© *The Journal of Risk and Insurance*, 2009, Vol. 76, No. 1, 53-85

Asymmetric Information, Long-Term Care Insurance, and Annuities: The Case for Bundled Contracts

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Abstract

This article examines the markets for long-term care insurance and annuities when there is asymmetric information and there are costs of administering contracts. Individuals differ in terms of their risk aversion. Risk-averse individuals take more care of their health and are relatively high risk in the annuities market and relatively low risk in the long-term care insurance market. In the long-term care insurance market, both separating and partial-pooling equilibria are possible. However, in the stand-alone annuity market, only separating equilibria are possible. We show, consistent with the extant empirical research, that in the presence of administration costs the more risk-averse individuals may buy relatively more long-term care insurance and more annuity coverage. Under the same assumptions, we show that equilibria exist with bundled contracts that Pareto dominate the outcomes with stand-alone contracts and are robust to competition from stand-alone contracts. The remaining empirical puzzle is to explain why bundled contracts are such a small share of the voluntary annuity market.

Introduction

This article is concerned with markets for stand-alone annuities and long-term care insurance and with markets for bundled contracts. The article first develops a simple unified treatment of market equilibrium for stand-alone annuities and long-term care insurance. This provides the basis for the main objective of the article, which is to investigate equilibria in markets for contracts that bundle the two types of insurance and to examine the efficiency properties of such equilibria relative to outcomes in stand-alone insurance contracts.

Finkelstein and Poterba (2002, 2004) have investigated the implications of asymmetric information for the supply and demand for annuities. They state, "Findings of differential mortality experience for annuitants who purchase different types of policies are consistent with standard models of adverse selection in which individuals have

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private information about their risk type and this private information influences their choice of insurance contract" (2004). Finkelstein and Poterba (2004) go on to note that risk-averse individuals may both take more care of their health and live longer and demand more mortality related insurance. In the case of long-term care insurance, Finkelstein and McGarry (2006) provide an innovative analysis that highlights the role of asymmetric information. Their results show that individuals who undertake a greater degree of preventive health activity (more cautious individuals) are less likely to enter a nursing home. They also indicate that those who undertake more preventive health activity are also more likely to have more long-term care insurance, which runs counter to the standard model.[1] This article argues that in order to explain the above empirical findings in a model of asymmetric information, two key ingredients are necessary: more risk-averse individuals take more care of their health and have a longer life expectancy and a lower probability of needing long-term care, and insurance is unfair. To do this, a framework is proposed that extends the competitive insurance market model of Rothschild and Stiglitz (1976) along similar lines to de Meza and Webb (2001). This forms the basis for the subsequent analysis of the market for bundled insurance contracts.

In the first part of the article we present two separate asymmetric information insurance games, played simultaneously by the same population of two types of individual. The two types differ in their risk aversion, which is private information.[2] Moreover, risk aversion is correlated with health status, which determines both longevity and the probability of a healthy old age. Here, for simplicity, we assume that this is exogenous.

The two markets considered trade stand-alone contracts in long-term care insurance and annuities. The long-term care insurance market may be characterized by either a separating equilibrium or by a partial-pooling equilibrium. In the annuity market, partial-pooling equilibria do not emerge, the market retains the properties of Rothschild and Stiglitz (1976), so that for certain population proportions a separating equilibrium exists. The article shows that for certain values of administration costs and thereby unfairness in pricing insurance, the more risk-averse type who take more care of their health obtain more of both types of insurance, even though their probability of using long-term care insurance is lower than that of the less risk-averse type. Hence, the empirical observations of Finkelstein and Poterba (2004) and Finkelstein and McGarry (2006) are consistent with simultaneous separating equilibria in the two markets. This characterization of equilibria in the markets for stand-alone contracts provides the benchmark from which to examine the market for bundled insurance contracts.

[1] We note that Cawley and Philipson (1999) find that in life insurance markets, where the insured are insuring against early death, rather than long life in annuity markets, those individuals who buy larger policies do not display above-average mortality risk. The finding that the correlation between risk and insurance demand is negative in the insurance market is investigated by Hemenway (1990), de Meza and Webb (2001), and Chiappori et al. (2006).

[2] The purpose of the present article is to examine competitive insurance markets with unobservable risk preferences. Laffont and Rochet (1988), for example, examine the case of monopoly, in which the insurance company offers a nonlinear insurance pricing scheme conditioned on the risky assets held by individuals.

We next turn to a discussion of bundled contracts. In contrast to the literature above, Murtaugh, Spillman, and Warshawsky (2001) examined the implications of the positive correlation between mortality and disability for the benefits of combining long-term care insurance with an immediate annuity, purchased with a lump sum of accumulated saving. Looking at the histories of a sample of 16,587 people dying in 1986, they simulated pools of persons likely, or eligible to purchase annuities and long-term care insurance at ages 65 and 75. They found that by offering bundled products to pools of individuals, combining say low life expectancy with a higher than average likelihood of needing long-term care, significantly reduced premiums by 3–5 percent relative to the stand-alone products. They showed that the bundled product could have a significant impact on the proportion of people participating in the market. In particular, they found that minimal underwriting, excluding only those who would be eligible for disability benefits at purchase, would increase the potential market to 98 percent of 65-year-olds, compared with only 77 percent under "current" long-term care insurance underwriting practice.

In the final and main part of the article we consider the market for bundled insurance products. Unlike Murtaugh, Spillman, and Warshawsky (2001), we consider deferred and not immediate annuities. The key insight is that individuals who take care are relatively low risk in the long-term care insurance market but high risk in the annuity market, with the opposite being the case for those who take less care. This raises the possibility that combining an annuity and long-term care insurance, purchased when young (not on becoming old), makes the individual incentive-compatibility conditions easier to satisfy and means that bundled contracts may be preferable to stand-alone contracts.[3] Along similar lines to the analysis in Fluet and Pennequin (1997), we investigate the existence of equilibria in bundled insurance products and find that separating and partial-pooling equilibria exist.[4] It is shown that the allocations with bundled products Pareto dominate the equilibrium allocation in stand-alone contracts. This outcome is shown to be robust to the existence of the stand-alone insurance products and to insurance pricing being unfair.

The structure of the article is as follows. The sections "The Basic Problem With Stand-Alone Insurance Contracts" and "Stand-Alone Contracts Equilibrium" develop the basic analysis with stand-alone insurance contracts. In the absence of administration costs we outline the equilibrium possibilities in the markets for long-term care insurance and then for annuities. We then introduce positive administration costs and show that with empirically plausible cost structures, the model generates patterns of equilibrium demand in the two markets that are consistent with observed behavior. The section "Bundled Contracts" develops the model to consider the market for bundled contracts and shows the equilibrium possibilities. The section "Welfare Properties of the Equilibrium With Bundled Contracts" compares the welfare properties of the market for bundled contracts to the allocations with stand-alone contracts. The

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3 Bodie (2003) argued that combining the two types of insurance coverage may mitigate the adverse selection that occurs in the demand for each of the two products on a stand-alone basis.

4 Partial pooling equilibria are not considered by Fluet and Pennequin (1997). They also abstract from a consideration of the role of administration costs.

final section of the article draws some conclusions and addresses the question of why the market for bundled contracts is so underdeveloped.

THE BASIC PROBLEM WITH STAND-ALONE INSURANCE CONTRACTS

We begin by outlining the basic problem of individual choice over stand-alone long-term care insurance and annuities that are supplied by competitive insurance companies. The framework is kept simple by considering only a single source of uncertainty, health status, which affects individuals' mortality and heath given survival. This in turn is linked to the fundamental unobservable, which is risk preference. Throughout the article we limit attention to just two types of individual. So we rule out the possibility that there are two types of risk-averse individual who have correspondingly high and low heath status, which could in turn be linked to wealth differences.[5]

This section of the article first outlines the individuals' decision problems and the nature of type heterogeneity. It then describes the basic nature of insurance supply.

Individuals' Preferences and Opportunities

There is a continuum of individuals who come in two types, indexed by i. A proportion α are high risk aversion denoted by R, and the remaining proportion are low risk aversion, denoted by r. The individual's type is private information. Each individual knows both his risk preferences and health status, which are correlated. Health status determines longevity and the chance of ill-health in old age.[6] Individuals are born at date $t = 0$ and live to either date $t = 1$ or $t = 2$. The probability of survival to date $t = 2$ is given by $0 < p_i < 1, i = R, r$, and the probability of a healthy old age is given by $0 < q_i < 1, i = R, r$. In the case of ill health in old age, the individual suffers a cost of long-term care equal to X.

Individuals' preferences over consumption, C_{it}, at dates $t = 1$, and $t = 2$ are given by

$$V_i = U_i(C_{1i}) + p_i\left[q_i U_i\left(C_{2i}^h\right) + (1 - q_i))U_i\left(C_{2i}^l\right)\right], \tag{1}$$

with $U_i' > 0, U_i'' < 0$.[7] Here, C_{2i}^h is consumption in the state of health at date $t = 2$, and C_{2i}^l is consumption in the state of ill health at date $t = 2$. This specification of

[5] Smart (2000) and Wambach (2000) consider a single insurance market with four types differentiated exogenously along two dimensions: risk aversion and accident probability.

[6] Brown (2000) notes that "if the insurance company could accurately determine the expected mortality of each individual applicant, it could price each policy in a manner that was appropriate and profitable. In practice, life-insurance companies estimate mortality for life-insurance contracts, using medical exams and health histories to separate individuals into risk classes. This estimation is almost never done for annuity products, however."

[7] The utility specification assumes that the utility function is the same for the states of health and ill health. Of course the consumption bundle of an ill person may differ significantly from that of a healthy one. This would suggest that the utility function should be state dependent. Although this is realistic and will have an effect on equilibrium, the principal results are the same, so we abstract from this consideration. For a model that allows for different utility functions in the states of health and ill health, see Philipson and Becker (1998). In particular, the marginal utility of consumption in the healthy state is higher than in the unhealthy state.

preferences is consistent with that in Yaari (1965), in which the probability of survival to date $t = 2$ takes the same form as discounting.

Both types' utility functions have constant absolute risk aversion,

$$a_i = (-U_i''(C_{ti})/U_i'(C_{ti})) = \text{a positive constant}, \; i = R, r,$$

moreover, $a_R > a_r$. Our results also apply for the more general case of decreasing absolute risk aversion, with a similar ordering (see Wambach, 2000).[8]

We assume that type R are healthier than type r, so that $p_R > p_r$ and $q_R > q_r$. Note that this does not mean that type R will have a lower expected utilization of long-term care as it is possible that $p_R(1 - q_R) > p_r(1 - q_r)$. However, in what follows we assume that:

Assumption 1: $p_R(1 - q_R) < p_r(1 - q_r)$.

This means that there is a negative correlation between life expectancy and the date $t = 0$ probability of needing long-term care. So it is those of the less risk averse/less healthy group who live longer who have the highest probability of needing long-term care. The assumption has empirical support; for example, Lakdawalla and Philipson (2002) document a decline in per capita nursing home output in the United States during the 1980s and 1990s. This, they argue, is explained by an improvement in the health status of the elderly.[9] Subsequently we will see that when this assumption is combined with some degree of unfairness in the pricing of insurance, our model generates predictions consistent with empirical findings for the purchase of stand-alone contracts. However, it should not be taken to mean that the opposite case is uninteresting or intractable.

The above assumption implies a link between risk aversion and risky behaviour. Using data from the Health and Retirement Study (HRS), Cutler, Finkelstein, and McGarry (2008) measure risk tolerance using the following variables: smoking, having 3+ alcoholic drinks per day, job-based mortality risk, receipt of preventive health services, and seat belt usage. The authors find a negative relationship between individuals who engage in risky behavior (i.e., smoking, drinking, and those working in a high-risk occupation) and the percent who purchase various types of insurance, and a positive relationship between those who engage in risk-reducing behavior (i.e., preventive medical care, seat belt usage) and the purchase of insurance. They also examined the relationship between the behavior measures and a proxy for risk aversion based on respondents' reported willingness to engage in various hypothetical

8 The assumption of constant absolute risk aversion means there are no wealth effects. The presence of wealth effects does lead to some algebraic complication without adding additional insights here. For example, with constant relative risk aversion, the slope and curvature expressions for indifference curves are more complex and wealth dependent. In Wambach's (2000) formulation, individuals have the same utility function that exhibits decreasing absolute risk aversion. Wealth differences then account for differences in risk aversion.

9 These authors also explore the role that longer life expectancy and improvements in health in old age have on informal caregiving among the elderly as a substitute for formal long-term care.

income gambles. They find that the two are moderately negatively related, which is consistent with the prior analysis of Barsky et al. (1997). That study finds that risk aversion measured from experiments is negatively related to risky behaviours, including smoking, drinking, failing to have insurance, and holding stocks rather than Treasury bills. These relationships are both statistically and quantitatively significant, although measured risk aversion explains only a relatively small fraction of the variation of the studied behaviors.

The next issue we address is heterogeneity within a particular type. We find in analyzing the equilibrium of the model that there emerges the possibility that insurance companies earn positive profits in supplying long-term care insurance. To ensure zero-profit equilibria we assume, as in de Meza and Webb (2001), that there is a small amount of heterogeneity in types' probabilities of needing long-term care, specifically:

Assumption 2: *There are small differences in the probability of individuals needing long-term care, so that for each member of each group, this probability is* $q_i + \varepsilon, i = R, r$, *where* ε *is distributed uniformly over the compact interval* $[0, \overline{\varepsilon}]$. $\overline{\varepsilon}$ *falls within a neighborhood of transaction costs, such that it is too costly to offer separate contracts within a group based upon such small differences.*

Here the distribution of heterogeneity for each group is the same. However, this is not strictly necessary, indeed for our analysis it is only necessary that this heterogeneity exists for the type r.

Now we outline individuals' opportunity sets. All individuals are assumed to have the same initial wealth of $W > 0$. They can purchase an annuity to cover old age consumption and insurance to cover a fraction $0 \leq \lambda \leq 1$ of the cost of long-term care if they need it. At the planning date, $t = 0$, the individual allocates his assets of W between consumption at dates $t = 1$ and $t = 2$. Resources are used to purchase an annuity at a premium P that pays Y_2 if the individual survives and long-term care insurance at a premium Q that covers a fraction λ of the cost of care, X, in the event of poor health in old age.[10] We assume that long-term care insurance is bought before the realization of life expectancy. We could also allow individuals access to a savings account (storage) but for simplicity and without significant loss we assume that they do not do any such saving. This means that there will be efficiency losses under asymmetric information but the qualitative results remain the same. However, we will comment on this issue further at the end of the section "Stand-Alone Contracts Equilibrium."

Formally, in the absence of saving, individuals' consumption opportunities are given by:

$$C_{1i} = W - P_i - Q_i, \tag{2}$$

$$C_{2i}^h = Y_{2i}, \text{ in the event of survival and good health,} \tag{3}$$

[10] It is relatively easy to add saving other than annuity purchase to the model. However, if we do so it is important that this is not observed by insurance companies. If it were, then it can itself partly overcome the problems arising from asymmetric information.

and

$$C^{l}_{2i} = Y_{2i} - X + \lambda_i X, \text{ in the event of survival and poor health,}$$

where $i = R, r$.

Insurance Supply

Insurance contracts are supplied by insurance companies engaged in competition. Contracts comprise a premium and a level of insurance coverage, either long-term care coverage or an annuity payment. In offering insurance contracts, insurance companies know market characteristics but cannot identify individuals' types directly. Throughout the analysis we assume that individuals' second-period health status is verifiable.

To allow for a richer set of equilibrium outcomes we introduce empirically observed costs of supplying insurance contracts. Administration costs may have a proportional and a fixed element, but for simplicity and without significant loss we abstract from proportional costs.[11]

Assumption 3: *There is a nonnegative fixed cost of administering each copy of each insurance contract.*

The fixed cost of administering long-term care insurance contracts is denoted by $k_L \geq 0$, so that insurance is unfair if $k_L > 0$. An insurance company's expected profit from a long-term care insurance contract $(Q, \lambda X)$ is given by,

$$\Pi_L = Q - p^e(1 - q^e)\lambda X - k_L \geq 0, \tag{4}$$

where p^e and q^e are the insurance company's forecasts of the applicants' life expectancy and health in old age, respectively. Annuity contracts incur a fixed cost of $k_A \geq 0$. An insurance company's expected profit from a contract (P, Y_2) is then given by

$$\Pi_A = P - p^e Y_2 - k_A \geq 0. \tag{5}$$

STAND-ALONE CONTRACTS EQUILIBRIUM

We now consider the nature of equilibrium when individuals can participate in both the long-term care insurance market and the annuity market. The study of the simultaneous play in the stand-alone long-term care insurance and the annuity market games serves three purposes. First, it shows that separating and partial-pooling equilibria are possible in the long-term care insurance market but that only separating equilibria are possible in the annuity market. Second, it allows us to characterize an

[11] Adding proportional administration costs will enrich the set of equilibrium possibilities. Lui and Browne (2007) have recently examined some of the possibilities that arise in a model similar to a single-market version of the present model in which there is a proportional element in administration costs.

equilibrium that is consistent with various empirical findings in the literature that we noted in the Introduction. Third, it provides a basis from which to understand the market for contracts that bundle the two types of insurance together. But at this stage of the analysis we assume that the market for bundled contracts is closed.

The model consists of two market games: an annuity game and a long-term care insurance game. In playing each game we assume that players take the outcome of the other game as given. We assume that the insurance companies in the two games do not exchange information. They know that individuals participate in both markets but do not know the specific outcome obtained in the other market. That is, we assume that individuals cannot provide verifiable information on the product purchased in one market to improve the product terms they are offered in the other market, should it be in their interests to do so.[12]

The Game

In order to understand the principal factors at work in establishing various equilibrium outcomes, we initially assume that administration costs in both markets are zero and only later relax this assumption. Each market is characterized by a two-stage Nash game. At stage 1, each of two insurance companies engaged in price-quantity competition offer a menu of insurance contracts. At stage 2, individuals apply for one contract from one insurance company in this market, given the insurance contract in the other market. If both insurance companies offer the same contract, each applicant tosses a fair coin to determine the insurance company to which he applies (if he applies at all).

Competitive separating equilibria arise in the markets for the stand-alone annuity and long-term care insurance when profitable contracts can be offered that only attract one type. In each market a pair of contracts are offered that satisfy incentive-compatibility (or self-selection) constraints for each type and must be profitable for insurance companies on a stand-alone basis. Let $A_i = (P_i, Y_{2i})$ denote the annuity contract for type $i = R, r$, and $L_i = (Q_i, \lambda_i X)$ denote the long-term care insurance contract for each type. The utility of each type at each contract offer is written conditional upon the contract taken by this type in the other market, denoted by an asterisk. The incentive-compatibility (or self-selection) constraints in the long-term care insurance game are

$$V_R(L_R \mid A_R^*) \geq V_R(L_r \mid A_R^*), \tag{6}$$

and

$$V_r(L_r \mid A_r^*) \geq V_r(L_R \mid A_r^*). \tag{7}$$

In the annuity market game these are

$$V_R(A_R \mid L_R^*) \geq V_R(A_r \mid L_R^*), \tag{8}$$

[12] Laffont and Rochet (1988) examine this type of conditioning in a model with heterogeneous risk types and observable holdings of risky assets.

and

$$V_r\left(A_r \mid L_r^*\right) \geq V_r\left(A_R \mid L_r^*\right). \tag{9}$$

The insurance companies' profitability conditions for long-term care insurance contracts are given in (4) and those for annuity contracts in (5).

Long-Term Care Insurance Market Equilibrium

We first examine a separating equilibrium in the long-term care insurance market. In such an equilibrium, each insurance company offers two long-term care insurance contracts: $L_R = (Q_R, \lambda_R X)$ and $L_r = (Q_r, \lambda_r X)$. These contracts must satisfy (6) and (7) and must be profitable and satisfy (4) for each type. Belief consistency requires that if the contract is expected to only attract type R, then $q^e = q_R$ and $p^e = p_R$ and so on. Competition implies that equilibrium long-term care insurance contracts earn zero expected profit.

Given Assumption 1, from (4) as an equality for each type, under complete information the long-term care insurance premia for maximum coverage against the loss X for the two types have the following ordering, $\overline{Q}_R < \overline{Q}_r$. Note that this follows independently of the amount of annuities purchased by either type. Denote the complete-information contracts by $(\overline{L}_R, \overline{L}_r)$. Then we have the following simple property of our model:

Property 1: $V_r(\overline{L}_R \mid A_r^*) > V_r(\overline{L}_r \mid A_r^*)$.

This property precludes the complete information allocation from existing as an equilibrium under asymmetric information, since type r prefer the complete information contract for type R to that for its own type, violating condition (7).

In demonstrating the results we use diagrams drawn in $(Q, \lambda X)$ contract space. In the figures (see, e.g., Figure 1) the curves F_R and F_r are zero-profit offer curves for long-term care contracts for the two types, derived from (4), with administration costs set equal to zero. They are upward sloping, reflecting the fact that more coverage requires a higher premium, with the relative slopes reflecting Assumption 1. The indifference curves for types R and r are labeled I_R and Ir, respectively. As we demonstrate below, they are upward sloping and convex. The first property means that higher premium must be compensated with more coverage. The second property follows intuitively from the fact that risk averse agents reject fair bets. Finally, contract offers are represented by points.

The next step is to examine formally the indifference curves of the two types in $(Q, \lambda X)$ space. The slopes of the indifference curves are given by $d(\lambda_i X)/dQ_i = U'_i(C_{1i})/p_i(1 - q_i)U'_i(C^l_{2i}) > 0$ with $i = R, r$. Given Assumption 1, the type R curves are steeper at complete insurance, $\lambda = 1$, than those of the type r, $1/p_R(1 - q_R) > 1/p_r(1 - q_r)$. The change in the slope of the curves is given by $d^2\,(\lambda_i X)/dQ_i^2 = (d(\lambda_i X)/dQ_i)[a_i(d(\lambda_i X)/dQ_i - 1)]$. With less than or equal to full insurance, $C^l_{2i} \leqslant C_{1i}$, then $d(\lambda_i X)/dQ_i > 1$ and so $d^2\,(\lambda_i X)/dQ_i^2 > 0$ for $i = R, r$. Then given that $a_R > a_r$, $d^2\,(\lambda_R X)/dQ_R^2 > d^2\,(\lambda_r X)/dQ_r^2$. This in turn means that type R and type r indifference curves cut twice and there exists a unique tangency points between the two types' indifference curves.

In Figure 1, at the contract L'_R on F_R, $d(\lambda_R X)/dQ_R > d(\lambda_r X)/dQ_r$, then the type R indifference curve passing through L'_R (denoted by I'_R) cannot also cut the type r indifference curve passing through L'_R (denoted by I_r) to the right of F_R, where expected profits are positive if only type R demand the contracts. We call this case single-crossing in the region of nonnegative profits. However, we cannot in this case rule out the possibility that the indifference curve I'_R cuts the type r indifference curve a second time to the left of F_R and that there exists a tangent point, T, to the left of F_R. In the subsequent Figure 2, the contract L'_R on F_R leaves the r type just indifferent to the complete insurance contract at $\overline{L}_r$. If at $L'_R, d(\lambda_R X)/dQ_R < d(\lambda_r X)/dQ_r$ then as at $\overline{L}_r$ (also on F_R) $d(\lambda_R X)/dQ_R > d(\lambda_r X)/dQ_r$, the type R indifference curve through L'_R (denoted by I^*_R) must cut $\lambda X = 1$ to the left of $\overline{L}_r$. This in turn means that I^*_R must cut the type r indifference curve passing through L'_R (denoted by I_r) from below at a point between L'_R and $\overline{L}_r$. Hence, there exists a tangent point, T, between a type R indifference curve and the type r indifference curve passing through $L'_R(I_r)$ that lies to the right of F_R. We summarize these two cases in the following property:

Property 2: *(i) Single-crossing in the region of nonnegative profits and the tangency point, T, if it exists, lies to the left of* F_R*; (ii) Double crossing, T lies to the right of* F_R *.*

Each of the two cases in this property generate different equilibrium allocations, which are analyzed below.

Separating Equilibria. The natural starting point for our equilibrium analysis is the consideration of separating equilibria. Suppose therefore that Property 2(i) applies, so that in the region of nonnegative profits we have single-crossing. Then the analysis is analogous to Rothschild and Stiglitz (1976). In stating the proposition we need to define an (endogenous) upper threshold α^*, above which separating contract offers are Pareto dominated by full-pooling offers. That is, above this threshold, given the difference between types' probabilities of an unhealthy old age, the proportion of type R is so high that the cross subsidy to the poorer type r is smaller than the costs of under-insurance that are incurred by the type R is achieving separation.

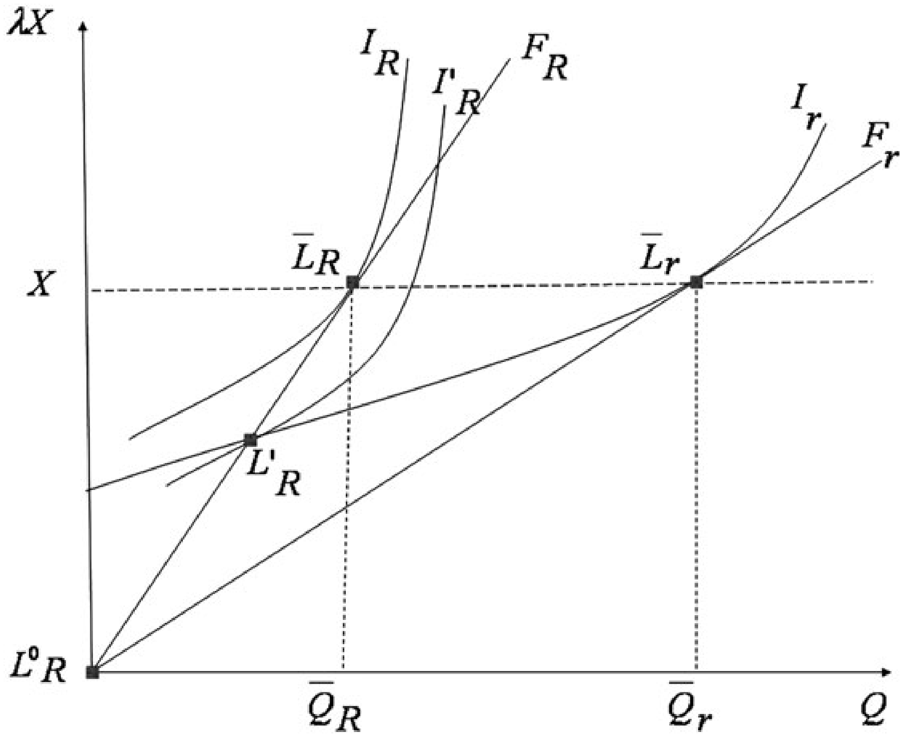
Proposition 1: *Given Property 2(i), if the proportion of type R lies in the interval* $0 < \alpha \leq \alpha^*$*, there exists a separating equilibrium with complete long-term care insurance for type r and partial coverage for type R.*

Proof: Here we refer to Figure 1. Property 1 means that the separating equilibrium pair of contracts maximizes (1) for type R subject to (2), (3), (6), and (7) and the two zero-profit conditions in (4). The constraint (6) is binding and the constraint (7) is slack. The contract taken by type r, $\overline{L}_r$, has complete insurance. The contract L'_R sets $\lambda_R < 1$ but at the highest level consistent with the contract not being demanded by type r. This contract lies at the intersection of the type r indifference curve passing through $\overline{L}_r$ and the zero-profit offer curve F_R. It is then straightforward to check that there do not exist any profitable out of equilibrium offers that attract one of the types.

The restriction that $\alpha \leq \alpha^*$ means that the type R are better of at he contract L'_R than at a pooled contract in which they cross-subsidize type r. With α in this range any

FIGURE 1

Separating Equilibrium in the Long-Term Care Insurance Market With No Administration Costs, in Which Type r Receives the Contract $\overline{L}_r$ and Type R Obtain the Contract L'_R



profitable pooling offers will lie far enough below F_R, so as to be unattractive to type R. Q.E.D.

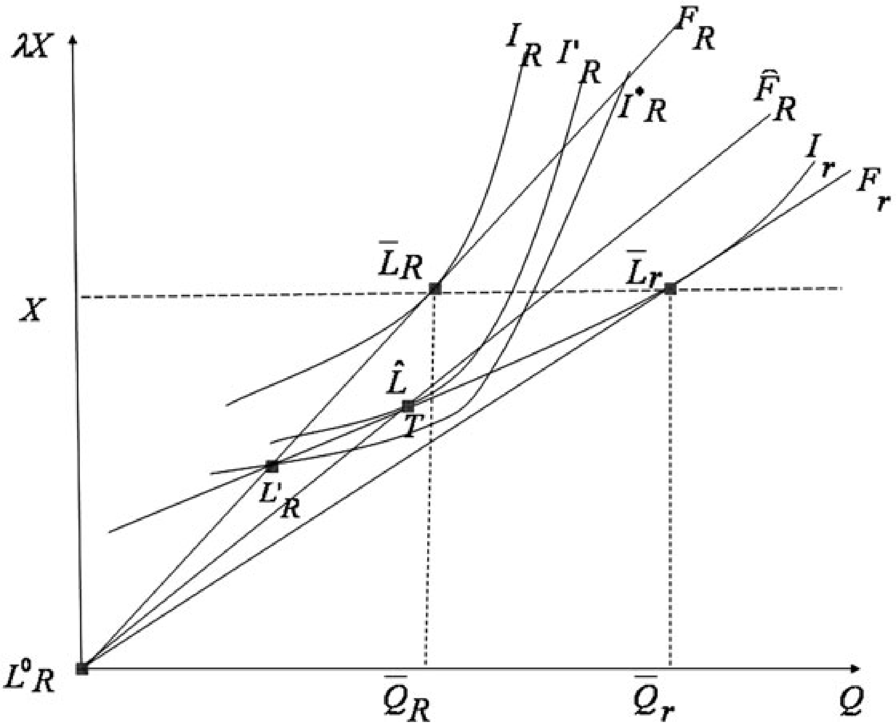
In the Appendix, Problem A, the $\lambda_R < 1$ property is demonstrated formally. The algebraic treatment allows us to see how λ_R depends upon parameters, which is useful later for comparison with bundled contracts.

The above result shows that under certain conditions, with price-quantity competition, limited coverage contracts allow the long-lived and healthier in old age type R to avoid paying an adverse-selection subsidy, which would exist if types were simply pooled and there was no link between price and quantity. Since the less healthy type r's incentive constraint binds, they then obtain complete coverage. However, this allocation, without administration costs, does not fit the empirical findings of Finkelstein and McGarry (2006) discussed earlier.

Partial-Pooling Equilibria. Now turn to the case in Property 2(ii). This property means that the tangency point, T, lies in the region of nonnegative profits to the right of F_R, so that the Rothschild and Stiglitz (1976) outcome is no longer possible. However, there now exists the possibility of partial-pooling equilibria. In analyzing this type of equilibrium, Assumption 2 plays a crucial role. This assumption means that contracts that would be positive profit if they were only demanded by type R can improve

FIGURE 2

Partial-Pooling Equilibrium in the Long-Term Care Insurance Market With No Administration Costs, in Which Some Type r Obtain the Contract $\overline{L}_r$ and the Remainder Pool with the Type R and Obtain the Contract $\widehat{L}$



their terms by a small amount and attract the right proportion of type r and just break even. The fraction of the type r in the interval $[0, \overline{\varepsilon}]$, who are just attracted to a contract for type R by a slight improvement in the terms will depend upon the degree of improvement. Moreover, whether they are from the low or high end of this interval will depend upon whether the contract for type r has, respectively, more or less insurance than the contract for type R.

In stating the next proposition we define an upper threshold α^{**}, above which the mix of partial pooling and separating contract offers are Pareto dominated by full-pooling offers, which attract all type r.

Proposition 2: *Given Property 2(i), if the proportion of type R lies in the interval* $0 < \alpha \leq \alpha^{**}$*, there exists a partial-pooling equilibrium in which some type r have complete long-term care insurance and all type R and the remaining fraction of type r obtain a contract with partial coverage.*

Proof: This case is illustrated in Figure 2. Property 1 means that potential separating equilibrium contracts maximize (1) for type R subject to (2), (3), (6), and (7) and the two nonnegative profit conditions in (4). The pair of contracts satisfy (7) strictly and (6) weakly. That is, $V_r(L_r \mid A_r^*) = V_r(L_R \mid A_r^*)$ and $V_R(L_R \mid A_R^*) > V_R(L_r \mid A_R^*)$. The contract L_r satisfies the nonnegative profit condition in (4) for type r strictly. This contract gives the type r individuals complete coverage for their type, $\overline{L}_r$, with $\lambda_r = 1$. Because of Property 2(ii), type R take the contract at $\widehat{L}$ (just above the tangent point T on the type r indifference curve through $\overline{L}_r$) with incomplete coverage, $\lambda_R < 1$. To see this consider the contract at T, this satisfies $V_R(T \mid A_R^*) \geq V_R(\overline{L}_r \mid A_R^*)$. If only this type demand this contract, as it lies to the right of F_R, it is positive profit. To render the contract zero profit, its terms are set just superior to the tangent contract so some fraction $0 < \delta_r < 1$ of type r, given ε, prefer this contract. As $\overline{L}_r$ has full insurance coverage, it will be the lowest ε that are attracted. When sufficient type r are attracted to the contract $\widehat{L}$, the zero-profit (partial pooling) offer curve derived from (4), $\widehat{F}$, is flatter, so that it passes through this contract, which is then zero profit. It is then straightforward to check that there do not exist any profitable out-of-equilibrium offers that attract one of the types.

The condition that $\alpha \leq \alpha^{**}$ means that there are sufficient type r to eliminate the positive profits at any tangency contract. Type R are clearly better off at the contract $\widehat{L}$ than at a contract in which they pool with all of type r. With α in this range, any profitable full-pooling offers will lie far enough below F_R, so as to be unattractive to type R. Q.E.D.

In the Appendix, Problem B demonstrates the $\lambda_R < 1$ property for the case of a partial-pooling equilibrium.

The intuition for the above result is as follows: The equilibrium in Proposition 1 does not exist because type R's demand for insurance is too high; hence, an allocation with partial pooling will dominate the putative equilibrium separating pair of contracts. This requires that there are sufficient type r to eliminate positive profits in any partial pooling contract. Moreover, in any partial-pooling equilibrium, the marginal type r must be held to the same level of utility as under complete fair insurance. The limiting case of a partial-pooling equilibrium has all type r demanding the pooling contract but with the marginal type r being indifferent to fair insurance, and the contract earning zero-expected profit. If $\alpha > \alpha^{**}$, the only zero-profit pooling contracts involve full pooling and can be shown to be unstable. However, the equilibrium allocation that we have described, without administration costs, is inconsistent with the findings of Finkelstein and McGarry (2006).

Annuities Market Equilibrium. Now consider the market for annuities.[13] Although straightforward, it is necessary to examine this so as to see how it is affected by the coexistence of long-term care insurance and also in order to understand some of the properties of the market for bundled contracts examined later in the article.

First we consider a separating equilibrium. Here we refer to Figure 3, in which the annuity contract offer curves for types R and r derived from (5) as an equality and with administration costs set equal to zero are given by H_R and H_r, respectively. For the same reasons as in the long-term care insurance market, these are upward sloping with the slope of the former being greater because $1/p_r > 1/p_R$. In a similar vein, and

[13] The basic analysis below is similar to that in Eckstein et al. (1985).

demonstrated formally below, the indifference curves, denoted by I_R and I_r have a positive slope and are convex.

In this market each insurance company offers a pair of contracts $A_R = (P_R, Y_{2R})$ and $A_r = (P_r, Y_{2r})$ that satisfy (8) and (9) and the two zero-profit conditions derived from (5). We denote the complete-information equilibrium annuity contracts by $(\overline{A}_R, \overline{A}_r)$. Note that the amount of annuity coverage that each type demands in this market under complete information depends upon the contract obtained in the long-term care insurance market.[14] Let type R be indifferent to the full-information contract for their type, $\overline{A}_R$, and the contract on the zero-profit contract curve for type r, H_r, at A'_r. If the full-information contract for type r on H_r, $\overline{A}_r$, lies to the right of A'_r, then the following holds:[15]

Property 3: $V_R(\overline{A}_r \mid L^*_R) > V_R(\overline{A}_R \mid L^*_R)$.

This condition precludes the existence of a complete information allocation under asymmetric information, because the longer lived type R prefer the complete information contract for type r to that for its own type. Hence, it is only if this property holds that the market has a selection problem.

We first consider the indifference curves of the two types in (P, Y_2) space. The slopes of the two types' indifference curves are given by $dY_{2i}/dP_i = U'_i(C_{1i})/p_i[q_i U'_i(C^h_{2i}) + (1 - q_i)U'_i(C^u_{2i})] > 0$ with $i = R, r$. It is easy to show that, given $a_R > a_r, dY^2_{2R}/dP^2_R > dY^2_{2r}/dP^2_r$. Now consider the relative slopes of the types' indifference curves. The type r indifference curves are steeper at full insurance than those of type $R, 1/p_r > 1/p_R$. Moreover, at contract A'_r on $H_r, dY_{2r}/dP_r > dY_{2R}/dP_R$. This follows from the convexity of indifference curves in this dimension and the fact that the type r' s strictly prefer A'_r to $\overline{A}_R$, while type R are indifferent between these two contracts. Finally, as can be seen in Figure 3, the fact that $dY^2_{2R}/dP^2_R > dY^2_{2r}/dP^2_r$ means that there is no second crossing of these indifference curves to the left of A'_r, however, there will be a second crossing to the right of A'_r. Hence, as can be seen in Figure 3, the indifference curve of type R, I'_R, is tangent to I'_r to the right of A'_r at M.

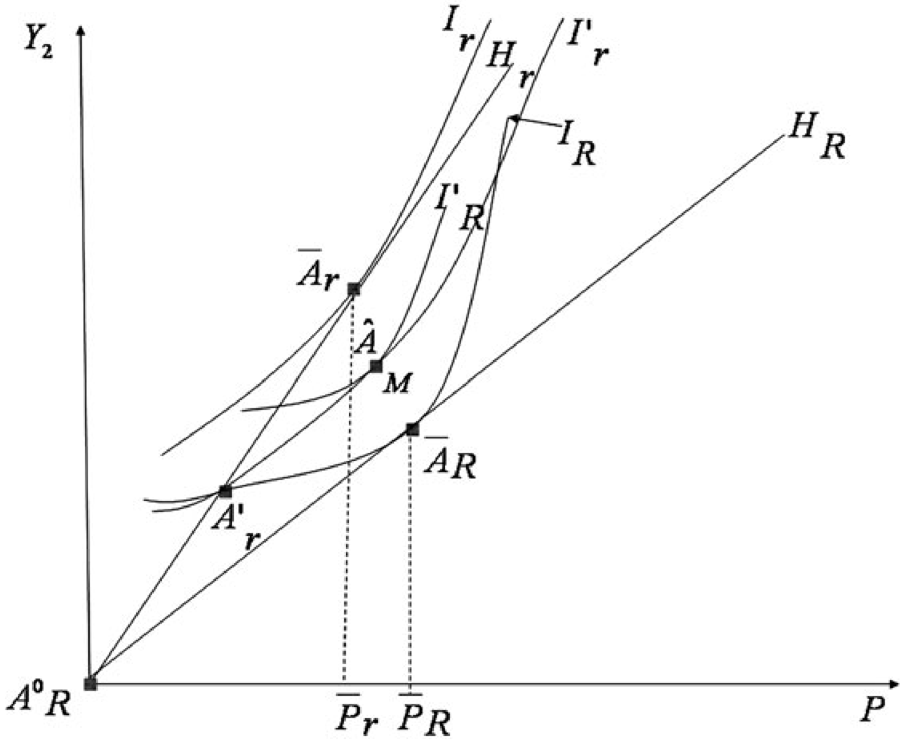
In stating the next proposition we define the threshold α^{***}, below which separating contract offers are Pareto dominated by full-pooling offers. In this market, as it is type r who obtain the best fair insurance, it is when the proportion of this type is high (α is low), that they are better off pooling with type R than incurring the costs of separation.

Proposition 3: *If the proportion of type R lies in the interval $\alpha^{***} \leq \alpha$, there exists a separating equilibrium in the annuity market in which type R have full insurance coverage and type r receive partial coverage.*

[14] We note that Brown, Davidoff, and Diamond (2005) show how, in an incomplete markets setting, uninsured medical expenses affect the demand for annuities.

[15] This statement is important, as it is necessary for adverse selection. The location of the type R indifference curve, I_R, through both $\overline{A}_R$ and A'_r in relation to the contract $\overline{A}_r$ summarizes all of the relevant information relating to type R's global problem for considering equilibrium in this market.

FIGURE 3
Separating Equilibrium in the Annuities Market With No Administration Costs, in Which Type r Obtain the Contract A'_r and Type R Obtain the Contract $\overline{A}_R$



Proof: This proposition is illustrated in Figure 3. Property 3 means that a potential separating equilibrium pair of contracts maximizes (1) subject to (2), (3), (8), and (9) and the two zero-profit conditions in (5). The contract $\overline{A}_R$ involves complete annuity coverage and leaves type R indifferent to the partial coverage contract for type r, A'_r, that is, $V_R(\overline{A}_R \mid L_R^*) = V_R(A'_r \mid L_R^*)$. The indifference curve of type R through $\overline{A}_R$ cuts the zero-profit offer curve for type r, H_r, at the contract A'_r, which has incomplete annuity coverage, $Y_{2r} < \overline{Y}_{2r}$. The contract A'_r satisfies $V_r(A'_r \mid L_r^*) > V_r(\overline{A}_R \mid L_r^*)$ and is the least cost separating contract for type r. This contract pair is readily seen to be robust to out of equilibrium offers.

In particular, we consider the contract on I'_r at the tangent point just above M. denoted by $\widehat{A}$. The question arises as to whether this contract can attract sufficient type r from the contract A'_r, as well as all type R and constitute a partial pooling equilibrium. So suppose that the contract $\widehat{A}$ was offered alongside A' and attracts only a fraction of the type r such that it makes zero profit. Then there must be a contract slightly better than $\widehat{A}$, which would attract all of the type r so that $\widehat{A}$ will not be offered. This new offer will then also attract all of the type R. However, such a full-pooling offer is not an equilibrium as there exist contract offers that cause the type r and not the type R to deviate, rendering it unprofitable.

In this Nash formulation there remains the problem that if α is low, $\alpha < \alpha^{***}$, using standard Rothschild Stiglitz arguments (1976), full-pooling offers can destabilize the separating pair of contracts. However, when α lies above this threshold, the cross-subsidy from type r to type R in pooling is too high for type r and there are no profitable pooling offers that dominate the separating offers. Q.E.D.

In the Appendix, Problem C demonstrates formally the $Y_{2r} < \overline{Y}_{2r}$ property.

In this market, the above result shows that under certain conditions, with price-quantity competition, limited coverage contracts allow the lower life expectancy individuals to avoid paying an adverse selection subsidy that would exist if types were simply pooled and there was no link between price and quantity.[16] The allocation that we have found in the model without administration costs, in this case is consistent with the findings of Finkelstein and Poterba (2002, 2004) discussed earlier.

Positive Administration Costs

As was noted in the Introduction, empirically observed administration costs enrich the set of equilibrium possibilities and as we will demonstrate, generate outcomes that are consistent with empirical findings. First, we consider the effect of positive administration costs in the market for long-term care insurance. Consider the case in Proposition 1. Given that the tangency point T in Figure 1 lies to the left of F_R so that we have single-crossing in the region of nonnegative profits, then there are two possibilities when administration costs are introduced. If in Figure 1 the type R indifference curve through L'_R cuts the type r indifference curve again to the left of F_R (at some premium $Q > 0$), then that type are willing to pay a higher fixed fee for the insurance coverage at L'_R than are the type r. Then at some threshold level of administration costs, k^r_L, insurance is sufficiently unfair that the type r switch to no insurance at the null contract L^0_r. In Figure 4 the distance from the origin to E measures administration costs. As can be seen, at the level of costs in this figure, the more risk-averse, more-careful type R demand positive coverage at the contract L'^*_R, which is the best contract satisfying (4) that is not demanded by type r.

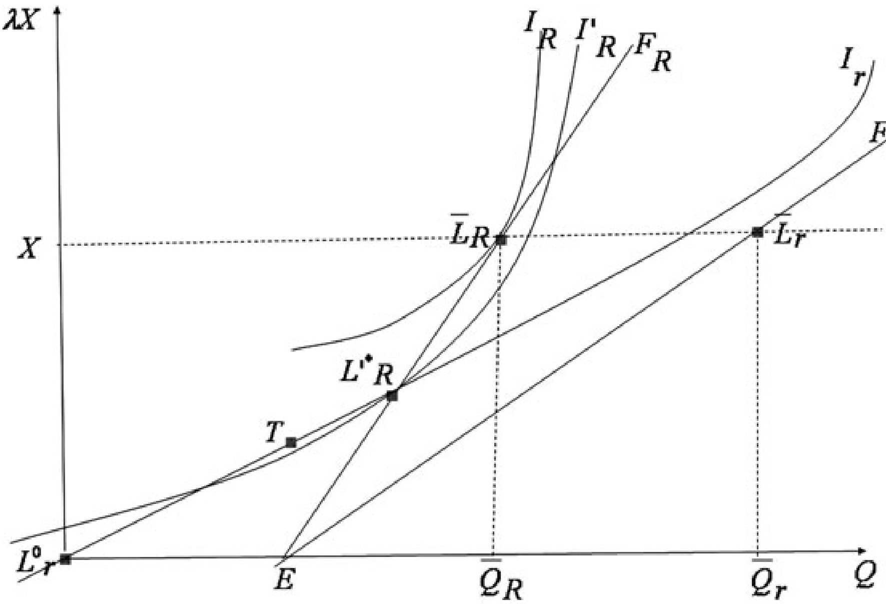
On the other hand, as shown in Figure 5, if type r are willing to pay a higher fixed fee for the insurance coverage at L'^*_R than are the type R, then there exists some threshold level of administration costs k^R_L at which type R drop out of the market first and receive L^0_R and the type r receive full insurance at the contract $\overline{L}_r$. In either of the above cases, if the administration cost exceeds some higher threshold, both types drop out of the market.

Now consider the case in Proposition 2. If the administration cost $k_L > k^{r*}_L$ (distinct from k^r_L in the first case we considered in this section), type r prefer no insurance at the null contract L^0_r rather than the partial coverage contract, L'^*_R in the first case. But for some range of costs, as illustrated in Figure 6, the more risk averse, more

[16] Of course the problem of buying multiple copies of annuities from different insurance companies is less of an issue in annuities markets than it is in life insurance or motor insurance. The moral hazard problems are quite different!. However, if this undermines price quantity competition in the annuity market so that only linear (price only) contracts are traded, with asymmetric information, welfare will be lower.

FIGURE 4

Separating Equilibrium in the Long-Term Care Insurance Market With Administration Costs (the Distance From the Origin to E), in Which Type r Drop Out of the Market and Obtain the Null Contract L_r^0 and Type R Obtain the Contract L'^*_R



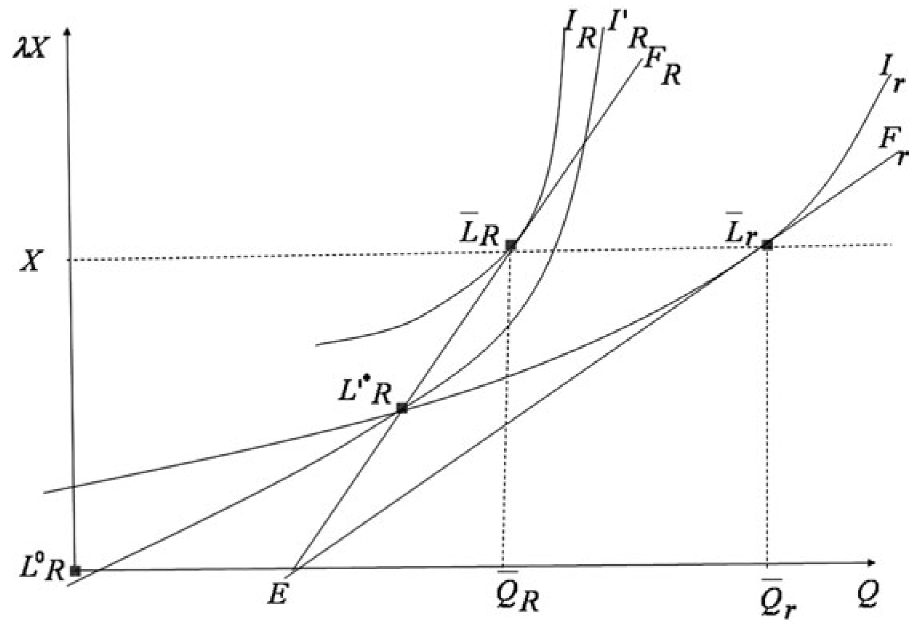
careful type, demand insurance at the partial-pooling contract $\widehat{L}^*$ (slightly better than the tangent contract at T on I_r^0 to the right of F_R) also demanded by some fraction of the type r, who prefer this to L_r^0. As L_r^0 involves no insurance, it is the higher ε type r that are attracted to the partial-pooling contract. As can be seen in Figure 6, at the higher level of administration costs, the tangency point T is relatively further to the right along the zero-profit partial-pooling offer curve, $\widehat{F}$, and involves more coverage to compensate for the increased unfairness per unit of coverage. Finally, if the administration cost exceeds some higher threshold, both types drop out of the market.

It should be noted that this last result is modified if there are proportional administration costs and hence some element of proportional unfairness in insurance pricing. We can then show that for some ranges of fixed and proportional costs, both types choose a positive level of long-term care insurance coverage, but type R demand relatively more coverage than type r.[17]

The implications of positive administration costs for the equilibrium in the annuity market are very simple. In the separating equilibrium characterized in Proposition 3, at the contract demanded by the type r, A'_r, we have single-crossing with $dY_{2r}/dP_r > dY_{2R}/dP_R$, and to the left of this contract the type R indifference curve

[17] Evidence (Association of British Insurers) suggests that for most types of insurance administration costs are invariant to the amount of cover.

FIGURE 5
Separating Equilibrium in the Long-Term Care Insurance Market With Administration Costs, in Which Type r Obtain the Contract $\overline{L}_r$ and Type R Obtain the Contract L'^{*}_{R}



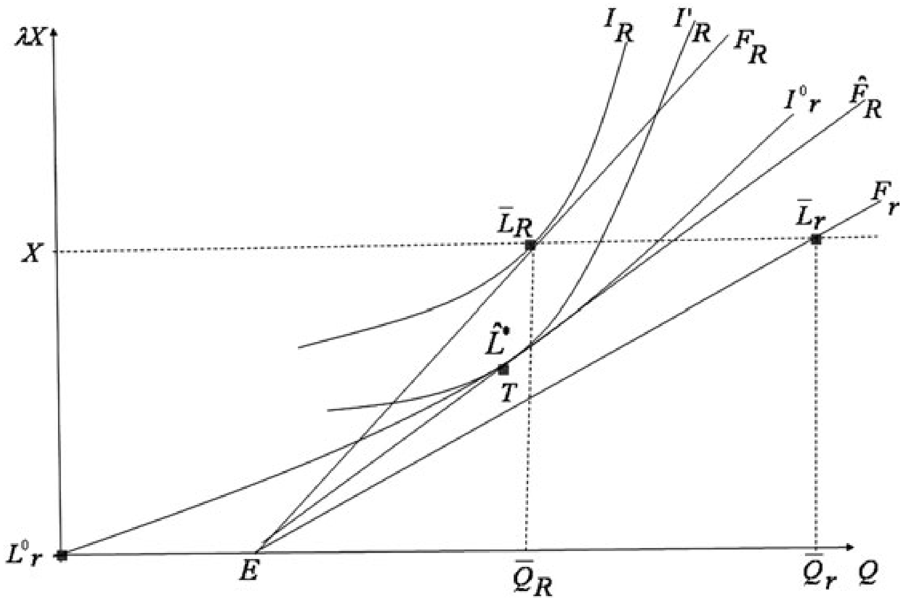
at A'_r remains below that of type r. Hence, comparing this contract to no insurance, we see that the level of administration costs that causes the type r to drop out of the market, k^r_A, is strictly less than that for type R.

Thus, we see for the long-term care insurance market that in the cases in Figures 4 and 6 with $k_L > k^r_L$ and $k_L > k^{r*}_L$, respectively, in equilibrium the high-risk-aversion, careful type, have a higher than average demand for long-term care insurance but also have a lower than average utilization of long-term care. This prediction is consistent with the empirical findings of Finkelstein and McGarry (2006).

In the annuity market administration costs cause type r to drop out of the market first. Thus if these costs are less than this threshold $k_A < k^r_A$ the allocation in Proposition 3 is retained. That is, type R has a relatively higher demand for annuities, which is consistent with the findings of Finkelstein and Poterba (2004).

The above prediction rests upon the relative magnitudes of administrative costs in the two markets. In the annuity market empirical evidence for the United States, such as that in Brown, Mitchell, and Poterba (2002) shows, using the life-expectancy of the average sixty-five-year old in the population, that loadings are about 15 percent of premium. However, once selection effects have been taken into account, this figure drops to below 10 percent. For long-term care insurance, Brown and Finkelstein (2007) report loadings of approximately 18 percent of premium, which would appear to be even higher once selection effects, which run in the

FIGURE 6
Partial Pooling Equilibrium in the Long-Term Care Insurance Market With Administration Costs, in Which Some Type r Obtain the Null Contract L_r^0 and the Remainder Pool With Type R at the Contract $\widehat{L}^*$



opposite direction to those in the annuity market, are removed.[18] If cost differences are not sufficient to support the empirical finding, then if the probability difference between types is relatively bigger in the annuity market, the same possibility emerges.

Equilibrium With Dual Stand-Alone Contracts

As both of our market games are played simultaneously, we have the consistency requirement that in equilibrium the constraint value of utility for a type in one market must equal the optimal value in the other. For type R, we have $V_R(A_R \mid L_R^*)$ and $V_R(L_R \mid A_R^*)$. In this case consistency requires that $A_R = A_R^*$ and $L_R = L_R^*$. For type r, we have, respectively, $V_r(L_r \mid A_r^*)$ and $V_r(A_r \mid L_r^*)$ and in this case consistency requires that $A_r = A_r^*$ and $L_r = L_r^*$. This in turn means that the pairs of contracts satisfy global incentive-compatibility conditions. That is neither type is better off misrepresenting itself in either market.

18 It should be noted that there are big gender biases in these loadings. Loadings of 44 percent are reported for men and better than actuarially fair prices (negative loadings) for women. The finding that the take up rate on this type of insurance by women is so low leads Brown and Finkelstein (2007) to suggest that the explanation of low levels of coverage may be to do with demand factors rather than supply.

Impact of Savings Accounts

In presenting the above analysis we did not allow individuals to save other than through annuity purchase. We do not propose to introduce saving accounts in a formal way here but do comment on the implications of simple unproductive saving. The saving account should be included in the individual's decision problem, with the saving decision being made simultaneously with the decision to buy long-term care insurance and annuities. The impact of saving though is quite intuitive and can be understood (in the main) in the absence of administration costs on a market-by-market basis. In the annuities market, we have seen that the costs of asymmetric information are borne by the type r, with the cost of underannuitizing being borne by a reduction in second-period consumption. With the introduction of savings accounts, the selection mechanism remains the same but saving allows the cost of selection to be spread between the first and second periods of life and will have the effect of further reducing annuity purchase by this group.

Turning to the case of long-term care insurance. In this market, in the separating equilibrium of Proposition 1, the costs of asymmetric information are borne by the type R, again through a reduction in second-period consumption. With the introduction of savings accounts, the selection mechanism is still the same but saving allows the cost of selection to be spread between the first and second periods of life and in this case will have the effect of further reducing long-term care insurance purchase by this group. In the case of the partial-pooling equilibrium in Proposition 2, matters are somewhat more complex. Then it can be shown that for type R, the costs of selection are still partially spread through saving but long-term care insurance coverage will still be higher than in the case of full separation. The complexity arises in analyzing the decision of the type r, who must compare fair coverage to the terms of the pooled contract and some level of saving.

In the same way as we have rehearsed the arguments with saving but no administration costs, so we can do the same for the cases with administration costs. As we have argued above, it is these cases that are of principal empirical interest, particularly in the case of long-term care insurance.

BUNDLED CONTRACTS

In the above we assumed that individuals only have access to stand-alone long-term care insurance and annuity contracts traded in separate markets. However, in the market place there exist financial instruments that are packages of long-term care insurance linked to a single-premium deferred annuity, namely, a life-care annuity. The idea is that some or all of the earnings on the annuity pay for long-term care. Thus, an annuity that would normally yield x percent might only yield y percent $< x$ percent when combined with long-term care insurance.[19]

[19] One advantage of this arrangement is that long-term care insurance premiums are paid with tax-deferred earnings but since they are expensed inside the policy, premiums become tax-free. Another advantage could be the perception that no money is lost to a long-term care insurance policy that may never be used. A possible perceived disadvantage is that the money is tied up. Removing money will kill the long-term care insurance.

The key contribution here is to show that contracts that combine or bundle the two types of insurance can be supported as a separating equilibrium that Pareto dominates the equilibrium in stand-alone contracts. Moreover, separating equilibria will exist for a wider range of values of the population proportion parameter, α, than the stand-alone contracts equilibrium. This result arises because the selection effects in the stand-alone long-term care insurance and annuity markets work in opposite directions. An individual who takes care of his health will be relatively low risk in the long-term care insurance market but high risk in the annuity market, with the opposite being the case for those who take less care. The terms of bundled contracts are set so as to exploit this asymmetry and satisfy a single pair of incentive (self-selection) constraints.

Instead of separate markets for stand-alone contracts, we consider a formulation in which there is only one competitive market for packages of insurance contracts that bundle annuities and long-term care insurance. Contract offers take the general form $S = (B, Y_2^B + \lambda^B X)$, where B is a single premium. The payment $\lambda^B X$ is only made if the individual survives and is in poor health, whereas the payment Y_2^B is paid simply on survival. In the absence of unproductive storage (saving), the individuals' budget sets are now given by

$$C_{1i} = W - B_i, \tag{10}$$

$$C_{2i}^h = Y_{2i}^B, \quad \text{if the individual survives and is in good health,} \tag{11}$$

and

$$C_{2i}^l = Y_{2i}^B - X + \lambda_i^B X, \quad \text{if the individual survives and is in poor health,}$$

with $i = R, r$.

Bundled contracts satisfy the nonnegative profit condition

$$\Pi_B = B - p^e\left(Y_2^B + (1-q^e)\lambda^B X\right) - k_B \geq 0, \tag{12}$$

where, as before, p^e and q^e are the insurance company's forecasts of the applicants' life expectancy and health in old age, respectively. The administration cost for bundled contracts, $k_B \geq 0$. This is assumed to be no more than the sum of the administration costs of the two stand-alone contracts ($k_B \leq k_L + k_A$), where the inequality is likely to be strict.

Equilibrium

In presenting the basic analysis here we assume that markets for stand-alone contracts are closed and show the existence of both partial-pooling and separating equilibrium in bundled contracts but focus attention on the latter.

Denoting the bundled contracts for the two types by $S_i = (K_i, Y_{2i}^B + \lambda_i^B X)$, $i = R, r$, the incentive-compatibility constraints for the two types are given by

$$V_R(S_R) \geq V_R(S_r), \tag{13}$$

and

$$V_r(S_r) \geq V_r(S_R). \tag{14}$$

In the following and without loss we abstract from the role of administration costs and briefly comment on their implications later. Under complete information (we drop the incentive-compatibility conditions), the annuity payments on the bundled contract for each type, Y_{2i}^B, are given by $Y_{2i}^B = \overline{Y}_{2i}$, $i = R$, r, and the amounts of long-term care coverage for each type, $\lambda_i^B, i = R, r$, are given by $\lambda_i^B = \overline{\lambda}_i = 1$. We can solve (12) for each type for the complete-information premia $\overline{B}_R$ and $\overline{B}_r$. Then $\overline{B}_R - \overline{B}_r = (p_R\overline{Y}_{2R} - p_r\overline{Y}_{2r}) + (p_R - p_r)X - (p_Rq_R - p_rq_r)X \lessgtr 0$. There are two cases: (a) If $(p_R - p_r)$ is large relative to $(q_R - q_r)$ and $\overline{Y}_{2R} - \overline{Y}_{2r}$ is large relative to X, then $\overline{B}_R - \overline{B}_r > 0$; (b) If $(p_R - p_r)$ is small relative to $(q_R - q_r)$ and $\overline{Y}_{2R} - \overline{Y}_{2r}$ is small relative to X, then $\overline{B}_R - \overline{B}_r < 0$. Complete information contracts denoted by $(\overline{S}_R, \overline{S}_r)$ then satisfy the following:

Property 4: *(i) If $\overline{B}_R - \overline{B}_r > 0$, then for the complete information contracts $V_R(\overline{S}_r) > V_R(\overline{S}_R)$. (ii) $\overline{B}_R - \overline{B}_r < 0$, then for the complete information contracts $V_r(\overline{S}_R) > V_r(\overline{S}_r)$.*

A necessary condition for a partial-pooling equilibrium in this market is that the indifference surfaces of type R are such as to permit a tangent to those of type r, a common supporting hyper-plane, in the region of positive profits. Evaluating the utility function (1) for each type at the bundled contract $(B, Y_2^B + \lambda^B X)$, we can determine the slopes of the utility surfaces in the different dimensions of the contract (annuity and long-term care insurance), which are the same as for the stand-alone contracts. A tangency of the surfaces is a mathematical possibility. This is seen immediately by considering the case where type r are close to risk neutral and their surface is almost flat. However, this is not sufficient for a partial-pooling equilibrium. There are two cases to consider, which we do below.

Suppose that Property 4(i) obtains, so that if type r were to be offered fair insurance terms, under asymmetric information this will be attractive to type R and will not be profitable. Now consider a putative partial-pooling equilibrium with bundled contracts at a tangency between the type R and type r indifference surfaces that lies below the fair-offer surface for type r. Suppose that the contract close to the tangency point partially pools some type r and all type R. However, at this contract, insurance companies could reduce the annuity terms and improve the long-term care terms on the contract, so as to make it slightly more favorable to type r, without attracting any type R, and so make positive profits, thereby destabilizing the putative equilibrium. Hence, the only possible equilibria are separating equilibria at which the slope of the indifference surface in the annuity dimension for type $r, dY_{2r}^B/dB_r$, exceeds that for type $R, dY_{2R}^B/dB_R$. However, in the second case, when Property 4(ii) obtains and in which type R would be offered better fair-insurance terms than type r, the tangency contract lies below the fair-offer surface for type R. It then follows that the above offer that is only attractive to type r will make losses.

Proposition 4: *Only if Property 4(ii) obtains are partial-pooling equilibria possible with bundled contracts.*

To establish the existence of a partial-pooling equilibrium we have to restrict the range of α. Define a value of α, denoted by $\widehat{\alpha}^{**}$ and note in parallel to Proposition 2, that if $\alpha \leq \widehat{\alpha}^{**}$ profitable full-pooling offers are not possible.

In the remainder of this section we focus attention on separating equilibria but note that the results presented in the next section in Propositions 7 and 8 carry over to partial-pooling bundled contracts. The structures of the following arguments are kept simple by noting that separating equilibria involve contracts at the intersection of the two types utility surfaces and the zero-profit offer surfaces. In particular, the contract for the better type will involve incomplete insurance and lie at the intersection of the zero-profit offer surface for its type and the incentive-constraint for the poorer type. Because of the two cases in Property 4, which type's incentive constraint binds varies.

First consider the case where Property 4(i) obtains. In this case the incomplete insurance contract is on the fair-offer surface for type r, which has slope $1/p_r$ in the annuity dimension and slope $1/p_r(1-q_r)$ in the long-term care dimension. In stating the proposition we define the upper threshold $\widehat{\alpha}^{***}$, below which separating contract offers are Pareto dominated by full-pooling offers.

Proposition 5: *If Property 4(i) holds and $\alpha \geq \widehat{\alpha}^{***}$, there exists a separating equilibrium in bundled contracts in which type R obtain complete insurance, while type r obtain incomplete long-term care coverage and incomplete annuity coverage. In this case, the burden of separation is in incomplete annuity coverage.*

Proof: Given Property 4(i), the bundled contracts are chosen to maximize (1) for type r subject to (10), (11), (13), (14), and the two nonnegative profit conditions derived from (12). In the least cost separating equilibrium, (13) binds but (14) does not. The solution gives complete insurance for type R at the contract $\overline{S}_R$ on the fair-offer surface for that type and partial coverage to type r at S_r' on the fair-offer surface for that type, which is the best contract on the surface for their type that does not attract type R.

Since (13) must bind, the equilibrium involves distortions from the first-best. The utility surface of type R passing through $\overline{S}_R$ must cut the fair-offer surface for type r from below at S'_r. However, by Assumption 1, $p_R(1-q_R)$ is relatively small; hence, although the contract for type r sets $Y_{2r}^B < \overline{Y}_{2r}$ and $\lambda_r^B < 1$, the burden of the deviation from first-best is in setting Y_{2r}^B low. This is because the cut in Y_{2r}^B has a relatively bigger impact on type R (the opposite is true for cuts in λ_r^B). It is then easily checked that the proposed allocation is robust to out of equilibrium contract offers.

Finally, as $\alpha \geq \widehat{\alpha}^{***}$, the cross-subsidy to type R in any potentially profitable pooling offer is high, so that type r prefer the separating contract for their type. Q.E.D

In the Appendix, Problem D, we demonstrate formally that $Y_{2r}^B < \overline{Y}_{2r}$ and $\lambda_r^B < 1$ and shows how the burden of deviation from the first-best is shared between the two effects.

The intuition for the above result is straightforward. The conditions of the proposition mean that in the market for bundled insurance products, the tendency is for the sufficiently numerous type r to subsidize the type R, because the difference in survival probabilities tends to dominate the difference in health status probabilities. The bundled contract has two dimensions along which to achieve separation. The attractive feature of the bundled contract for type r to type R is the cross-subsidy through the annuity element. By setting this sufficiently low, with an offsetting improvement in the long-term care element, which is relatively unattractive to type R, the contract will be unattractive to them. As we will show below, this bundling effect in the incentive constraint reduces the cost of achieving separation relative to the cost with stand-alone contracts.

Now consider the model when Property 4(ii) obtains. Then the incomplete insurance contract is on the fair-offer surface for type R, which has slope $1/p_R$ in the annuity dimension and slope $1/p_R(1-q_R)$ in the long-term care dimension.

In stating the next proposition we define the upper threshold $\widehat{\alpha}^*$, above which separating contract offers are Pareto dominated by full-pooling offers.

Proposition 6: *If Property 4(ii) holds and if $\alpha \leq \widehat{\alpha}^*$, there exists a separating equilibrium in which type r achieve complete insurance, while type R obtain incomplete annuity coverage and incomplete long-term care insurance. In this case the burden of separation is in incomplete long-term care insurance.*

Proof: Given Property 4(ii), the bundled contracts are chosen to maximize (1) for type R subject to (10), (11), (13), (14), and the two zero-profit derived from (12). In the least-cost separating equilibrium, (14) binds but (13) does not. The solution gives complete insurance to type r at the contract $\overline{S}_r$ and partial coverage to type R at S'_R, which is the best contract on the fair-offer-surface for type R that does not attract type r.

To achieve separation there are distortions from the first-best. In this case, the utility surface of type r passing through $\overline{S}_r$ must cut the fair-offer surface for type R from below at S'_R. However, by Assumption 1, $p_r(1-q_r)$ is relatively large, so that although the contract for type R sets $Y_{2R}^B < \overline{Y}_{2R}$ and $\lambda_R^B < 1$, the burden of the deviation from first-best is in setting λ_R^B low. This is because the cut in λ_R^B has a relatively bigger impact on type r (the opposite is true for cuts in Y_{2R}^B). In this case again, the above set of contract offers is robust to out-of-equilibrium contract offers.

Finally, as $\alpha \leq \widehat{\alpha}^*$, the cross-subsidy to type r in any profitable pooling contract offer is high, so that type R prefer the separating contract. Q.E.D.

The Appendix, Problem E, demonstrates $Y_{2R}^B < \overline{Y}_{2R}$ and $\lambda_R^B < 1$ formally and shows how the burden of deviation from the first-best is shared between the two effects.

The intuition for this result is that in this case in the market for bundled insurance products, there is a tendency for the sufficiently numerous type R to subsidize the type r because the difference in health status probabilities tends to dominate the difference in survival probabilities. The attractive feature of the bundled contract for type R to type r is the cross-subsidy through the long-term care element, so by setting this sufficiently low, with an offsetting improvement in the annuity element, the contract is unattractive to type r. This bundling effect in the incentive constraint, as in Proposition 5, reduces the cost of achieving separation relative to the equilibrium with stand-alone contracts.

Before explicitly examining the welfare properties of the equilibrium with bundled contracts we comment briefly how the more flexible bundled contracts achieve separation relative to the stand-alone contracts. In the case of a separating equilibrium with stand-alone contracts, all of the costs of separation in the long-term care insurance market are borne by type R by setting λ_R low (through the term $\mu p_r(1-q_r)U_r'(Y_{2R}^* - X + \lambda_R X)X$ in equation (A4) in Problem A in the Appendix). In turn, all of the costs of separation in the annuity market are borne by type r, by setting Y_{2r} low (determined by the term $\widehat{\mu}[p_R[q_R U_R'(Y_{2r}) + (1-q_R)U_R'(Y_{2r} - X + \lambda_{2r}^* X)]$ in Equation (C4) in Problem C in the Appendix). The bundled contracts potentially dominate the stand-alone contracts because they achieve separation through greater flexibility in the allocation of income between dates and states. As can be seen, for example, in Proposition 6, this is done by setting both λ_R^B and Y_R^B at less than the full-information levels.

Propositions 5 and 6 demonstrate quite different equilibrium outcomes linked to the two regimes described in Property 4. Hence, when bundled products are traded, outcomes will depend crucially upon differences in probabilities of survival and health in old age, as well as differences in risk aversion. However, again administration costs will affect the results and their impact can be examined along similar lines to that for the stand-alone products.

WELFARE PROPERTIES OF THE EQUILIBRIUM WITH BUNDLED CONTRACTS

In this section we compare the efficiency properties of the equilibria with bundled contracts to the equilibria with only stand-alone contracts. Having done this we allow stand-alone insurance contracts to be offered alongside bundled contracts and assess the robustness of the equilibrium in bundled contracts to such offers.

In our model, an individual who takes care will be relatively low risk in the long-term care insurance market but high risk in the annuity market, with the opposite being the case for those who take less care. The terms of bundled contracts are set to exploit this asymmetry. On a particular dimension of the contract, if one type is high risk the other is low risk, and coverage is set accordingly. For example, if type R are the high demand for total insurance group, the bundled contract offers a high level of annuity coverage, which they value highly and type r value relatively less. At the same time the contract offers less long-term care insurance that is of relatively less value to type R but this makes the contract less attractive to type r, who have a higher demand for this element of insurance. We now show that the bundled contracts are strictly welfare superior to the allocation in stand-alone contracts.

Proposition 7: *The equilibria in Propositions 5 and 6 Pareto dominate the allocations with stand-alone contracts.*

Proof: First consider the case in Proposition 5. With the bundled contracts, the cost of separation is borne by setting $Y_{2r}^B < \overline{Y}_{2r}$ and $\lambda_r^B < 1$, with type R obtaining complete insurance. Hence, type R is strictly better off with the bundled contract than the pair of stand-alone contracts (L_R^*, A_R^*), in which they obtain only partial long-term care insurance: $V_R(S'_R) > V_R(L^*_R, A^*_R)$. This implies that the incentive constraint (13) is easier to satisfy than that in (8), in which the right-hand side is conditional upon incomplete long-term care insurance. Type r can achieve separation with a contract that places the entire burden on cutting Y^B_{2r} to a value below $\overline{Y}_{2r}$, but by less than in the equilibrium with stand-alone contracts. But in addition, the optimal separating bundled contract shares the cost of separation with a cut in λ^B_r, so that Y^B_{2r} is cut by less (an option that is not available with the stand-alone contracts), thereby raising further the utility of type r with the bundled contract. Thus, this type must also be better off.

Now consider the case in Proposition 6. With the bundled contract, type r obtain complete insurance and are thus better off than with stand-alone contracts, which deliver incomplete annuity coverage. Hence, $V_r(S'_r) > V_r(L^*_r, A^*_R)$, so that (14) is easier to satisfy than (7), in which the right-hand side is conditional upon incomplete annuity coverage. Type R can achieve separation with a contract that places the entire burden on cutting λ^B_R, but by less than with the stand-alone contract. However, there is also an added gain, in that the optimal separating contract shares the cost with a cut in Y^B_{2R} and so cuts λ^B_R by less. Thus, this type must also be better off. Q.E.D.

The above argument shows that for a given value of α, separating equilibria are more likely with bundled contracts. That is, without a formal demonstration, for some range of values, below the lower cutoff value of α for a separating equilibrium in the stand-alone annuity market shown in Proposition 3, there will be equilibrium separation with bundled contracts, shown in Proposition 5, so $\widehat{\alpha}^{***} < \alpha^{***}$. Similarly, for some range of values of α, above the upper cutoff for separating equilibrium in the stand-alone long-term care insurance market shown in Proposition 1, there will be equilibrium separation in bundled contracts, shown in Proposition 6, so $\widehat{\alpha}^{*} > \alpha^{*}$.

Finally, we consider the implications of allowing markets for stand-alone contracts to operate side-by-side with markets for bundled contracts. The problem is to test whether the equilibrium offers in Propositions 5 and 6 are stable with respect to offers of stand-alone contracts. Suppose therefore that there is an equilibrium in the stand-alone markets and consider the opening of the market for bundled contracts. Then because of the results in Proposition 7, bundled contract offers can be made that are profitable and cause each type to deviate from the respective stand-alone configuration. However, as we show below, the opposite is not so straightforward.

Suppose that there is an equilibrium in bundled contracts and consider the possibility of stand-alone contracts being offered. Then we need to establish that an individual type cannot be induced to deviate to a pair of individually profitable offers of stand-alone contracts, as this would destabilize the putative equilibrium in bundled contracts. In presenting the following argument, we retain the game structure used in the body of the article. We assume that an individual of a particular type, in responding to the offer of a stand-alone contract in one market anticipates the offer it will get in the other stand-alone market. Then we can now demonstrate the following:

Proposition 8: *The equilibria in Propositions 5 and 6 are stable with respect to offers of stand-alone contracts.*

Proof: We only consider the case in Proposition 6, as essentially the same argument as below applies to the case in Proposition 5. In this case type r achieve complete insurance with the bundled contract and so will not deviate to profitable offers in stand-alone contracts for their type. Thus, we only need consider type R. Then in principle, insurance companies could independently offer this type a pair of stand-alone contracts, which give it maximum insurance and which are individually profitable if only taken by this type. However, given Property 4(ii), this configuration will attract type r, and so will be unprofitable and will not be offered. Hence, in this case, offers of stand-alone insurance coverage must offer type R less than maximum coverage. The best pair of individually profitable stand-alone contracts for type R that will not attract type r, leaves type r indifferent between this pair of contracts and the bundled contract for their type. But if this type is indifferent, the incentive-compatibility constraint for type r in the market for bundled contracts is unchanged from the case where the stand-alone markets are closed. However, because the incentive compatibility constraint for type r with bundled contracts, (14), is easier to satisfy than the pair of constraints with stand-alone contracts, (7) and (9), type R will strictly prefer the bundled contract for their type to this pair of stand-alone contracts. Thus, there does not exist a profitable pair of stand-alone contracts that attracts type R. Q.E.D.

This last proposition means that we do not need to propose banning stand-alone contracts to ensure that the desirable outcomes in Propositions 5 and 6 obtain. Indeed this proposition has the implication that the market for bundled contracts should crowd out stand-alone insurance.[20] This raises the puzzle of why the market for bundled contracts is not larger, which we discuss further below.

CONCLUSION

This article has presented a unified theoretical framework that yields predictions consistent with the empirical observations of Finkelstein and McGarry (2006) for the market for long-term care insurance and Finkelstein and Poterba (2002, 2004) for the annuity market. It shows that when the selection effects for the two types of individual in the markets for stand-alone contracts work in opposite directions, with an individual who is high risk in one market being low risk in the other, there exists the possibility of equilibrium in a single market for bundled contracts. Moreover, it is shown that this equilibrium allocation Pareto dominates the stand-alone contracts allocations.

If the low-risk aversion type are on the margin of participation in the stand-alone long-term care insurance market and certainly participate in the stand-alone annuity market, then they will definitely participate in the market for bundled contracts. Thus, combining the two types of insurance has the potential to increase the attractiveness of insurance relative to the stand-alone contracts and make insurance affordable to more potential buyers. However, although these contracts are marketed currently in both the United States and the United Kingdom, they make up less than 10 percent of the

[20] Since in reality both stand-alone and bundled contracts are marketed, we need a richer model to explain the coexistence of the two types of contract. The most natural additional factor that could explain this is differential liquidity.

voluntary annuity market. Brown and Finkelstein (2009) provide an extensive review of most of the explanations for the small size of the long-term case insurance. On the supply side these include high loadings and pricing not fairly reflecting utilization of long-term care by gender but conclude that these effects cannot explain observed coverage levels. Turning to demand-side factors, they discuss limited rationality or financial knowledge as a possible explanation of limited coverage and other explanations. One is that of Brown and Finkelstein (2007), who argue that state provision of care, through Medicaid in the United States, crowds out private provision. This argument applies mainly to poorer individuals who have exhausted their financial wealth but also affects richer groups through an implicit tax. The above arguments also apply to bundled contracts. However, even when controlling for these effects the market for bundled products remains puzzlingly small. Two possible explanations are offered by Brazell, Brown, and Warshawsky (2007). One is that current underwriting practices in the United States restrict access to these products by individuals with certain health histories. Second, the products have been tax disadvantaged but as a result of the Pension Protection Act of 2006, from 2010, this will no longer be the case.

The failure of the market to supply the bundled contracts may though be because of complex intergenerational gaming of the type discussed by Pauly (1990), in which parents may not buy this type of insurance, so as to ensure care from their children, who see their inheritance as being dependent upon such care. Alternatively, the market may be limited because individuals are concerned (as in the basic annuity market) about "liquidity," which may be relevant if they are undecided about, among other things, the bequest they wish to leave.[21] This will be linked to individuals learning about longevity (or indeed health status), who must then choose the timing of annuity purchase and precautionary saving (see, e.g., Brugiavini, 1993). This may also lead to a reduced demand for bundled contracts.[22]

From a welfare perspective, the result in Proposition 8 means that we do not advocate the banning of stand-alone contracts as a part of a program of encouraging the development of the market for bundled contracts. However, if the market for bundled contracts has not developed because of costs, the argument stemming from the article is that retirement programs should not be designed in isolation from long-term care insurance programs.[23]

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[21] The bundled contracts expose insurance providers to two types of risk, namely, longevity risk and to possible increases in the cost of long-term care. However, this argument applies equally to the stand-alone contracts. Philipson and Becker (1998) point to the moral hazard problem in providing mortality contingent claims as limiting the market. However, in the current model there is limited scope for this effect.

[22] In order to address the problem of the timing of annuities purchase and precautionary saving within the current framework, the model needs to be modified. In particular, individuals' lifetimes have to be divided into three periods with them learning about their health status and longevity at the end of the first period. At a more general level, Brown, Davidoff, and Diamond (2005) discuss the implications of market incompleteness for annuity demand and welfare.

[23] Chen (1994), for example, has argued that Social Security benefits could include a basic long-term care benefit. He argues that the state might develop markets for bundled contracts to take pressure off Medicare, or state disability benefit. However, the issue of the optimal,

APPENDIX

Problem A (Proposition 1): The contract L_R solves:

$$\max\left\{U_R\left(W-Q_R-P_R^*\right)+p_R\left[q_R U_R\left(Y_{2R}^*\right)+(1-q_R)U_R\left(Y_{2R}^*-X+\lambda_R X\right)\right]\right\} \tag{A1}$$

subject to

$$\begin{aligned}&U_r(W-Q_R-P_R)+p_R\left[q_R U_r\left(Y_{2R}^*\right)\right.\\&\qquad\left.+(1-q_R)U_r\left(Y_{2R}^*-X+\lambda_R X\right)\right]\leq V_r\left(L_r\mid A_r^*\right),\end{aligned} \tag{A2}$$

and

$$Q_R\geq p_R(1-q_R)\lambda_R X+k_L. \tag{A3}$$

Let $\mu\geq 0$ and $\phi\geq 0$ be the multipliers on the respective constraints. Then choose λ_R and Q_R and the multipliers to solve the problem. The first-order conditions are

$$\begin{aligned}&p_R(1-q_R)U_R'\left(Y_{2R}^*-X+\lambda_R X\right)X\\&\quad-\mu p_r(1-q_r)U_r'\left(Y_{2R}^*-X+\lambda_R X\right)X-\phi p_R(1-q_R)X=0,\end{aligned} \tag{A4}$$

$$-U_R'\left(W-Q_R-P_R^*\right)+\phi=0, \tag{A5}$$

and the two constraints. Inspection shows that (A2) binds so that $\mu>0$, implying that $\lambda_R<1$.

Problem B (Proposition 2): The optimization problem is to choose the contract at T to solve:

$$\max\left\{U_R\left(W-Q_R-P_R^*\right)+p_R\left[q_R U_R\left(Y_{2R}^*\right)+(1-q_R)U_R\left(Y_{2R}^*-X+\lambda_R X\right)\right]\right\} \tag{B1}$$

subject to

$$\begin{aligned}&U_r\left(W-Q_R-P_R^*\right)+p_R\left[q_R U_r\left(Y_{2R}^*\right)\right.\\&\qquad\left.+(1-q_R)U_r\left(Y_{2R}^*-X+\lambda_R X\right)\right]\leq V_r\left(L_r\mid A_r^*\right),\end{aligned} \tag{B2}$$

and

$$Q_R\geq p_R(1-q_R)\lambda_R X+k_L. \tag{B3}$$

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welfare-maximizing levels of mandatory annuitization of retirement wealth and long-term care insurance is replete with problems of implementation. On the other hand, it is worth remarking that government transfers such as means-tested benefit and public provision of long-term care may provide incentives for some individuals to draw down their savings early and to take less preventative action than otherwise.

Let $\mu \geq 0$ and $\phi \geq 0$ be the multipliers on the respective constraints, noting that (B3) does not bind, so that $\phi = 0$. Then choose λ_R and Q_R and μ to satisfy

$$p_R(1-q_R)U_R'\big(Y_{2R}^* - X + \lambda_R X\big)X - \mu p_r(1-q_r)U_r'\big(Y_{2R}^* - X + \lambda_R X\big)X = 0, \quad \text{(B4)}$$

and the two constraints. Inspection shows that (B2) binds so that $\mu > 0$, implying that $\lambda_R < 1$. As the contract $\widehat{L}$ is close to T, this is also a property of this contract.

Problem C (Proposition 3): The contract A_r solves the following problem:

$$\max\big\{U_r\big(W - Q_r^* - P_r\big) + p_r(q_r U_r(Y_{2r}) + (1-q_r)U_R\big(Y_{2r} - X + \lambda_r^* X\big)\big\} \quad \text{(C1)}$$

subject to

$$\begin{aligned} &U_R\big(W - Q_r^* - P_r\big) + p_R\big[q_R U_R(Y_{2r}) \\ &\quad + (1-q_R)U_R\big(Y_{2r} - X + \lambda_r^* X\big)\big] \leq V_R\big(A_R \mid L_R^*\big), \end{aligned} \quad \text{(C2)}$$

and

$$P_r \geq p_r Y_{2r} + k_A. \quad \text{(C3)}$$

Let $\widehat{\mu} \geq 0$ and $\widehat{\phi} \geq 0$ be the multipliers on the respective constraints. Then choose Y_{2r} and P_r and the multipliers to solve the problem. The first-order conditions are

$$\begin{aligned} &p_r\big(q_r U_r'(Y_{2r}) + (1-q_r)U_r'\big(Y_{2r} - X + \lambda_r^* X\big)\big) \\ &\quad + \widehat{\mu}\big[p_R\big[q_R U_R'(Y_{2r}) + (1-q_R)U_R'\big(Y_{2r} - X + \lambda_{2r}^* X\big)\big] - \widehat{\phi} q_r = 0, \end{aligned} \quad \text{(C4)}$$

$$-U_r'\big(W - Q_r^* - P_r\big) + \widehat{\phi} = 0, \quad \text{(C5)}$$

and the two constraints. Inspection shows that (C2) binds and $\widehat{\mu} > 0$ so that $Y_{2r} < \overline{Y}_{2r}$.

Problem D (Proposition 5): The contract S_r solves the following problem:

$$\max\big\{U_r(W - B_r) + p_r\big(q_r U_r\big(Y_{2r}^B\big) + (1-q_r)U_r\big(\widetilde{Y}_{2r} - X + \lambda_r^B X\big)\big\} \quad \text{(D1)}$$

subject to

$$U_R(W - B_r) + q_R\big[p_R U_R\big(Y_{2r}^B\big) + (1-p_R)U_R\big(Y_{2r}^B - X + \lambda_r^B X\big)\big] \leq V_R\left(S_R^*\right), \quad \text{(D2)}$$

and

$$B_r \geq p_r Y_{2r}^B - p_r(1-q_r)\lambda_r^B X + k_B. \quad \text{(D3)}$$

Let $\theta \geq 0$ and $\gamma \geq 0$ be the multipliers on the respective constraints. Then choose λ_r^B, Y_{2r}^B, and B_r and the multipliers to solve the problem. The first-order conditions are:

$$p_r(1-q_r)U_r'\left(Y_{2r}^B - X + \lambda_r^B X\right)X \\ -\theta p_R(1-q_R)U_R'\left(Y_{2r}^B - X + \lambda_r^B X\right)X - \gamma p_r(1-q_r)X = 0, \tag{D4}$$

$$p_r\left(q_r U_r'\left(Y_{2r}^B\right) + (1-q_r)U_r'\left(Y_{2r}^B - X + \lambda_r^B X\right)\right) \\ +\theta\left[p_R\left[q_R U_R'\left(Y_{2r}^B\right) + (1-q_R)U_R'\left(Y_{2r}^B - X + \lambda_{2r}^B X\right)\right] - \gamma p_r = 0, \tag{D5}$$

$$-U_r'(W - B_r) + \gamma = 0, \tag{D6}$$

and the two constraints. To achieve separation, Equation (D2) must bind, $\theta > 0$, then we see that there are distortions from the first-best (which applies when $\theta = 0$) in both (D4) and (D5). However, as we have assumed (Assumption 1) that $p_R(1-q_R)$ is relatively small, the distortion in (D4) is small relative to that in (D5), in which the term $p_R q_R U'_R(Y_{2r}^B)$ is relatively large. Thus, although the contract for type r sets $Y_{2r}^B < \overline{Y}_{2r}$ and $\lambda_r^B < 1$, the burden of the deviation from first-best is in setting Y_{2r}^B low. This is because the cut in Y_{2r}^B has a relatively bigger impact on type R (the opposite is true for cuts in λ_r^B).

Problem E (Proposition 6): The contract S_R solves

$$\max\left\{U_R(W - B_R) + p_R\left(q_R U_R\left(Y_{2R}^B\right) + (1-q_R)U_R\left(Y_{2R}^B - X + \lambda_R^B X\right)\right\}\right. \tag{E1}$$

subject to

$$U_r(W - B_R) + p_R\left[q_R U_r\left(Y_{2R}^B\right) + (1-q_R)U_r\left(Y_{2R}^B - X + \lambda_R^B X\right)\right] \leq V_r\left(S_r^*\right), \tag{E2}$$

and

$$B_R = p_R Y_{2R}^B - p_R(1-q_R)\lambda_R^B X. \tag{E3}$$

Let $\widehat{\theta} \geq 0$ and $\widehat{\gamma} \geq 0$ be the multipliers on the respective constraints. Then, choose λ_R^B, Y_{2R}^B, and B_R and the multipliers to solve the problem. The first-order conditions are:

$$p_R(1-q_R)U_R'\left(Y_{2R}^B - X + \lambda_R^B X\right)X \\ -\widehat{\theta} p_r(1-q_r)U_r'\left(Y_{2R}^B - X + \lambda_R^B X\right)X - \widehat{\gamma} p_R(1-q_R)X = 0, \tag{E4}$$

$$p_R\left(q_R U_R'\left(Y_{2R}^B\right) + (1-q_R)U_R'\left(Y_{2R}^B - X + \lambda_R^B X\right)\right) \\ +\widehat{\theta}\left[p_r\left(q_r U_r'\left(Y_{2R}^B\right) + (1-q_r)U_r'\left(Y_{2R}^B - X + \lambda_R^B X\right)\right] - \widehat{\gamma} p_R = 0, \tag{E5}$$

$$-U_R'(W - B_R) + \widehat{\gamma} = 0, \tag{E6}$$

and the two constraints. As $\widehat{\theta} > 0$, then we can see that there are distortions from the first-best in both (E4) and (E5). However, by Assumption 1, $p_r(1 - q_r)$ is relatively large, the distortion in (E4) is small relative to that in (E5) in which the term $p_r q_r U'_R(Y^B_{2r})$ is relatively small. Thus, although the contract for type R sets $Y^B_{2R} < \overline{Y}_{2R}$ and $\lambda^B_R < 1$, the burden of the deviation from first-best is in setting λ^B_R low. This is because the cut in λ^B_R has a relatively bigger impact on type r (the opposite is true for cuts in Y^B_{2R}).

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