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Dynamic Prevention in Short-Term Insurance Contracts

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Abstract

This article looks at the dynamic properties of insurance contracts when insurers have a better technology at preventing catastrophic losses than the insured. When the prevention technology is irreversible and its benefits last for all future periods although its cost is borne in the period in which it is made, a hold-up problem occurs because the insured can change insurer after his initial insurer has invested in prevention. Investment in prevention is then delayed compared to the first best outcome. When the audit cost must be incurred by the insured when he wants to change insurer, the incumbent insurer has an informational advantage so that he can keep his client over the entire investment horizon, even though long-term contracts are not possible. This does not avoid the delay in investment, however.

Introduction

Typical articles on moral hazard and prevention assume implicitly if not explicitly that agents for whom prevention is valuable are aware of the best available practices. This is not necessarily the case, however. For example, a firm may not have the knowledge of the best available practices to reduce pollution (the firm produces golf carts and emits a toxic mercury by-product) or to manage computer hacker risk (the firm sells furniture and has all inventory and merchandise movement computerized). A firm's

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core competence may be so different from the competence needed to reduce risk that it needs outside help. Gains from trade then exist if prevention technology specialists sell their expertise to firms that need it. In some instances, the insurance industry is the best provider of prevention technology, especially in the case where the insurer is the underwriter.

Being faced with many risks of the same type in the same industry, insurers learn a great deal more than firms about the most efficient way to reduce risk (see Mayers and Smith, 1982; Doherty, 1997, for similar arguments). This is especially true for risks that are very rare so that only entities that deal with many possible exposures can correctly assess the quality of the prevention technology. Put another way, insurers have a comparative advantage in preventing catastrophes. Information technology management provides a good example. Firms generally have no special knowledge of their exposure to computer network and client database risk. Insurers, on the other hand, given their enduring financial and operational exposure to such risks, have acquired more efficient technological knowledge to prevent catastrophes. An insurer's claim specialist department[1] often provides risk assessments and offers consulting services and solutions for their clients. Insurers can also implement and service the prevention technology for their corporate clients (see van Santen, 1997; De Marcellis, 2000).

A second important aspect to consider regarding investments in prevention is that firms could be unable to finance the purchase of the prevention technology. For instance, information asymmetries and imperfect commitment could restrict access to credit and prevent firms from raising outside capital. This is especially true for small businesses that rely on bank debt rather than external equity as their principal financing source because they lack enough collateral to break the financing constraint they face (see Bester, 1987; Binks and Ennew, 1996). Even though bank finance helps reduce capital constraints, smaller firms with more risky returns are nevertheless more subject to capital constraints than larger firms (see Evans and Jovanovic, 1989; Holtz-Eakin, Joulfaian, and Rosen, 1994a,b).

A 1994 report by the U.S. Small Business Administration (SBA) stresses that many small businesses do not have banking relationships or enough collateral to secure the necessary funding for future development. In particular, the reports states that

> Banks recent measures to minimize their environmental risk have had a heavy impact on small businesses that handle dangerous chemicals or produce contaminated waste. Large companies often have variety of assets to offer as collateral to cover any potential environmental liability that small businesses do not have. (p. 62)

In line with these arguments, we will assume that firms are exogenously constrained on credit. Introducing credit constraints allows us to represent well the situation of a small risky firm for which borrowing is more difficult. Indeed, as the SBA (1994) report states,

[1] One may also think of health insurance. Health providers (i.e., doctors) are usually better equipped to prevent diseases than their patients.

> One of the most difficult obstacles is that (environmental friendly) equipment for which the loans are requested does not increase business operating revenues . . . Instead (the installation of air pollution prevention equipment) cash flows negatively and the debt burden is increased. (p. 65)

The more the firm needs to borrow, as would arise in case of a value-reducing accident, the harder it is to obtain financing, thus fueling the corporate demand for insurance. We, therefore, exogenously impose credit constraints even though we assume a competitive insurance market to lighten a model that is aimed at observing the dynamics of prevention. A model of endogenous credit constraints à la Stiglitz and Weiss (1981) would only complicate the framework—albeit making it more realistic—without helping us understand the origin of the inefficiency of investment in prevention. Our model introduces insurance companies with large cash holdings and technological know-how that finance the investment in prevention.

In this article, we analyze the relationship between a firm and an insurer-provider of prevention in a dynamic context. The firm faces an insurable catastrophic risk in each of a finite number of periods. Managing potential catastrophic losses[2] implies insurance that reduces the event's impact on cash flows, and investment in prevention that reduces the event's frequency. Unfortunately, the firm does not possess the prevention technology (as is often the case as reported in SBA, 1994; Porter and van der Linde, 1995) or even know the extent to which it faces an environmental risk. The DuPont Chemicals case reported by Parkinson (1990) is a perfect example. Before conducting an extensive audit of its environmental liability sources, DuPont thought that it only produced 40 possible waste streams; the external audit identified 497 waste streams.

As a result, firms must find help to invest in the proper prevention technology. We let the insurer be the owner of such prevention technology so that the firm purchases the prevention technology as well as insurance against the environmental risk from the same entity. Investment in prevention cannot be observed costlessly by any outside party. Hence, an insurer knows the risk type of a firm only if he has worked with it in the previous period. A firm that wants to change insurer must make its risk type known, which we model as entering in a round of costly auditing. For example, the firm must hire an outside consultant who will assess the risk type of the firm. According to Quazi (1999), some insurance companies require environmental audits before agreeing to sell an insurance policy to a firm.

The U.S. Small Business Administration (1994) report highlights this particular difficulty for small businesses that face prohibitively high costs of hiring environmental consultants and engineers. Further support for this assumption is found in Porter and van der Linde (1995), who write that

> Companies are still inexperienced in measuring their discharges, understanding the full cost of incomplete utilization of resources and toxicity, and conceiving new approaches to minimize discharges or eliminate hazardous substances. (p. 99)

[2] A catastrophic loss is defined in our model as a loss that will cause the firm to go bankrupt.

The premise or our article is similar to that of Caillaud, Dionne, and Jullien (2000) who show that although firms are risk neutral, financing requirements make it appear as if firms are risk averse so that those with financial needs and exposure to value-reducing risks choose a financial contract that combines debt and insurance. Insurance covers bankruptcy risk and debt provides the required capital to grow. In our model, insurers provide insurance and prevention technology to a firm whose access to capital is limited.

We assume that the prevention technology is highly specific and complex so that it is unverifiable by parties from outside the industry. This triggers potential opportunistic behaviors on the part of the firm. As a consequence, the insurer does not want to fully finance the investment since he has no guarantee that the firm will not *take the money and run* to a competitor. This is especially true if the prevention technology is additive so that investment in prevention today reduces the likelihood of catastrophes in the future. Once the prevention technology is in place, every future insurer will benefit from it since risk is reduced. When there is only one period, the problem is trivial: the insurance contract is effective for the same exact time period as the prevention technology. This means that the insurer will invest in an optimal level of prevention on behalf of the firm. The problem with catastrophic risks is that insurance contracts are short-lived whereas investment in prevention is long-lived. The insurer is faced with the fact that investing in prevention today reduces the risk of the firm in future periods no matter who insures the firm then.[3]

When investments are nonverifiable but offer long-term benefits, insurers face a hold-up problem: what prevents the firm from changing insurer? As is known from the literature on transaction-cost economy (see Williamson, 1979) the solution to the hold-up problem would be to bind both parties in a long-term contract (LTC). Given the firm's preference for full insurance and smoothing of its income, the optimal solution would have an insurer invest the long-term optimal amount in prevention in the first period and the firm pay each period, to this same insurer, full insurance-fair premiums plus a payment for the initial investment. The nonverifiability of investment, however, generates imperfect commitment and makes binding LTCs impossible. Joskow (1985, 1987, 1988) argues that when the hold-up problem cannot be solved using LTCs, because commitment is imperfect and future uncertain, vertical integration can be the solution. We do not consider this possibility here since, even if the insurer's investment in the firm is specific, the relationship between an insurer and the firm is not, so that transaction costs make it unprofitable for an insurer to purchase all its clients.

In some instances, an efficient LTC can be implemented by a series of short-term, renegotiable contracts (see Rey and Salanié, 1990, 1996; Malcomson and Spinnewyn, 1988). This, however, supposes that parties can at least commit to two-period contracts. This is not the case in our model where one important clause of the contract, the level of investment in prevention, is nonverifiable. Parties in the insurance contract can commit to execute payments specified in the insurance contract (premium and indemnity) that are spot in nature. They do not, however, commit to more than

[3] In that sense, our prevention technology has some basic characteristics of a public good because once an insurer makes the investment, every future insurer benefits from it.

one-period relationships, which means that they can change counterpart at the start of any period. This approach is similar to that of Rey and Salanié (1990) who show that if parties have conflicting interests, then, in the absence of asymmetric information, they can attain the long-term allocation through a sequence of short-term contracts. This is not the case in our setting where a contract stipulates not only payments but also levels of investment in prevention that benefit both parties. Opening short-term contracts to renegotiation is not an issue in our environment. The long-lasting investment in prevention being nonverifiable, it is impossible to stipulate clauses that would condition exit penalties and renegotiation rules.

With no binding contracts and no vertical integration we show that the hold-up situation results in suboptimal investment and delays in prevention investment. We also show that a better informed insurer[4] uses this informational advantage to extract a rent from the firm. Competition for this rent, however, drives an insurer's profit to zero in the initial period.

The standard literature on hold-up is static in the sense that the bargaining occurs only once. We innovate over Rogerson (1992) and Gul (2001) by presenting a dynamic hold-up problem where insurers offer standard insurance contracts that generate zero profit in expectation. Other models on dynamic insurance have typically centered around the repetition of adverse selection (see Cooper and Hayes, 1987; Laffont and Tirole, 1987; Dionne and Doherty, 1994) and moral hazard (see Rogerson, 1985; Chiappori et al., 1994) problems, whereas, in our case, the information problem is dealt with through verification.

We present the basic model in the following section. The first section presents the dynamics of investment in two extreme benchmark cases of insurer-firm relationship: the full commitment contract that is the efficient solution to the long-term relationship and the case where a new competitive insurer is hired in each period. The second section builds on this last solution to find the optimal long-term relationship with noncommitment. The third section presents a simulated example that illustrates our theoretical results. The final section concludes.

THE BASIC MODEL

A firm lives for $T+1$ periods denoted $t = 0, \ldots, T$. In each period, the firm receives a nonrandom revenue W and faces a potential catastrophic loss $L \geq W$. The probability of occurrence of the catastrophic loss L can be reduced by self-protection performed through investments in some prevention technology. We denote x_t the amount invested in prevention in period t and $X_t = \sum_{\tau=0}^{t} x_\tau$ the accumulated amount invested in prevention from period 0 through t. We assume that there is no depreciation in prevention so that any amount invested in period t is still in use in the subsequent periods. The introduction of a depreciation rate would not alter the results.[5] Moreover, it is not possible to remove any part of the technology; that is, investment x_t is

[4] We define an informed insurer at the beginning of period t as the one that was in a relationship with the firm in period $t-1$.

[5] It is easy to verify that assuming that the investment depreciates at rate $(1-\gamma) < 1$ would be equivalent to having a discount rate equal to $\delta\gamma$ in our problem, leaving the essence of the results unaffected.

irreversible. We can view this prevention technology as an organizational design that (1) is costly to implement, (2) runs fully without new investments or alterations, and (3) cannot be undone.

The loss L occurs in period t with probability $\Pi(X_t)$. More prevention is better, which means that the loss probability function is decreasing in the amount of prevention: $\Pi'(X_t) < 0$. The probability function is convex, $\Pi''(X_t) > 0$, so that the marginal decrease in the probability of loss is reduced with each additional dollar invested in prevention. Finally, some prevention is always desirable as $\Pi'(0) = -\infty$.

Because of bankruptcy costs, we can view the firm as being risk averse and thus in need of insurance.[6] Let us define $V(d_t)$ as the per period value of the firm over the dividend d_t it can distribute to its stock holders. Because of bankruptcy costs and financial distress, we let $V'(d_t) > 0$ and $V''(d_t) < 0$. In states where bankruptcy occurs, the firm benefits from limited liability so that the dividend is zero. By normalizing $V(0) = 0$, the intertemporal value of the firm, in the absence of an insurance contract, is

$$\mathcal{V} = \sum_{t=0}^{T} \delta^t \left(\prod_{\tau=0}^{t} (1 - \Pi(X_\tau)) \right) V(\underbrace{W - x_t}_{d_t}),$$

where the firm goes bankrupt in any period t with probability $\Pi(X_t)$, and where each period is discounted by some factor $\delta \leq 1$.

Since the firm has a concave value function, trades with a risk-neutral insurer are Pareto improving. Purchasing insurance against the loss L is value increasing because it eliminates the possibility of bankruptcy. As much as risk-averse agents (here firms) wish to purchase insurance each period against occurrences of L, they also wish to smooth income (here dividends) over time.

There are N potential insurers on the market competing for the contract with this firm.[7] At every period, there is an insurance contract running between the firm and an insurer controlling not only premium and coverage, but also prevention. The accumulated amount of prevention, X_t, is known to the firm at all times, but cannot be observed by outsiders unless the firm conducts an audit[8] at cost a that is independent of previous investment in prevention and constant over time. We let the audit cost be borne by the firm in line with the U.S. Small Business Administration (1994) report, which states that "the consulting and engineering costs of environmental audits are almost always borne by the (firm)" (p. 64).

We suppose that if an insurer remains with the firm for more than one period, it gains proprietary knowledge of the firm's level of risk. Since investment in prevention

[6] See Mayers and Smith (1982), MacMinn (1987), Smith and Stulz (1985), and Caillaud, Dionne, and Jullien (2000) for similar arguments. Another reason why firms may be viewed as risk averse is that the managers who run the firm are undiversified and thus need protection from adverse shocks; see Stulz (1984), Campbell and Kracaw (1987), and DeMarzo and Duffie (1995).

[7] N does not have to be large to entail competition among insurers.

[8] An outsider in period t is defined as any insurance company that did not transact with the firm in period $t - 1$.

is controlled by the insurance contract, the firm's stock of prevention technology at the beginning of period t, X_{t-1}, is known to the period $t-1$ insurer. This is a sensible assumption since outsiders may not be able to observe changes inside the firm. Although an audit makes the prevention technology known to all insurers, only $N-1$ insurers benefit from such an information dissemination, since the Nth insurer (the incumbent) already knows this information prior to the audit being conducted.

We also implicitly assume that an insurer forgets the level of prevention as soon as he no longer insures the firm and that every market participant also forgets the outcome of the audit after one period. Our setting is justified by the organization of the insurance market dealing with highly specific risks, such as technological risk. Although the firm pays for the audit, it does not conduct it per se because it has no experience in the risk. Rather, audits are performed with the help of a specialized intermediary who receives bids from insurers once the results of the audit have been made public. The firm can remain anonymous and only receive new offers from outsiders through the intermediary. This structure ensures that an audit is not conducted by each potential insurer in each round but by one auditor only. The competitiveness of offers by potential outside insurers is checked by this intermediary who protects the anonymity of the firm because the profitability of its own activities depends on it.

Insurance contracts specify contingent transfers from the firm to the insurer, p_t^i, $i = n, l$ where l denotes the loss state and n denotes the no-loss state. We assume that the firm has no access to the credit market. Any investment in prevention is either internally financed or paid for by the insurer. Hence, transfer p_t^i embodies the insurance premium for period t on top of payments for investment in prevention, net of the potential indemnity payment. Contracts also specify an amount of investment in prevention x_t made prior to Nature's move.

Since the firm faces a specific risk outside its core competence, it does not have the knowledge to assess it, let alone invest in prevention. A possible situation is that no costly audit is performed so that no prevention is done. In that case, the firm that faces a generic probability of loss equal to $\bar{\Pi}$ purchases full insurance at price $\bar{p}$ that allows any insurer to break even in each period. Firm value in period 0 is then simply $\sum_{t=0}^{T} \delta^t V(W - \bar{p})$. Because we are interested in how prevention is affected by the absence of long-term commitment, we assume throughout the article that the audit cost a is small enough so that this situation is suboptimal.

BENCHMARKS

We first want to study two benchmark cases that will be compared, in terms of investment delay and firm value, to the main results of our article. These two benchmarks correspond to the case where an LTC can be signed by the firm and the insurer and to the case where the firm pays for an audit in each period in order to be able to change insurer in every period. We will then show in the next section how these benchmarks compare to the optimal insurance relationship where the incumbent insurer convinces the firm to renew its policy instead of conducting an audit and going on the open market for a new insurer.

The LTC

If the firm and an insurer can commit for the entire horizon, an optimal LTC can be found in period 0 that prescribes transfers for all future contingencies. Such a contract stipulates a sequence of transfers $\{p_t^n, p_t^l\}_{t=0}^T$ and investments $\{x_t\}_{t=0}^T$ that maximizes the firm's value over the T periods under the constraint that the insurer's expected discounted payoff is at least equal to zero. Since the contract provides insurance against catastrophic losses, the firm no longer faces states of the world in which it goes bankrupt. The binding LTC solves:

$$\max_{\{x_t, p_t^n, p_t^l\}_{t=0}^T} \quad \sum_{t=0}^{T} \delta^t \left(\left(1 - \Pi\left(\sum_{\tau=0}^{t} x_\tau \right) \right) V(W - p_t^n) + \Pi\left(\sum_{\tau=0}^{t} x_\tau \right) V(W - p_t^l) \right)$$

$$\text{such that } x_t \geq 0 \quad \forall t = 0, \ldots, T \tag{1}$$

$$-a + \sum_{t=0}^{T} \delta^t \left\{ -x_t + \left(1 - \Pi\left(\sum_{\tau=0}^{t} x_\tau \right) \right) p_t^n + \Pi\left(\sum_{\tau=0}^{t} x_\tau \right) (p_t^l - L) \right\} \geq 0 \tag{2}$$

Note that the firm and the insurer share the same discount factor δ.

The set of constraints (1) prevents disinvestment in the prevention technology, ensuring that the total amount invested is nondecreasing. This is a set of T irreversibility constraints. Constraint (2) is the insurer's participation constraint.[9] The insurer is thus sure to receive an expected payoff of zero over the life of the LTC. Note that we subtract the cost of investment in prevention from the insurer's payoff. It is charged to the firm through the transfers p_t^l and p_t^n. As a result, the timing of the transfers can be different from the timing of the investment in prevention.

Let us associate Lagrange multipliers λ_t, $t = 0, \ldots, T$, to the irreversibility constraints and μ to the insurer's participation constraint. First order conditions with respect to premiums p_t^n and p_t^l are:

$$\mu = V'(W - p_t^n) = V'(W - p_t^l) \quad \text{for all } t = 0, \ldots, T.$$

This means that the optimal LTC offers full insurance and complete smoothing through a constant premium p^{**}. Then, the problem writes

$$\max_{\{x_t, p\}_{t=0}^T} \quad \sum_{t=0}^{T} \delta^t V(W - p)$$

$$\text{such that } x_t \geq 0 \quad \forall t = 0, \ldots, T$$

$$-a + \sum_{t=0}^{T} \delta^t \left\{ -x_t + p - \Pi\left(\sum_{\tau=0}^{t} x_\tau \right) L \right\} \geq 0.$$

[9] An audit must be run in the first period. In an LTC setting, it is equivalent to include the audit cost a in the insurer's participation constraint or in the firm's objective since the insurer acts as a financier for this expense also.

Solving this simplified problem for p and the sequence of investments $\{x_t\}_{t=0}^{T}$ yields the optimal LTC[10] that is described in the following proposition.

Proposition 1: *The LTC offers full insurance and complete smoothing of dividends through a constant transfer p^{**}. An amount $x_0 = X^{**}$ is invested in prevention in the initial period such that $1 + \Sigma_{i=0}^{T} \delta^i \Pi'(X^{**})L = 0$. No investment is made in the subsequent periods.*

Proof: All proofs are relegated to the Appendix.

The amount X^{**} invested in period 0 equates the marginal cost of investment with its marginal profitability over the next T periods. Since investing today helps decrease the accident probability in each subsequent period, the marginal benefit of prevention takes into account the decrease of the expected loss in each future period, discounted by factor δ. Hence, the longer the horizon (the higher T), the greater the optimal investment X^{**} made in period 0.

After period 0, however, the irreversibility constraints are binding since the marginal benefit of investment is shrinking with the number of periods left until the end of the horizon. Although the firm and the insurer would like to resell the technology when the weight of future profitability decreases, the irreversibility constraints forbid it so that no investment occurs after the initial period, i.e., $x_t = 0$ for $t = 1, \ldots, T$. In other words, the full commitment LTC is such that the risk neutral insurer offers financing to the firm so that the investment in period 0, X^{**}, is paid evenly by the firm in every future period.

Changing Insurer Every Period

In this section, we determine the equilibrium sequence of competitive one-period contracts, that is, the extreme opposite of a full commitment LTC. An audit is conducted every period, so that the market is informed about the firm's risk and competition is open for insurers every period. We suppose for now that the informed insurer makes no attempts to try and keep the contract for more than one period. This is an extreme and improbable case on which we build in the section on "The Long-Term Nonbinding Agreement" where the incumbent insurer offers a contract that the firm will surely accept, thus saving on the audit cost. For now, however, the firm's problem in period 0 is the following:

$$\max_{\{p_t^n, p_t^l, x_t\}_{t=0}^{T}} \quad \sum_{t=0}^{T} \delta^t \left\{ \left(1 - \Pi\left(\sum_{\tau=0}^{t} x_\tau\right)\right) V\left(W - a - p_t^n\right) + \Pi\left(\sum_{\tau=0}^{t} x_\tau\right) V\left(W - a - p_t^l\right) \right\}$$

$$\text{such that} \quad x_t \geq 0 \quad \forall t = 0, \ldots, T \tag{3}$$

$$-x_t + \left(1 - \Pi\left(\sum_{\tau=0}^{t} x_\tau\right)\right) p_t^n + \Pi\left(\sum_{\tau=0}^{t} x_\tau\right) \left(p_t^l - L\right) \geq 0 \quad \forall t = 0, \ldots, T. \tag{4}$$

The firm chooses a sequence of transfers and investments in prevention that maximizes its expected value over its lifetime. Since an audit is conducted every period

[10] The same results would hold if the firm could finance its prevention technology through the long-term credit market.

at a cost of a, the firm is able to contract with any insurer every period, which means that a participation constraint (4) must hold each period.

Applying multipliers μ_t, $t = 0, \ldots, T$ to the $T + 1$ insurers' participation constraints (4), we find the following first-order conditions with respect to p_t^n and p_t^l:

$$\delta^t V'\big(W - a - p_t^n\big) = \delta^t V'\big(W - a - p_t^l\big) = \mu_t \quad t = 0, \ldots, T.$$

Hence, the firm chooses full insurance each period with premium $p_t = p_t^n = p_t^l$. Perfect smoothing is not achieved, however, since the firm's marginal utility evolves over time with μ_t. Competition on the insurance market ensures that the transfer p_t is equal to the fair price of insurance plus the cost of investment (i.e., $p_t = x_t + \Pi(X_t)L$) so that the firm's dividend writes $d_t = W - a - x_t - \Pi(X_t)L$, with $X_t = \sum_{\tau=0}^{t} x_\tau$.

The optimal contract determines a sequence of investments that must be optimal over the horizon. The choice of an optimal investment in t depends on past choices included in the state variable X_{t-1} as well as on future impact for the firm's value. The optimal sequence of investments must be such that the subsequence starting in any period is also optimal. Therefore, the optimization problem can be written with a Bellman formulation accounting for the recursiveness of the problem. Let us define, for $t = 0, \ldots, T - 1$,

$$\mathcal{V}_t(X_{t-1}) = \max_{x_t \geq 0} V(W - a - x_t - \Pi(X_{t-1} + x_t)L) + \delta \mathcal{V}_{t+1}(X_{t-1} + x_t), \tag{5}$$

$$\text{with} \quad X_{-1} = 0,$$

$$\text{and} \quad \mathcal{V}_T(X_{T-1}) = \max_{x_T \geq 0} V(W - a - x_T - \Pi(X_{T-1} + x_T)L).$$

The Bellman function $\mathcal{V}_t$ gives the maximum firm value in t given X_{t-1} has been invested in prevention in the past and investment will be chosen optimally in the future. The transition equation, $X_t = X_{t-1} + x_t$, and the irreversibility condition, $x_t \geq 0$, constraining the problem are included in the writing of the Bellman equation.

Each period problem gives a first order condition with respect to x_t and an envelope condition. For $t = 0, \ldots, T - 1$, we apply multiplier λ_t to the irreversibility constraint in t and obtain:

$$-(1 + \Pi'(X_t)L)V'(d_t) + \delta \mathcal{V}'_{t+1}(X_t) + \lambda_t = 0, \tag{6}$$

$$\mathcal{V}'_t(X_{t-1}) = -\Pi'(X_t)L\,V'(d_t) + \delta \mathcal{V}'_{t+1}(X_t). \tag{7}$$

First-order condition (6) describes the optimal choice of x_t as a trade off between current marginal cost of investment $V'(d_t)$ and current and future marginal benefits $-\Pi'(X_t)L\,V'(d_t) + \delta \mathcal{V}'_{t+1}(X_t)$. The envelope condition (7) gives a decomposition of the direct effect of a small increase in the state variable as an immediate increase of dividend plus a future increase of welfare, both due to a decrease of the insurance premium. This last condition makes the link between marginal values in consecutive periods and allows us to use the recursiveness of the problem to solve for an optimal investment schedule. The following proposition describes the optimal sequence of investments.

Proposition 2: *If the firm audits and signs a short-term competitive insurance contract in each period, the path of investments is such that there is a date $\hat{t} \in \{1, \ldots, T\}$ after which no investment is made, that is, $x_t = 0$ for all $t \geq \hat{t}$.*

In every period the marginal (current) cost of investing in prevention is compared to its marginal (current and future) benefit, each of these amounts being weighted by the marginal utility in the period. Although the marginal cost is always equal to 1, the marginal benefit decreases over time because the discounted sum of future expected losses decreases as time passes. Hence, the optimal amount of prevention decreases with the number of future periods to come. Given there is no further investment and perfect equality of dividends after period $\hat{t}$, if investment were reversible, the rule for the optimal level of prevention in any period $t \geq \hat{t}$ would be

$$1 + \left(\sum_{i=0}^{T-t} \delta^i \right) \Pi'(X_t)L = 0.$$

It follows that as soon as the left-hand member in the above expression is positive in the beginning of one period, the existing amount of prevention is more than sufficient for the remaining periods, even if it may not have been for the period before. Moreover, if at some point in time, investment is not profitable, then it cannot be profitable afterward.[11] We denote by $\hat{t}$ the first period in which this happens.

Note that $\hat{t}$ does not have to be period 1. There is, then, delay in investment compared to the LTC solution since positive investments can be conducted during some periods at the start of the horizon. When long-term commitment between the parties is not possible, investment cannot be financed by the first insurer who would spread the reimbursement of X_0 over the $T - 1$ next periods. For period 1 through T, on competitive markets, insurance must be priced at the expected loss. Hence, the firm has to pay for X_0 in the first period. Since it has no access to credit markets and since it is concerned with the smoothing of its dividend, it would choose in $t = 0$ an amount lower than X^{**}. There may then remain incentives to increase the level of investment in the following periods.

Let us denote by X^*, defined by $1 + \Pi'(X^*)L = 0$, the optimal level of prevention in a static environment. X^* is the minimum level of investment performed in any period, it must then be that $X_0 \geq X^*$. In the last period, the optimal prevention level would be equal to X^*. Since at least X^* has been invested in the past, no more investment is performed in T and we find that $x_T = 0$.

The optimal path of investments depends on the periodic comparison between marginal cost and benefits as well as on the trade-off between dividend smoothing and prevention. Using conditions (6) and (7) for periods t and $t + 1$, we obtain the condition for optimal smoothing

[11] See Claim 2 in the Appendix.

$$1 + \Pi'(X_t)L = \frac{\delta V'(d_{t+1})}{V'(d_t)} - \frac{\delta\lambda_{t+1} - \lambda_t}{V'(d_t)}. \tag{8}$$

Since there is some investment in period $t < \hat{t}$, the irreversibility constraint does not bind ($\lambda_t = 0$). The ratio $V'(d_{t+1})/V'(d_t)$ increases with $1 + \Pi'(X_t)L$ as long as X_t increases. After period $\hat{t}$, when there is no further investment, competition between insurers imposes the same premium in each period. The firms's earnings are then constant, implying a ratio of marginal utilities equal to 1.

Proposition 3: *The level of prevention obtained in $\hat{t}$ is less than X^{**}.*

With a sequence of short-term contracts, investment in prevention is not only delayed but also less than what it would be in an LTC. There is thus an clear trade-off between early investment and smoothing. The firm is better off delaying investment on the first periods at the cost of facing a higher probability of loss in every period thereafter. As the number of remaining periods decreases, the marginal benefit of investment decreases. Therefore, investments stop before level X^{**} has been reached.

In this section, we assume that the firm makes its type known to all in every period. This entails an auditing cost that reduces the value of the firm. The firm may be better off, however, if it could secure an arrangement with the insurer instead of incurring the auditing cost in each period. We study this precise case in the next section.

THE LONG-TERM NONBINDING AGREEMENT

Since audits are costly, conducting one every period is necessarily sub optimal. The incumbent insurer should take advantage of the information he has at the end of period $t - 1$ to propose a contract that would be cheaper than conducting a costly audit in period t. It then follows that the transfer and investment schedule described in the previous section may not be optimal if agents behave rationally.

In this section, we present the optimal long-term nonbinding agreement (NBA) between a firm and its insurer. The agreement is nonbinding in the sense that the parties can exit the relationship without penalty. In any period, the insurer may decide not to recontract with the firm and the firm may decide to change insurer, provided it incurs the audit cost. Hence, a long-term relationship defines, in period 0, a self-enforcing sequence of insurance and prevention transfers, that is, a sequence that both parties will want to comply with in the future. The solution for the optimal non binding long-term insurance relationship is not trivial. Contracts offered in each period depend on the value of the relationship in the future as well as on the amount invested in prevention in the past.

The optimal sequence of investments and transfers maximizes the firm's value over the T periods with a participation constraint for each insurer involved. The optimal sequence must be such that the subsequence starting in any period is also optimal. Starting from the sequence of competitive short-term contracts described in the previous section, we apply backward induction to construct a subgame perfect Nash equilibrium whereby in each period the incumbent insurer offers the firm a contract before it decides to enter a round of costly auditing (i.e., before it makes its type public). We use a Bellman equation for every period to find the competitive insurance premium and the condition for optimal investment. We then show that the incumbent insurer can propose a premium higher than the competitive one and still keep the contract in every period because he allows the firm to save the audit cost. Allowing the firm to keep the same insurer changes the premium and, then, the dividend, from a scalar amount only, so that the condition for optimal investment is not affected. Only the value of the state variable changes compared to the solution of the "Changing Insurer Every Period" benchmark. Hence, the result of Proposition 2 still applies; there is delay in investment.

Proposition 4: *With no LTC available, the optimal relationship is such that the firm obtains insurance and prevention through a long-term NBA with a single insurer that is such that:*

(i) *given the sequence of investments* $\{x_t\}_{t=0}^{T}$, *transfers are*

$$\begin{aligned} p_0 &= x_0 + \Pi(X_0)L - \delta a, \\ p_t &= x_t + \Pi(X_t)L + (1-\delta)a \quad \textit{for all } 0 < t < T, \\ p_T &= \Pi(X_T) + a, \end{aligned}$$

(ii) *and there is a date* $\tilde{t} \in [0, T]$ *such that* $x_t = 0$ *for all* $t \geq \tilde{t}$.

In the proof of Proposition 4 we show that the outcome of the sequence of competitive short-term contracts can be recursively improved. This result obtains by the observation that, for a given schedule of investments, an insurer can make periodic positive profits and still offer the same lifetime utility to the firm. Keeping the same insurer all along allows the firm to economize on audits since the incumbent in period $t-1$ knows the amount of prevention at the beginning of period t. Given that the amount saved by the firm on an audit is a, the insurer makes a positive profit each period by offering a contract that gives the same dividend profile as the one the firm would obtain after an audit has been performed.[12]

Since the insurance market is competitive, every insurer in period t is willing to give the firm a rebate (of δa) to receive the contract and secure a rent (of a) the next period. Transfers, then, include a loading $(1-\delta)a$. In the first period, an initial audit has to be conducted before the firm chooses an insurer so that the initial insurer cannot receive rent a. Since the insurer is certain to keep the contract until period T, however, he is willing to offer a rebate in period 0 equal to the discounted value of the future profits he will be able to receive as the informed insurer, δa. It follows that the initial period's insurer makes a zero net expected profit over its entire relationship.

The loading $(1-\delta)a$ affects the firm's dividend by a scalar amount. The condition for optimal investment is not affected, however. The optimal sequence of investments in prevention is still driven by condition (8) for optimal smoothing

[12] The optimal relationship is necessarily a subgame perfect Nash equilibrium in the sense that accepting the incumbent insurer's offer is always the firm's best response at the same time as it is optimal for the insurer to make this offer to the firm, given that the firm accepts it.

$$(1 + \Pi'(X_t)L) = \frac{\delta V'(d_{t+1})}{V'(d_t)} + \frac{\lambda_t - \delta\lambda_{t+1}}{V'(d_t)},$$

where dividends are now $d_t = W - (1-\delta)a - x_t - \Pi(X_t)L$ for $t = 1, \ldots, T-1$. The firm pays the loading $(1-\delta)a$ that is less than the audit cost a so that dividends are higher than in the sequence of competitive short-term contracts by an amount δa. Because the optimal sequence of dividends d_t is a function of the ratios $\delta V'(d_{t+1})/V'(d_t)$—which change when δa is saved by the firm—the investment sequence when the firm keeps the same insurer and economize on audit costs is different than when the firm recontracts each period on the market. Although the sequence is different, a cutoff still rules the sequence of investment in prevention. That is, the proof of Proposition 2 is still valid and there exists a $\tilde{t}$ such that $x_t = 0$ for all $t > \tilde{t}$. The path of investments is obviously conditioned by an income effect due to the increase in dividends after $\tilde{t}$. This income effect affects how investment is delayed from period to period. The size of the income effect is more or less important, depending on the function V.

Since the firm could possibly change insurer each period, it is not possible for the insurer winning the contract in period 0 to dissociate the investment in prevention from its payment by the firm. Optimally, the insurer would invest everything in period 0 and offer financing to the firm through future insurance premiums. However, this is impossible since, once the investment is performed, any investor can propose another contract with the profile p_t described above, that is accepted by the firm and leaves the first insurer uncompensated for its initial investment. There is, then, delay in the investment in prevention.

TWO EXAMPLES

We present two example of how investment in prevention is delayed in our models. The first example uses a constant relative risk aversion (CRRA) value function for the firm: $V(d_t) = \ln(d_t + 1)$. The second example uses a constant absolute risk aversion (CARA) value function for the firm: $V(d_t) = 1 - e^{-d_t}$. The per period income of the firm is $W = 10$, whereas the potential loss is $L = 10$. In both examples, the firm is bankrupt in the event of an uninsured loss since $V(d) = 0$. The cost of auditing is $a = 2$. The discount factor is $\delta = 0.9$. The prevention technology impacts the probability of loss according to the following formula: $\Pi(X) = 0.05 + \frac{0.2}{X+1}$. Obviously, the probability of loss is always comprised between 5 percent and 25 percent. There are $T + 1 = 6$ periods in the first example and $T + 1 = 12$ periods in the second. Table 1 presents the results of our CRRA example whereas Table 2 presents the results for our CARA example.

CRRA Value Function

As predicted in the theoretical models, investment in prevention is delayed when the insurer and the policyholder are unable to commit to a long-term relationship as is evident in the last two columns of Table 1.

Also, as predicted, the firm's expected value is greatest when the firm and the insurer are able to commit to an LTC. It is interesting to note that the first-period transfers are much smaller than subsequent transfers in both the LTC and the long-term NBA cases. This means that the insurer is financing the insured's investment in prevention.

TABLE 1

Investment in Prevention and Premium Paid by the Firm in the Three Contracting Environments: Results Under a CRRA Value Function

	Long-Term Contract (LTC)	Changing Every Period (CEP)	Nonbinding Agreement (NBA)
x_0	2.060	1.666	1.776
x_1	0	0.171	0.124
x_2	0	0	0
x_3	0	0	0
x_4	0	0	0
x_5	0	0	0
X_5	2.060	1.837	1.900
p_0	0.020	2.916	1.196
p_1	2.020	1.376	1.513
p_2	2.020	1.205	1.390
p_3	2.020	1.205	1.390
p_4	2.020	1.205	1.390
p_5	2.020	1.205	3.190
EV	10.285	9.354	10.260

Note: $x_0, \ldots, x_5$ are each period's investment in prevention, the stock of prevention is given by $X_t = X_{t-1} + x_t$, and $p_0, \ldots, p_5$ are each period's transfers from the firm to the insurer. EV is the firm's expected value. With no investment in prevention, the firm is still fully insured in each period, but its expected discounted value is $\ln(W - L\Pi(0) + 1)(\frac{1-\delta^6}{1-\delta}) = 10.03$.

This investment is repaid over time by the insured through higher premiums. This financing cannot occur by definition when the policyholder changes insurer every period (CEP). In the case of the NBA, this illustrates the fact that insurers offer a rebate in period 0 to gain and then keep new clients.

It is interesting to note the increase in the transfer made by the policyholder to the insurer in the last period ($p_5 = 3.190$) of the NBA case. Whereas the transfer equaled 1.390 in periods 2 through 4, it jumps to 3.190 in the last period. The difference between the two values is 1.8, which is exactly equal to the difference between the rent collected by the insurer in period 5 (i.e., $a = 2$) and the rent collected by the insurer in each of the periods 2 through 4 (i.e., $(1 - \delta)a = (1 - 0.9) \times 2 = 0.2$). This 1.8 is also equal to the amount the insurer pays to the policyholder in the initial period to secure his business.

CARA Value Function

The results we obtain when we use a CARA value function and increase the number of periods (Table 2) are qualitatively the same as in the case of the CRRA value function.

In the LTC the first-period transfer is negative, meaning that the insurer is investing more than the amount he collects in premium. The insurer is again financing the insured's investment in prevention, ensuring the highest possible expected value for the firm. We also note that the increase in the transfer made by the policyholder to the insurer in the last period under the NBA is exactly equal to $\delta a = 1.8$, which is the

TABLE 2
Investment in Prevention and Premium Paid by the Firm in the Three Contracting Environments: Results Under a CARA Value Function

	LTC	CEP	NBA		LTC	CEP	NBA
x_0	2.788	0.951	0.957	p_0	−0.305	2.476	0.679
x_1	0	0.546	0.559	p_1	1.695	1.847	2.054
x_2	0	0.344	0.391	p_2	1.695	1.548	1.779
x_3	0	0.240	0.291	p_3	1.695	1.389	1.616
x_4	0	0.131	0.219	p_4	1.695	1.254	1.504
x_5	0	0.001	0.157	p_5	1.695	1.123	1.417
x_6	0	0	0.103	p_6	1.695	1.122	1.347
x_7	0	0	0.055	p_7	1.695	1.122	1.291
x_8	0	0	0.017	p_8	1.695	1.122	1.251
x_9	0	0	0	p_9	1.695	1.122	1.234
x_{10}	0	0	0	p_{10}	1.695	1.122	1.234
x_{11}	0	0	0	p_{11}	1.695	1.122	3.034
X_{11}	2.788	2.213	2.748	EV	7.1739	7.1635	7.1734

Note: $x_0, \ldots, x_{11}$ are each period's investment in prevention, the stock of prevention is given by $X_t = X_{t-1} + x_t$, and $p_0, \ldots, p_{11}$ are each period's transfers from the firm to the insurer. EV is the firm's expected value. With no investment in prevention, the firm is still fully insured in each period, but its expected discounted value is $[1 - e^{-(W - L\Pi(0))}](\frac{1-\delta^6}{1-\delta}) = 7.1717$.

difference between the rent $a = 2$ collected by the insurer in period 11 and the rent $(1 - \delta)\, a = 0.2$ collected by the insurer in every other period.

It is particularly interesting to note also for how long investment in prevention is delayed when the value function is of the type $V(d_t) = 1 - e^{-d_t}$. Indeed, the example shows that investment in prevention under NBA occurs in eight more periods than under the first best situation and three more periods than under CEP. The CARA function is characterized by a relative risk aversion that is increasing in wealth. Hence, the firm chooses to sacrifice a greater share of its dividend to increase prevention and spreads this expense over a longer length of time.

The delay in prevention could be even more detrimental to society than to the firm itself if we believe that the actual damage to society of the accident is larger than the firm's value. In other words, it could be that the firm is asked to pay only some value $L = W$, but that society bears an amount $\bar{L}$ on top of that. By delaying the investment in prevention, due perhaps to a constrained credit market, society is bearing too much risk compared to what would be economically optimal.

CONCLUSIONS

We showed in this article that financially constrained firms that do not have access to LTCs delay their investment in prevention. This occurs even if the firm's insurer is the same in every period where the firm faces the catastrophic risk. Although the investment path has no impact on the shape of the insurance contract (i.e., the firm is fully insured in each period), the firm is unable to smooth its income perfectly across periods. This is due to the availability of short-term contracts that induce the firm to pay in each period for that period's marginal investment in prevention. This investment path contrasts with the full commitment case where the socially optimal level of investment is done in the initial period and where the firm pays for this investment over its entire life, thus perfectly smoothing its income over time.

As we show in the "Benchmarks" section, it is clear that an insurer who knows that he can hold the contract on the entire horizon will want to invest a socially optimal amount X^{**} in the first period. The implementation of this policy depends on the enforceability of an LTC between the insurer and the firm. If the firm can contract with competitive insurers when the investment has been paid by the initial insurer, then the initial insurer will not invest X^{**} in the first period; instead, the initial insurer will delay investment.

The theory of prevention we developed predicts, in a world where only short- term insurance contracts exist, that: (1) although insurance contracts are signed on an annual basis, firms rarely change insurer, (2) insurers offer a better deal for the insurance/prevention package early in the relationship, and (3) insurance premiums have a tendency to rise with time as the investment in prevention amortizes. The last two predictions are similar to the lowballing results found by Kunreuther and Pauly (1985) and D'Arcy and Doherty (1990) in the insurance literature. In these results, competing insurers are unable to offer price–quantity menus of contracts because of information asymmetries between insurers. As a result insurers systematically undercharge new business against the implicit acknowledgement of over-charges in the future.

A direct application of our models can be found in the case of environmental hazard insurance where the government sets the clean up fee of environmental damages, particularly as it comes to small- and medium-size businesses. Indeed, together with the insurance policy, insurance companies offer consulting services for the prevention of specific risks through specialized departments.[13] Moreover, small businesses are more likely to be unaware of the environmental risks they face, not to mention being unaware of the technological tools available to mitigate these risks (see U.S. Small Business Administration, 1994).

Another application is to the amount of prevention offered to patients by medical providers. If patients are able to switch from one health insurer to another, we would possibly observe a delay in the amount of preventive medicine offered by health providers. This then raises the question of whether the U.S. health system, where individuals may not always have the same health insurer over their lifetime, produces socially inefficient health prevention incentives. On the other hand, a health system like in Canada, France, and Germany may provide more efficient long-term incentive to invest a socially optimal amount in prevention.

[13] A perfect example of this appears in the recent news release by Chubb (dated October 3, 2006). It states that the company has been using an infrared thermography technology to spot moisture, electrical, and foundation problems in commercial properties the company insures.

APPENDIX

Proof of Proposition 1:[14] The first order conditions with respect to x_t gives

$$\lambda_t = \delta^t \mu \left(1 + \sum_{j=0}^{T-t} \delta^j \Pi' \left(\sum_{\tau=0}^{t+j} x_\tau\right) L\right), \quad \forall t = 0 \cdots T. \tag{A1}$$

Since the insurer's participation constraint is binding at the solution, we have

$$p^{**} = \frac{\sum_{t=0}^{T} \delta^t \left(\Pi\left(\sum_{\tau=0}^{t} x_\tau\right) L + x_t\right) + a}{\sum_{t=0}^{T} \delta^t}.$$

With this constant premium, the firm's value $\sum_{t=0}^{T} \delta^t V(W - p^{**})$ is maximized by the sequence $\{x_t\}_{t=0}^{T}$ that minimizes p^{**}. We have

$$\frac{\partial p^{**}}{\partial x_t} = \frac{\delta^t}{\sum_{t=0}^{T} \delta^t} \left(1 + \sum_{j=0}^{T-t} \delta^j \Pi' \left(\sum_{\tau=0}^{t+j} x_\tau\right) L\right)$$

and

$$\frac{\partial^2 p^{**}}{\partial x_t^2} > 0.$$

From first order condition (A1), since λ_t cannot be negative, we have $\partial p^{**}/\partial x_t \geq 0$ and p^{**} is minimized when all x_t are minimized. For x_0 such that $\partial p^{**}/\partial x_0 = 0$, we have $1 + \sum_{j=0}^{T} \delta^j \Pi'(X_{t+j})L = 0$ and $\partial p^{**}/\partial x_t > 0$ for $t > 0$. Therefore, the solution is:

$$\begin{aligned} x_0 &= X^{**} \text{ such that } 1 + \left(\sum_{j=0}^{T} \delta^j\right) \Pi'(X^{**})L = 0, \\ x_t &= 0 \text{ for all } t > 0. \end{aligned}$$

□

Proof of Proposition 2: We present two claims that lead to Proposition 2.

Claim 1: $x_T = 0$ *and* $1 + \Pi'(X_T)L > 0$.

[14] We thank an anonymous referee for helping simplify this proof.

Proof: With no more periods to come, the problem in T writes

$$\mathcal{V}_T(X_{T-1}) = \max_{x_T \geq 0} V(W - a - x_T - \Pi(X_{T-1} + x_T)L). \tag{A2}$$

The first-order condition with respect to x_T and the envelope condition are, respectively

$$-(1 + \Pi'(X_T)L)V'(d_T) + \lambda_T = 0, \tag{A3}$$

$$\mathcal{V}'_T(X_{T-1}) = -\Pi'(X_T)LV'(d_T). \tag{A4}$$

Suppose $x_T > 0$ so that $\lambda_T = 0$ and $(1 + \Pi'(X_T)L) = 0$. This also implies $X_T > X_{T-1}$ and, then, $0 = 1 + \Pi'(X_T)L > 1 + \Pi'(X_{T-1})L$. Writing the first-order condition (6) in $T - 1$ accounting for the envelope condition (A4) in T, we have that $-(1 + \Pi'(X_{T-1})L)V'(d_{T-1}) - \Pi'(X_T)LV'(d_T) + \lambda_{T-1} = 0$. This is impossible if $1 + \Pi'(X_{T-1})L < 0$. Hence, λ_T and $(1 + \Pi'(X_T)L)$ have to be positive and $x_T = 0$.

Claim 2: *Given $x_\tau = 0$ for all $\tau > t$, a necessary condition for no investment in t is*

$$1 + \left(\sum_{i=0}^{T-t} \delta^i\right) \Pi'(X_{t-1})L \geq 0.$$

Proof: Starting from (A4), the envelope condition in T, and incorporating it in the envelope condition (7) in $T - 1$ and so on until t, we find

$$\mathcal{V}'_t(X_{t-1}) = -\sum_{i=0}^{T-t} \delta^i \Pi'(X_{t+i})LV'(d_{t+i}), \quad \forall t = 0, \ldots, T. \tag{A5}$$

The first-order condition for x_t, using (A5), writes

$$-(1 + \Pi'(X_t)L)V'(d_t) - \sum_{i=1}^{T-t} \delta^i \Pi'(X_{t+i})LV'(d_{t+i}) + \lambda_t = 0. \tag{A6}$$

Suppose the solution in all $\tau > t$ is $x_\tau = 0$ so that $d_\tau = W - a - \Pi(X_t)L$. Then, this first-order condition becomes

$$-(1 + \Pi'(X_t)L)V'(d_t) - \left(\sum_{i=1}^{T-t} \delta^i\right) \Pi'(X_t)LV'(d_{t+1}) + \lambda_t = 0.$$

If $x_t = 0$ is the solution at t, then $d_{t+1} = d_t$ and the condition writes

$$-\left(1 + \left(\sum_{i=0}^{T-t} \delta^i\right) \Pi'(X_{t-1})L\right) V'(d_t) + \lambda_t = 0.$$

Hence, a necessary condition for no investment (CNI) in t, given there is no investment after t, is:

$$1+\left(\sum_{i=0}^{T-t}\delta^i\right)\Pi'(X_{t-1})L \geq 0.$$

Note that if $1+\sum_{i=0}^{T-t}\delta^i\Pi'(X_{t-1})L \geq 0$ for any t, then $1+\sum_{i=0}^{T-(t+\tau)}\delta^i\Pi'(X_{t-1})L > 0$ for all $\tau = 1\cdots T-t$. Hence, if the condition for no investment holds in t (given there is no investment in $t+1,\ldots,T$), it holds in all subsequent periods.

Finally, since there is no investment in T, the condition applies in $T-1$, and in the preceding period if $X_{T-1}=0$, and so on. □

Since $x_0 > 0$ and $x_T = 0$, by continuity, there exists a $\hat{t} \in \{1,\ldots,T\}$ such that $1+\sum_{i=0}^{T-\hat{t}}\delta^i\Pi'(X_{\hat{t}-1})L \geq 0$ and $1+\sum_{i=0}^{T-(\hat{t}-1)}\delta^i\Pi'(X_{\hat{t}-1})L < 0$. This completes the proof of Proposition 2. □

Proof of Proposition 3: Note that period $\hat{t}$ is chosen such that the condition for no investment holds in $\hat{t}$ and not in $\hat{t}-1$:

$$1+\left(\sum_{i=0}^{T-(\hat{t}-1)}\delta^i\right)\Pi'(X_{\hat{t}-2})L < 0 \quad \text{and} \quad 1+\left(\sum_{i=0}^{T-\hat{t}}\delta^i\right)\Pi'(X_{\hat{t}-1})L \geq 0.$$

Denote $\hat{X} \geq X_0$ the minimum prevention level such that no investment is performed in $\hat{t}$. This level is such that $1+(\sum_{i=0}^{T-\hat{t}}\delta^i)\Pi'(\hat{X})L = 0$ while the full commitment level is such that $1+(\sum_{i=0}^{T}\delta^i)\Pi'(X^{**})L = 0$. Hence, $(\sum_{i=0}^{T-\hat{t}}\delta^i)\Pi'(\hat{X})L = (\sum_{i=0}^{T}\delta^i)\Pi'(X^{**})L$ so that $\hat{X} < X^{**}$. □

Proof of Proposition 4:

(i) Independently of the contracting framework and the schedule of investment, the firm always chooses full insurance at the optimum. Let us denote $\tilde{p}_t$, the premium offered on the competitive market in t.

In period T, finding an insurer on the competitive market leads to problem (A2) when the firm CEP. $\mathcal{V}_T(X_{T-1})$ is the optimal firm value in T given past prevention X_{T-1} (see proof of Claim 1). The binding insurer's participation constraint means that insurance is fairly priced on the competitive market, $\tilde{p}_T = x_T + \Pi(X_T)L$. Conditions (A3) and (A4) imply that $x_T = 0$ because a positive investment in T would suppose that suboptimal amounts have been invested in the past. Hence, $X_T = X_{T-1}$.

Incumbent insurer in T (that had the contract in $T-1$) knows the firm's level of risk at the start of period T. By accepting the incumbent insurer's contract for period T, the firm saves the audit cost. Everything else being equal, the incumbent insurer can charge a above the fair premium, that is, $p_T = \Pi(X_T)L + a$, and still maintain his relationship with the firm. The firm is indifferent between this contract and the one it could obtain on the competitive market after an audit. We suppose it accepts the incumbent's contract.

The insurer who obtains the contract in period $T-1$ will be able to retain the contract in period T, charge premium $p_T = \Pi(X_T)L + a$, and thus secure a payoff of a in the end period. In period $T-1$, the problem on the competitive market can be characterized with

$$\mathcal{W}_{T-1}(X_{T-2}) = \max_{x_{T-1}, p_{T-1}} V(W - a - p_{T-1}) + \delta V(W - a - \Pi(X_{T-2} + x_{T-1})L)$$

$$\text{such that} \quad x_{T-1} \geq 0$$

$$p_{T-1} - x_{T-1} - \Pi(X_{T-2} + x_{T-1})L + \delta a \geq 0.$$

A binding participation constraint implies that the market insurance premium in $T-1$ is $\tilde{p}_{T-1} = x_{T-1} + \Pi(X_{T-1})L - \delta a$, for all possible choice of x_{T-1}.

The incumbent insurer in $T-1$, can offer a contract that allows the firm to save on the audit cost. Offering the premium $p_{T-1} = a + \tilde{p}_{T-1}$, he can maintain his relationship with the firm. The discounted sum of periods $T-1$ and T profits for the insurer is $\mathcal{P}_{T-1} = a - \delta a + \delta(a) = a$.

Suppose the same insurer can insure the firm in each period after t, with transfers equal to $P_\tau = a + \Pi(X_\tau)L - \delta a$, for all $\tau = t+1, \ldots, T$. That insurer pockets a rent of $(1-\delta)a$ every period that leaves him in $t+1$ with a discounted profit $\mathcal{P}_{t+1} = \sum_{i=0}^{T-t-2} \delta^i (1-\delta)a + \delta^{T-t-1} a = a$. In period t, the optimal contract for the firm is the one obtained on the competitive market as a solution to

$$\mathcal{W}_t(X_{t-1}) = \max_{x_t, p_t} V(W - a - p_t) + \delta \mathcal{W}_{t+1}(X_{t-1} + x_t)$$

$$\text{such that} \quad x_t \geq 0$$

$$p_t - x_t - \Pi(X_{t-1} + x_t)L + \delta \mathcal{P}_{t+1} \geq 0.$$

The binding participation constraint for the insurer gives the competitive premium in t as $\tilde{p}_t = x_t + \Pi(X_t)L - \delta a$.

The incumbent insurer in t, can offer the same contract to the firm with premium $p_t = a + \tilde{p}_t$ and still remain the insurer until T, without affecting the value of x_t.

In period 0, an audit must be conducted since there is no incumbent insurer with the information on the firm's risk. Since the insurer that receives the contract in $t = 0$ can remain in relation with the firm until T and secure a payoff valued δa in period 0, competition pushes all investors to propose a rebate of δa in period 0. The transfer in $t = 0$ is then $p_0 = x_0 + \Pi(X_0)L - \delta a$, and the discounted total payoff from the contract for the insurer is $\mathcal{P}_0 = 0$.

(ii) Since the offer made by the incumbent insurer leaves the firm with the same dividend as if it insures with a new insurer, the prevention decision is unaffected by the choice of insurer. The optimal choice of investment in any period t is given by the following program:

$$\mathcal{W}_t(X_{t-1}) = \max_{x_t \geq 0} V(W - (1-\delta)a - x_t - \Pi(X_{t-1} + x_t)L) + \delta \mathcal{W}_{t+1}(X_{t-1} + x_t).$$

The addition of δa does not alter the marginal effect of investment on the firm's dividend, then, the optimal choice of x_t is given by the same first-order condition (A6) as in the proof of Claim 2:

$$-(1 + \Pi'(X_t)L)V'(d_t) - \sum_{i=1}^{T-t} \delta^i \Pi'(X_{t+i})L V'(d_{t+i}) + \lambda_t = 0,$$

with

$$\begin{aligned} d_t &= W - (1-\delta)a - x_t - \Pi(X_t)L, \quad t = 1, \dots, T-1, \\ d_T &= W - a - \Pi(X_{T-1})L. \end{aligned}$$

If $\lambda_\tau > 0$ for $\tau > t$, then condition (A6) writes

$$\begin{aligned} &-(1 + \Pi'(X_t)L)V'(d_t) - \left(\sum_{i=1}^{T-t+1} \delta^i \right) \Pi'(X_{t+1})L V'(d_{t+1}) \\ &\quad - \delta^{T-t} \Pi'(X_{t+1})L V'(d_T) + \lambda_t = 0, \end{aligned}$$

with $d_{t+1} > d_T$. If $x_t = 0$ then $\lambda_t \geq 0$, and $d_t = d_{t+1} > d_T$ so that $V'(d_t) = V'(d_{t+1}) < V'(d_T)$. We must then have that

$$1 + \left(\sum_{i=0}^{T-t} \delta^i \right) \Pi'(X_t)L > 0.$$

Since $x_0 > 0$ and $x_T = 0$, by continuity, there exists a $\tilde{t}$ such that this condition holds in $\tilde{t}$ and not in $\tilde{t} - 1$. □

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