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MULTIDIMENSIONAL CREDIBILITY WITH TIME EFFECTS: AN APPLICATION TO COMMERCIAL BUSINESS LINES

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ABSTRACT

This article considers Danish insurance business lines for which the pricing methodology has been dramatically upgraded recently. A costly affair, but nevertheless, the benefits greatly exceed the costs; without a proper pricing mechanism, you are simply not competitive. We show that experience rating improves this sophisticated pricing method as much as it originally improved pricing compared with a trivial flat rate. Hence, it is very important to take advantage of available customer experience. We verify that recent developments in multivariate credibility theory improve the prediction significantly, and we contribute to this theory with new robust estimation methods for time (in-)dependency.

INTRODUCTION

In this article, credibility theory and experience rating mean more or less the same thing; however, strictly speaking, credibility theory describes a theoretical model with a latent risk variable, whereas experience rating is the act of including observed experience in the rating process. This latter act is sometimes carried out in non-life insurance companies without a consistent theoretical model behind it. But since all experience rating in this article is based on a theoretical model, we can more or less use the two expressions interchangeably. Credibility theory has a long tradition in actuarial science; we show in this article that there is indeed a good reason for this. In our concrete application to Danish commercial business lines, we show that the use of experience rating is as important as the use of pricing as such. In other words, we double the quality of the price rating by the inclusion of credibility theory in the rating process. We also consider the recently developed method of multivariate experience

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rating in which the latent risk parameter is allowed to be multidimensional such that each dimension represents one cover from the business line (see Englund et al., 2008). We also introduce a method to estimate a time effect of this model. We show that this more general version of credibility theory gives better results than the results from the classical one-dimensional credibility theory. We follow the standard approach of actuarial practitioners and only use frequency information in our credibility approach. However, the severity of experience claims should contain some valuable information as well, indicating that there might be even more to gain from credibility theory if a robust and stable credibility method is developed incorporating severity information in the experience rating.

An early beginning of credibility theory appeared in Mowbray (1914) and Whitney (1918). After the elegant approach presented by Bühlmann (1967) and Bühlmann and Straub (1970), a large number of extensions have been derived. References can be made to Jewell (1974), Hachemeister (1975), Sundt (1979, 1981), and Zehnwirth (1985). See also Halliwell (1996), Greig (1999), and Bühlmann and Gisler (2005) for more comprehensive surveys.

Evolutionary models are not new in credibility theory. The idea is that recent claim information is more valuable than old claim information. This approach was introduced in the 1970s for one-dimensional credibility models (see Gerber and Jones, 1975a,b; De Vylder, 1976). Much of the work on the time-dependent models focused on credibility formulas of the updating type. These recursive estimators were introduced by Mehra (1975) for credibility applications and further developed by De Vylder (1977), Sundt (1981), and Kremer (1982). For the time dependence in this article, we use a multivariate generalization of the recursive credibility estimator of Sundt in which the risk parameter itself is modeled as an autoregressive process.

The article is organized as follows. In "Multidimensional Credibility Theory," we state the credibility model and the estimators in our multidimensional setup. The model is generalized in "Evolutionary Effects" to incorporate an evolutionary effect, and a recursive credibility estimator is stated. The article is finished with the results of an empirical data study, in "An Application to Danish Commercial Business Lines," from which we, in "Discussion and Conclusions," draw conclusions concerning the predictability of the credibility estimators.

MULTIDIMENSIONAL CREDIBILITY THEORY

In this section, we repeat the multivariate credibility model of Englund et al. (2008) and define a new, more robust variance estimator inspired by the elegant approach of Bühlmann and Gisler (2005). We consider only frequencies of claims.

The Multidimensional Credibility Model

We observe insurance claims N_{ijk} with expected number of claims λ_{ijk} for individual $i \in (1, \ldots, I)$, calendar year $j \in (1, \ldots, J)$, and coverage $k \in (1, \ldots, K)$. A dot, "$\cdot$", indicates summation over that index. Define the standardized number of claims as a vector of k dimensions, $\mathbf{F}_{ij} = \mathbf{N}_{ij}(\lambda_{ij})^{-1}$.

Model Assumptions:

(i) *Given the vector of individual risk parameters* $\mathbf{\Theta}_i$*, the insurance claims* $\mathbf{N}_{ij}$ *are independent and Poisson-distributed, with expected value* $E[\mathbf{N}_{ij} \mid \mathbf{\Theta}_i] = \lambda_{ij}\mathbf{\Theta}_i$*. The expected value and (co-) variance of* $\mathbf{F}_{ij}$*, conditional on* $\mathbf{\Theta}_i$*, are:*

$$E[\mathbf{F}_{ij} \mid \mathbf{\Theta}_i] = \mathbf{\Theta}_i,$$

$$\mathrm{Cov}\left[\mathbf{F}_{ij}, \mathbf{F}_{ij}^T \middle| \mathbf{\Theta}_i\right] = \mathbf{S}_i, \textit{with diagonal elements } \mathrm{Var}[\mathbf{F}_{ijk} \mid \mathbf{\Theta}_i] = \frac{\sigma_k^2(\mathbf{\Theta}_i)}{\lambda_{ijk}}, \textit{ and}$$

$$\mathrm{Cov}\left[\mathbf{\Theta}_i, \mathbf{\Theta}_i^T\right] = \mathbf{T}.$$

(ii) *The pairs* $(\mathbf{\Theta}_1, \mathbf{N}_{1j}), (\mathbf{\Theta}_2, \mathbf{N}_{2j}), \ldots, (\mathbf{\Theta}_I, \mathbf{N}_{Ij})$ *are independent, and* $\mathbf{\Theta}_1, \mathbf{\Theta}_2, \ldots, \mathbf{\Theta}_I$ *are independent and identically distributed with* $E[\mathbf{\Theta}_i] = [E[\Theta_{01}], E[\Theta_{02}], \ldots, E[\Theta_{0K}]]^T = \mathbf{\Theta}_0$.

The Multidimensional Credibility Estimator

The credibility estimators can be seen as projections in the Hilbert space of all square-integrable random variables (see, e.g., Zehnwirth, 1985). Hence, the best linear, unbiased estimator of the multidimensional latent risk parameter $\mathbf{\Theta}_i$, given the observed experience, is:

$$\hat{\mathbf{\Theta}}_i = E[\mathbf{\Theta}_i] + \mathrm{Cov}[\mathbf{\Theta}_i, \mathbf{F}_{i\cdot}]\mathrm{Cov}\left[\mathbf{F}_{i\cdot}, \mathbf{F}_{i\cdot}^T\right]^{-1}(\mathbf{F}_{i\cdot} - E[\mathbf{F}_{i\cdot}]), \tag{1}$$

where

$$\begin{aligned}\mathrm{Cov}[\mathbf{\Theta}_i,\mathbf{F}_{i\cdot}] &= E[\mathrm{Cov}[\mathbf{\Theta}_i, \mathbf{F}_{i\cdot}] \mid \mathbf{\Theta}_i] + \mathrm{Cov}[E[\mathbf{\Theta}_i \mid \mathbf{\Theta}_i], E[\mathbf{F}_{i\cdot} \mid \mathbf{\Theta}_i]] \\ &= 0 + \mathrm{Cov}\left[\mathbf{\Theta}_i, \mathbf{\Theta}_i^T\right] = \mathbf{T},\end{aligned}$$

and

$$\begin{aligned}\mathrm{Cov}\left[\mathbf{F}_{i\cdot}, \mathbf{F}_{i\cdot}^T\right] &= E\left[\mathrm{Cov}\left[\mathbf{F}_{i\cdot}, \mathbf{F}_{i\cdot}^T\right] \middle| \mathbf{\Theta}_i\right] + \mathrm{Cov}\left[E[\mathbf{F}_{i\cdot} \mid \mathbf{\Theta}_i], E[\mathbf{F}_{i\cdot} \mid \mathbf{\Theta}_i]^T\right] \\ &= E\left[\mathbf{S}_i\right] + \mathrm{Cov}\left[\mathbf{\Theta}_i, \mathbf{\Theta}_i^T\right] = \mathbf{S}\mathbf{W}_i^{-1} + \mathbf{T}.\end{aligned}$$

The vector of standardized number of claims is $\mathbf{F}_{i\cdot} = [F_{i\cdot 1}, F_{i\cdot 2}, \ldots, F_{i\cdot K}]^T$, where $F_{i\cdot k} = (\sum_{j=1}^J \lambda_{ijk})^{-1} \sum_{j=1}^J \lambda_{ijk} F_{ijk}$. The weight matrix $\mathbf{W}_i$ is a diagonal matrix, with $\lambda_{i\cdot k} = \sum_{j=1}^J \lambda_{ijk}$ as the kth diagonal element. The credibility weight α_i is estimated by $\widehat{\boldsymbol{\alpha}}_i = \mathbf{T}(\mathbf{S}\mathbf{W}_i^{-1} + \mathbf{T})^{-1} = \mathbf{T}\mathbf{W}_i(\mathbf{T}\mathbf{W}_i + \mathbf{S})^{-1}$. This estimator differs from Bühlmann and Gisler (2005) but is able to use information from additional coverages to calculate the individual risk parameter, even if we lack information in a specific (inactive) coverage. The resulting credibility estimator can be seen as both a weighted sum of individual and collective claim information and a linear regression:

$$\begin{aligned}\hat{\mathbf{\Theta}}_i &= \widehat{\boldsymbol{\alpha}}_i \mathbf{F}_{i\cdot} + (\mathbf{I} - \widehat{\boldsymbol{\alpha}}_i)\mathbf{\Theta}_0 \\ &= \mathbf{\Theta}_0 + \widehat{\boldsymbol{\alpha}}_i(\mathbf{F}_{i\cdot} - \mathbf{\Theta}_0).\end{aligned} \tag{2}$$

Here, **I** is the identity matrix. Note that the one-dimensional credibility estimator is a trivial special case of Equation (2).

Estimation of the Parameters

The estimation of the parameters $\mathbf{\Theta}_0$, $\mathbf{T}$, and $\mathbf{S}$ is inspired by the elegant estimators presented by Bühlmann and Gisler (2005) for a different but related multivariate credibility problem. We estimate the elements of $\mathbf{\Theta}_0$ by $\hat{\boldsymbol{\theta}}_{0k} = (\sum_{i=1}^{I} \widehat{\boldsymbol{\alpha}}_{ik})^{-1} \sum_{i=1}^{I} \widehat{\boldsymbol{\alpha}}_{ik} F_{i\cdot k}$. The estimator of $\mathbf{S}$ is a diagonal matrix, with $\hat{\sigma}_k^2$ as kth diagonal:

$$\hat{\sigma}_k^2 = \frac{1}{J_{\cdot k} - I_k} \sum_{i=1}^{I} \sum_{j=1}^{J} \lambda_{ijk} \left(F_{ijk} - F_{i\cdot k}\right)^2.$$

$J_{\cdot k}$ is the sum of the number of yearly observations for all I_k individuals with an active coverage k. Note that we assume the occurrence of claims to be Poisson-distributed and that this, theoretically, gives us $\sigma_k^2 = \theta_{0k}$. The estimation of θ_{0k} precedes the estimation of σ_k^2, because of the estimation of the credibility weights, wherefore, we use two different estimators for this. The preliminary estimator of $\mathbf{T}$, $\widetilde{\mathbf{T}}$, has diagonal

$$\widetilde{\tau}_{kk}^2 = c_k \left(\frac{1}{I_k - 1} \sum_{i=1}^{I} \frac{\lambda_{i\cdot k}}{\lambda_{\cdot\cdot k}} (F_{i\cdot k} - F_{\cdot\cdot k})^2 - \frac{\sigma_k^2}{\lambda_{\cdot\cdot k}} \right), \ F_{\cdot\cdot k} = \left(\sum_{i=1}^{I} \lambda_{i\cdot k} \right)^{-1} \sum_{i=1}^{I} \lambda_{i\cdot k} F_{i\cdot k},$$

and for $k \neq k'$:

$$\widetilde{\tau}_{kk'}^2 = \frac{c_k}{I_{kk'} - 1} \sum_{i=1}^{I} \frac{\lambda_{i\cdot k}}{\lambda_{\cdot\cdot k}} (F_{i\cdot k} - F_{\cdot\cdot k})(F_{i\cdot k'} - F_{\cdot\cdot k'}).$$

Here, $I_{kk'}$ is all individuals with both coverage k and k' active and $\lambda_{\cdot\cdot k} = \sum_{i=1}^{I} \lambda_{i\cdot k}$ and $\lambda_{i\cdot k} = \sum_{j=1}^{J} \lambda_{ikj}$. The parameter c_k in the formulas above takes the expression $c_k = (I-1)(\frac{I}{\lambda_{\cdot\cdot k}} \sum_{i=1}^{I} \lambda_{i\cdot k}(1 - \frac{\lambda_{i\cdot k}}{\lambda_{\cdot\cdot k}}))^{-1}$.

Since $\widetilde{\tau}_{kk'}^2$ may be less than 0, the final diagonal estimator is defined as $\overline{\tau}_{kk'}^2 = \max\{\widetilde{\tau}_{kk'}^2, 0\}$. If $\widetilde{\tau}_{kk'}^2 \leq \widetilde{\tau}_{kk}^2 \widetilde{\tau}_{k'k'}^2$, we again follow the suggestions in Bühlmann and Gisler (2005) and replace $\widetilde{\tau}_{kk'}^2$ by

$$\overline{\tau}_{kk'}^2 = signum \left(\frac{\widetilde{\tau}_{kk'}^2 + \widetilde{\tau}_{k'k}^2}{2} \right) min \left(\frac{\left| \widetilde{\tau}_{kk'}^2 + \widetilde{\tau}_{k'k}^2 \right|}{2}, \sqrt{\widetilde{\tau}_{kk}^2 \widetilde{\tau}_{k'k'}^2} \right). \tag{3}$$

Even the corrected estimator $\overline{\mathbf{T}}$ resulting from Equation (3) is not necessarily positive semidefinite, for $K > 2$. Therefore, to make the credibility estimation meaningful and achieve the positive semidefiniteness, we adjust the estimator one last time. We compute the eigenvalue decomposition of $\overline{\mathbf{T}}$, put a floor of zero on the diagonal eigenvalue matrix, and compose the new eigenvalue matrix and reconstruct the final estimator $\hat{T}$. This estimator is more robust to calculate than the original estimator of Englund et al. (2008). The only parameter left to estimate now is λ_{ijk}, which we assume to be estimated from some pricing model developed by the company. This can be done at various levels of sophistication.

EVOLUTIONARY EFFECTS

A time-independent credibility estimator implies that the risk parameter for each coverage and policyholder is constant over time and the estimator will, therefore, treat old and new claim information equally. However, this might be quite insufficient in some cases, e.g., the abilities of a car driver are not constant. Hence, instead of assuming that the risk characteristics are given once and for all by the parameter $\mathbf{\Theta}_i$, we now suppose that the risk characteristics of year s are given by an unknown parameter $\mathbf{\Theta}_{is}$, and that the dependence between $\mathbf{\Theta}_{is}$ and $\mathbf{\Theta}_{it}$ decreases as $|s - t|$ increases. This is done by modeling the risk parameter as a stationary process, more specifically, as an autoregressive process. The interpretation of this approach is that new claim information will affect the claim prediction more than old claim information. We apply the same time dependency model as in Englund et al. (2008), since it is intuitive and easy to interpret. When it comes to estimation, we follow the recursive estimation principle of Sundt (1981), which is more stable and easier to implement than the estimator of Englund et al.

The Time-Dependent Model and Estimator

As a model for the time-dependent credibility estimator, we generalize the model stated in "The Multidimensional Credibility Model" by introducing a new index and thereby incorporate a time dependence and a correlation structure of the latent risk parameter.

We use the same model as in Englund et al. (2008); i.e., we assume the insurance claims N_{ijk} to be independent and Poisson-distributed, given Θ_{ijk}, with expected value $E[\,N_{ijk} \mid \Theta_{ijk}] = \lambda_{ijk}\Theta_{ijk}$ and the process $\{\Theta_{ijk}\}_{\,j=1,\dots,J}$ to be an autoregressive process with lag 1, with the covariance structure $\mathrm{Cov}[\Theta_{ijk}, \Theta^T_{i'j'k'}] = \tau^2_{kk'}(\rho_{kk'})^{|j-j'|}$ if $i = i'$ and $\mathrm{Cov}[\Theta_{ijk}, \Theta^T_{i'j'k'}] = 0$ otherwise, and with $\tau^2_{kk'} = \tau^2_{k'k}$, $\rho_{kk'} = \rho_{k'k}$, and $|\rho| \leq 1$.

The recursive estimator of the latent risk parameter is the result of a straightforward generalization of a theorem that is found and proved in Sundt (1981) for the one-dimensional case and in Bühlmann and Gisler (2005) for the multidimensional case. The estimator is

$$\hat{\mathbf{\Theta}}_{i(j+1)} = \mathbf{a}(\hat{\boldsymbol{\alpha}}_{ij}\mathbf{F}_{ij} + (\mathbf{I} - \hat{\boldsymbol{\alpha}}_{ij})\hat{\mathbf{\Theta}}_{ij}) + (\mathbf{I} - \mathbf{a})\mathbf{\Theta}_0, \tag{4}$$

where $\mathbf{F}_{ij} = [F_{ij1}, F_{ij2}, \dots, F_{ijK}]^T$, as previously noted. Here, $\mathbf{a}$ is a diagonal matrix of elements in [0, 1) driving the stationary autoregressive processes. The credibility weight is $\hat{\alpha}_{ij} = \mathbf{T}_{ij}\,\mathbf{W}_{ij}(\mathbf{T}_{ij}\,\mathbf{W}_{ij} + \mathbf{S})^{-1}$, which is the same as in Equation (2) but with the annual index j. The matrix $\mathbf{T}_{i(j+1)}$ is defined as

$$\mathbf{T}_{i(j+1)} = \mathbf{a}^2(\mathbf{I} - \hat{\boldsymbol{\alpha}}_{ij})\mathbf{T}_{ij} + (\mathbf{I} - \mathbf{a}^2)(\mathbf{T}_1 + \mathbf{S}).$$

The starting values for the recursion are $\hat{\mathbf{\Theta}}_{i1} = \hat{\mathbf{\Theta}}_0$ and $\mathbf{T}_1 = \hat{\boldsymbol{T}}$, as in "The Multidimensional Credibility Estimator." The elements in **a** are estimated by performing a minimization of the residual sum of squares (see "The Results") based on the one-dimensional, time-dependent credibility estimator for each coverage. Note that the estimator deals with individual missing values automatically.

AN APPLICATION TO DANISH COMMERCIAL BUSINESS LINES

In this section, we present an extensive study of 18 different estimators of each line of business in the four-dimensional commercial data set. We calculate the simple flat rate, which no actuaries would recommend in practice. The comparison of the performance of, respectively, the flat rate and the sophisticated estimate $\hat{\lambda}_{ijk}$ gives us the possibility to evaluate the extra information that can be extracted from the observed experience. The remaining 16 estimators are credibility estimators with and without time effect based on varying dimensions of the data set: one, two, three, or four dimensions.

The Data Set

The data set includes four coverages, *Fire*, *Glass*, *Other*, and *Water*, and consists of insurance information for 2,842 policyholders. We have available the estimated (expected) claim frequency, the duration ω_{ijk}, and the number of reported claims n_{ijk}, for each policyholder, coverage, and year. The estimated number of claims, $\hat{\lambda}_{ijk}$, is the product of the estimated claim frequency and the duration. The estimated claim frequency was originally determined via a Poisson regression, based on a large number of covariates. We have up to 8 years of information. Only few policyholders have 8 years of information for all four coverages. In Table 1, we present a comparison of the expected and the reported number of claims for the different coverages. The number of expected claims is lower than that reported in *Glass* and *Other*, whereas it is the opposite situation for *Fire* and *Water*. However, the total number of expected claims is rather close to the total number of reported ones.

The Results

Let us first consider the situation without time effect. For every coverage, for example, *Fire*, we have one credibility estimator based on all four dimensions, three credibility estimators based on three dimensions, three credibility estimators based two dimensions, and one credibility estimator based on one dimension, that is, eight estimators. We also consider the same eight credibility estimators with time effect; therefore,

TABLE 1
The Expected and the Reported Total Number of Claims in the Different Coverages

Insurance Coverage	Total Number of Expected Claims	Total Number of Reported Claims
Building - Fire	809	787
Building - Glass	7,455	7,797
Building - Other	1,980	2,004
Building - Water	2,763	2,731

we have a total of 16 credibility estimators for each coverage. In the following, we evaluate their performance and compare them to the flat rate and to the sophisticated rating that does not take advantage of experience rating.

1d CE: The one-dimensional credibility estimator, influenced only by claims occurring in that coverage;

2d CE: The two-dimensional credibility estimator, influenced by claims occurring in that and one other coverage;

3d CE: The three-dimensional credibility estimator, influenced by claims occurring in that and two other coverages; and

4d CE: The four-dimensional credibility estimator, influenced by claims occurring in all four coverages.

A time-dependent credibility estimator is denoted with a "t," e.g., 3d tCE for the three-dimensional, time-dependent credibility estimator.

We use the residual sum of squares (SS) as our performance measure:

$$SS = \sum_{i=1}^{I} \left(\hat{N}_{ijk} - n_{ijk}\right)^2, \quad k = 1, 2, 3, 4,$$

where $\hat{N}_{ijk}$ is the estimated number of claims for individual i in coverage k, i.e., $\hat{N}_{ijk} = \widehat{\bar{\lambda}}_k$, $\hat{N}_{ijk} = \hat{\lambda}_{ijk}$, or $\hat{N}_{ijk} = \hat{\lambda}_{ijk}\hat{\theta}_{ik}$. The estimate $\widehat{\bar{\lambda}}_k$ is calculated with the estimator $\bar{\lambda}_k = \sum_{i=1}^{I}\sum_{j=1}^{J} n_{ijk}(\sum_{i=1}^{I}\sum_{j=1}^{J} \omega_{ijk})^{-1}$, which the is the so-called mean value estimator (MVE), also called the flat rate. This estimator is introduced to get a better notion of the improvement in prediction received using any of the credibility estimators. With the MVE, every individual is considered having equal risk, which means that the estimator is one of the simplest types possible. It is neither affected by covariates nor by individual claim record. The estimate of the latent risk parameter $\hat{\theta}_{ik}$ is calculated with any of the credibility estimators, and n_{ijk} is the reported number of claims in the validation data set, which consists of all individual information in a specific year $j = J + 1$, for each policyholder. The results are presented in Figure 1. In this figure, the SS values, normalized (divided) with the SS value for the present estimator $\hat{\lambda}_{ijk}$, for 18 different estimators are plotted, which from the left are the mean value estimator $\bar{\lambda}_k$, the present estimator $\hat{\lambda}_{ijk}$, and the 16 different credibility estimators $\hat{\lambda}_{ijk}\hat{\theta}_{ik}$. In Table 2, we present a sheet to make the interpretation of Figure 1 easier.

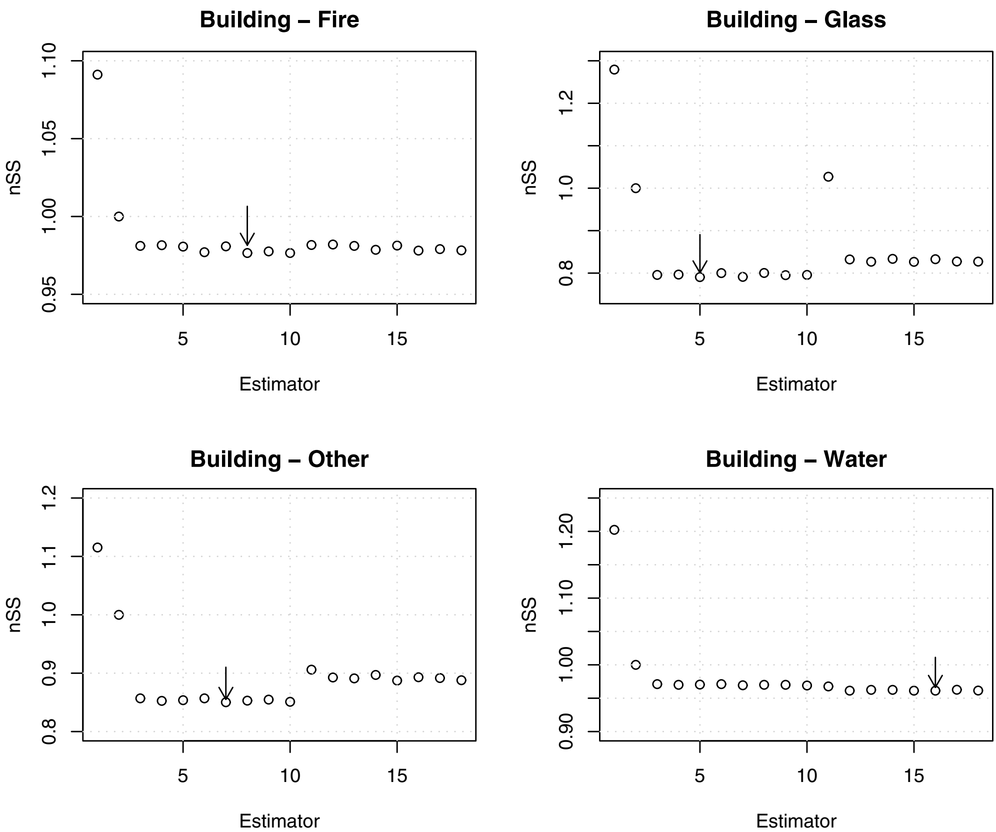
Table 3 contains the resulting SS values represented in Figure 1.

The numbers on the horizontal axis of the subfigures in Figure 1 are keyed to Table 2.

For the four-dimensional case, we present the estimated covariance matrix $\hat{\boldsymbol{T}}$ to show how the claim experience is connected between the different coverages, which corresponds to the correlation matrix $\mathbf{C}$:

$$\hat{\boldsymbol{T}} = \begin{bmatrix} 2.708 & 0.1537 & 0.2315 & 0.1744 \\ 0.1537 & 1.692 & 0.5838 & 0.2394 \\ 0.2315 & 0.5838 & 1.444 & 0.1015 \\ 0.1744 & 0.2394 & 0.1015 & 1.025 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0.0718 & 0.1171 & 0.1047 \\ 0.0718 & 1 & 0.3735 & 0.1818 \\ 0.1171 & 0.3735 & 1 & 0.0835 \\ 0.1047 & 0.1818 & 0.0835 & 1 \end{bmatrix}.$$

FIGURE 1
The Normalized Sum of Squares (nSS) Values for the Different Estimators



According to Figure 1, we see that a credibility estimator is the best choice for claim prediction in every one of the four coverages. However, the optimal choice of estimator, for each specific coverage, varies, and a universal answer to which is the best estimator can, therefore, not be found according to Figure 1. The best estimators for the four different coverages are:

Fire: The three-dimensional estimator with *Glass* and *Water* as additional coverages.

Glass: The two-dimensional estimator with *Other* as additional coverages.

Other: The three-dimensional estimator with *Fire* and *Glass* as additional coverages.

Water: The three-dimensional, time-dependent estimator with *Fire* and *Other* as additional coverages.

DISCUSSION AND CONCLUSIONS

The conclusion of our empirical study is that experience rating is extremely useful for pricing. Although this hardly is a surprising conclusion, it might be surprising that we are able to present a situation in which the inclusion of experience rating gives

TABLE 2
Table to Help Interpret Figure 1

x-Axis	Building - Fire	Building - Glass	Building - Other	Building - Water
1	MVE	MVE	MVE	MVE
2	Present	Present	Present	Present
3	1d CE	1d CE	1d CE	1d CE
4	2d CE (Glass)	2d CE (Fire)	2d CE (Fire)	2d CE (Fire)
5	2d CE (Other)	2d CE (Other)	2d CE (Glass)	2d CE (Glass)
6	2d CE (Water)	2d CE (Water)	2d CE (Water)	2d CE (Other)
7	3d CE (Glass, Other)	3d CE (Fire, Other)	3d CE (Fire, Glass)	3d CE (Fire, Glass)
8	3d CE (Glass, Water)	3d CE (Fire, Water)	3d CE (Fire, Water)	3d CE (Fire, Other)
9	3d CE (Other, Water)	3d CE (Other, Water)	3d CE (Glass, Water)	3d CE (Glass, Other)
10	4d CE (All)	4d CE (All)	4d CE (All)	4d CE (All)
11	1d tCE	1d tCE	1d tCE	1d tCE
12	2d tCE (Glass)	2d tCE (Fire)	2d tCE (Fire)	2d tCE (Fire)
13	2d tCE (Other)	2d tCE (Other)	2d tCE (Glass)	2d tCE (Glass)
14	2d tCE (Water)	2d tCE (Water)	2d tCE (Water)	2d tCE (Other)
15	3d tCE (Glass, Other)	3d tCE (Fire, Other)	3d tCE (Fire, Glass)	3d tCE (Fire, Glass)
16	3d tCE (Glass, Water)	3d tCE (Fire, Water)	3d tCE (Fire, Water)	3d tCE (Fire, Other)
17	3d tCE (Other, Water)	3d tCE (Other, Water)	3d tCE (Glass, Water)	3d tCE (Glass, Other)
18	4d tCE (All)	4d tCE (All)	4d tCE (All)	4d tCE (All)

Notes: MVE stands for mean value estimator, present for the present estimator, and (t)CE for credibility estimators with and without time dependence. The coverage names in the parentheses show the additional coverages used.

TABLE 3
Table Showing the Resulting SS Values of the Validation Analysis

Estimator	1	2	3	4	5	6	7	8	9
Building - Fire	138.97	127.38	124.96	125.02	124.91	124.45	124.93	124.37	124.51
Building - Glass	2637.1	2061.2	1640.2	1641.7	1629.6	1648.8	1630.9	1649.7	1639.1
Building - Other	830.12	744.38	637.94	634.70	635.61	637.86	632.83	634.87	636.18
Building - Water	376.82	313.42	304.38	304.07	304.12	304.38	303.81	304.04	304.07
	10	11	12	13	14	15	16	17	18
Building - Fire	124.39	125.02	125.08	124.96	124.65	124.99	124.56	124.71	124.59
Building - Glass	1640.0	2116.7	1715.6	1704.3	1718.2	1703.5	1716.7	1705.3	1704.7
Building - Other	633.54	674.46	664.46	663.24	667.76	660.48	664.66	663.69	661.05
Building - Water	303.70	303.30	301.28	301.68	301.71	301.28	301.28	301.74	301.31

Note: The values are represented in Figure 1.

an extra improvement of the same order of magnitude as the improvement obtained from leaving the trivial flat rate and entering sophisticated rating principles without experience rating. We also conclude that our multivariate credibility approach indeed is capable of improving the quality of estimation compared with the classical one-dimensional credibility theory. However, the most important thing is to use experience rating; the multivariate approach is just an extra improvement. Note that adding the time effect does not generally improve prediction in our case. The reason seems to be that our average duration of information is too short to divide it into old and new observations. However, adding a time effect does not harm our prediction either. We, therefore, expect that adding a time effect would improve prediction in our case if, for example, an average of 10 years of observed history were available.

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