

ASSESSING THE RISK POTENTIAL OF PREMIUM PAYMENT OPTIONS IN PARTICIPATING LIFE INSURANCE CONTRACTS

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ABSTRACT

Most life insurance contracts embed the right to stop premium payments during the term of the contract (paid-up option). Thereby, the contract is not terminated but continues with reduced benefits and often provides the right to resume premium payments later, thus increasing the previously reduced benefits (resumption option). In our analysis, we start with a basic contract with two standard options, namely, an interest rate guarantee and annual surplus participation. Next, in addition to the features of the basic contract, a paid-up and resumption option is included in the framework. The valuation process is not based on assumptions about a particular policyholders' exercise strategy but instead assesses the risk potential from the insurer's viewpoint by providing an upper bound for any possible exercise behavior. This approach provides important information to the insurer about the potential hazard of offering the paid-up and resumption option. Further, the approach allows an analysis of the impact of guaranteed interest rate, annual surplus participation, and investment volatility on the values of the premium payment options.

INTRODUCTION

In most life insurance contracts, the policyholder has the right to stop premium payments at any time during the term of the contract (*paid-up option*). This feature is contained, for example, in level premium whole life insurance contracts in the U.S. market or endowment policies in the European market. The paid-up option differs from a surrender option in that the contract is not terminated but continues with

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reduced benefits. Additionally, the contract can provide the right to resume premium payments later and thus to increase the previously reduced benefits (*resumption option*). Flexible premium payments are offered in universal life insurance contracts sold in the United States and can also be found in European insurance markets. The aim of this article is to study the risk potential of premium payment options within participating life insurance contracts that include two standard options—an interest rate guarantee and a guaranteed annual surplus participation.

Life insurance contracts often embed various types of implicit options. Concern over embedded options was intensified in 2000 when the British life insurer Equitable Life had to stop taking new business due to an improper hedging of provided options. The growing interest in the field of valuation and risk management of embedded options in life insurance contracts is also demonstrated by the increasing number of scientific contributions to this field: Briys and de Varenne (1994) treat the bonus option and the insolvency option in a contingent claim framework. Hansen and Miltersen (2002) and Tanskanen and Lukkariinen (2003) conduct a fair pricing of contracts with a guaranteed interest rate and different annual surplus participation schemes. Ballotta, Haberman, and Wang (2006) consider guarantees in participating life insurance contracts commonly offered in the United Kingdom while studying the effect of the default option on fair pricing. Grosen and Jørgensen (2000) add to the analysis of guaranteed interest rate and surplus participation by also taking the surrender option into account. In Grosen and Jørgensen (2002), the insurer's insolvency option and regulatory intervention are considered. Bacinello (2003a,b) analyzes the surrender option in an Italian life insurance contract with single and periodic premiums, including mortality risk; Bacinello (2005) performs this analysis for unit-linked contracts; Albizzati and Geman (1994) analyze the value of a surrender option in French life insurance contracts while establishing the concept of exercise probabilities.

Among the literature on paid-up options, Herr and Kreer (1999) model a life insurance contract with surrender and paid-up options with underlying stochastic interest rates; however, the surplus participation rate of the contract is assumed to be deterministic. A drawback of the model is that it only implicitly accounts for mortality risk. Instead, annual fictitious risk premiums for the death benefit are calculated, which deterministically reduce the cash flow of the insurer. Thus, the options can be exercised until maturity without taking early death of the insured into account. Steffensen (2002) establishes a very general framework including surrender and paid-up option and suggests applying optimal intervention theory. Linnemann (2003, 2004) compares actuarial approaches to the valuation of the paid-up and surrender option in participating life insurance contracts and studies the effect on actuarial reserving. Kling, Russ, and Schmeiser (2006) perform an analysis of paid-up options for government-subsidized pension products in Germany based on different assumptions about the policyholder's exercise behavior.

The impact of guaranteed interest rate, stochastic annual surplus participation, and investment volatility on the values of the premium payment options have not been studied in the context of participating life insurance contracts to date. Hence, in a first step, we provide a framework that includes a basic endowment policy with

two typical standard options, namely, interest rate guarantee and surplus participation. In a second step, we include different forms of premium payment options; besides the paid-up and resumption option, we also consider the flexible premium payment option, where the policyholder is free to stop and resume payments at any point in time. Combining the paid-up option with the resumption option leads to a complex path-dependent structure as the policyholder's decision to resume payments depends on the exercise date of the paid-up option. Thereby, one needs to take into account that the benefits are reduced after premium payments are stopped, which has an effect on the policyholder's decision to exercise the option. Hence, the interaction between financial and mortality factors, periodic premium payments and the premium payment options leads to a complex payoff distribution for the contract.

The derived framework can be evaluated under different assumptions about an insured's exercise behavior given a variety of option prices. We concentrate on the evaluation of a behavioral-independent risk potential by providing an upper bound for different premium payment options in participating life insurance contracts. Hence, we focus on the potential hazard of such options from the insurer's viewpoint. In addition, we assess the sensitivity of the values of the premium payment options to variation in the contract parameters.

A key result of the article is that the guaranteed interest rate has a crucial impact on the value of the premium payment options in the contract. In general, offering more favorable contract conditions (e.g., higher guaranteed interest rate, higher surplus participation rate, or higher investment volatility), *ceteris paribus*, leads to an increase in the basic contract's value. However, the paid-up option value decreases at the same time and therefore exhibits a counterbalancing effect. Lowering the guaranteed interest rate for fair basic contracts with the same market value (using risk-neutral valuation) leads to a dramatic increase in the pure paid-up option value, even though the participation rate is raised at the same time in order to ensure a fair basic contract. Furthermore, for fair contracts, this risk associated with premium payment options cannot be effectively reduced by reducing volatility of the contract's underlying investment portfolio.

The remainder of the article is organized as follows. In the following section, we present our model framework for the basic contract and introduce the paid-up and resumption option. The next section contains numerical results for the situation when the paid-up and resumption options are exercised at maximum value. In the subsequent section, we study the sensitivity of the option values to several contract parameters. We conclude in the last section.

MODEL FRAMEWORK

The Basic Contract

In the basic setup, the contract only contains two standard options: a guaranteed interest rate and annual surplus participation. The premium payments B_{t-1} are paid annually at the beginning of the t th policy year given the insured remains alive until maturity T . In case of death during the t th year of the contract (between time $t-1$ and t), the policyholder's heirs receive the death benefit Y_t at the end of the year. In case

of survival until maturity T , the insurer pays out the accumulated policy reserve V_T , which also includes participation in the annual surplus of the life insurer's investment portfolio. Let x be the age of the insured at inception of the contract. In our analysis, the death benefit Y and the annual premium payments B are constant. Death and survival probabilities are derived from a mortality table. In the insurance business, the initial guaranteed death benefit Y , which in our case is also the (minimum) benefit the insured will receive in the case of survival until maturity, is calculated according to the actuarial equivalence principle (see, e.g., Linnemann, 2003). This principle requires that the expected payments to the insured must be equal to the expected premium payments from the insured. Hereby, the guaranteed interest rate is used as the annually compounded interest rate for discounting future benefits and premiums (see Bacinello, 2003a, pp. 464–466). In our case, this can be written as

$$B \sum_{t=0}^{T-1} {}_t p_x \cdot (1+g)^{-t} = Y \left(\sum_{t=0}^{T-1} {}_t p_x \cdot q_{x+t} \cdot (1+g)^{-(t+1)} + {}_T p_x \cdot (1+g)^{-T} \right). \quad (1)$$

Accordingly, ${}_t p_x$ stands for the probability of an x -year-old policyholder surviving for the next t years, and ${}_t q_x (= 1 - {}_t p_x)$ is the corresponding probability that an x -year-old will die within the next t years. Hence, $q_{x+t} ({}_x p_{x+t})$ gives the probability that an $(x+t)$ -year-old policyholder will die within the next year (survive one more year). Hence, one can calculate the initial guaranteed benefit Y from

$$Y = \frac{B \sum_{t=0}^{T-1} {}_t p_x (1+g)^{-t}}{\sum_{t=0}^{T-1} {}_t p_x q_{x+t} (1+g)^{-(t+1)} + {}_T p_x (1+g)^{-T}}. \quad (2)$$

The policy reserve at the beginning of the t th year, V_{t-1} , and the premium payment B annually earn the greater of the guaranteed interest rate g or a fraction α of the annual surplus $(S_t/S_{t-1} - 1)$ of the insurer's investment portfolio. This type of contract is called a cliquet-style guarantee since the fraction of the surplus in turn becomes part of the guarantee. In our model, the life insurer's investment portfolio follows a geometric Brownian motion given a complete, perfect, and frictionless market.¹ In case of death during the t th year, the benefit Y is paid. If the policyholder survives, the new reserve is given with V_t . It is assumed that mortality risk is entirely diversifiable, that is, there is no systematic risk from mortality for the insurer. V_t can be recursively computed by the following relation (see Bacinello, 2001, p. 293),

¹ Let W_t (with $0 \leq t \leq T$) be a standard Brownian motion on a probability space (Ω, Φ, P) and Φ_t (with $0 \leq t \leq T$) be the filtration generated by the Brownian motion. In this setting, let S_t be defined as $dS_t = \mu S_t dt + \sigma S_t dW_t^P$ with deterministic asset drift μ , volatility σ , and a P -Brownian motion W^P . Under the risk-neutral unique equivalent martingale measure Q , the drift of the process changes to the risk-free interest rate r , and the solution of the stochastic differential equation (with Q -Brownian motion W^Q) is then given by $S_t = S_{t-1} \exp(r - 0.5\sigma^2 + \sigma(W_t^Q - W_{t-1}^Q))$ (see, e.g., Harrison and Kreps, 1979).

$$(V_{t-1} + B)(1 + \max[g, \alpha(S_t/S_{t-1} - 1)]) = p_{x+t-1} \cdot V_t + q_{x+t-1}Y, \quad (3)$$

which can be written as

$$V_t = ((V_{t-1} + B)(1 + \max[g, \alpha(S_t/S_{t-1} - 1)]) - q_{x+t-1} \cdot Y) / p_{x+t-1},$$

with $V_0 = 0$. The accumulated payoff of the basic contract P_T at maturity T is determined by the payments to the insured less the premium payments, compounded with the risk-free interest rate r .

$$P_T = \sum_{t=0}^{T-1} Y \cdot {}_t p_x \cdot q_{x+t} \cdot e^{r(T-t-1)} + {}_T p_x \cdot V_T - \sum_{t=0}^{T-1} B \cdot {}_t p_x \cdot e^{r(T-t)}. \quad (4)$$

The first term in Equation (4) represents the death benefit, the second term is the benefit payable at maturity in case of survival, and the last term contains the expected premium payments to the insurer, compounded to maturity T . The net present value Π_0 of the contract payoff P_T at time $t = 0$ is calculated by using risk-neutral valuation. Furthermore, we assume independence between mortality and financial risks. With E_t^Q denoting the conditional expected value with respect to the probability measure Q under the information available in t , one gets

$$\Pi_0 = E_0^Q(e^{-rT} P_T).$$

The contract is fair if its net present value equals 0,

$$\Pi_0 = 0. \quad (5)$$

Hence, we say that a contract is “fair” if the present value of benefits under the risk-neutral martingale measure is equal to the present value of premiums paid by the policyholder (see, e.g., Doherty and Garven, 1986, p. 1034). For given benefit Y , we can then calibrate the guaranteed interest rate and annual surplus participation (g, α) parameters to obtain fair contracts satisfying Condition (5). This requirement implies that the two options in the basic contract (guarantee and surplus participation) are covered by the annual premiums.

Exercising the Paid-Up Option Once Without Resumption Option

Next, in addition to the features of the basic contract, we include a paid-up and resumption option in the framework. Let us first consider the exclusive paid-up option without the option to resume payments. The policyholder has the right to exercise annually until maturity and the option is therefore a Bermuda style option (see, e.g., Hull, 2005, p. 531). After exercising the paid-up option at time $\tau \in \{1, \dots, T - 1\}$ and given the policyholder is still alive, the benefits are adjusted according to a mechanism

explained below. For now, let $Y^{(\tau)}$ denote the adjusted constant death benefit and $V_T^{(\tau)}$ be the new survival benefit if the paid-up option is exercised in τ .

In actuarial practice, it is common to calculate $Y^{(\tau)}$ by taking the reserve V_τ , present at the time the policyholder stops premium payments, as single premium for a new contract (see, e.g., Linnemann, 2004, p. 86). Taking into account the current age of the insured, the adjusted benefit (which is also the minimum survival benefit) can be obtained analogous to Equations (1) and (2).

$$V_\tau = Y^{(\tau)} \left(\sum_{t=\tau}^{T-1} {}_{t-\tau}p_{x+\tau} \cdot q_{x+t} \cdot (1+g)^{-(t-\tau+1)} + {}_{T-\tau}p_{x+\tau} \cdot (1+g)^{-(T-\tau)} \right).$$

$$Y^{(\tau)} = \frac{V_\tau}{\sum_{t=\tau}^{T-1} {}_{t-\tau}p_{x+\tau} \cdot q_{x+t} \cdot (1+g)^{-(t-\tau+1)} + {}_{T-\tau}p_{x+\tau} \cdot (1+g)^{-(T-\tau)}}. \quad (6)$$

The reserve's development over time and the adjusted survival benefit can be calculated in analogy to Equation (3). Accordingly, $V_t^{(\tau)}$ stands for the reserve in $t = \tau + 1, \dots, T$ if the policyholder uses the paid-up option in τ . The adjusted survival benefit is denoted by $V_T^{(\tau)}$. Hence, one gets the relation

$$V_{t-1}^{(\tau)} (1 + \max[g, \alpha(S_t/S_{t-1} - 1)]) = p_{x+t-1} \cdot V_t^{(\tau)} + q_{x+t-1} \cdot Y^{(\tau)},$$

which can be transformed to

$$V_t^{(\tau)} = (V_{t-1}^{(\tau)} (1 + \max[g, \alpha(S_t/S_{t-1} - 1)]) - q_{x+t-1} \cdot Y^{(\tau)}) / p_{x+t-1},$$

where $t = \tau + 1, \dots, T$ and $V_\tau^{(\tau)} = V_\tau$.

The accumulated payoff of the contract including the paid-up option at maturity T is given in the case the paid-up option is exercised in τ by

$$P_T^{(\tau)}(Pa) = \sum_{t=0}^{\tau-1} Y \cdot {}_t p_x \cdot q_{x+t} \cdot e^{r(T-t-1)} + \sum_{t=\tau}^{T-1} Y^{(\tau)} {}_t p_x \cdot q_{x+t} \cdot e^{r(T-t-1)} \\ + {}_T p_x \cdot V_T^{(\tau)} - \sum_{t=0}^{\tau-1} B \cdot {}_t p_x \cdot e^{r(T-t)}. \quad (7)$$

Analogous to Equation (4), the first term in Equation (7) represents the original death benefit until the exercise date of the paid-up option, the second term is the death benefit after exercising the paid-up option, and the third term is the adjusted survival benefit at time T . Since premium payments are stopped at time τ , the last term contains the premiums until time τ , compounded to time T . The value of the paid-up option $\Pi_0^{(\tau)}(Pa)$ can then be residually determined by the difference of the contract value

including the paid-up option (see Equation (7)) less the value of the basic contract without the paid-up option (see Equation (4)), which can be transformed to

$$\Pi_0^{(\tau)}(Pa) = E_0^Q(e^{-rT} [P_T^{(\tau)}(Pa) - P_T]) = E_0^Q(e^{-r\tau} \cdot {}_{\tau}p_x \cdot C_{\tau}^{(\tau)}(Pa)), \quad (8)$$

where

$$\begin{aligned} C_{\tau}^{(\tau)}(Pa) = & \sum_{t=\tau}^{T-1} (Y^{(\tau)} - Y) \cdot {}_{t-\tau}p_{x+\tau} \cdot q_{x+t} \cdot e^{-r(t-\tau+1)} \\ & + (V_T^{(\tau)} - V_T) \cdot {}_{T-\tau}p_{x+\tau} \cdot e^{-r(T-\tau)} + \sum_{t=\tau}^{T-1} B \cdot {}_{t-\tau}p_{x+\tau} \cdot e^{-r(t-\tau)}. \end{aligned} \quad (9)$$

As demonstrated by Equations (8) and (9), the paid-up option gives the policyholder the right to exchange the payoff of the basic contract from τ to T , thus obtaining a different contract with adjusted benefits. Obviously, this option can be exercised only if the policyholder is still alive at the possible exercise dates.

The Resumption Option

We now analyze the impact of combining the paid-up option with the possibility of later resuming premium payments on the contract. The premium payments to be resumed are the same as they were before the paid-up option was exercised. The resumption option is also Bermudan since it can be exercised annually on dates $v \in \{\tau + 1, \dots, T - 1\}$. Hence, we can see that the policyholder receives a combined option since he or she always receives a resumption option when exercising the paid-up option. This combination of the options thus leads to a complex path dependence. As in the case of the paid-up option, we calculate the present value of the contract payoff, assuming that the policyholder stops premium payments at time $\tau \in \{1, \dots, T - 1\}$ and resumes payments at time $v \in \{\tau + 1, \dots, T - 1\}$, given the policyholder is still alive. In this case, the insurance benefits are readjusted with the adjusted survival benefit $V_T^{(\tau, v)}$ and the constant death benefit $Y^{(\tau, v)}$. The death benefit $Y^{(\tau, v)}$ is calculated analogously to Equation (6),

$$Y^{(\tau, v)} = \frac{V_v^{(\tau)} + \sum_{t=v}^{T-1} B \cdot {}_{t-v}p_{x+v} \cdot (1+g)^{-(t-v)}}{\sum_{t=v}^{T-1} {}_{t-v}p_{x+v} \cdot q_{x+t} \cdot (1+g)^{-(t-v+1)} + {}_{T-v}p_{x+v} \cdot (1+g)^{-(T-v)}}, \quad (10)$$

and the adjusted survival benefit $V_T^{(\tau, v)}$ can be calculated in analogy to Equation (3), that is,

$$(V_{t-1}^{(\tau, v)} + B)(1 + \max[g, \alpha(S_t/S_{t-1} - 1)]) = p_{x+t-1} \cdot V_t^{(\tau, v)} + q_{x+t-1} \cdot Y^{(\tau, v)},$$

$$V_t^{(\tau, v)} = ((V_{t-1}^{(\tau, v)} + B)(1 + \max[g, \alpha(S_t/S_{t-1} - 1)]) - q_{x+t-1} \cdot Y^{(\tau, v)}) / p_{x+t-1},$$

where $t = v + 1, \dots, T$ and $V_v^{(\tau, v)} = V_v^{(\tau)}$.

The payoff of the contract including the combined paid-up and resumption option can be obtained from:

$$\begin{aligned}
 P_T^{(\tau, \nu)}(PR) = & \sum_{t=0}^{\tau-1} Y \cdot {}_t p_x \cdot q_{x+t} \cdot e^{r(T-t-1)} + \sum_{t=\tau}^{\nu-1} Y^{(\tau)} \cdot {}_t p_x \cdot q_{x+t} \cdot e^{r(T-t-1)} \\
 & + \sum_{t=\nu}^{T-1} Y^{(\tau, \nu)} \cdot {}_t p_x \cdot q_{x+t} \cdot e^{r(T-t-1)} + {}_T p_x \cdot V_T^{(\tau, \nu)} \\
 & - \sum_{t=0}^{\tau-1} B \cdot {}_t p_x \cdot e^{r(T-t)} - \sum_{t=\nu}^{T-1} B \cdot {}_t p_x \cdot e^{r(T-t)}.
 \end{aligned}$$

Thus, the net present value of the combined paid-up and resumption option, $\Pi_0^{(\tau, \nu)}(PR)$, is

$$\Pi_0^{(\tau, \nu)}(PR) = E_0^Q(e^{-rT} [P_T^{(\tau, \nu)}(PR) - P_T]) = E_0^Q(e^{-r\tau} \cdot {}_\tau p_x \cdot C_\tau^{(\tau, \nu)}(PR)),$$

where

$$\begin{aligned}
 C_\tau^{(\tau, \nu)}(PR) = & \sum_{t=\tau}^{\nu-1} (Y^{(\tau)} - Y) \cdot {}_{t-\tau} p_{x+\tau} \cdot q_{x+t} \cdot e^{-r(t-\tau+1)} \\
 & + \sum_{t=\nu}^{T-1} (Y^{(\tau, \nu)} - Y) \cdot {}_{t-\tau} p_{x+\tau} \cdot q_{x+t} \cdot e^{-r(t-\tau+1)} \\
 & + (V_T^{(\tau, \nu)} - V_T) \cdot {}_{T-\tau} p_{x+\tau} \cdot e^{-r(T-\tau)} + \sum_{t=\tau}^{\nu-1} B \cdot {}_{t-\tau} p_{x+\tau} \cdot e^{-r(t-\tau)}. \quad (11)
 \end{aligned}$$

Valuation and Exercise Behavior

Valuation of premium payment options is connected to the policyholder's exercise behavior. Thus, given the framework described above, various valuation techniques can be applied. A discussion and analysis of the different exercise schemes for German government-subsidized pension products is presented in Kling, Russ, and Schmeiser (2006). One way to evaluate the options is to consider a policyholder who follows an exercise strategy that maximizes his or her option value given the information available at the exercise date. This approach leads to an optimal stopping problem that can be solved using, for example, Monte Carlo simulation methods, as is done, for example, in Andersen (1999) and Douady (2002). In this case, the random variable τ that describes when to exercise the option is an admissible strategy as it only uses information known at the present time; that is, τ has to be adapted to the filtration τ_t (with $0 \leq t \leq T$). As a second approach, one can consider the concept of exercise probabilities, which is appropriate for assessing the value of the payment options for a given portfolio of insurance contracts. Here, the insurer could estimate the empirical exercise probabilities from historical data. However, this procedure might

result in a misleading assessment of risks if future exercise behavior is different from past behavior. In this case, a reasonable model for the expected exercise behavior should be implemented. For example, Albizzati and Geman (1994) analyze the value of a surrender option in French equity-linked contracts while establishing a concept of exercise probabilities. Even in a case in which the insurer offers the premium payment options free of charge, it is possible that a positive net present value could be generated for the insurer if policyholders do not ever exercise the option in a profitable manner.

We do not base our pricing on certain assumptions about particular policyholder exercise strategies but instead aim to assess the behavioral-independent risk potential so as to provide an upper bound for any possible exercise scenario, particularly the ones described above. This is done by considering a policyholder who exercises the option at its maximum value.² This approach points out the potential hazards of offering the paid-up and resumption option. Thus, in the following, option *value* refers to the value of the premium payment options calculated using this approach.

Hence, let Π^{Opt} denote the net present value of the contract from the policyholder's viewpoint. For a contract including a paid-up option that can be used only once and that does not allow resumption of payments, the procedure can be written as

$$\Pi^{Opt} = E_0^Q \left(\max_{\tau \in \{1, \dots, T-1\}} \left[\tau p_x e^{-r\tau} \max \left(0, C_\tau^{(\tau)}(Pa) \right) \right] \right), \quad (12)$$

where $C_\tau^{(\tau)}(Pa)$ is the payoff stream of the paid-up option (see Equation (9)). In the case where the paid-up option has a value, Π^{Opt} will be positive, given the data of the basic scenario.

Next, the contract allows the policyholder to exercise the paid-up option once and to resume payments later. Hence, in analogy to Equation (12), an upper bound is obtained by

$$\Pi^{Opt} = E_0^Q \left(\max_{\substack{\tau \in \{1, \dots, T-1\} \\ \nu \in \{\tau+1, \dots, T\}}} \left[\tau p_x e^{-r\tau} \max \left(0, C_\tau^{(\tau, \nu)}(PR) \right) \right] \right). \quad (13)$$

Here, $C_\tau^{(\tau, \nu)}(PR)$ is the payoff stream of the combined paid-up and resumption option (see Equation (11)). This type is labeled the "resumption" contract in the following analysis. In our model, the value of the resumption option includes the value of the paid-up option. The value of the pure resumption option can be defined as the difference between the resumption option value (see Equation (13)) and the paid-up option value (see Equation (12)).

² Kling, Russ, and Schmeiser (2006) have also considered this case among other strategies for German subsidized pension products. Thereby, it is assumed that the insurer receives the information about the exercise at maturity of the contract.

The focus of this analysis is on the maximum risk exposure from the insurer's perspective. However, the policyholder's view of the resumption option contains aspects of interest as well, such as the impact of the insured's health status during the contract's duration on the value of the resumption option. Thus, a person in good health can purchase a new or different policy at a possibly lower rate, which would reduce the value of the resumption option substantially. For an unhealthy person, on the other hand, the option to resume the payments into the contract could be very valuable.

Moreover, we derive option values for the case where the contract allows flexible premium payments. In general, this flexibility is included, for example, in universal life insurance policies offered in the U.S. market (see Trieschmann, Hoyt, and Sommer, 2005, p. 325). Hence, after paying the first premium, the policyholder is free to stop and resume payments at any time t (with $t = 1, \dots, T - 1$). This type of contract is referred to as "flexible." The derivation of these values is done analogously to the framework shown in this section for the resumption option. The option value Π^{Opt} is then calculated in line with Equation (13).

NUMERICAL RESULTS

Input Data

In our numerical examples we consider both a 30-year-old and a 50-year-old male policyholder who enters into a contract with a term of 15 years. The contracts are taken out in calendar year 2005; that is, the 30-year-old policyholder was born in 1975 and the 50-year-old was born in 1955. In the case of the 30-year-old male policyholder, we also analyze contract duration of 30 years. We extend the analysis in Appendix A with numerical examples for a 30-year-old male policyholder who enters into a contract with a term of 10 years. Additionally, the case of a 30-year-old female policyholder with a contract term of 30 years is being considered.

To obtain the numerical results, Monte Carlo simulation is utilized with antithetic variables. This method results in variance reduction by generating negatively correlated variables such that large outputs are accompanied and counterbalanced with small outputs (see Glasserman, 2004, p. 205). Average population mortality data are derived from Bell and Miller's (2002) cohort life table.

In the numerical analysis, the risk-free interest rate $r = 4$ percent and the annual premium payments³ $B = \$1,000$. To calculate the adjusted death benefit whenever the policyholder exercises the paid-up option, the reserve—present at the time premium payments are stopped—is needed (see Equations (6) and (10)). From Equation (3) it can be seen that the size of the reserve in τ is stochastic.

Further Procedure and Table Description

Tables 1–3 show option values, Π^{Opt} , as well as the present value of the expected premium payments, Π^{Pre} . Both values refer to $t = 0$. Furthermore, the ratio $R = \Pi^{Opt} / \Pi^{Pre}$ shows how much the option value makes up in terms of the premiums actually paid on the contract so as to allow for comparing contracts with different volumes.

³ In our framework, the contract values are linear with respect to the premium amount: for example, for $B = \$500$, the option values decrease by one-half compared to $B = \$1,000$.

TABLE 1
Results for a 30-Year-Old Male Policyholder With Contract Term of 15 Years

Contract Figures		Results	Basic	Paid-Up	Resumption	Flexible
$\sigma = 20\%$	$g = 3\%$	Π^{Opt}	\$0	\$236	\$243	\$247
	$\alpha = 22.42\%$	Π^{Pre}	\$11,337	\$4,992	\$5,209	\$5,142
	$Y = \$18,706$	R	0%	4.73%	4.66%	4.80%
	$g = 1\%$	Π^{Opt}	\$0	\$478	\$498	\$507
	$\alpha = 34.66\%$	Π^{Pre}	\$11,337	\$5,140	\$5,408	\$5,338
	$Y = \$15,953$	R	0%	9.30%	9.21%	9.50%
$\sigma = 10\%$	$g = 3\%$	Π^{Opt}	\$0	\$208	\$215	\$218
	$\alpha = 39.70\%$	Π^{Pre}	\$11,337	\$5,142	\$5,374	\$5,307
	$Y = \$18,706$	R	0%	4.05%	4.00%	4.11%
	$g = 1\%$	Π^{Opt}	\$0	\$406	\$425	\$432
	$\alpha = 57.16\%$	Π^{Pre}	\$11,337	\$5,315	\$5,590	\$5,516
	$Y = \$15,953$	R	0%	7.64%	7.60%	7.83%

Notes: $B = 1,000$ = annual premium in \$; σ = asset volatility; g = guaranteed interest rate; α = annual participation rate; Y = initial death benefit; Π^{Opt} = option value in \$ given exercise at maximum value; Π^{Pre} = corresponding present value of the expected premium payments in \$; R = ratio in% = Π^{Opt} / Π^{Pre} .

TABLE 2
Results for a 50-Year-Old Male Policyholder With Contract Term of 15 Years

Contract Figures		Results	Basic	Paid-Up	Resumption	Flexible
$\sigma = 20\%$	$g = 3\%$	Π^{Opt}	\$0	\$184	\$191	\$195
	$\alpha = 22.43\%$	Π^{Pre}	\$10,497	\$4,738	\$4,912	\$4,769
	$Y = \$16,577$	R	0%	3.88%	3.89%	4.09%
	$g = 1\%$	Π^{Opt}	\$0	\$383	\$400	\$410
	$\alpha = 34.66\%$	Π^{Pre}	\$10,497	\$4,860	\$5,089	\$4,957
	$Y = \$14,461$	R	0%	7.88%	7.86%	8.27%
$\sigma = 10\%$	$g = 3\%$	Π^{Opt}	\$0	\$162	\$169	\$173
	$\alpha = 39.71\%$	Π^{Pre}	\$10,497	\$4,871	\$5,059	\$4,912
	$Y = \$16,577$	R	0%	3.33%	3.34%	3.52%
	$g = 1\%$	Π^{Opt}	\$0	\$325	\$341	\$350
	$\alpha = 57.16\%$	Π^{Pre}	\$10,497	\$5,008	\$5,247	\$5,112
	$Y = \$14,461$	R	0%	6.49%	6.50%	6.85%

Notes: $B = 1,000$ = annual premium in \$; σ = asset volatility; g = guaranteed interest rate; α = annual participation rate; Y = initial death benefit; Π^{Opt} = option value in \$ given exercise at maximum value; Π^{Pre} = corresponding present value of the expected premium payments in \$; R = ratio in% = Π^{Opt} / Π^{Pre} .

In a first step, the parameters g (guaranteed interest rate) and α (fraction of the annual return of the investment portfolio) in the basic contract are calibrated in order to get fair contract conditions given risk-neutral valuation. This implies that the implicit option contained in the basic contract—guaranteed interest rate g and annual participation rate α —are covered by the annual premium payments. In the following tables this is called the “basic” contract.

TABLE 3

Results for a 30-Year-Old Male Policyholder With Contract Term of 30 Years

Contract Figures		Results	Basic	Paid-Up	Resumption
$\sigma = 20\%$	$g = 3\%$	Π^{Opt}	\$0	\$488	\$506
	$\alpha = 22.53\%$	Π^{Pre}	\$17,072	\$7,627	\$8,024
	$Y = \$43,879$	R	0%	6.40%	6.31%
	$g = 1\%$	Π^{Opt}	\$0	\$1,020	\$1,063
	$\alpha = 34.82\%$	Π^{Pre}	\$17,072	\$7,603	\$8,068
$\sigma = 10\%$	$Y = \$32,641$	R	0%	13.42%	13.18%
	$g = 3\%$	Π^{Opt}	\$0	\$428	\$445
	$\alpha = 39.84\%$	Π^{Pre}	\$17,072	\$7,792	\$8,224
	$Y = \$43,879$	R	0%	5.49%	5.41%
	$g = 1\%$	Π^{Opt}	\$0	\$866	\$905
	$\alpha = 57.35\%$	Π^{Pre}	\$17,072	\$7,798	\$8,292
	$Y = \$32,641$	R	0%	11.10%	10.91%

Notes: $B = 1,000$ = annual premium in \$; σ = asset volatility; g = guaranteed interest rate; α = annual participation rate; Y = initial death benefit; Π^{Opt} = option value in \$ given exercise at maximum value; Π^{Pre} = corresponding present value of the expected premium payments in \$; R = ratio in% = Π^{Opt} / Π^{Pre} .

In a second step, additional options are included in the contract, that is, the paid-up option (paid-up), the resumption option (resumption), and the flexible payments option (flexible). The premium payment options are then evaluated as described in the previous section.

The tables also show the initial death benefit Y at inception of the contract, calculated under the assumption that all premiums are paid as long as the policyholder is alive. When exercising the paid-up or resumption option, the death benefit is adjusted according to Equations (6) and (10).

Numerical Results Given a Contract Term of 15 Years

We begin with a 30- and a 50-year-old male policyholder each taking out a contract with a term of 15 years. Let us first study the calibration effects on the contract figures in Table 1 given the case of a 30-year-old male policyholder. The initial death benefit for a guaranteed interest rate of $g = 3$ percent is $Y = \$18,706$. To obtain a fair contract for an asset volatility of $\sigma = 20$ percent (Table 1), $g = 3$ percent requires an annual surplus participation rate $\alpha = 22.42$ percent. Lowering the guaranteed rate to 1 percent leads to a higher participation rate of 34.66 percent. The same effect can be observed when σ is lowered to 10 percent (hence, $\alpha = 39.70$ percent), which is due to the impact of volatility on the assets. A decrease in σ lowers the chances of higher investment returns and, therefore, the participation coefficient needs to be increased to satisfy the condition of a fair contract. Concerning the calibration, we also note that the annual surplus participation rate α shows almost no effect when, *ceteris paribus*, the insured's age (Table 1 vs. Table 2) or the time to maturity (Table 1 vs. Table 3) varies.

The results in Table 1 show that the options can reach a significant value. For example, the value can reach up to \$507 in case of the flexible payments option, that is, 9.50 percent of the present value of the expected premium payments given $\sigma = 20$ percent and $g = 1$ percent (Table 1). As one would anticipate, offering more freedom in paying the premiums, for example, offering the resumption option on top of the paid-up option, increases the excess value. In the example where $\sigma = 20$ percent and $g = 1$ percent, the paid-up option is worth \$478; for the resumption option it is \$498. Interestingly, taking into account the premiums paid in the respective cases, the option value's portion of the present value of the expected premium payments is more than 9 percent for both the paid-up and the resumption options. Compared to the basic contract, the present value of the expected premium payments is substantially reduced whenever the contract includes a paid-up option since—on average—fewer premiums are paid into the contract.

Table 1 illustrates that the paid-up option is the most valuable part of the additional contract options analyzed. Hence, the increase of the option value through an additional resumption or flexible payments option is, in general, moderate, which can be explained by the reasons for making a contract paid-up in the first place, namely, unattractive contract conditions compared to investing, for example, in the money market. In general, decreasing values of g and α lead to increases in policyholder exercise behavior. This in turn leads to an increase in the values of the premium payment options and, at the same time, to a decrease in the basic contract value.

We now study the impact of the model parameters, first noting the drastic effect of the guaranteed interest rate. In our example, the option values are more than doubled when g is decreased from 3 percent to 1 percent, even though α is raised to keep the basic contract fair. Taking, for example, $\sigma = 10$ percent in Table 1, we find that the resumption option increases from \$215 to \$425, illustrating that the guaranteed interest rate is a key driver for the value of the additional premium payment options. The associated death benefit decreases from \$18,706 to \$15,953 when g is changed from 3 percent to 1 percent.

The impact of the investment portfolio's volatility on the option values is minor compared to the effect of the guaranteed interest rate. As can be seen in Table 1 in the case of the resumption option for $g = 1$ percent, changing σ from 20 percent to 10 percent leads to a decrease in the option value from \$498 to \$425. Hence, in this example, reducing volatility does not effectively reduce the risk for the insurer. On the contrary, as will be shown in the sensitivity analysis (see the next section), a decrease in the volatility lowers the value of the basic contract and thus increases the paid-up option value. In the example above, changing σ from 10 percent to 20 percent leads to a reduction in the annual surplus participation coefficient in order to keep the contract fair. Since these parameters have counterbalancing effects on the paid-up option value, one can identify α as being dominant compared to asset volatility. This is reasonable in light of the fact that the annual surplus participation parameter controls how much of the investment portfolio's volatility is transferred to the policyholder's payoff.

To analyze the effect of mortality on the values of premium payment options, let us now look at the numerical results for the case of a 50-year-old policyholder as set out in Table 2.

When comparing the outcomes of a 15-year contract for the 30-year-old and the 50-year-old, one can observe a decrease in the option values for the older policyholder as well as a decrease in the ratio R . For example, take the flexible payments option for a contract with $\sigma = 20$ percent and $g = 1$ percent. For the 30-year-old policyholder with a 15-year contract (Table 1), the value is \$507, and for the 50-year-old (Table 2) it is \$410, which makes a difference of around 25 percent. Furthermore, the ratio R decreases from $R = 9.50$ percent (30-year-old policyholder) to $R = 8.27$ percent (50-year-old policyholder). This result is due to the higher mortality for a 50-year-old compared to a 30-year-old policyholder since the premium payment options can be exercised only if the policyholder is still alive. Hence, the options are less valuable for those who are older at entry. An analogous observation regarding the impact of mortality factors can be made when comparing female and male policyholders (see Table A3 in Appendix A) and for the same reason since female policyholders have on average lower death probabilities.

Numerical Results Given a Contract Term of 30 Years

In this section, we extend the contract term to 30 years, omitting the flexible payments option due to computational complexity. Table 3 gives an overview of the numerical results.

First, it can be seen that longer contracts have significantly higher option values for the paid-up and resumption options. Comparing the results for $T = 15$ (Table 1) and $T = 30$ years (Table 3) in the case of a 30-year-old male policyholder, we find that the values have roughly doubled: e.g., given $\sigma = 20$ percent and $g = 1$ percent, the value of the paid-up option has increased from \$478 to \$1,020. Moreover, the ratio R increases by roughly 20 percent, an increase due to increased opportunities for exercising the option.⁴

In Appendix A, we extend our numerical analysis to shorter contract terms (30-year-old and 50-year-old male policyholder with a contract term of 10 years; see Tables A1 and A2) and include female policyholders (age 30 with contract term of 30 years) to further illustrate effects of contract term and mortality. Shorter contract durations decrease, *ceteris paribus*, the option value. On the other hand, option values are higher for policyholders who are younger at entry and for females because the options can be exercised only if the policyholder is alive and thus the options are more valuable for clients with a lower mortality. Particularly interesting is the interaction between contract parameters and their effects on option values; for example, the guaranteed interest rate g in combination with the annual participation rate α has a much stronger impact on option value than does asset volatility.

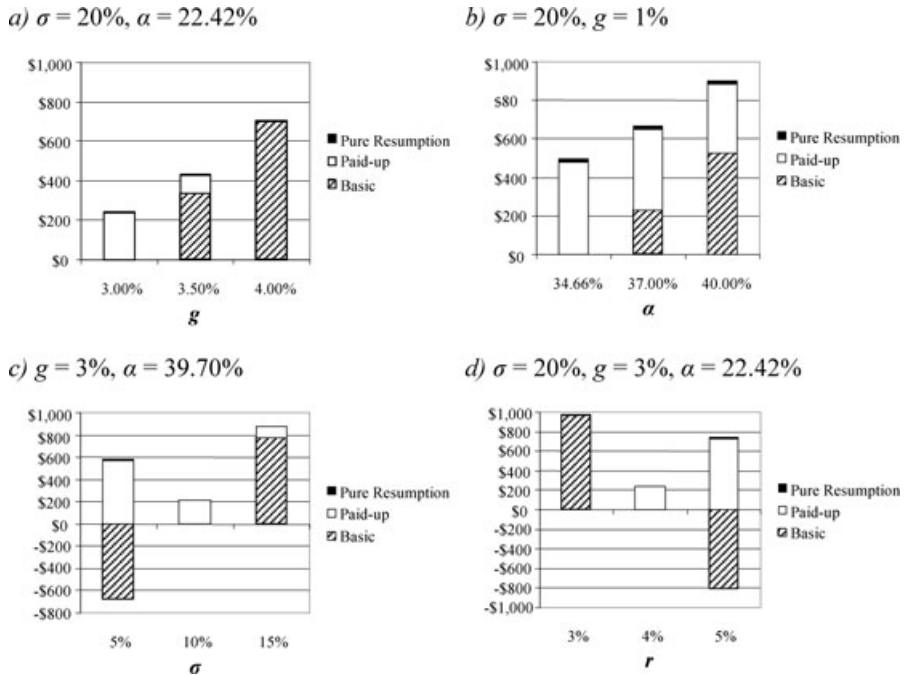
SENSITIVITY OF OPTION VALUES

The sensitivity of option values in participating life insurance contracts with respect to contract parameters is examined in this section. In particular, we isolate the effect

⁴ This result further illustrates the complex interaction between different policy options contained in insurance contracts, which can also feature counterbalancing effects. With respect to paid-up options, the insurer might find it preferable to offer contracts with a shorter duration, whereas, in general, policy persistence is desirable from the insurer's perspective.

FIGURE 1

Additional Option Values in \$ at $t = 0$ Given Exercise at Maximum Value With Respect to the Guaranteed Interest Rate g , Annual Surplus Participation Rate α , Asset Volatility σ , and Risk-Free Interest Rate r for a 30-Year-Old Male Policyholder With Contract Term of 15 Years.



Notes: $B = 1,000$ = annual premium in \$; g = guaranteed interest rate; α = annual participation rate; σ = asset volatility; r = risk-free interest rate.

of the guaranteed interest rate, the surplus participation rate, asset volatility, risk-free interest rate, and the term of the contract on the value of the basic contract, the paid-up option, and the resumption option. The flexible premium payment option is omitted in this section as it only marginally contributes to the total contract value.

Figure Description

Figure 1 shows the values Π^{Opt} of the basic contract (basic), which includes the guaranteed interest rate and the annual surplus participation, the pure paid-up option (paid-up), and the pure resumption option (pure resumption). For presentation purposes, we define the pure resumption option as the difference between the resumption option value (see Equation (13)) and the pure paid-up option value (see Equation (12)). In the following analysis we consider a 30-year-old male policyholder with a contract term of 15 years. In Appendix B we extend the analysis concerning the sensitivity of option values in participating life insurance contracts by providing three more examples. Figure 1 contains results for changes in the guaranteed interest rate (Graph *a*), annual surplus participation (Graph *b*), asset volatility (Graph *c*), and risk-free interest rate (Graph *d*).

Figure 1 illustrates that offering more favorable contract conditions (e.g., higher guaranteed interest rate, higher surplus participation rate, or higher investment volatility), *ceteris paribus*, leads to an increase in the basic contract's value. At the same time, the paid-up option value decreases and therefore has a counterbalancing effect. Furthermore, in contrast to the paid-up option, the pure resumption option value contributes only marginally to the total value of the resumption option. This pattern can be explained by the reason for stopping the premium payments in the first place: a policyholder would make the contract paid up only if contract conditions are unattractive, a situation that would rarely make resuming payments beneficial and thus leads to fewer exercises. Individual graphs are discussed in more detail below.

Varying the Guaranteed Interest Rate

Graph *a* in Figure 1 shows the main results for changes in the guaranteed interest rate. We start with the situation given in Table 1, where asset volatility $\sigma = 20$ percent and annual surplus participation $\alpha = 22.42$ percent, $g = 3$ percent. The net present value of the basic contract equals 0. Hence, the two options given in the basic contract—guaranteed interest rate and the annual surplus participation—are covered by the annual premiums. The paid-up option value is \$236, and the resumption option value is \$243. The resumption option value when considered separately (pure resumption) can then be obtained residually: $\$243 - \$236 = \$7$.

Increasing the guaranteed interest rate to 3.5 percent (4 percent), *ceteris paribus*, results in an increase in value of the basic contract. In addition to the situation where $g = 3$ percent, at inception the policyholder has to pay \$336 (\$697) for the two options in the basic contract in order to make the contract fair. At the same time, the paid-up (and pure resumption) option value declines. For $g = 4$ percent (= the riskless rate of return r), the value of the paid-up option (and thus the resumption option) is almost 0.

Varying the Annual Surplus Participation Rate

In Graph *b* in Figure 1, the sensitivity of the option values with respect to the annual surplus participation rate α is studied. Given the data in Table 1, the net present value of the basic contract is 0 for $\alpha = 34.66$ percent and $g = 1$ percent. Raising the surplus participation to $\alpha = 37$ percent and $\alpha = 40$ percent *ceteris paribus*, increases the value of the basic contract and lowers the paid-up option value. This effect is similar to the observations for the guaranteed interest rate. Again, the impact of the pure resumption option is negligible.

Varying the Asset Volatility

The impact of the volatility of the contract's underlying investment portfolio is depicted in Graph *c* in Figure 1. Given $g = 3$ percent, $\alpha = 39.70$ percent, and $\sigma = 10$ percent, the basic contract again has 0 net present value (Table 1). This leads to the situation shown in Figure 1: increasing the volatility to 15 percent, all else equal, raises the value of the basic contract and reduces the paid-up option value. Lowering the volatility to 5 percent leaves the basic contract with a negative net present value, given an increased value for the paid-up option. This illustrates that the risk resulting from the paid-up option cannot be lessened by reducing the volatility of the underlying investment portfolio.

Varying the Risk-Free Interest Rate and the Contract Term

In Graph *d*, we start with the same situation as in Graph *a*, namely, the situation in Table 1, where asset volatility $\sigma = 20$ percent and annual surplus participation $\alpha = 22.42$ percent, $g = 3$ percent, and the risk-free interest rate is $r = 4$ percent. As observed in Graph *a*, when decreasing the risk-free interest rate to $r = 3$ percent and thus making it equal to the guaranteed interest rate, the paid-up option value tends to 0, whereas the value of the basic contract rises. In contrast, an increase in the risk-free interest rate to 5 percent leads to higher paid-up option value, and to a negative net present value of the basic contract.

Furthermore, in the examples provided in Appendix B, Figure B1, we demonstrate that time to maturity has a rather moderate effect on the option values given in the basic contract, whereas the paid-up option value increases enormously, comparatively, in long-term contracts.

SUMMARY

This article evaluates the paid-up and resumption options in participating life insurance contracts starting with a basic contract that features a guaranteed interest rate and annual surplus participation, as well as a guaranteed death benefit. First considered are the input parameters that lead to fair basic contracts, that is, with a net present value of 0. In a second step, the value of additional premium payment options is derived: the paid-up option, the resumption option, and the flexible payments option. In the case of a pure paid-up option, the insured has the right to stop premium payments at any time; the contract is not terminated but continues with reduced benefits. If (in addition) a resumption option is offered, the policyholder is allowed to resume the payments once after exercising the paid-up option. Under the flexible payments option, the insured is free to stop and resume payments at multiple points in time. In addition to valuating the premium payment options, we assess the sensitivity of the option values to variation in the (basic) contract parameters, that is, guaranteed interest rate, annual surplus participation, and investment volatility.

Valuation of the model framework provided in the section “Model Framework” can be done in various ways by using different types of valuation techniques, depending on assumptions about the exercise behavior. This article focuses on the analysis of the upper bound of the option price, that is, the risk potential. For the insurer, information about this is valuable in analyzing the potential hazard of the embedded options. Employing the ratio of option value to present value of the expected premium payments (the contract’s volume) made it feasible to compare different contracts. Our numerical results show that especially the paid-up option can be of substantial value. However, there is only a very moderate increase in option value when more flexibility is offered on top of the paid-up option (resumption or flexible payments). Even though the (pure) option to resume payments has a positive value, the paid-up option remains the most important from the insurer’s perspective.

The sensitivity analysis showed that offering more favorable contract conditions (e.g., higher guaranteed interest rate, higher surplus participation rate, or higher investment volatility), *ceteris paribus*, leads to an increase in the basic contract’s value. However, the paid-up option value decreases at the same time, which is

a counterbalancing effect. In contrast to the paid-up option, the pure resumption option leads to only minor additional value.

We find that the guaranteed interest rate g is the key driver for the value of the premium payment options in the fair contract. Lowering g leads to a tremendous increase, especially for the pure paid-up option, even though the participation rate α is raised at the same time in order to keep the basic contract fair. In contrast to the dramatic impact of the guaranteed interest rate, investment portfolio volatility has very little effect on the paid-up option value in fair contracts.

For fair basic contracts, a decrease in volatility requires higher annual surplus participation, which overall leads to a decrease in the paid-up option value. Hence, in general, this risk cannot be effectively lessened for fair basic contracts by reducing the volatility of the contract's underlying investment portfolio. Furthermore, our findings show a substantial increase in the option values for longer contract terms. For example, a 30-year contract can result in a paid-up option value that makes up more than 10 percent of the present value of expected premium payments (as a measure of the contract volume). Mortality also has an impact on paid-up and resumption option values. Our numerical results show that a higher life expectancy (e.g., younger age at entry, being female) increases the paid-up, resumption, and flexible option values.

Life insurance contracts embed various types of implicit options. Concern regarding embedded options was intensified when the British life insurer Equitable Life had to close to new business in 2000 due to an improper hedging of embedded options (see Penrose, 2004). This article focuses on paid-up and resumption options offered on top of a contract that already includes a guaranteed interest rate and annual surplus participation. The substantial value of these options shows the necessity of appropriate pricing and adequate risk management.

APPENDIX A

TABLE A1

Results for a 30-Year-Old Male Policyholder With Contract Term of 10 Years

Contract Figures		Results	Basic	Paid-Up	Resumption	Flexible
$\sigma = 20\%$	$g = 3\%$	Π^{Opt}	\$0	\$137	\$140	\$142
	$\alpha = 22.41\%$	Π^{Pre}	\$8,334	\$3,724	\$3,833	\$3,797
	$Y = \$11,666$	R	0%	3.68%	3.65%	3.74%
	$g = 1\%$	Π^{Opt}	\$0	\$277	\$287	\$291
	$\alpha = 34.66\%$	Π^{Pre}	\$8,334	\$3,902	\$4,050	\$4,021
	$Y = \$10,458$	R	0%	7.10%	7.09%	7.24%
$\sigma = 10\%$	$g = 3\%$	Π^{Opt}	\$0	\$121	\$124	\$126
	$\alpha = 39.70\%$	Π^{Pre}	\$8,334	\$3,859	\$3,976	\$3,942
	$Y = \$11,666$	R	0%	3.13%	3.12%	3.20%
	$g = 1\%$	Π^{Opt}	\$0	\$235	\$245	\$249
	$\alpha = 57.17\%$	Π^{Pre}	\$8,334	\$4,026	\$4,195	\$4,175
	$Y = \$10,458$	R	0%	5.84%	5.84%	5.96%

TABLE A2

Results for a 50-Year-Old Male Policyholder With Contract Term of 10 Years

Contract Figures		Results	Basic	Paid-Up	Resumption	Flexible
$\sigma = 20\%$	$g = 3\%$	Π^{Opt}	\$0	\$120	\$123	\$125
	$\alpha = 22.42\%$	Π^{Pre}	\$7,967	\$3,621	\$3,711	\$3,659
	$Y = \$10,972$	R	0%	3.31%	3.31%	3.42%
	$g = 1\%$	Π^{Opt}	\$0	\$245	\$254	\$259
	$\alpha = 34.67\%$	Π^{Pre}	\$7,967	\$3,777	\$3,911	\$3,871
	$Y = \$9,918$	R	0%	6.49%	6.50%	6.69%
$\sigma = 10\%$	$g = 3\%$	Π^{Opt}	\$0	\$106	\$109	\$111
	$\alpha = 39.70\%$	Π^{Pre}	\$7,967	\$3,744	\$3,847	\$3,796
	$Y = \$10,972$	R	0%	2.83%	2.83%	2.92%
	$g = 1\%$	Π^{Opt}	\$0	\$208	\$217	\$221
	$\alpha = 57.17\%$	Π^{Pre}	\$7,967	\$3,899	\$4,050	\$4,013
	$Y = \$9,918$	R	0%	5.33%	5.36%	5.51%

Notes: $B = 1,000$ = annual premium in \$; σ = asset volatility; g = guaranteed interest rate; α = annual participation rate; Y = initial death benefit; Π^{Opt} = option value in \$ given exercise at maximum value; Π^{Pre} = corresponding present value of the expected premium payments in \$; R = ratio in % = Π^{Opt} / Π^{Pre} .

TABLE A3

Results for a 30-Year-Old Female Policyholder With Contract Term of 30 Years

Contract Figures		Results	Basic	Paid-Up	Resumption
$\sigma = 20\%$	$g = 3\%$	Π^{Opt}	\$0	\$529	\$548
	$\alpha = 22.53\%$	Π^{Pre}	\$17,418	\$7,743	\$8,157
	$Y = \$46,155$	R	0%	6.83%	6.72%
	$g = 1\%$	Π^{Opt}	\$0	\$1,086	\$1,132
	$\alpha = 32.95\%$	Π^{Pre}	\$17,418	\$7,724	\$8,192
	$Y = \$33,771$	R	0%	14.06%	13.82%
$\sigma = 10\%$	$g = 3\%$	Π^{Opt}	\$0	\$465	\$483
	$\alpha = 37.96\%$	Π^{Pre}	\$17,418	\$7,889	\$8,344
	$Y = \$46,155$	R	0%	5.89%	5.79%
	$g = 1\%$	Π^{Opt}	\$0	\$922	\$964
	$\alpha = 57.35\%$	Π^{Pre}	\$17,418	\$7,913	\$8,421
	$Y = \$33,771$	R	0%	11.65%	11.45%

Notes: $B = 1,000$ = annual premium in \$; σ = asset volatility; g = guaranteed interest rate; α = annual participation rate; Y = initial death benefit; Π^{Opt} = option value in \$ given exercise at maximum value; Π^{Pre} = corresponding present value of the expected premium payments in \$; R = ratio in % = Π^{Opt} / Π^{Pre} .

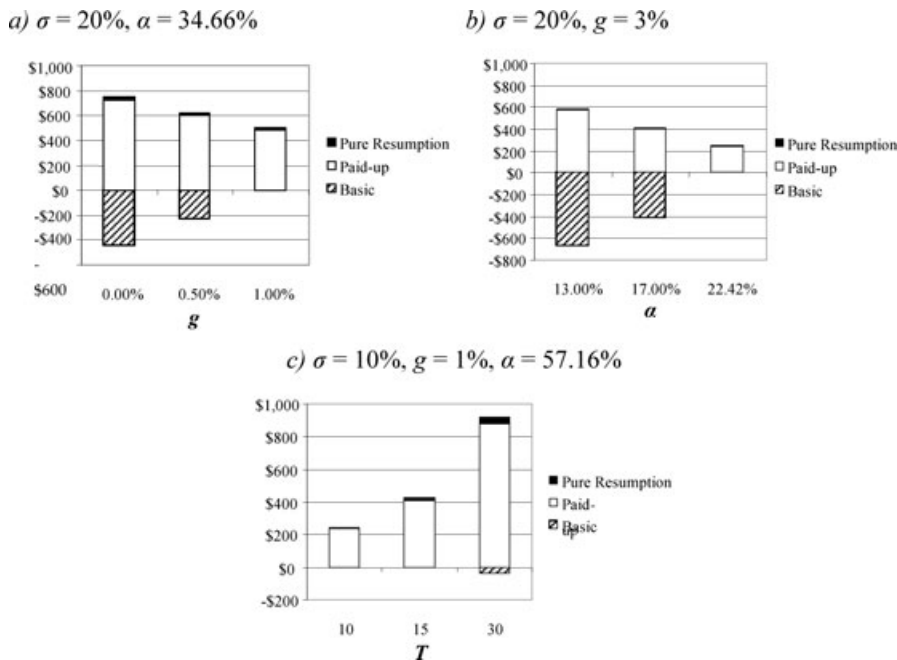
APPENDIX B

Description (a)

In Graph *a* in Figure B1, the annual surplus participation is changed to $\alpha = 34.66$ percent. In this case, $g = 1$ percent corresponds to a fair basic contract (Table 1). Decreasing

FIGURE B1

Additional Option Values at $t = 0$ With Respect to the Guaranteed Interest Rate g , Annual Surplus Participation Rate α , and Contract Term T for a 30-Year-Old Male Policyholder With Contract Term of 15 Years.



Notes: $B = 1,000$ = annual premium in \$; g = guaranteed interest rate; α = annual participation rate; σ = asset volatility; T = contract term.

the guaranteed interest rate to 0.5 percent and 0 percent leads, *ceteris paribus*, to the basic contract having a negative net present value from the policyholder's viewpoint. In this situation, the paid-up option gains significant value. The increasing values of the paid-up option, given $g = 0.5$ percent and 0 percent, can more than compensate for the negative development of Π^{Opt} of the basic contract.

Description (b)

In Graph *b*, we return to a guaranteed interest rate $g = 3$ percent; hence, the basic contract is fair for $\alpha = 22.42$ percent (Table 1). Decreasing the annual surplus participation to 17 percent and 13 percent, respectively, leads to negative net present value for the basic contract and to an increasing paid-up option value. A similar pattern can be observed in Figure B1 in the case of a decreasing guaranteed interest rate.

Description (c)

Graph *c* shows the option values with respect to changes in the contract's time to maturity. Here, $\sigma = 10$ percent, $g = 1$ percent, and $\alpha = 57.16$ percent, thus obtaining 0 net present value for a basic contract with a term of 15 years (Table 1). Interestingly, the term of the contract seems to have almost no effect on the value of the basic contract,

but does have a heavy impact on the value of the paid-up option and at least a small effect on the pure resumption option. Only when the contract's term is increased to 30 years does the basic contract's value become negative. This result is supported by Table 3: the fair contract conditions for this example would, *ceteris paribus*, require an annual participation rate of 57.35 percent (30 years) instead of 57.16 percent (15 years).

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