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On the Role of Patience in an Insurance Market With Asymmetric Information

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Abstract

We analyze a two-period competitive insurance market that is characterized by the simultaneous presence of moral hazard and adverse selection with regard to consumer time preferences. It is shown that there exists an equilibrium in which patient consumers use high effort and buy an insurance contract with high coverage, whereas impatient consumers use low effort and buy a contract with low coverage or even remain uninsured. This finding may help to explain why the opposite of adverse selection with regard to risk types can sometimes be observed empirically.

Introduction

Empirically, personal discount rates vary to a significant degree among people. For example, Warner and Pleeter (2001) use data from the U.S. military downsizing program of the early 1990s to estimate the discount rates of separatees who could choose between an annuity and a lump-sum payment. Their estimates of discount rates range from 0 percent to over 30 percent. Frederick, Loewenstein, and O'Donoghue (2002) survey articles that try to estimate the annual discount rate of individuals. Across and within the various studies, there is a tremendous variance in results, which take values from zero to infinity, with some results even being negative. These findings seem to underline the relevance of accounting for different time preferences among consumers. However, this issue, to our knowledge, has not received attention so far in the context of insurance markets.

Since the typical insurance contract requires insurees to pay the premium up front for several periods (e.g., months), there exists a role for time preference in the consumers'

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ex ante valuation of the contract. The aim of this article is to analyze the effects of the differences in the personal discount rate of individuals in a competitive insurance market. For this purpose, we employ a two-period model with both moral hazard and adverse selection in the spirit of de Meza and Webb (2001). However, it is assumed that the informational asymmetry is not with regard to risk aversion but with regard to the individuals' personal discount rate, which can either be high (impatient) or low (patient). This corresponds to a low (impatient) or high (patient) discount factor. The discount factor will be used for modeling purposes throughout this article.

We show that there exists a separating equilibrium in which patient consumers use high effort and buy an insurance contract with an unfair premium. In contrast, impatient consumers use low effort and buy a contract with lower cover than the patient consumers or even prefer to remain uninsured. In this sense, accounting for the differences in the personal discount rate helps to explain the opposite of adverse selection ("advantageous" selection, as it is called by de Meza and Webb).

This finding contrasts with traditional models of adverse selection in insurance, for example, as represented by Rothschild and Stiglitz (1976), which predict that it is the high risks who are more keen on buying insurance.

However, there is some evidence that does not seem to fit these predictions. Dionne, Gourieroux, and Vanasse (2001) criticize an empirical study by Puelz and Snow (1994) that finds that adverse selection is a relevant problem for automobile insurance. They show that under refined estimation methods, the result cannot be confirmed. Cawley and Philipson (1999) analyze whether data from life insurance are consistent with the adverse selection hypothesis. They report that in several regards the data exhibit the opposite of the expected pattern. For example, there is a negative covariance between risk and quantity. This suggests that it is actually the low risks who are inclined to buy more insurance.

This article draws on de Meza and Webb (2001) who develop a model that can explain the opposite of adverse selection in a competitive insurance market. This follows from adverse selection on risk aversion in the simultaneous presence of moral hazard. Consumers are split into timid and bold people, whereby the bold ones are less risk-averse. They can choose between a high or a low level of unobservable precautionary effort in order to avoid the loss. de Meza and Webb show that for certain parameter values a unique separating equilibrium exists in which timid individuals buy insurance and employ high effort whereas bold ones remain uninsured and employ low effort. In contrast to de Meza and Webb, adverse selection in the present model occurs regarding the consumers' personal time preferences.

This work is linked to the literature on multidimensional adverse selection (Smart, 2000; Villeneuve, 2003; Wambach, 2000). Those models can also lead to contracts with unfair premiums, in which, however, the higher risks buy more coverage. In these models with multidimensional adverse selection, undercutting the unfair equilibrium contract will attract (loss-making) high risks, whereas in our work, undercutting attracts (loss-making) impatient types who exert low effort.

Theoretical models of asymmetric information in the insurance market can be grouped according to whether they employ pure adverse selection (e.g., Rothschild and Stiglitz, 1976), pure moral hazard (e.g., Arnott and Stiglitz, 1988), or a combination of both (e.g., de Meza and Webb, 2001, or the present model). Both pure theories on their own suggest that in equilibrium a higher cover implies a higher risk, thus leading to a positive correlation between cover and loss probability. In a recent article, Chiappori et al. (2006) establish sets of conditions under which this positive correlation property follows also from models with simultaneous moral hazard and adverse selection. The present model satisfies one set of these conditions that require that the consumers know their loss probabilities, and that their risk aversion be identical and publicly known. Interestingly, there can still be a negative correlation between loss probability and cover. The reason for this discrepancy is that different personal discount rates translate into different curvatures of patient and impatient types' present value indifference curves. This is equivalent to different risk aversions in a one-period model.

The article is structured as follows. In "The Model," we describe the model. "Equilibria" analyzes different equilibrium configurations. In "Effects of Including a Capital Market," the effects of including a capital market that allows the transfer of wealth across time are analyzed. We conclude in "Conclusion."

The model

The model we employ in this section consists of a game with four stages throughout two periods:

1. Insurance companies make an irrevocable offer of a contract that specifies both premium P and indemnity I.[1]
2. Clients buy at most one contract from one insurance company. When buying insurance, a client has to pay the premium P up front.
3. The consumer decides which unobservable effort level e to choose in order to avoid the loss.
4. The loss occurs or not, and the indemnity is paid out in case of a loss.

Stages 1 to 3 take place in period 1, whereas stage 4 takes place in period 2.

Insurers

There are two or more risk-neutral insurers in the market who compete in contracts. There is informational asymmetry regarding the patience type of consumers: insurers know the distribution of patience types in society, but they cannot identify the patience type of an individual who wants to buy a contract. Furthermore, the consumers' utility function, the two available effort levels, and the resulting loss probabilities are also known to the insurers. This enables them to conjecture the effort level employed by insurees of each type under any contract correctly, even though effort is unobservable.

For simplicity, we assume that the market interest rate is zero.

[1] We follow the well-established precedent in this literature by assuming that each insurer offers only a single contract. In the final section, multicontract offers are discussed.

Consumers

It is assumed that the consumers' *ex ante* expected utility EU can be described by

$$EU = U(w - P) + \delta[[1 - s(e)]U(w) + s(e)U(w - L + I)] - c(e),$$

with U being a concave, time-additive utility function with exogenously given risk aversion. Premium is denoted by P, and indemnity by I. Furthermore, there is period income w, personal discount factor δ, effort e with $e \in \{e_l, e_h\}$, loss probability $s(e)$ with $s(e_h) < s(e_l)$, costs of effort $c(e)$ with $c(e_l) < c(e_h)$, and loss L. To simplify notation, we define $s_n := s(e_n)$ and $c_n := c(e_n)$ for $n \in \{h, l\}$.

The level of effort is chosen and paid for in the first period, whereas the benefit of the effort accrues in the future. This way of modeling effort is especially adequate when it is a technical necessity to make precautionary provisions before or at the very beginning of the insurance contract. For example, when building a house, the agent can decide whether or not to use fire-retardant materials. This investment will carry on its beneficial effect over the policy period of a fire insurance. Another example is travel health insurance. Here, the insuree can decide whether or not to get vaccinated before departure. Sometimes, an audit conducted by the insurer in case a loss is reported allows to infer the effort level employed to a certain extent. However, a perfect inference seems to be unlikely in many cases. Then, the two effort levels in this model can be interpreted as the residual consumer's discretion with regard to effort that cannot be detected in an audit anymore. Further examples are health or life insurance in which individuals decide earlier in their life whether to live a healthy lifestyle, do sports, etc. The benefits of this behavior will come in the future.

The population shall be split up in a fraction γ of patient people with a high discount factor δ_p and a fraction $1 - \gamma$ of impatient people with a low discount factor δ_i, with $\delta_i < \delta_p$.

Via the implicit function theorem, the slope of the consumers' indifference curves IC in the premium-indemnity space (Figure 1) can be verified to be

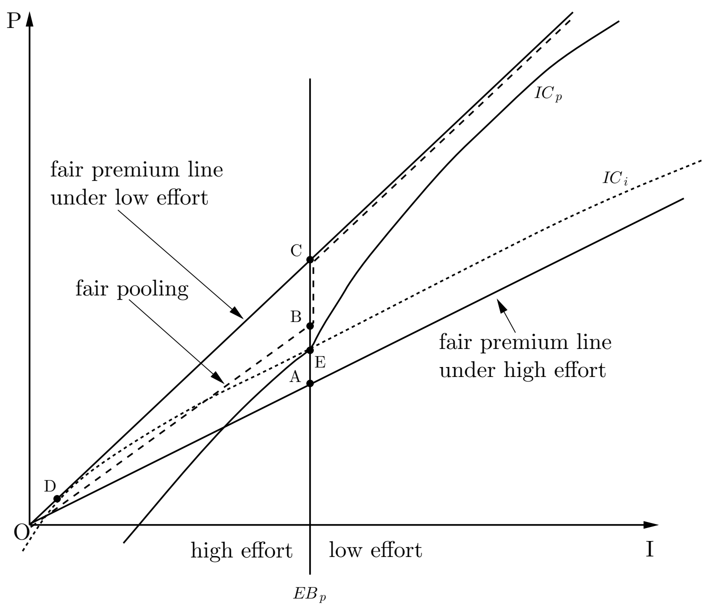
$$S(\delta,e,P,I) := \left.\frac{dP}{dI}\right|_e = \frac{\delta s(e)U'(w - L + I)}{U'(w - P)} > 0. \tag{1}$$

For extremely low patience with $\delta = 0$, the indifference curve is a flat horizontal line. With increasing δ, the indifference curve becomes steeper at every given point in the premium-indemnity space (Figure 1).

Given the same effort level e, the indifference curve of a patient type IC_p is steeper than the indifference curve of an impatient type IC_i for any contract $\{P, I\}$. This can be seen by computing the derivative of Equation (1) with respect to δ:

$$\left.\frac{\partial S(\delta,e,P,I)}{\partial \delta}\right|_e = \frac{s(e)U'(w - L + I)}{U'(w - P)} > 0.$$

FIGURE 1
Contract Space in a Two-Period Competitive Insurance Market in Which Consumers Differ in Patience. There Is a Unique Separating Equilibrium in Which the Impatient Types Employ Low Effort and Buy Zero-Profit Contract *D* With Low Indemnity, Whereas the Patient Types Employ High Effort and Buy Profit-Making Contract *E* With High Indemnity



Combining S with the fact that P is a function of I on an indifference curve, it can be shown that indifference curves are concave under a constant effort level e:

$$\frac{dS}{dI} = \frac{\delta s(e)[U''(w-L+I)U'[w-P(I)] - U'(w-L+I)U''[w-P(I)][-P'(I)]]}{[U'(w-P(I))]^2} < 0.$$

In the equilibrium analysis, we sometimes make the following assumption concerning the slopes of the indifference curves.

Assumption 1: *Even under high effort e_h, a patient type (δ_p) has a steeper indifference curve than an impatient type (δ_i) under low effort e_l for any contract $\{P, I\}$.*

According to Equation (1), this requires that $\delta_p s_h > \delta_i s_l$. This "single crossing" condition implies that indifference curves of the patient and impatient type only cross once, even if they employ different effort levels. The typical loss probability in most lines of insurance can be considered relatively small in comparison to the typical discount rate, as suggested by the literature cited above. Therefore, the condition does not seem overly restrictive as long as a low level of precautionary effort does not cause s_l to skyrocket too much. Furthermore, the possibility of an audit may impose a lower bound on e_l. Later on, we also discuss the situation under which this assumption is not satisfied.

Effort Border

Depending on parameter values, there may be a border line in the premium-indemnity space that describes when a certain patience type is exactly indifferent between high and low effort. To determine this effort border *EB*, we compute a consumer's advantage in expected utility from employing high effort:

$$A(\delta) := EU_h - EU_l = \delta[s_l - s_h][U(w) - U(w - L + I)] - (c_h - c_l).$$

If $A(\delta)$ is positive, it pays to employ high effort, whereas otherwise, the individual is better off employing low effort.

In order to obtain a condition under which the individual is indifferent between high and low effort, we set $A(\delta)$ equal to 0 and obtain:

$$U(w - L + I) = U(w) - \frac{c_h - c_l}{\delta(s_l - s_h)}. \tag{2}$$

When the individual switches from high effort e_h to low effort e_l (e.g., while P is constant and I increases marginally), it becomes apparent from Equation (1) that the indifference curve exhibits a kink at the effort border, as its slope becomes steeper. The indifference curve of the patient type IC_p in Figure 1 gives an example.

According to Equation (2), the position of the kink is determined by the following factors:

- The position of the kink does not depend on P, so the curve of kinks (effort border) is a linear vertical line in the premium-indemnity space.
- A larger δ moves the kink to the right in the premium space so that patient types employ high effort for levels of indemnity for which impatient types would employ low effort.
- A larger difference in loss probabilities for low and high effort ($s_l - s_h$) and a larger loss L move the kink to the right in the premium-indemnity space.
- A larger cost difference between high and low effort ($c_h - c_l$) moves the kink to the left. This is also the case for a larger w if $I < L$, because U is concave.

If the circumstances are such that the effort border of a patience type is located at an indemnity level of zero or below, the individuals of this patience type never employ high effort.

EQUILIBRIA

We assume that at stage 2 consumers buy the best contract available. If two or more insurers offer the same optimal contract, clients randomize with equal probability. Effort is chosen accordingly. Thus, we can concentrate on the decision of the insurer at stage 1.

Characteristics of Different Equilibria

Before analyzing equilibria of the model, we outline the properties of separating and pooling equilibria, following de Meza and Webb (2001). The outside option contract (no insurance) is denoted by O.

Separating Equilibrium. A separating equilibrium is characterized by the following four properties, whereby C_n^* with $n \in \{i, p\}$ stands for the contract chosen by type n in equilibrium.

Incentive Compatibility. There must be no incentive for an impatient type to buy the contract for a patient type and vice versa:

$$EU_i(C_i^*) \geq EU_i(C_p^*),$$
$$EU_p(C_p^*) \geq EU_p(C_i^*).$$

Effort Incentives. Since effort is unobservable, the individual employs high effort only if this is advantageous in terms of a higher expected utility:

$$e(\delta_n) = \begin{cases} e_h, & \text{if } A(\delta) \geq 0 \\ e_l, & \text{if } A(\delta) < 0. \end{cases}$$

Participation. Consumers cannot be forced to buy insurance, but they insure themselves voluntarily if this results in a higher expected utility than the outside option of remaining uninsured:

$$EU_n(C_n^*) \geq EU_n(O) \quad \text{for} \quad n \in \{i, p\}.$$

Profit Maximization. By offering either C_i^* or C_p^*, each insurer maximizes his profit, given that both C_i^* and C_p^* are offered by his competitors.

Pooling Equilibrium. A pooling equilibrium C^* is characterized by the following three properties:

Effort Incentives

$$e(\delta_n) = \begin{cases} e_h, & \text{if } A(\delta) \geq 0 \\ e_l, & \text{if } A(\delta) < 0. \end{cases}$$

Participation

$$EU_n(C^*) \geq EU_n(O) \quad \text{for} \quad n \in \{i, p\}.$$

Profit Maximization. Given that C^* is offered by his competitors, each insurer maximizes his profit by offering C^*.

The situations that arise from the various parameter constellations can be grouped according to which types employ high effort at least in some area of the contract space. Possible answers are none, only the patient types, or both types. Since the case in which no one ever employs high effort is not very interesting, we concentrate on the latter two cases. In this subsection, we analyze the simpler case in which the impatient types never exerts high effort. The other case is analyzed in the following section.

Impatient Types Do Not Employ High Effort Under Any Contract

The analysis in this subsection is characterized by the following property:

P-1: *The impatient types do never employ high effort, but there is a region for the patient types in the contract space where they employ high effort. Formally:*

$$\delta_i \leq \frac{c_h - c_l}{(s_l - s_h)(U(w) - U(w - L))} < \delta_p.$$

That is to say that the effort border of the impatient types EB_i is not visible in the premium-indemnity space, as it is located at a weakly negative indemnity. However, the effort border of the patient types EB_p is located at a strictly positive indemnity, as shown in Figure 1.

Separating Equilibrium. The resulting equilibria can be seen most easily by proceeding diagrammatically. Figure 1 depicts the contract space (premium-indemnity space). The concave lines are indifference curves of the patient types (IC_p) and the impatient types (IC_i), respectively. At some critical indemnity level, there is the patient types' effort border EB_p. To the left of it, they voluntarily employ high effort, as their expected utility is higher than under low effort. To the right of it, they employ low effort. Switching to the low effort level makes the loss probability jump from s_h to s_l. Therefore, the patient types' indifference curves have a kink at their effort border because insurance is now more valuable again.

The insurance company's fair premium lines for high and low effort are straight lines with slope s_h and s_l, respectively, because it is assumed that the insurer is risk neutral and that the market interest rate is zero. The fair pooling line describes all contracts that yield zero profit for the insurer if they are bought by the whole population of patient and impatient people. This line jumps at the effort border line, as here, the risk of the patient types jumps from s_l to s_h.

Regarding equilibrium, we distinguish between the two cases. This is because the relevant indifference curve of the impatient types can cut the patient types' effort border below (Figure 1) or above (Figure 3) the fair pooling line (point B).

Proposition 1: *Assume P-1 holds and that the impatient types' discount factor δ_i is such that IC_i through D (or O, if impatient types prefer being uninsured) (Figure 1) cuts EB_p above A but below B at some point E, and IC_p through E is always below the fair pooling line. If Assumption 1 holds, then there exists the following unique separating equilibrium: The patient types employ high effort and buy the unfair contract E. If $U'(w) < \delta_i\, U'(w - L)$, the* *impatient types employ low effort and buy contract D with a fair premium, which has lower cover than contract E. Otherwise, the impatient types remain uninsured.*[2,3]

Proof: The patient types are strictly better off with contract E in comparison to being uninsured. The impatient types are indifferent between contract E and contract D (or not being insured at all), and thus, as in Rothschild and Stiglitz (1976), it is assumed that they buy D.

Impatient types prefer to buy some insurance at their fair premium when $-s_l U'(w - s_l I) + s_l\, \delta_i\, U'(w - L + I) \geq 0$ at $I = 0$. This condition is expressed in the proposition.

Now, we challenge contract E by considering deviations to the left of EB_p. Offering contracts above IC_p does not make sense. This is because below IC_i, only impatient types are attracted, which results in losses, and contracts above IC_i will not attract any customer. By offering a contract below IC_p, an insurer could attract all patient types. However, because of Assumption 1, such a contract would also attract all impatient types. Since the fair pooling line is above the deviating contract, it necessarily leads to losses.

Deviations to the right of EB_p are also loss making. This is because in this region, both types employ low effort, and the patient types cannot even be attracted away from contract E by offers on the fair premium line under low effort.

The patient types prefer contract E to any other contract that lies on IC_i through D (or O, if impatient types prefer to be uninsured) and is on or above the applicable fair premium line. Therefore, there can be no other separating equilibrium. A pooling equilibrium would have to be on or above the fair pooling line. Since IC_p through E is always below the fair pooling line by assumption, there cannot be a pooling equilibrium that would not be destroyed by D (or O) and E. Uniqueness follows. Q.E.D.

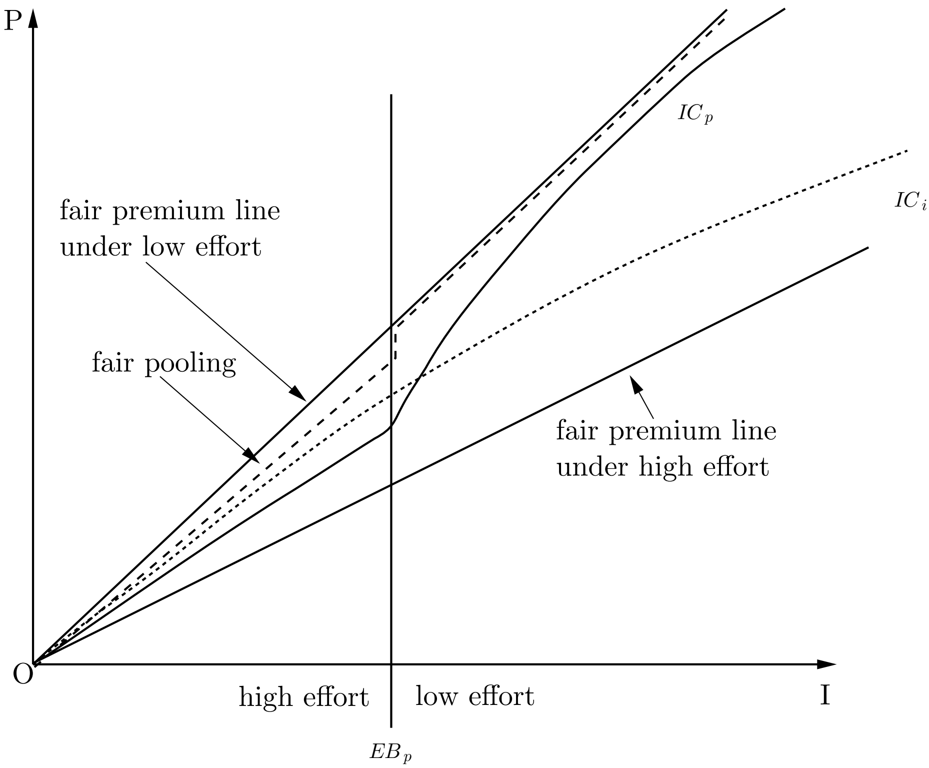
Situation With Assumption 1 Not Being Satisfied. An interesting question to analyze at this point is what happens when Assumption 1 does not hold. This means that the indifference curve of the impatient type under low effort is steeper than the indifference curve of the patient type under high effort. Suppose that the impatient types do not want to buy insurance even for their fair premium (IC_i does not touch the fair premium line under low effort). Then, there are two possibilities with regard to the fair pooling line: it can either cross the indifference curve of the patient types through O (IC_p in Figure 2) or not. In the first case, a contract on the fair pooling line dominates O, but it can always be destroyed by a profit-making deviating offer that only attracts patient types. In the end, no equilibrium exists. In the second case, which

[2] If IC_p through E cuts the low effort fair premium line, then in equilibrium, the patient types will obtain a contract on the low effort fair premium line. In that case, both types obtain their optimal contract, given that they exert low effort. Although the contract of the patient type is no longer marginally distorted, the adverse selection problem might still be relevant, as long as without the other type being present the low type would obtain contract A. We thank an anonymous referee for pointing this out to us.

[3] Allowing for a fixed fee of entry, as in Smart (2000), would lead to a sufficient number of firms sharing the market for contract E to dissipate the associated profits.

FIGURE 2

If Assumption 1 Is Not Satisfied, IC_i Through O Does Not Cross the Fair Premium Under Low Effort, and the Fair Pooling Line Does Not Cross IC_p, Then the Equilibrium Contract Is O

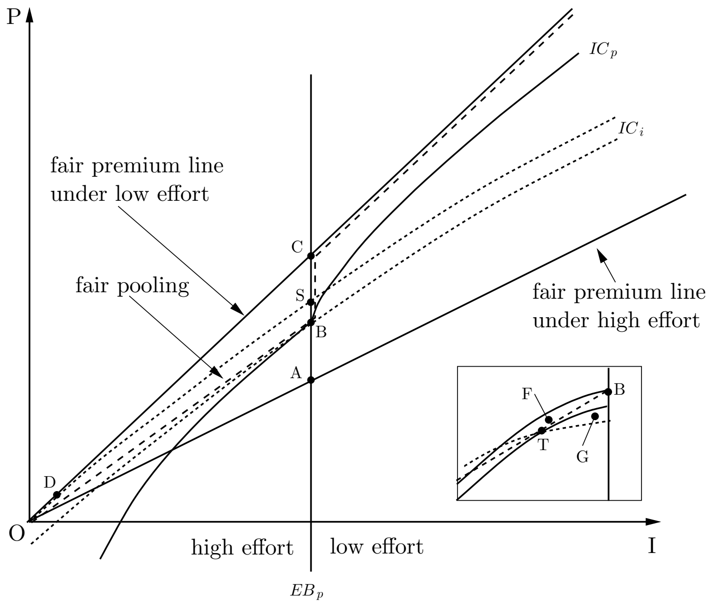


is shown in Figure 2, O is an equilibrium. This shows that consumer heterogeneity with regard to time preference and simultaneous moral hazard may contribute to the fact that some risks are uninsurable. A similar result is obtained by Chiu and Karni (1998) in the context of private unemployment insurance. The authors demonstrate that adverse selection on the employees' preference for leisure together with moral hazard regarding the employees' effort of working hard can explain the absence of private unemployment insurance as an equilibrium outcome.

Pooling Equilibrium.

Proposition 2: *Assume P-1 holds and that the impatient types' discount factor δ_i is such that IC_i through D (or O, if impatient types prefer being uninsured) (Figure 3) cuts EB_p above B but below C, and IC_p through B is always below the fair premium line under low effort. Under Assumption 1, there exists a unique zero-profit pooling equilibrium in which the impatient types employ low effort, the patient types employ high effort, and both types buy contract B if IC_p at point B is steeper than the fair pooling line to the left of EB_p (condition C1). Otherwise, there exists no equilibrium in pure strategies.*

FIGURE 3
Unique Zero-Profit Pooling Equilibrium in Which Both Types Buy Contract *B*. The Impatient Types Employ Low Effort, Whereas the Patient Types Employ High Effort



Proof: Suppose first that condition *C*1 holds. Both types are strictly better off buying contract *B* instead of remaining uninsured. Now, we consider deviating contracts to the left of EB_p: offering a deviating contract above IC_p through *B* does not make sense, as no patient types can be attracted. Offering a profitable deviating contract below IC_p through *B* is loss making since, by condition *C*1, such contracts are below the fair pooling line and would also be bought by all impatient types (Assumption 1).

Deviating contracts to the right of EB_p are also loss making. This is because in this region, both types employ low effort and cannot even be attracted away from *B* by contracts on the fair premium line under low effort.

The patient types prefer contract *B* to all other contracts on or above the fair pooling line. Thus, there cannot be another pooling equilibrium. It must be that in a candidate for a separating equilibrium, the impatient types get contract *D* (or *O*, if impatient types prefer being uninsured) and are indifferent between their own contract and the contract of the patient types, which would be *S*. Because of incentive compatibility, the separating contract for the patient types must be on or above IC_i through *D* (or *O*). Therefore, both types will prefer contract *B* to the candidate for a separating contract and *B* is a unique pooling equilibrium.

Now, we consider the situation in which condition *C*1 does not hold, which is depicted in the box in Figure 3. In this case, there is a point *T* to the left of EB_p at which IC_p is a tangent to the fair pooling line. Offering a contract such as *F* close to *T* that attracts both types but still is above the fair pooling line destabilizes *B*. However, because of Assumption 1, *T* is also not a stable equilibrium since a profitable deviating contract such as G can be offered. This reasoning holds true for any contract on and above the fair pooling line. Since IC_p is below the fair premium line under low effort, there cannot be a pooling equilibrium to the right of EB_p either.

Suppose S is part of a candidate for a separating equilibrium. All insurers offering S earn a profit per contract that is represented by the distance $\overline{AS}$. If there are too many insurers offering S, a deviation slightly to the southwest of S can be profitable for an individual insurer. This is the case if the pooling profit earned by serving all consumers is greater than the share in profit by serving only patient consumers. The consequence is that no equilibrium is possible because of the single crossing property (Assumption 1) in the area to the left of EB_p. Even if there are not too many insurers offering S, new entrants in the insurance market are attracted, and S is destabilized. Thus, a separating equilibrium cannot exist. In the end, neither a pooling nor a separating equilibrium exists if condition $C1$ does not hold. Q.E.D.

Impatient Types Employ High Effort Under Some Contracts

This case shall be characterized by the following properties P-2 and P-3 that ensure that both types want to buy insurance.

P-2: *Both the impatient as well as the patient types employ high effort in some region of the contract space. Formally:*

$$\frac{c_h - c_l}{(s_l - s_h)(U(w) - U(w-L))} < \delta_i (< \delta_p).$$

The parameters in Equation (2) take values for which the indifference curves of both types feature a kink.

P-3: *The impatient types are interested in buying insurance at their fair premium under high effort.*

This property requires the slope of the impatient types' indifference curve without insurance ($P = 0$ and $I = 0$) not to be flatter than the fair premium line under high effort. This is captured by the following condition:

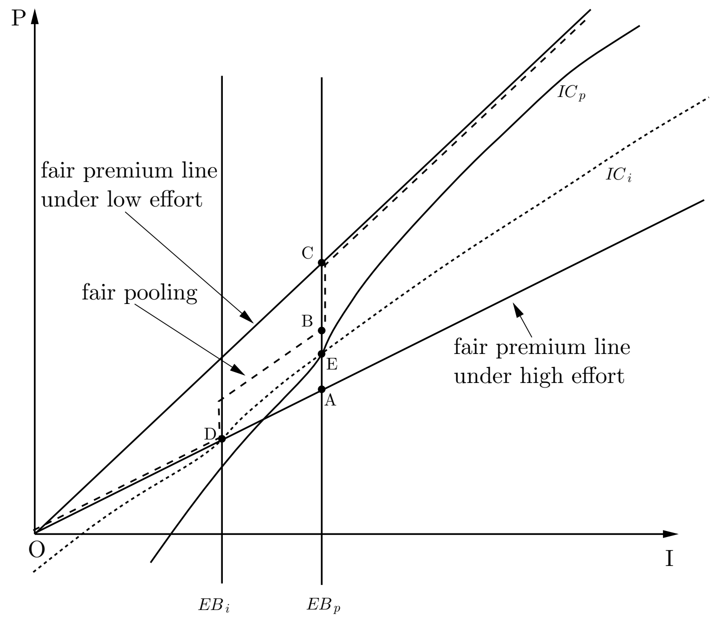
$$s_h < \frac{\delta_i s_h U'(w-L)}{U'(w)}.$$

Proposition 3: *Assume that P-2 and P-3 hold and that the impatient types' discount factor δ_i is such that IC_i through D (Figure 4) cuts EB_p above A but below B at some point E, and IC_p through E is always below the fair pooling line. If Assumption 1 holds, then there exists a unique separating equilibrium in which both types employ high effort. The patient types buy contract E with an unfair premium and the impatient types buy contract D at the fair premium.*

Proof: Both types are strictly better off than without insurance. The impatient types are indifferent between their own contract D and the patient types' contract E. Therefore, it is assumed that the impatient types buy their own contract D. Deviations from

FIGURE 4

Both Types Feature a Kink in Their Indifference Curves. There Is a Separating Equilibrium in Which Both Types Employ High Effort. The Patient Types Buy Contract *E* With an Unfair Premium, Whereas the Impatient Types Buy Contract *D* With a Fair Premium



D are not an issue, as *D* maximizes the impatient types' expected utility subject to a zero underwriting profit by construction.

Deviations from *E* to the left of EB_p: it does not pay to offer a contract above IC_p through E. This is because above IC_i through *D*, no customer can be attracted. Below it, only impatient types who employ low effort can be attracted, which leads to losses. This also holds true for all points on IC_i through *D*, with the only exception being *D*, since IC_i does not cross the fair pooling line to the right of EB_i as we look at the case in which *E* is below *B*. However, offering *D* instead of *E* is not attractive either, because it involves a zero underwriting profit. Below IC_p through *E*, both customer types are attracted because of Assumption 1. However, since this area is below the fair pooling line, deviating offers below IC_p through *E* cause losses as well.

To the right of EB_p, profitable offers are not possible either. In this region, not even offers on the fair premium line can attract any patient customers away from *E*.

The patient types prefer contract *E* to any other contract that lies on IC_i through *D* and is on or above the applicable fair premium line. Therefore, another separating equilibrium cannot exist.

A pooling equilibrium would have to be on or above the fair pooling line. Since IC_p through *E* is always below the fair pooling line, there cannot exist a pooling equilibrium that would not be destroyed by *E*. Uniqueness follows. Q.E.D.

EFFECTS OF INCLUDING A CAPITAL MARKET

So far, the analysis abstracted from the existence of a capital market in which consumers can transfer wealth across time by either saving or borrowing. Insurance served as the only means of saving by allowing to exchange an immediate premium payment into a future expected value of an indemnity payment. In order to analyze the effect of a capital market, we augment the model as follows: a new variable t describes the amount consumers want to save ($t > 0$) or borrow ($t < 0$) in period 1, whereby a new parameter $r > 0$ represents the market interest rate on savings and loans for the time between period 1 and period 2. The consumers' *ex ante* expected utility EU is given by

$$EU = U(w - P - t) + \delta[[1 - s(e)]U(w + rt) + s(e)U(w - L + I + rt)] - c(e).$$

A positive market interest rate will also have an effect on the insurers. Their fair premium lines will become flatter because the expected loss payment can be discounted with r.

In the analysis so far without a capital market, an indifference curve described all premium-indemnity combinations that represent a particular expected utility level. This continues to be the case in the new situation with a capital market. However, now the expected utility of each premium-indemnity combination on the indifference curve must be calculated under optimal saving, which could either be saving or borrowing.

Optimizing wealth transfer t for a particular premium-indemnity combination yields the following first-order condition:

$$\delta r[(1 - s)U'(w + rt) + sU'(w - L + I + rt)] = U'(w - P - t).$$

It becomes clear that the optimal wealth transfer equalizes the expected marginal utility in both periods. When assuming a concave utility function, impatient types with a low δ have a tendency toward lower savings or even toward borrowing.

Starting from a point on an indifference curve, increasing the indemnity by a certain amount will increase expected utility. Therefore, the premium must be increased by a certain amount as well to reduce expected utility back to the original level. The sensitivity of expected utility with regard to indemnity and premium variations determines the shape of the indifference curve. For a given optimal transfer t^*, the according derivatives are:

$$\frac{dEU}{dI} = \delta s U'(w - L + I + r\,t^*),$$
$$\frac{dEU}{dP} = -U'(w - P - t^*).$$

Assuming a concave utility function, the above equations show that saving ($t > 0$) will make the indifference curve flatter in the neighborhood of any premium-indemnity combination in comparison to a situation without a capital market. This is because the additional amount of money rt^* in period 2 reduces the marginal utility of additional indemnity. Therefore, a lower premium increase suffices to compensate for a given higher indemnity. This effect is increased further as the negative impact of additional premium on expected utility in period 1 is amplified by a higher marginal utility of money due to saving t^*. In the same line of reasoning, borrowing ($t < 0$) makes indifference curves steeper in comparison to a situation without a capital market.

Capital Market Without Credit Constraint

In the example given in Figure 5, IC_i and IC_p represent indifference curves of the impatient and the patient types without a capital market. It can be seen, that IC_{it}, which is an indifference curve of the impatient types with the possibility of wealth transfer across periods (both saving and borrowing are possible), is much steeper than IC_i. This is because the impatient types want to borrow along the complete indifference curve, as is indicated in the lowest graph of Figure 5. On the other hand, IC_{pt}, which is an indifference curve of the patient types with the possibility of wealth transfer across periods, starts out flatter than IC_p, as the patient types initially want to save (medium graph of Figure 5). However, for intermediate and high indemnity levels, even the patient types prefer to borrow, which makes IC_{pt} steeper than IC_p in this region.

As a consequence, the existence of a capital market without credit constraint renders the outcome of the kinds of equilibria described in the previous section less probable. The reason for this is that impatient types want to save less or borrow more than patient types in the first period. This increases the steepness of the impatient types' indifference curves relative to the patient types' indifference curves and thus works against Assumption 1.

Capital Market With Credit Constraint

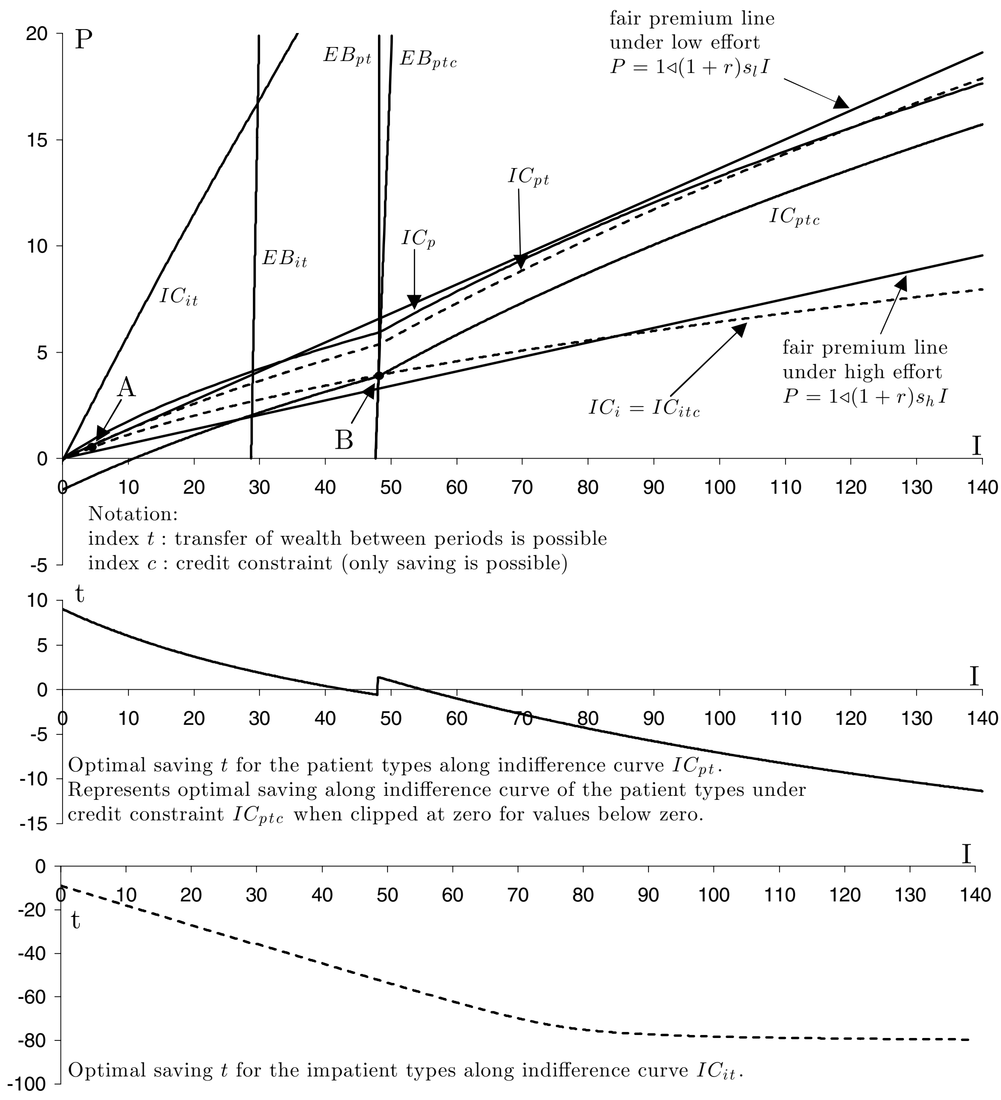
The extent of the destabilizing effect of a capital market on the equilibria described in the previous section is reduced significantly when there exists a credit constraint; i.e., only saving is possible. In the example of Figure 5, the indifference curve of the impatient types under a wealth transfer possibility with credit constraint IC_{itc} is unchanged from the shape of the indifference curve of the impatient types without a capital market IC_i. Therefore, only potential saving of the patient types, which tends to make their indifference curves flatter, works against Assumption 1. In the situation shown in Figure 5, at point B, the indifference curve of the patient types under credit constraint IC_{ptc} is still steeper than the indifference curve of the impatient types IC_{itc} to the left of the according effort border of the patient types EB_{ptc}. In the end, as shown in Figure 5, a separating equilibrium in which patient types exert high effort and buy contract B with a high indemnity whereas impatient types employ low effort and buy contract A with low indemnity is still possible.

CONCLUSION

In this article, we employ a two-period competitive insurance model that is characterized by the simultaneous presence of moral hazard and adverse selection. Moral hazard is modeled along the traditional lines and is assumed to occur with regard to

FIGURE 5

Indifference Curves and Optimal Saving in an Example With a Capital Market. The Details of the Specification Are as Follows: $\delta_p = 0.9$, $\delta_i = 0.3$, $r = 1.1$, $U(x) = \sqrt{x}$, $w = 100$, $L = 90$, $s_h = 0.075$, $s_l = 0.15$, $c_h = 0.16$, $c_l = 0$



unobservable precautionary effort, which can be either high or low. However, adverse selection occurs with regard to the personal discount rate of consumers, which can be high or low as well. It is assumed that consumers decide whether or not to buy insurance in the first period. If so, they have to pay the premium up front and decide about the precautionary effort level they wish to employ. In the second period, the consumer faces the risk of a loss. This setup is meant to capture the fact that real-world insurance contracts usually require the consumer to prepay the premium for several periods (e.g., 12 months), which creates a role for the consumer's personal discount rate in the *ex ante* valuation of contracts.

The different time preferences among consumers open the door for separating equilibria in which the patient types employ a high effort level and buy high cover whereas impatient types employ low effort and buy little or even no cover. Since high effort implies low risk and vice versa, this result is equivalent to saying that the low risks are more fond of buying insurance. Thus, the prevailing outcome is the opposite of the traditional adverse selection theory, in which adverse selection takes place with regard to consumer risk types. Furthermore, it is possible that the equilibrium contract for the patient types entails an unfair premium. In this case, undercutting the premium would attract all the impatient types who employ a low precautionary effort. This would result in a loss for the insurer. The insight gained by this approach might also shed some light on the coexistence of discount insurers and full-service insurers. Discount insurers could screen the market for more patient consumers by paying out indemnities only with a delay in comparison to full-service insurers.

The model used in this article assumed that if insurers deviate from the equilibrium, they only offer one contract each. However, in reality, insurers are offering menus of contracts that allow for the possibility of cross-subsidization of contracts. Allowing insurers to deviate with an offer of more than one contract in our model would lead to the following results.

In the case of a separating equilibrium (Figure 1), the equilibrium will break down. First note that offering two contracts just "below" D and E will attract all customers away from the equilibrium contracts and will lead to a profit for the deviating insurer. The insurer will make losses with the contract for the impatient types while making profits with the contract for the patient types. Cross-subsidizing contracts by themselves are never an equilibrium, as any insurer would only offer the profit-making contract (if all others offer the pair of cross-subsidizing contracts). Thus, no equilibrium exists. In the case of a pooling equilibrium (Figure 3), a similar result can be obtained. Offering a contract just"below" B and a second one that is on the indifference curves of the impatient type going through this contract can lead to a profit-making pair of contracts in which, again, the contract for the impatient type makes a loss, whereas the contract for the patient type makes a profit. As before, such a pair of contracts cannot be an equilibrium.

There is a huge literature on the equilibrium nonexistence problem in insurance markets with adverse selection. It is probably fair to say that, so far, there has been no general agreed upon way on how to deal with this problem. We will discuss here two possible responses to this problem and outline the corresponding equilibria these will lead to.

The first is the so-called WMS equilibrium approach (Wilson, 1977; Miyazaki, 1977; Spence, 1978), which (for single contract offers) has found a game-theoretic basis in the work by Hellwig (1987). In this approach, cross-subsidizing equilibrium contracts are possible, as for any deviating menu of contract, the insurers can react by withdrawing their contracts. Thus, if someone intends to cream skim the market by offering the profit-making contract only, the others will withdraw their contracts so that the deviating insurer is left with all customers and makes a loss. In our model, a WMS equilibrium is given by contract pairs of the form described in the previous paragraph: the patient type receives a contract on his effort border line, which is such that he employs high effort and the insurer make profits with this contract. The impatient type obtains a contract on his indifference curve going through the contract for the patient type. Insurers make losses with this contract, which are such that they, on average, exactly equal the profits made with the patient type. Thus, the result that the higher risks buy the contract with smaller coverage remains to hold, whereas the results on the profit-making contract and on pooling contracts do not hold anymore.

A second approach to the equilibrium nonexistence problem is the Riley equilibrium, the consequences of which have found a game-theoretic basis in the work by Inderst and Wambach (2001). Riley assumes that if deviating contracts are offered, the other insurers can respond by offering new contracts themselves. This makes offering cross-subsidizing deviating contracts unattractive, as the other insurers will respond by offering only the profit-making contract. Therefore, the deviator is left with all the high risks and only a few, if any, low risks. In Inderst and Wambach (2001), it is assumed that insurers are capacity constrained. In that case, deviating by offering cross-subsidizing contracts might lead to a loss, as only the high risks will come to buy their new (loss making) contract from the deviating insurer. Although low risks would also prefer their new contract, they would profit less than the high risks. If the deviating insurer is capacity constrained, the low risks have thus less incentive to take on the risk of not being served. In our model, the Riley equilibrium takes the form as discussed in the previous section (in Figures 1 and 3). Separating contracts with unfair premium as well as pooling contracts remain to exist in equilibrium. Any cross-subsidizing deviation will become unattractive if other insurers can react or if only the impatient types approach such a deviator. This, in turn, justifies why throughout the article we concentrated on the simpler analysis of allowing for single contract deviations only.

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