for t_L in a neighborhood of n and a relatively smooth behavior of $F_L(t_L)$, the derivative in Equation (6) will be positive as well. So, there are at least some actuarially fair contracts that a low-risk type will not purchase. Now, consider inequality (ICH), which is binding in equilibrium. The greater η_H , the higher the premium for the high risks, and the longer the probationary period for the low risks. Hence, for high values of η_H , the critical value of t_1 will be at a point where the derivative in Equation (6) is positive.

When instead a *limited term* is chosen, low-risk type will attain expected utility

$$E_L(0, t_2, P_L) = (\eta_L - F_L(t_2))U(W - P_L - D) + (1 - \eta_L + F_L(t_2))U(W - P_L).$$

Hence, the derivative with respect to t_2 for an actuarially fair contract is

$$\begin{aligned} \frac{dE_L(0, t_2, (F_L(t_2))D)}{dt_2} &= f_L(t_2)D \cdot \left\{ \frac{U(W - P_L) - U(W - P_L - D)}{D} - (\eta_L - F_L(t_2)) \right. \\ &\quad \left. \times U'(W - P_L - D) - (1 - \eta_L + F_L(t_2))U'(W - P_L) \right\}, \quad (8) \end{aligned}$$

and for $t_2 = 0$, we get

$$\begin{aligned} \left(\frac{dE_L(0, t_2, F_L(t_2)D)}{dt_2} \right)_{t_2=0} &= f_L(0)D \cdot \{U'(W - P_L - \beta(0)) \\ &\quad - \eta_L U'(W - P_L - D) - (1 - \eta_L)U'(W - P_L)\}, \end{aligned}$$

which is negative for high values of η_L , and thus Equation (8) is also negative in a neighborhood of 0. Obviously, the reasoning from above concerning the conditions on η_H applies also in this case.

(ii) *Probationary period*: Under the conditions stated, a sufficient condition arises if, for any t_1 , the slope of the iso-profit curve for pooling contracts,

$$-\frac{dP_L}{dt_1} = (\rho f_H(t_1) + (1 - \rho)f_L(t_1))D,$$

is at least as great as the marginal rate of substitution for the low-risk type:

$$\begin{aligned} -\frac{dP_L}{dt_1} &= \frac{f_L(t_1)(U(W - P_L) - U(W - P_L - D))}{F_L(t_1)U'(W - P_L - D) + (1 - F_L(t_1))U'(W - P_L)} \\ &= \frac{f_L(t_1)U'(W - P_L - \beta(t_1))}{F_L(t_1)U'(W - P_L - D) + (1 - F_L(t_1))U'(W - P_L)} D. \end{aligned}$$

Now, given that $U'(W - P_L - \beta(t_1)) < U'(W - P_L - D)$, this leads to the sufficient condition

$$\rho > \frac{(1 - F_L(t_1))(U'(W - P - D) - U'(W - P))}{\left(\frac{f_H(t_1)}{f_L(t_1)} - 1 \right) (U'(W - P) + F_L(t_1)(U'(W - P - D) - U'(W - P)))},$$

which makes sense as long as $\frac{f_H(t_1)}{f_L(t_1)} > 1$.

Limited term: Exactly the same conditions for existence apply.

Q.E.D.

This result—that the low-risk type might not prefer to purchase any insurance—might not seem very surprising, given the finding of Proposition 1 that the low-risk types tend to be offered a probationary period when they are not particularly prone to suffer an accident at the beginning (and the converse goes for the limited term: it tends to be offered when they are not particularly prone to suffer an accident at the end of the term). However, the finding that no insurance becomes particularly attractive for high values of η_L might require some further explanation. The effect that seems to be driving this result is the peculiarity of the marginal utility with respect to changes in the length of the probationary period (Equation (5)) or the limited term (Equation (8)). This marginal utility consists of one positive and one negative part. The positive part is related to the reduction in the premium rate that the individual enjoys as a result of an increase in the time deductible. There is also a negative effect, reflecting the reduction in utility because the individual now has less coverage. However, only the first, positive effect is proportional to the overall length time deductible—and whenever this deductible becomes long enough, the positive effect of increasing the time deductible will dominate to such an extent that the individual will decide to purchase no insurance. This is also the reason why the same condition applies for the probationary period and the limited term in the same way: both deductibles converge to η_L when the period of noncoverage grows long.

Concerning the conditions for existence, we find that they are similar to the findings for the monetary deductible: that the proportion of high risks must be relatively high, since otherwise, the separating equilibrium can be challenged by a pooling contract.

THE WILSON FORESIGHT

If firms behave with the Wilson foresight, an equilibrium, based on maximal welfare for the low risks, always exists. In this section, we show that such an equilibrium may be of a pooling type.

Optimal Pooling Contract

Miyazaki (1977) and Spence (1978) have shown that for the monetary deductible as a screening device, the equilibrium is always of a separating type. From the insurer's point of view, a pooling strategy without full coverage will always be inferior to offering a pair of different contracts, with the original pooling contract designed for the low risks and full coverage for the high risks.

Furthermore, a pooling strategy can never involve complete coverage. In the case of pooling, one of the risk types would pay a loaded premium. As discussed, for example, in Arrow (1963) and Pashigian, Schkade, and Menefee (1966), full insurance cannot be optimal if a premium is loaded. This point is also stressed in Eeckhoudt et al. (1988).

In this subsection, we will concentrate on the latter conclusion. We show that, using time deductibles as screening device, the full coverage pooling contract might be the pooling contract that provides the low-risk type with the highest expected utility. In the next subsection, we proceed to show that this contract might even be the equilibrium contract.

Assume that the insurer can only offer a pooling contract (t_1, t_2, P) . We denote the objective function by $\hat{V}_L(t_1, t_2, P)$, which is defined as the expected utility for a low-risk type agent resulting from offering such a contract. Then, we have

$$\begin{aligned}\hat{V}_L(t_1, t_2, P) &= (\eta_L - F_L(t_2) + F_L(t_1))U(W - P - D) \\ &\quad + (1 - \eta_L + F_L(t_2) - F_L(t_1))U(W - P).\end{aligned}\quad (9)$$

Such contracts satisfy the binding non-profit constraint:

$$P = ((1 - \rho)(F_L(t_2) - F_L(t_1)) + \rho(F_H(t_2) - F_H(t_1)))D. \quad (10)$$

So, we can express P as a function of t_1 and t_2 . We will use the notation $P(t_1, t_2)$ and, consequently, $\hat{V}_L(t_1, t_2)$ to denote $\hat{V}_L(t_1, t_2, P)$ in Equation (9) expressed as a function of t_1 and t_2 only:

$$\begin{aligned}\hat{V}_L(t_1, t_2) &= (\eta_L - F_L(t_2) + F_L(t_1))U(W - P(t_1, t_2) - D) \\ &\quad + (1 - \eta_L + F_L(t_2) - F_L(t_1))U(W - P(t_1, t_2)).\end{aligned}\quad (11)$$

We analyze the function (11) by taking its derivatives with respect to t_1 and t_2 . This returns:

$$\begin{aligned}\frac{d\hat{V}_L(t_1, t_2)}{dt_1} &= -f_L(t_1)(U(W - P(t_1, t_2)) - U(W - P(t_1, t_2) - D)) \\ &\quad - \frac{dP(t_1, t_2)}{dt_1} \left((\eta_L - F_L(t_2) + F_L(t_1))U'(W - P(t_1, t_2) - D) \right. \\ &\quad \left. + (1 - \eta_L + F_L(t_2) - F_L(t_1))U'(W - P(t_1, t_2)) \right) \\ &= f_L(t_1) \left\{ \left(1 + \rho \left(\frac{f_H(t_1)}{f_L(t_1)} - 1 \right) \right) D \begin{pmatrix} (\eta_L - F_L(t_2) + F_L(t_1)) \\ \times U'(W - P(t_1, t_2) - D) \\ + (1 - \eta_L + F_L(t_2) - F_L(t_1)) \\ \times U'(W - P(t_1, t_2)) \end{pmatrix} \right. \\ &\quad \left. - (U(W - P(t_1, t_2)) - U(W - P(t_1, t_2) - D)) \right\}.\end{aligned}\quad (12)$$

In accordance with the mean value theorem

$$\frac{U(W - P(t_1, t_2)) - U(W - P(t_1, t_2) - D)}{D} = U'(W - P(t_1, t_2) - \beta(t_1, t_2)), \quad (13)$$

for some $\beta(t_1, t_2) \in [0, D]$, $t_1, t_2 \in [0, n]$. Note that, compared with the previous subsection, P is a function of t_1 and t_2 , so β is a function of t_1 and t_2 as well. Consequently

$$U'(W - P(t_1, t_2) - D) \leq U'(W - P(t_1, t_2) - \beta(t_1, t_2)) \leq U'(W - P(t_1, t_2)). \quad (14)$$

So we can rewrite Equation (12) as

$$\frac{d\hat{V}_L(t_1, t_2)}{dt_1} = f_L(t_1)D \left\{ \left(1 + \rho \left(\frac{f_H(t_1)}{f_L(t_1)} - 1 \right) \right) \left(\begin{array}{l} (\eta_L - F_L(t_2) + F_L(t_1)) \\ \times U'(W - P(t_1, t_2) - D) \\ + (1 - \eta_L + F_L(t_2) - F_L(t_1)) \\ \times U'(W - P(t_1, t_2)) \end{array} \right) \right. \\ \left. - U'(W - P(t_1, t_2) - \beta(t_1, t_2)) \right\}. \quad (15)$$

For $t_1 = 0$, Expression (15) reduces to

$$\left(\frac{d\hat{V}_L(t_1, t_2)}{dt_1} \right)_{(t_1=0)} = f_L(0)D \left\{ \left(1 + \rho \left(\frac{f_H(0)}{f_L(0)} - 1 \right) \right) \left(\begin{array}{l} (\eta_L - F_L(t_2)) U'(W - P(0, t_2) - D) \\ + (1 - \eta_L + F_L(t_2)) U'(W - P(0, t_2)) \end{array} \right) \right. \\ \left. - U'(W - P(0, t_2) - \beta(0, t_2)) \right\}. \quad (16)$$

For low values of η_L , this expression will be negative whenever $\rho = 0$ or $\frac{f_H(0)}{f_L(0)} = 1$, irrespective of the value of t_2 . By continuity, it is therefore negative at $t_1 = 0$ for ρ and $\frac{f_H(0)}{f_L(0)}$ in a neighborhood of 0 and 1, respectively. This implies that for such values of ρ or $\frac{f_H(0)}{f_L(0)}$, the pooling contract without a probationary period is at least not the worst among all the pooling contracts. Recall that, for the monetary deductible as a screening device, the above expression is always positive, so complete coverage can never constitute an equilibrium.

Concerning the limited term t_2 , the corresponding derivative is

$$\frac{d\hat{V}_L(t_1, t_2)}{dt_2} = f_L(t_2) (U(W - P(t_1, t_2)) - U(W - P(t_1, t_2) - D)) \\ - \frac{dP(t_1, t_2)}{dt_2} \left(\begin{array}{l} (\eta_L - F_L(t_2) + F_L(t_1)) U'(W - P(t_1, t_2) - D) \\ + (1 - \eta_L + F_L(t_2) - F_L(t_1)) U'(W - P(t_1, t_2)) \end{array} \right) \\ = f_L(t_2) \left\{ \begin{array}{l} (U(W - P(t_1, t_2)) - U(W - P(t_1, t_2) - D)) \\ - \left(1 + \rho \left(\frac{f_H(t_2)}{f_L(t_2)} - 1 \right) \right) D \left(\begin{array}{l} (\eta_L - F_L(t_2) + F_L(t_1)) \\ \times U'(W - P(t_1, t_2) - D) \\ + (1 - \eta_L + F_L(t_2) - F_L(t_1)) \\ \times U'(W - P(t_1, t_2)) \end{array} \right) \end{array} \right\}. \quad (17)$$

For $t_2 = n$, expression (17) reduces to

$$\begin{aligned} & \left(\frac{d\hat{V}_L(t_1, t_2)}{dt_2} \right)_{(t_2=n)} \\ &= f_L(n)D \left\{ - \left(1 + \rho \left(\frac{f_H(n)}{f_L(n)} - 1 \right) \right) \left(F_L(t_1)U'(W - P(t_1, n) - D) \right. \right. \\ & \quad \left. \left. + (1 - F_L(t_1))U'(W - P(t_1, n)) \right) \right\}. \end{aligned}$$

For low values of η_L , this expression will be positive whenever $\rho = 0$ or $\frac{f_H(n)}{f_L(n)} = 1$, irrespective of the value of t_1 . By continuity, it is therefore negative at $t_2 = n$ for ρ and $\frac{f_H(n)}{f_L(n)}$ in a neighborhood of 0 and 1, respectively. This implies that for such values of ρ or $\frac{f_H(n)}{f_L(n)}$, the pooling contract without a limited term is at least not the worst among all the pooling contracts.

Furthermore, note that if η_L is small, then $F_L(t)$ is small for each $t \in [0, n]$ and hence, again by continuity, we have for each (t_1, t_2) such that $0 \leq t_1 \leq t_2 \leq n$:

$$\begin{aligned} U'(W - P(t_1, t_2) - \beta(t_1, t_2)) &\geq (\eta_L - F_L(t_2) + F_L(t_1))U'(W - P(t) - D) \\ &\quad + (1 - \eta_L + F_L(t_2) - F_L(t_1))U'(W - P(t)). \end{aligned} \quad (18)$$

This inequality implies that $\frac{d\hat{V}_L(t_1, t_2)}{dt_1}$ is negative and $\frac{d\hat{V}_L(t_1, t_2)}{dt_2}$ is positive everywhere for ρ and $\frac{f_H(t)}{f_L(t)}$ in a neighborhood of 0 and 1, respectively. It follows that the best pooling strategy may be indeed to provide full coverage.

Of course, the insurer's choice is not restricted to pooling contracts; the general case in which the insurer can offer any pair of policies is dealt with in the next section.

The Equilibrium

Obviously, the low risks' opportunities for a higher welfare are enhanced if, unlike the previous subsection, its choice is not restricted to pooling contracts. We now derive properties of the Wilson equilibria. In this part, the analysis is considerably simplified by the fact that a Nash equilibrium satisfies all the constraints of a Wilson equilibrium. Hence, it follows that the Nash equilibrium—if it exists—will also be the Wilson equilibrium, unless a Pareto improvement can be attained by letting the low-risk type cross-subsidize the high-risk type. Cross-subsidization in the other direction will not take place in equilibrium since the contract offered to the high-risk type in such a case would make nonnegative profits if offered with an actuarially fair premium instead.

It follows that all gains due to the possibility of cross-subsidization will accrue to the low-risk type. The reason is that among two Pareto-efficient separating contracts, the low-risk types will choose the contract that provides them with the highest utility, and once they withdraw from the alternative contract, this contract will be making

losses and will have to be withdrawn. Hence, Miyazaki's (1977) finding that if the Wilson equilibrium deviates from the Nash equilibrium, the former behaves as if it would be maximizing low-risk type's utility carries over to our case also.

Thus, we can use the following definition for the Wilson equilibrium.

Definition 2: *The Wilson equilibrium is a set of contracts (t_1^H, t_2^H, P_H) and (t_1^L, t_2^L, P_L) that maximizes low-risk type's expected welfare*

$$E_L(t_1^L, t_2^L, P_L)$$

subject to the following constraints:

$$E_H(t_1^H, t_2^H, P_H) - E_H(t_1^L, t_2^L, P_L) \geq 0, \quad (\text{ICH})$$

$$E_L(t_1^L, t_2^L, P_L) - E_L(t_1^H, t_2^H, P_H) \geq 0, \quad (\text{ICL})$$

$$\rho((F_H(t_2^H) - F_H(t_1^H))D - P_H) + (1 - \rho)((F_L(t_2^L) - F_L(t_1^L))D - P_L) \leq 0, \quad (\text{BE})$$

$$t_1^i \in [0, n], \quad i = L, H, \quad (\text{TC1})$$

$$t_2^i \in [0, n], \quad i = L, H, \quad \text{and} \quad (\text{TC2})$$

$$t_1^i \leq t_2^i, \quad i = L, H. \quad (\text{TC3})$$

The definition is similar to that of the Nash equilibrium (Definition 1), with the main difference that cross-subsidization is allowed. It should be noted that the above constraints allow for a pooling equilibrium since $t_1^L = t_1^H, t_2^L = t_2^H$, and $P_H = P_L$ are possible.

Finally, we combine the setting of this subsection with findings from the previous one and derive conditions under which a pooling equilibrium with full coverage is the Wilson equilibrium.

Proposition 3: *For low values of ρ and the ratio $\frac{f_H(t)}{f_L(t)} \approx 1, \forall t \in [0, 1]$, the Wilson equilibrium entails a pooling contract with full coverage.*

Proof: We have already established that full coverage pooling is the best pooling contract under these circumstances. For any premium P_L that satisfies the constraints

of Definition 2 above, the contract will be a pooling contract whenever

$$U(W - (\rho\eta_H + (1 - \rho)\eta_L)D) \geq (\eta_L - F_L(t_2) + F_L(t_1))U(W - P_L - D) \\ + (1 - \eta_L + F_L(t_2) - F_L(t_1))U(W - P_L). \quad (19)$$

We have established above that the Wilson equilibrium premium for low-risk type will always be at least as high as the actuarially fair premium. Hence, a sufficient condition for Equation (19) to hold is

$$U(W - (\rho\eta_H + (1 - \rho)\eta_L)D) \geq (\eta_L - F_L(t_2) + F_L(t_1))U(W - (F_L(t_2) \\ - F_L(t_1))D - D) + (1 - \eta_L + F_L(t_2) \\ - F_L(t_1))U(W - (F_L(t_2) - F_L(t_1))D).$$

Whenever $\rho = 0$, this inequality holds strictly for any $n > t_2 > t_1 > 0$ if the low-risk type is risk-averse (Lemma 1). It follows that it will also hold in a neighborhood of $\rho = 0$. Q.E.D.

CONCLUSIONS AND FINAL REMARKS

In this article, we have investigated the effectiveness of a time deductible as a screening device in an insurance market with adverse selection problems. In general, we find evidence that the time deductible is a relatively poor instrument, and this finding seems to be robust to varying assumptions on the extent of forward-looking behavior on the part of the insurance companies.

Accordingly, we find that the Cournot–Nash separating equilibrium, if it exists, may entail no insurance coverage at all for the low risks. This outcome is quite different from the equilibrium derived in Rothschild and Stiglitz (1976), in which low risks always get some degree of coverage. This degenerate equilibrium comes about if the low risks' probability of incurring an accident is large, and the difference between low and high risks in the incidence of accidents changes monotonically over time. The general conditions for existence of an equilibrium are quite similar to those found by Rothschild and Stiglitz (1976), i.e., that the proportion of high risks in the population is sufficiently high.

Moreover, we find that, in contradiction to the findings in Miyazaki (1977) and Spence (1978), a pooling contract with complete coverage may provide the low risks more welfare than any other pooling contract. Offering such a contract may even be more beneficial to this class of risk than offering any set of separating policies. A strategy involving a pooling full coverage contract can be an equilibrium if the low risk's probability of incurring an accident, the differences between the low risks' and the high risks' probabilities, and the proportion of high risks within the entire population are all relatively low.

In fact, our conclusions show consistency with the findings in Fluet (1992). Consider a contract with a probationary period only. Then, the monetary deductible is equal to the loss during that period and zero thereafter. Apparently, in many cases, this maximal difference between the deductibles does not work out very well for the individual.

Fluet has shown that any time-dependent monetary deductible for the low risks is always strictly positive. This finding, however, requires that the hazard rates for the two types are clearly ranked for all subperiods—which is more restrictive than our assumption of monotonic behavior of $\frac{f_H(t)}{f_L(t)}$. It is interesting to note that in case $b(t)$ is constant so that $F_H(t) = bF_L(t)$, Fluet establishes that the monetary deductible for the low risks is constant over time. This implies that the separating menu does not involve a time deductible. Fluet only considers the case of a Nash behavior in detail. However, he rightly points out in his concluding remarks that the low risks will purchase the same type of contract if firms behave with the Wilson foresight.

REFERENCES

- Arrow, K. J., 1963, Uncertainty and the Economics of Medical Care, *American Economic Review*, 53(5): 941-973.
- Crocker, K. J., and A. Snow, 1985, The Efficiency of Competitive Equilibria in Insurance Markets With Asymmetric Information, *Journal of Public Economics*, 26: 207-219.
- Eeckhoudt, L., J. F. Outreville, M. Lauwers, and F. Calcoen, 1988, The Impact of a Probationary Period on the Demand of Insurance, *Journal of Risk and Insurance*, 55: 219-228.
- Fluet, C., 1992, Probationary Periods and Time-Dependent Deductibles in Insurance Markets With Adverse Selection, in: G. Dionne, ed., *Contributions to Insurance Economics* (Boston/Dordrecht/London: Kluwer), pp. 359-375.
- Harris, M., and R. M. Townsend, 1981, Resource Allocation Under Asymmetric Information, *Econometrica*, 49(1): 33-64.
- Milgrom, P. R., 1981, Good News and Bad News: Representation Theorem and Applications, *Bell Journal of Economics*, 12(2): 380-391.
- Miyazaki, H., 1977, The Rat Race and Internal Labor Markets, *Bell Journal of Economics*, 18: 349-418.
- Pashigian, B. P., L. L. Schkade, and G. H. Menefee, 1966, The Selection of an Optimal Deductible for a Given Insurance Policy, *Journal of Business*, 39(1): 35-44.
- Rothschild, M. and J. Stiglitz, 1970, Increasing Risk I: A Definition, *Journal of Economic Theory*, 2: 225-243.
- Rothschild, M., and J. Stiglitz, 1976, Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information, *Quarterly Journal of Economics*, 90: 629-649.
- Spence, M., 1978, Product Differentiation and Performance in Insurance Markets, *Journal of Public Economics*, 10: 427-447.
- Spreeuw, J., 2005, The Probationary Period as a Screening Device: The Monopolistic Insurer, *Geneva Risk and Insurance Review*, 30: 5-14.
- Stiglitz, J., 1977, Monopoly, Non-Linear Pricing and Imperfect Information: The Insurance Market, *Review of Economic Studies*, 44: 407-430.
- Wilson, C., 1977, A Model of Insurance Markets With Incomplete Information, *Journal of Economic Theory*, 16: 167-207.

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