

WHEN IS A COINSURANCE-TYPE INSURANCE POLICY INFERIOR OR EVEN GIFFEN?

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ABSTRACT

This article derives the necessary and sufficient conditions for a coinsurance-type insurance policy covering a particular risk to be inferior and to be Giffen. Mossin's decreasing absolute risk aversion assumption for insurance to be inferior is avoided. The result generalizes Hoy and Robson and Briys, Dionne, and Eeckhoudt's results to the case with a continuum of states and relaxes their assumption of constant relative risk aversion. It is shown that knowledge about the distribution of risk can be used to relax assumptions on an utility function for a coinsurance-type insurance policy to be inferior and to be Giffen.

INTRODUCTION

Mossin's (1968) seminal article suggests that decreasing absolute risk aversion (DARA) implies that coinsurance-type insurance is inferior. His study has stimulated investigations in many related aspects of insurance demand (Schlesinger, 1981; Hoy and Robson, 1981; Dionne and Eeckhoudt, 1984; Borch, 1986; Briys, Dionne, and Eeckhoudt, 1989; Cleeton and Zellner, 1993; Schlesinger, 2000). Particularly, using a two-state model, Hoy and Robson (1981) show that DARA is insufficient for insurance to be Giffen. They show that under constant relative risk aversion, the coefficient of relative risk aversion must be greater than unity for coinsurance-type insurance to be Giffen. Using Cheng, Magil, and Shafer (1987) methodology, Briys, Dionne, and Eeckhoudt (1989) show that DARA is necessary and sufficient for insurance to be inferior. They also derive the necessary and sufficient condition under which insurance is non-Giffen. By reversing the condition and focusing on a two-state case, they confirm Hoy and Robson's (1987) result.

This article does not treat insurance as a generic good. Instead, it attempts to derive the conditions under which a coinsurance-type insurance policy covering a particular risk is inferior and under which it is Giffen. The analysis differs from that of Briys,

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Dionne, and Eeckhoudt (1989) in that in their proofs of necessity of their conditions for coinsurance-type insurance to be non-Giffen and to be inferior, they are allowed to choose a two-state distribution satisfying some specific restrictions for generating counterexamples. This strategy is inapplicable in this article as the distribution of the risk concerned is fixed *a priori*. Besides, it is shown that whereas Briys, Dionne, and Eeckhoudt's condition for insurance to be inferior can be reversed to yield the condition for insurance to be normal, reversing their condition for insurance to be non-Giffen to obtain that for insurance to be Giffen may encounter difficulties in general. On the contrary, the conditions derived in this article for a particular insurance policy to be Giffen can be reversed to obtain the corresponding conditions for the policy to be non-Giffen.

The main propositions derived in this article turn out to have some useful implications. For instance, they can be used to show that a condition weaker than DARA is sufficient for insurance to be inferior. They can also be used to generalize Hoy and Robson (1981) and Briys, Dionne, and Eeckhoudt's (1989) result. In particular, these authors show that under constant relative risk aversion, it is necessary for the coefficient of relative risk aversion to exceed unity for insurance to be Giffen. Briys, Dionne, and Eeckhoudt's proof applies only to the case with two states of nature (or with a discrete state space given extreme parameters) and relies exclusively on the reversibility of the condition for insurance to be non-Giffen. It is shown that their result can be extended to the case with a continuum of states and without the assumption of constant relative risk aversion.

Finally, sufficient conditions taking account of the curvature of the distribution function of the risk for a particular insurance policy to be inferior and to be Giffen are derived allowing restrictions on the utility function to be relaxed. Interpretations of these results under some common asymmetric bell-shaped loss distributions are discussed. These results are particularly useful because many recent empirical studies (e.g., Szpiro, 1983; Eisenhauer, 1997; Eisenhauer and Halek, 1999) reject the uniform DARA hypothesis. Other empirical studies (e.g., Skinner, 1988; Kuehlwein, 1991; Guiso, Jappelli, and Terlizzese, 1992; Parker, 1999) on precautionary saving reject a positive third-derivative of utility functions and, therefore, indirectly reject the uniform DARA hypothesis.

The rest of the article is organized as follows. The first section introduces the model. The second section derives the necessary and sufficient conditions under which an insurance policy covering a particular risk is inferior. The third section derives the necessary and sufficient conditions under which an insurance policy covering a particular risk is Giffen. The fourth section relates the results to those of the literature and derives some extensions. The fifth section investigates some implications and applications of the results. The final section concludes.

THE MODEL

Consider a risk-averse individual with initial wealth W_0 . He faces random loss $\tilde{L}$ with realization L , density function $f > 0$, distribution F , and support $[0, L]$, where $\tilde{L} \leq W_0$. In the presence of a coinsurance-type insurance policy, the individual's realized final wealth is given by

$$W(L) = W_0 - L + \alpha(L - \lambda E(\tilde{L})),$$

where $\alpha \in [0, 1]$ is the coverage rate, $\lambda \geq 1$ is the gross loading factor of insurance, and $\lambda E(\tilde{L})$ is the premium payment for full insurance with E being the expectation operator.¹

The individual chooses α to maximize his von Neumann–Morgenstern expected utility given by

$$H(\alpha; W_0, \lambda) = E[U(W_0 - \tilde{L} + \alpha(\tilde{L} - \lambda E(\tilde{L})))] \quad (1)$$

where utility U is a function of realized final wealth. The first-order condition for an optimum is given by

$$H_\alpha(\alpha^*; W_0, \lambda) = E[U'(W(\tilde{L}))(\tilde{L} - \lambda E(\tilde{L}))] = 0, \quad (2)$$

at $\alpha = \alpha^*$, where α^* is the optimal coverage rate. Integrating (2) by parts gives²

$$\begin{aligned} 0 &= H_\alpha(\alpha^*; W_0, \lambda) \\ &= U'(W(0)) \int_0^L (L - \lambda E(\tilde{L})) dF(L) \\ &\quad + \int_0^L U''(W(L)) W'(L) \left\{ \int_L^{\tilde{L}} (t - \lambda E(\tilde{L})) dF(t) \right\} dL, \end{aligned} \quad (3)$$

at $\alpha = \alpha^*$, where $W'(L) = -(1 - \alpha) \leq 0$. For the problem to be interesting, assume that $\alpha^* \in (0, 1)$, which holds when $\lambda > 1$ but is not too large. A necessary condition is that $\lambda E(\tilde{L}) < \tilde{L}$. The second-order condition is given by

$$H_{\alpha\alpha}(\alpha; W_0, \lambda) = E[U''(W(\tilde{L}))(\tilde{L} - \lambda E(\tilde{L}))^2] < 0,$$

which is always satisfied given $U'' < 0$.

Realizing that the concept of risk aversion alone does not guarantee that coinsurance-type insurance is normal or even non-Giffen, Mossin (1968), Hoy and Robson (1981), and Briys, Dionne, and Eeckhoudt (1989) investigate the relation between the degree of absolute risk aversion and the effects of changes in income and loading on insurance demand. Denote the Arrow–Pratt coefficient of absolute risk aversion by $A(W) = -U''(W)/U'(W)$.

¹ An insurance policy is said to be actuarially fair (unfavorable) if $\lambda = (>) 1$.

² The integration by parts is carried out from L to $\tilde{L}$ instead of from 0 to L because some results to be established later rely crucially on the signs of U' and $U''W'$ in (3) being the same. One can check that integrating by part from 0 to L gives

$$\begin{aligned} H_\alpha(\alpha; W_0, \lambda) &= U'(W(L)) \int_0^L (L - \lambda E(\tilde{L})) dF(L) \\ &\quad - \int_0^L U''(W(L)) W'(L) \left\{ \int_0^L (t - \lambda E(\tilde{L})) dF(t) \right\} dL. \end{aligned}$$

Note that the signs of U' and $-U''W'$ in the above equality are opposite.

Throughout this article, it is assumed that utility function U satisfies the following condition:

Condition (C): Utility function U exhibits $U' > 0$ and $U'' < 0$ with the coefficient of absolute risk aversion, A , being bounded from above. Moreover, U is twice continuously differentiable and thrice piecewise continuously differentiable.

Note that U is assumed to be thrice piecewise continuously differentiable so that $A'(W)$ is piecewise continuous and hence is defined for all W .

CHANGES IN INITIAL WEALTH

To see whether a coinsurance-type insurance policy is inferior, one needs to check the sign of $d\alpha^*/dW_0 = -H_{\alpha W_0}/H_{\alpha\alpha}$. Since $H_{\alpha\alpha} < 0$ due to the second-order condition, one has $d\alpha^*/dW_0 \leq 0$ if and only if

$$H_{\alpha W_0}(\alpha; W_0, \lambda) = E[U''(W(\tilde{L}))(\tilde{L} - \lambda E(\tilde{L}))] \leq 0. \quad (4)$$

Mossin (1968) shows that DARA (i.e., $A' < 0$) implies that (4) holds, regardless of the distribution of the risk concerned. Briys, Dionne, and Eeckhoudt (1989) have proved that DARA is necessary and sufficient for (4) to hold such that coinsurance-type insurance is inferior. This is restated with some rewording as follows:

Proposition 1 (Briys, Dionne, and Eeckhoudt's, 1989, Proposition 1 and Corollary 1): *Fixing W_0 and λ arbitrarily, for any U satisfying condition (C) and any F , $d\alpha^*/dW_0 \leq 0$ if and only if*

$$A(a) \geq A(\bar{W}_0) \geq A(b), \text{ for all } (a, b) \text{ with } a < \bar{W}_0 < b, \quad (5)$$

where $\bar{W}_0 = W_0 - \lambda E(\tilde{L})$ if and only if $A' \leq 0$.³

Note that (5) can be interpreted as requiring $A(W(L)) - A(\bar{W}_0)$ to have the single-crossing property. In Athey's (2002) terminology, this gives rise to the so-called "monotone comparative statics" result. Note also that the main inequalities in (5) can be reversed to yield the necessary and sufficient condition under which coinsurance-type insurance is normal, that is, $d\alpha^*/dW_0 > 0$ if and only if $A' > 0$.

Briys, Dionne, and Eeckhoudt's (1989) result is an extension of Cheng, Magill, and Shafer's (1987) result on the case of a simple portfolio choice to the case of coinsurance-type insurance. Following Cheng, Magill, and Shafer's strategy, Briys, Dionne, and Eeckhoudt have proved the necessity of their conditions (in particular, condition (5) in this article) by showing that there exists a two-state distribution for $\tilde{L}$ with arbitrary realizations L_a and L_b having probabilities $1 - \pi$ and π such that

$$\pi = \frac{U'(a)(a + L_a - W_0)}{-U'(b)(b + L_b - W_0) + U'(a)(a + L_a - W_0)} \quad (6)$$

³ To see the equivalence of the latter two conditions, simply realize that for any distribution with any two realizations a and b , $E(\tilde{L})$ (and hence $\bar{W}_0$) is arbitrary as a and b are arbitrary.

so that (4) is equivalent to (5).⁴ This implies that a violation of (5) results in a violation of (4). Here, a and b are assumed to be the realized final wealth levels corresponding to L_a and L_b , respectively.

Even though Briys, Dionne, and Eeckhoudt's (1989) results are theoretically interesting, it may not be relevant to a particular insurance company. A pertinent question that an insurance company will ask is, given a particular risk with a particular distribution F , whether there are conditions under which an insurance policy covering the risk is inferior. For example, a car insurance company is interested in knowing the condition under which a rise in an insured's initial wealth always raises his car insurance coverage. Since F is fixed *a priori* for a particular risk concerned, Briys, Dionne, and Eeckhoudt's proof of necessity, which relies on the freedom of setting F to be a two-state distribution satisfying (6), is no longer applicable. Therefore, the necessary condition for the new problem should be different from that stated in Proposition 1.

The following proposition gives the necessary and sufficient condition under which a coinsurance-type insurance policy covering a particular risk is inferior:

Proposition 1': Fixing W_0 , λ , and F arbitrarily, for any U satisfying condition (C), $d\alpha^*/dW_0 \leq 0$ if and only if there exists $\gamma \in \Re$ such that

$$\int_L^{\bar{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) \geq \gamma \int_L^{\bar{L}} (t - \lambda E(\tilde{L})) dF(t) \quad (7)$$

for all $L \in [0, \bar{L}]$.

Proof: The proof of necessity is tedious and is dedicated to the Appendix. To prove sufficiency, it suffices to check from (4) that

$$\begin{aligned} H_{\alpha W_0}(\alpha; W_0, \lambda) &= E[-A(W(\tilde{L}))U'(W(\tilde{L}))(\tilde{L} - \lambda E(\tilde{L}))] \\ &= \int_0^{\bar{L}} -A(W(L))U'(W(L))(L - \lambda E(\tilde{L})) dF(L) \\ &= \left[U'(W(L)) \int_L^{\bar{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) \right]_0^{\bar{L}} \\ &\quad - \int_0^{\bar{L}} U''(W(L))W'(L) \left\{ \int_L^{\bar{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) \right\} dL \\ &= -U'(W(0)) \int_0^{\bar{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) \\ &\quad - \int_0^{\bar{L}} U''(W(L))W'(L) \left\{ \int_L^{\bar{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) \right\} dL \\ &\leq -U'(W(0))\gamma \int_0^{\bar{L}} (t - \lambda E(\tilde{L})) dF(t) \end{aligned}$$

⁴ See Briys, Dionne, and Eeckhoudt's (1989) Proposition 1. Their Equation (8) is exactly Equation (6) in this article.

$$\begin{aligned}
& - \int_0^L U''(W(L))W'(L) \left\{ \gamma \int_L^L (t - \lambda E(\tilde{L})) dF(t) \right\} dL \\
& = -\gamma H_\alpha(\alpha; W_0, \lambda) = 0.
\end{aligned} \tag{8}$$

The third equality in (8) is obtained using integration by parts. The inequality is due to (7) and $W' < 0$. The last two equalities are due to first-order condition (3). $\square$

By reversing the inequality in (7) of Proposition 1', one immediately obtains the following necessary and sufficient conditions for an insurance policy to be a normal good:

Proposition 1'': *Fixing W_0 , λ , and F arbitrarily, for any U satisfying condition (C), $d\alpha^*/dW_0 \geq 0$ if and only if there exists $\gamma \in \Re$ such that*

$$\int_L^L A(W(t))(t - \lambda E(\tilde{L})) dF(t) \leq \gamma \int_L^L (t - \lambda E(\tilde{L})) dF(t) \text{ for all } L \in [0, \bar{L}].$$

Note that condition (7) seems even more complicated than condition (4). However, it will be shown in the sections "Relating to and Extension of Existing Literature" and "Some Important Implications" that Proposition 1' has some interesting implications. Note also that Proposition 1' is closely related to Gollier's (1995) Proposition 1 that deals with the comparative statics of changes in risk.⁵ More explicitly, Gollier's necessary and sufficient condition for a change in the distribution of a risky prospect $\tilde{x}$ from F with density f to G with density g to reduce a risk-averse individual's optimal choice α is given by

$$\int_{x_0}^x z_\alpha(x, \alpha) dG(t) = \int_{x_0}^x \left[\frac{g(x)}{f(x)} \right] z_\alpha(x, \alpha) dF(t) \leq \gamma \int_{x_0}^x z_\alpha(x, \alpha) dF(t), \tag{9}$$

where z is the payoff function with $z_x > 0$, and x is a realization of $\tilde{x}$ with support $[x_0, x_1]$. One can compare conditions (9) and (7). In particular, the term $\int_L^L (t - \lambda E(\tilde{L})) dF(t)$ in (7) is closely related to the concept of "location-weighted probability mass function" introduced by Gollier given by $\int_{x_0}^x z_\alpha dF(t)$ in (9). By replacing $\tilde{x}$ by $\tilde{L}$, $z(x, \alpha)$ by $W(L)$, z_α by $L - \lambda E(\tilde{L})$, and $g(x)/f(x)$ by $A(W(L))$, and taking account of the difference in the treatment between $\tilde{x}$ and $\tilde{L}$ due to integration by parts as discussed in footnote 2, one can easily compare the similarities and differences between the two sets of conditions.⁶

⁵ The proof of Gollier's (1995) Proposition 1 is simpler because it allows $U' = 0$ at the smallest realized wealth. This, however, violates condition (C) of this article making A undefined at that point. Moreover, the utility function constructed by Gollier has U'' being a discontinuous step function with $U''' = 0$ implying the special case of uniform increasing absolute risk aversion. Finally, whereas the payoff rises with the random prospect in Gollier's model, the payoff falls with the random loss in this article.

⁶ Note, however, that the proof of necessity of Proposition 1' is more complicated than that of Gollier's (1995) Proposition 1 because of the additional restrictions imposed on A that contains $U'' < 0$ and $U' > 0$.

CHANGES IN INSURANCE PREMIUM

To see whether a coinsurance-type insurance policy is a Giffen good, one simply check the sign of $d\alpha^*/d\lambda = -H_{\alpha\lambda}/H_{\alpha\alpha}$. Since $H_{\alpha\alpha} < 0$, one has $d\alpha^*/d\lambda \leq 0$ if and only if

$$H_{\alpha\lambda}(\alpha^*; W_0, \lambda) = E[-U''(W(\tilde{L}))\alpha^*E(\tilde{L})(\tilde{L} - \lambda E(\tilde{L})) - U'(W(\tilde{L}))E(\tilde{L})] \leq 0 \quad (10)$$

if and only if

$$E \left\{ \left[A(W(\tilde{L})) - \frac{1}{\alpha(\tilde{L} - \lambda E(\tilde{L}))} \right] U'(W(\tilde{L}))(L - \lambda E(\tilde{L})) \right\} \leq 0, \quad (11)$$

which is equivalent to

$$E \left\{ \left[A(W(\tilde{L})) - \frac{1}{W(\tilde{L}) - W_0 + \tilde{L}} \right] U'(W(\tilde{L}))(W(\tilde{L}) - W_0 + \tilde{L}) \right\} \leq 0. \quad (12)$$

Briys, Dionne, and Eeckhoudt (1989) have derived the necessary and sufficient conditions under which (12) holds and hence insurance is non-Giffen. Their result is stated with some rewording as follows:

Proposition 2 (Briys, Dionne, and Eeckhoudt's, 1989, Proposition 2): *Fixing W_0 and λ arbitrarily, for any U satisfying condition (C) and any F , $d\alpha^*/d\lambda \leq 0$ if and only if*

$$A(b) - \frac{1}{b - W_0 + L_b} \geq A(a) - \frac{1}{a - W_0 + L_a} \text{ for all } (a, b), \quad (13)$$

with $a < \bar{W}_0 < b$ and where $\bar{W}_0 = W_0 - \lambda E(\tilde{L})$, $b < W_0 - L_b$ and $a > W_0 - L_a$.

Using exactly the same approach for proving Proposition 1, Briys, Dionne, and Eeckhoudt (1989) prove the necessity of Proposition 2 by choosing a two-state distribution with realizations a and b and by fixing their probabilities $1 - \pi$ and π such that (6) is satisfied. They show that fixing the distribution this way, (10) is equivalent to (13). Therefore, a violation of (13) leads to a violation of (10).

Close inspection again reveals that Briys, Dionne, and Eeckhoudt's (1989) proof of necessity of Proposition 2 relies exclusively on the assumption that F can be chosen freely. When F is fixed for a particular risk *a priori*, their proof is no longer applicable. Therefore, an insurance company cannot employ Proposition 2 to find the necessary condition under which a fall in the price of a particular insurance policy on a particular risk reduces demand.

Another limitation of Proposition 2 is that whereas it is legitimate to reverse condition (5) in Proposition 1 to obtain the necessary and sufficient condition for insurance to be normal, reversing (13) in Proposition 2 to obtain conditions for insurance to be Giffen may be problematic in general. Unlike $A(W(L))$ in (5), the expressions in (13) having the following form

$$A(W(L)) - \frac{1}{W(L) - W_0 + L}$$

are clearly divergent at $W = \bar{W}_0$ (i.e., at $L = \lambda E(\tilde{L})$). The reason is that even though A may be monotonic in L , when L is in the neighborhood of $\lambda E(\tilde{L})$, the term $-(W(L) -$

$W_0 + L)^{-1}$ tends to negative infinity from the right and positive infinity from the left. When the inequality in (13) is reversed, it will always be violated when b tends to $\bar{W}_0$ or when a tends to $\bar{W}_0$ as A is bounded.

Facing this difficulty, Briys, Dionne, and Eeckhoudt (1989) have only attempted to reinvestigate Hoy and Robson's (1981) result for insurance to be a Giffen good under the special case of a two-state risk. When there are only two discrete states, as long as a and b (being the wealth levels in the bad and the good states) do not converge to $\bar{W}_0$, the two-way divergence problem of $-(W(L) - W_0 + L)^{-1}$ at $W = \bar{W}_0$ does not arise. They then show by reversing the inequality in (13) that given constant relative risk aversion, it is necessary for the coefficient of relative risk aversion to be greater than unity for insurance to be Giffen. Note that for a discrete state space with many states, a realized loss should not be too close to $\lambda E(\tilde{L})$; otherwise, one may need very extreme values for A' in order to reverse (13).

The following proposition states the necessary and sufficient condition for a coinsurance-type insurance policy covering a particular risk to be a Giffen good without restricting the risk to have a discrete state space:

Proposition 2': Fixing W_0 , λ , and F arbitrarily, for any U satisfying condition (C), $d\alpha^*/d\lambda \geq 0$ if and only if there exists $\gamma \in \Re$ such that

$$\int_L^{\bar{L}} [A(W(t))\alpha^*(t - \lambda E(\tilde{L})) - 1] dF(t) \geq \gamma \int_L^{\bar{L}} \alpha^*(t - \lambda E(\tilde{L})) dF(t) \quad (14)$$

for all $L \in [0, \bar{L}]$.

Proof: To prove sufficiency, it suffices to rewrite the left side of (11) using integration by parts as

$$\begin{aligned} & \int_0^{\bar{L}} [A(W(L))\alpha^*(L - \lambda E(\tilde{L})) - 1] U'(W(L)) dF(L) \\ &= U'(W(0)) \int_0^{\bar{L}} [A(W(L))\alpha^*(L - \lambda E(\tilde{L})) - 1] dF(L) \\ & \quad + \int_0^{\bar{L}} U''(W(L)) W'(L) \left\{ \int_L^{\bar{L}} [A(W(t))\alpha^*(t - \lambda E(\tilde{L})) - 1] dF(t) \right\} dL \\ & \geq U'(W(0)) \int_0^{\bar{L}} \gamma (L - \lambda E(\tilde{L})) dF(L) \\ & \quad + \int_0^{\bar{L}} U''(W(L)) W'(L) \cdot \left[\gamma \int_L^{\bar{L}} (t - \lambda E(\tilde{L})) dF(t) \right] dL \\ &= \gamma H_\alpha(\alpha; W_0, \lambda) = 0. \end{aligned} \quad (15)$$

Therefore, $d\alpha^*/d\lambda \geq 0$. It should be noted that (15) is analogous to (8). The proof of necessity follows straightforwardly that of Proposition 1' with $-A(W(L))(L - \lambda E(\tilde{L}))$ replaced by $A(W(L))\alpha^*(L - \lambda E(\tilde{L})) - 1$ and is thus omitted. $\square$

Again, Proposition 2' has some implications that will be discussed in the sections "Relating to and Extension of Existing Literature" and "Some Important Implications." By reversing the inequality in (14), one can immediately obtain from Proposition 2' the necessary and sufficient condition for an insurance policy to be non-Giffen as follows:

Proposition 2'' *Fixing W_0 , λ , and F arbitrarily, for any U satisfying condition (C), $d\alpha^*/d\lambda \leq 0$ if and only if there exists $\gamma \in \Re$ such that*

$$\int_L^{\bar{L}} [A(W(t))\alpha^*(t - \lambda E(\tilde{L})) - 1] dF(t) \leq \gamma \int_L^{\bar{L}} \alpha^*(t - \lambda E(\tilde{L})) dF(t) \text{ for all } L \in [0, \bar{L}].$$

RELATING TO AND EXTENSION OF THE EXISTING LITERATURE

This section attempts to relate the results developed in the previous sections to the literature and to provide some extension. The following lemma states the relation between condition (7) in Proposition 1' and DARA.

Lemma 1: *If $A' \leq 0$, then there exists $\gamma \in \Re$ such that (7) holds.*

Proof: Consider any $L < \lambda E(\tilde{L})$. One has $W(L) > W(\lambda E(\tilde{L}))$, which implies that $A(W(L)) \leq A(W(\lambda E(\tilde{L})))$ given $A' \leq 0$ such that

$$A(W(L))(L - \lambda E(\tilde{L})) \geq A(W(\lambda E(\tilde{L})))(L - \lambda E(\tilde{L})). \quad (16)$$

Next, consider any $L > \lambda E(\tilde{L})$. One has $W(L) < W(\lambda E(\tilde{L}))$, which implies that $A(W(L)) \geq A(W(\lambda E(\tilde{L})))$ as $A' \leq 0$ such that (16) again holds. Multiplying both sides of (16) by $f(L)$ and integrating give

$$\int_L^{\bar{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) \geq \int_L^{\bar{L}} A(W(\lambda E(\tilde{L})))(t - \lambda E(\tilde{L})) dF(t).$$

By setting $\gamma = A(W(\lambda E(\tilde{L})))$, one obtains (7). □

Lemma 1 and Proposition 1' give rise to the following well-known result:

Corollary 1 (Mossin, 1968): *Fixing W_0 and λ arbitrarily, for any U satisfying condition (C) and any F , if $A' \leq 0$, then $d\alpha^*/dW_0 \leq 0$.*

The proof of Lemma 1 clearly suggests that the condition of DARA can be weakened by requiring $A(W) - A(\bar{W}_0)$ to possess the single-crossing property. The following corollary utilizes condition (5), which is weaker than DARA given a fixed distribution F with a fixed value for $\lambda E(\tilde{L})$:

Corollary 2: *Fixing W_0 , λ , and F arbitrarily, for any U satisfying condition (C), $d\alpha^*/dW_0 \leq 0$ if*

$$A(a) \geq A(\bar{W}_0) \geq A(b) \text{ for all } (a, b) \text{ with } a < \bar{W}_0 < b,$$

where $\bar{W}_0 = W_0 - \lambda E(\tilde{L})$.

Proof: It suffices to realize that (16) holds on $[0, \bar{L}]$ under the stated condition. Again by setting $\gamma = A(W(\lambda E(\tilde{L})))$, one obtains (7). By Proposition 1', $d\alpha^*/dW_0 \leq 0$. □

Proposition 2' also has some implications. Denote the coefficient of relative risk aversion for utility function U by $R(W) = -W U''(W)/U'(W)$.⁷ A utility function is said to exhibit constant relative risk aversion if $R(W) = \beta$ for all W , where β is a constant. Realizing that the coefficient of absolute risk aversion alone does not give rise to unambiguous conditions under which insurance is Giffen, Hoy and Robson (1981) and Briys, Dionne, and Eeckhoudt (1989) have turned to the concept of relative risk aversion. They show that when a risk averse utility function exhibits constant relative risk aversion, a necessary condition for insurance to be Giffen is that the coefficient of relative risk aversion exceeds unity. As mentioned in the section "Changes in Insurance Premium," Briys, Dionne, and Eeckhoudt's proof relies crucially on reversing the inequalities in (13), which is valid only for the case with a discrete state space and is inapplicable to the case with a continuum of states.

The following corollary of Proposition 2' extends the results of Hoy and Robson (1981) and Briys, Dionne, and Eeckhoudt (1989, Proposition 3) by relaxing the assumption of constant relative risk aversion and by allowing for the case with a continuum of states:

Corollary 3: *Fixing W_0 and λ arbitrarily, for any U satisfying condition (C) and any F , $d\alpha^*/d\lambda \geq 0$ only if $R(W) > 1$ for some $W \in [W(\bar{L}), W(0)]$.*

Proof: Rewrite (14) as

$$\int_L^{\bar{L}} \{\alpha^*[A(W(t)) - \gamma](t - \lambda E(\bar{L})) - 1\} dF(t) \geq 0. \quad (17)$$

Rewrite the left-hand side of (17) as

$$\int_L^{\bar{L}} \{(R(W(t)) - 1) - A(W(t))(W_0 - t) - \gamma(t - \lambda E(\bar{L}))\} dF(t). \quad (18)$$

It can be checked that the integral in (18) is nonnegative for all $L \in [0, \bar{L}]$, only if $R(W) > 1$ for some W . To see this, it suffices to realize that $-A(W(L))(W_0 - L) < 0$ for all $L \in [0, \bar{L}]$ as $\bar{L} \leq W_0$ and that there exists $L \in [0, \bar{L}]$ at which $-\int_L^{\bar{L}} \gamma(L - \lambda E(\bar{L})) dF(t) \leq 0$ as $\int_L^{\bar{L}} (t - \lambda E(\bar{L})) dF(t)$ alternates in sign on $(0, \bar{L})$. Suppose by contradiction that $R(W) \leq 1$ for all W . Substituting these into (18) gives rise to a violation of (17) for some $L \in [0, \bar{L}]$. By Proposition 2', there exists utility function U satisfying condition (C) such that the insurance policy is non-Giffen. A contradiction! Since F is arbitrary and does not affect the specified condition, the result follows. $\square$

Mann (1991) suggests that under a two-state model, when relative risk aversion is not constant, it is necessary that $R'(W) < 0$ for insurance to be Giffen. Corollary 3 (particularly the expression in (18)) suggests that this result is an artifact of the two-state formulation. Corollary 3 also gives rise to the following well-known result:

⁷ Menezes and Hanson (1970) have a detailed exposition of the relation between absolute risk aversion and relative risk aversion. For some applications of the concept of relative risk aversion to the problem of production under risk, see Sandmo (1971), Katz (1983), and Briys and Eeckhoudt (1985).

Corollary 4 (*Robson and Hoy, 1981, and Briys, Dionne, and Eeckhoudt's, 1989, Proposition 3*): Fixing W_0 and λ arbitrarily, for any U satisfying condition (C) and exhibiting constant relative risk aversion and any F , $d\alpha^*/d\lambda \geq 0$ only if $\beta > 1$.

SOME IMPORTANT IMPLICATIONS

So far, the conditions for insurance to be inferior proposed in the literature only impose restrictions on the curvature of a utility function. Particularly, the uniform DARA assumption has often been invoked. Unfortunately, empirical findings on the validity of the uniform DARA hypothesis have been mixed. Based on life insurance demand data, Eisenhauer (1997) and Eisenhauer and Halek (1999) have found positive correlations between life insurance demand and wealth (assets) and hence have rejected the uniform DARA hypothesis, using Mossin's (1968) proposition (Corollary 1 in this article). Szpiro (1983) finds only limited support for the DARA hypothesis based on nonlife insurance demand data. On the contrary, based on a survey of Italian households related to hypothetical investment opportunities, Eisenhauer and Ventura (2003) have found evidence of DARA when households are divided into two wealth categories. Based on the data of bidding responses among Treasury bill dealers, Wolf and Pohlman (1983) have found evidence of DARA.

In another branch of research, empirical studies (e.g., Skinner, 1988; Kuehlwein, 1991; Guiso, Jappelli, and Terlizzese, 1992; Parker, 1999) have found no evidence of precautionary saving, meaning that absolute prudence $P = -U'''/U''$ is nonpositive, and hence $\text{sgn}(A') = \text{sgn}(A - P)$ cannot be negative, indirectly rejecting the DARA hypothesis.⁸

Propositions 1' and 2' suggest that one can take advantage of the knowledge about the interaction of utility U and distribution F specified in conditions (7) and (14) to obtain new conditions for a particular insurance policy to be inferior. This is especially useful in cases where the uniform DARA hypothesis is violated. The following lemma helps prove the next corollary that utilizes one's knowledge about the distribution of the risk concerned:

Lemma 2: *There exists $\gamma \in \Re$ such that (7) holds if*

$$\int_0^{\bar{L}} A(W(L))(L - \lambda E(\tilde{L})) dF(L) \geq 0. \quad (19)$$

Proof: Consider any $L \in (\lambda E(\tilde{L}), \bar{L}]$. $L - \lambda E(\tilde{L}) > 0$ implies that

$$\int_L^{\bar{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) > 0.$$

⁸ The presence of precautionary saving under risk is known to be represented by a positive third derivative of an individual's utility function (e.g., Kimball, 1990). Simple differentiation shows that $A'/A = A - P$, where $P = -U'''/U''$ is the coefficient of absolute prudence introduced by Kimball (1990).

Next, consider $L \in [0, \lambda E(\tilde{L})]$. $L - \lambda E(\tilde{L}) < 0$ implies that

$$\begin{aligned} & \int_L^{\tilde{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) \\ & > \int_L^{\tilde{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) + \int_0^L A(W(t))(t - \lambda E(\tilde{L})) dF(t) \geq 0. \end{aligned}$$

The first inequality is due to $L - \lambda E(\tilde{L}) < 0$; the last inequality is due to (19). Clearly, $\int_L^{\tilde{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) > 0$ for all $L \in (0, \tilde{L})$.

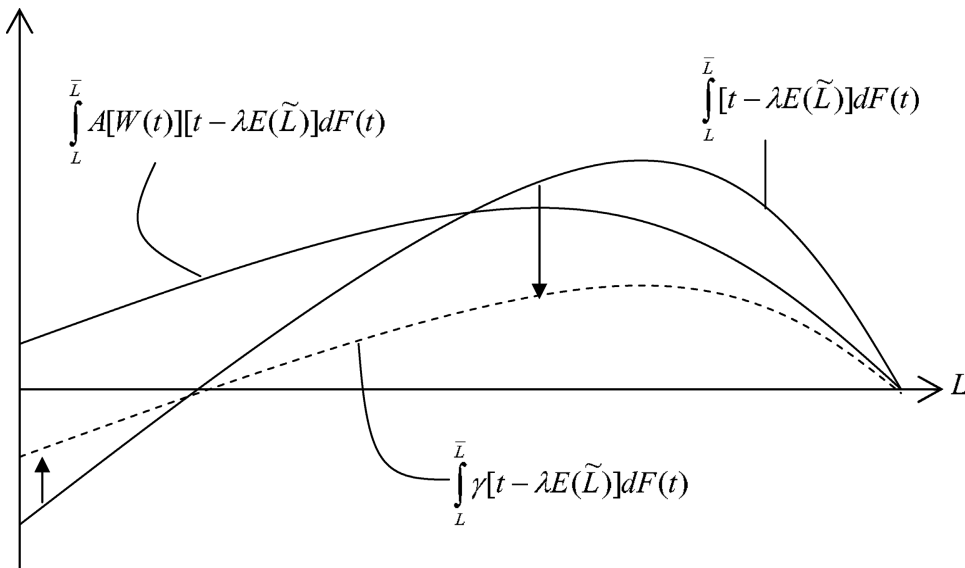
Since $\int_0^{\tilde{L}} (t - \lambda E(\tilde{L})) dF(t) < 0$ and $L - \lambda E(\tilde{L})$ has the single-crossing property, there exists $L^* \in (0, \tilde{L})$ such that $\int_L^{\tilde{L}} (t - \lambda E(\tilde{L})) dF(t) \geq 0$ for all $L \in [L^*, \tilde{L}]$ and $\int_L^{\tilde{L}} (t - \lambda E(\tilde{L})) dF(t) < 0$ for all $L \in (0, L^*]$. Set

$$\gamma = \inf_{L \in [L^*, \tilde{L}]} \left[\int_L^{\tilde{L}} A(W(t))[t - \lambda E(\tilde{L})] dF(t) \Big/ \int_L^{\tilde{L}} [t - \lambda E(\tilde{L})] dF(t) \right] > 0.$$

It can be checked that (7) holds on $[L^*, \tilde{L}]$. For any $L \in [0, L^*)$, since $\int_L^{\tilde{L}} (t - \lambda E(\tilde{L})) dF(t) < 0$, but $\int_L^{\tilde{L}} A(W(t))(L - \lambda E(\tilde{L})) dF(t) \geq 0$, one again obtains (7). $\square$

The intuition of Lemma 2 as an application of Proposition 1' is shown in Figure 1. (19) is a sufficient condition under which condition (7) of Proposition 1' holds.

FIGURE 1
Visualization of Lemma 2



$\int_0^L A(W(L))[L - \lambda E(\tilde{L})]dF(L) > 0$ guarantees that $\int_L^L A(W(t))[t - \lambda E(\tilde{L})]dF(t)$ is uniformly positive so that there exists γ sufficiently small satisfying (7).

Corollary 5: Fixing W_0 , λ , and F arbitrarily, for any U satisfying condition (C), $d\alpha^*/dW_0 \leq 0$ if

$$\frac{(1 - \alpha^*)A'}{A} \leq \frac{f'}{f} \quad (20)$$

and

$$\int_0^L (L - \lambda E(\tilde{L}))dL \geq 0. \quad (21)$$

Proof:⁹ It can be checked that (20) is equivalent to $d(A(W(L))f(L))/dL \geq 0$. The last inequality implies that for any $L \in [\lambda E(\tilde{L}), \bar{L}]$,

$$A(W(L))f(L) \geq A(W(\lambda E(\tilde{L})))f(\lambda E(\tilde{L}))$$

such that

$$A(W(L))f(L)(L - \lambda E(\tilde{L})) \geq A(W(\lambda E(\tilde{L})))f(\lambda E(\tilde{L}))(L - \lambda E(\tilde{L})). \quad (22)$$

Also, for any $L \in [0, \lambda E(\tilde{L})]$,

$$A(W(L))f(L) \leq A(W(\lambda E(\tilde{L})))f(\lambda E(\tilde{L}))$$

such that (22) again holds. Integrating both sides of (22) gives

$$\int_0^L A(W(L))(L - \lambda E(\tilde{L}))dF(L) \geq A(W(\lambda E(\tilde{L})))f(\lambda E(\tilde{L})) \int_0^L (L - \lambda E(\tilde{L}))dL \geq 0.$$

The last inequality is due to $A > 0$, $f > 0$, and (21). By Lemma 2 and Proposition 1', $d\alpha^*/dW_0 \leq 0$. $\square$

Mossin (1968) has explained the intuition of Corollary 1 in the section "Relating to and Extension of the Existing Literature." As absolute risk aversion decreases with wealth, a higher level of endowed wealth raises the wealth level in every loss state reduces the risk-aversion attitude of the individual and hence lowers the individual's demand for insurance coverage. Therefore, DARA renders insurance an inferior good regardless of the distribution of the risk. Note that uniform DARA is equivalent to $dA(W(L))/$

⁹ One can in fact prove Corollary 5 directly without utilizing Proposition 1' and Lemma 2. Readers who are interested in comparing the direct proof of Corollary 5 to that of Mossin's (1968) result (Corollary 1 in this article) to gain additional intuition can obtain the proof from the author.

$dL = (1 - \alpha^*) (dA/dW) \geq 0$ on $[0, \bar{L}]$. The proof of Corollary 5 suggests that by assuming a sufficiently small loading satisfying condition (21), one can apply Lemma 2. This allows one to replace the $dA/dL \geq 0$ condition by the $d(Af)/dL \geq 0$ condition (equivalently condition (20)).

The new condition allows some trade-off between absolute risk aversion and the probability density weight attached to it. Now, absolute risk aversion weighted by the probability density of loss falls either when absolute risk aversion is decreasing or when the density weight is decreasing in wealth (or equivalently, when the density weight is increasing in the realized loss, i.e., $f' \geq 0$). The trade-off between the change in absolute risk aversion and the change in probability density weight of loss as wealth changes is given by condition (20).

Note that, just as one can replace the uniform DARA condition (i.e., $dA/dL \geq 0$) by the single-crossing assumption on A , so one can replace condition (20) in Corollary 5 (i.e., $d(Af)/dL \geq 0$) by the single-crossing assumption on $A \cdot f$. This can be checked easily from the proof of Corollary 5. Note also that the term $f'/f = F''/F'$ used in both Corollaries 5 and 6 represents the curvature of F and is known as the maximum likelihood ratio in the statistics literature.¹⁰ It can be observed from condition (20) of Corollary 5 that when f' is positive for some L , absolute risk aversion is allowed to be increasing for the wealth levels corresponding to these L values.

The above suggests that one can take advantage of the knowledge about the distribution function to make the assumption on the utility function less restrictive. As an example, suppose the risk follows a beta distribution with parameters θ_1 and θ_2 , $E\tilde{L} = \theta_1/(\theta_1 + \theta_2)$, and $\bar{L} = 1$.¹¹ Since

$$\int_0^{\bar{L}} (L - \lambda E(\tilde{L})) dL = \frac{(\bar{L} - \lambda E(\tilde{L}))^2}{2} - \frac{(-\lambda E(\tilde{L}))^2}{2},$$

(21) is satisfied if gross loading $\lambda \leq (\theta_1 + \theta_2)/2\theta_1$. It is well known that with a beta distribution, f is unimodal with f' switching from positive to negative as L rises when both θ_1 and θ_2 are greater than unity. This together with (20) in Corollary 5 implies that for a coinsurance-type insurance policy to be inferior, U is allowed to exhibit increasing absolute risk aversion with A'/A subject to an upper bound at a relatively small L . At a relatively large L , U may need to exhibit DARA. This also suggests that the single-crossing property of $A(W) - A(\bar{W}_0)$ suggested in Corollary 2 is not needed.

A similar argument applies to the case of a gamma distribution with parameters θ_3 and θ_4 , and mean θ_3/θ_4 .¹² Since $\bar{L}$ tends to infinity under a gamma distribution, Condition (21) always holds. Besides, a gamma distribution is also unimodal when $\theta_3 > 1$, giving

¹⁰ The concept of maximum likelihood ratio has been used extensively in solving the principal-agent problem (Grossman and Hart, 1983; Rogerson, 1985; Jewitt, 1988).

¹¹ The density function of a beta distribution is given by $\frac{\Gamma(\theta_1 + \theta_2)}{\Gamma(\theta_1)\Gamma(\theta_2)} L^{\theta_1-1} (1-L)^{\theta_2-1}$, where Γ is the gamma function.

¹² A gamma distribution has density $L^{(\theta_3-1)} e^{-L/\theta_4} (\Gamma(\theta_3)\theta_4^{\theta_3})^{-1}$.

rise to $f' > 0$ when L rises initially such that absolute risk aversion is allowed to rise initially for (20) to hold.

A corollary similar to Corollary 5, stating sufficient conditions for a coinsurance-type insurance policy to be Giffen, can be proved with the help of the following lemma:

Lemma 3: *There exists $\gamma \in \Re - \{0\}$ such that (14) holds if $(1 - \alpha^*) A'/A \leq \alpha^* A$ and*

$$\int_0^L A(W(L)) \left[L - \lambda E(\tilde{L}) - \frac{1}{\alpha^* A(W(L))} \right] dF(L) > 0. \quad (23)$$

Proof: Observe that

$$\int_0^{\tilde{L}} A(W(L)) \left[L - \lambda E(\tilde{L}) - \frac{1}{\alpha^* A(W(L))} \right] dF(L) < \int_0^{\tilde{L}} A(W(L))(L - \lambda E(\tilde{L})) dF(L)$$

so that (23) implies that

$$\int_0^{\tilde{L}} A(W(L))(L - \lambda E(\tilde{L})) dF(L) > 0. \quad (24)$$

By Lemma 2, there exists $\gamma^\# \in \Re - \{0\}$ such that

$$\int_L^{\tilde{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) \geq \gamma^\# \int_L^{\tilde{L}} (t - \lambda E(\tilde{L})) dF(t) \quad (25)$$

for all $L \in [0, \tilde{L}]$. Now, $d[L - \lambda E(\tilde{L}) - (\alpha^* A(W(L)))^{-1}]/dL \geq 0$ if and only if $(1 - \alpha^*) A'/A \leq \alpha^* A$. Since both $L - \lambda E(\tilde{L})$ and $L - \lambda E(\tilde{L}) - (\alpha^* A(W(L)))^{-1}$ are increasing and become negative when L is sufficiently small, using arguments similar to that presented in Lemma 2, one can show that (24) and (23) imply

$$\int_L^{\tilde{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) > 0$$

and

$$\int_L^{\tilde{L}} A(W(t))[t - \lambda E(\tilde{L}) - (\alpha^* A(W(t)))^{-1}] dF(t) > 0$$

for all $L \in [0, \tilde{L}]$. Choosing $\delta > 0$ sufficiently small gives

$$\int_L^{\tilde{L}} A(W(t))[t - \lambda E(\tilde{L}) - (\alpha^* A(W(t)))^{-1}] dF(t) > \delta \int_L^{\tilde{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t) > 0$$

for all $L \in [0, \tilde{L}]$. Setting $\gamma = \gamma^\# \delta$ and using (25) gives

$$\int_L^{\bar{L}} A(W(t))[t - \lambda E(\bar{L}) - (\alpha^* A(W(t)))^{-1}] dF(t) \geq \gamma \int_L^{\bar{L}} (t - \lambda E(\bar{L})) dF(t)$$

for all $L \in [0, \bar{L}]$. □

Corollary 6: Fixing W_0, λ , and F arbitrarily, for any U satisfying condition (C), $d\alpha^*/d\lambda \geq 0$ if

$$\frac{(1 - \alpha^*)A'}{A} \leq \min \left\{ \frac{f'}{f}, \alpha^* A \right\} \quad (26)$$

and

$$\int_0^{\bar{L}} \left[L - \lambda E(\bar{L}) - \frac{1}{\alpha^* A(W(L))} \right] dL > 0. \quad (27)$$

Proof: Let $L^\#$ be the smallest L satisfying $L^\# = \lambda E(\bar{L}) + (\alpha^* A(W(L^\#)))^{-1}$. It can be checked that (26) implies that $d(A(W(L))f(L))/dL \geq 0$ and that $d(L - \lambda E(\bar{L}) - (\alpha^* A(W(L)))^{-1})/dL \geq 0$. These imply that for any $L \in [L^\#, \bar{L}]$,

$$A(W(L))f(L) \geq A(W(L^\#))f(L^\#)$$

and hence

$$\begin{aligned} & A(W(L))f(L) \left[L - \lambda E(\bar{L}) - \frac{1}{\alpha^* A(W(L))} \right] \\ & \geq A(W(L^\#))f(L^\#) \left[L - \lambda E(\bar{L}) - \frac{1}{\alpha^* A(W(L))} \right]. \end{aligned} \quad (28)$$

Also, for any $L \in [0, L^\#]$,

$$A(W(L))f(L) \leq A(W(L^\#))f(L^\#)$$

such that (28) again holds. Integrating both sides of (28) gives

$$\begin{aligned} & \int_0^{\bar{L}} A(W(L))(L - \lambda E(\bar{L}) - (\alpha^* A(W(L)))^{-1}) dF(L) \\ & \geq A(W(L^\#))f(L^\#) \int_0^{\bar{L}} (L - \lambda E(\bar{L}) - (\alpha^* A(W(L)))^{-1}) dL > 0. \end{aligned}$$

The last inequality is due to $A > 0$, $f > 0$, and (27). By Lemma 3 and Proposition 2', $d\alpha^*/d\lambda \geq 0$. □

Corollary 6 has similar implications as Corollary 5, except that (26) is more restrictive than (20). Unlike $L - \lambda E(\bar{L})$ that crosses zero once, there is no guarantee that $L - \lambda E(\bar{L}) - (\alpha^* A(W(L)))^{-1}$ crosses zero only once. The second part of (26) serves to

guarantee the single-crossing at zero of the latter expression. The first part of (26) is identical to (20) of Corollary 5, which assures that $d(Af)/dL \geq 0$. Lastly, (27) is more restrictive than (21) for a coinsurance policy to be Giffen.

As an example of how Corollary 6 can be applied, assume for simplicity that U has a constant coefficient of absolute risk aversion ζ and $\tilde{L}$ has a beta distribution. Expression (27) requires $\lambda \leq (2\alpha^* \zeta \theta_1)^{-1} (\alpha^* \zeta - 2)(\theta_1 + \theta_2)$. Since $\lambda > 1$, the coefficient of absolute risk aversion must be sufficiently large such that $\zeta > 2(\theta_1 + \theta_2)[\alpha^*(\theta_2 - \theta_1)]^{-1}$. Given a beta distribution (26) and (27) may hold even when absolute risk aversion increases at a relatively small L as long as $A(W(L))$ is relatively large, although (27) is more restrictive and hence more difficult to be satisfied than (21). In the case of a gamma distribution, (27) always holds as $\tilde{L}$ tends to infinity (unless A tends to zero). Since f is unimodal when parameter $\theta_3 > 1$, absolute risk aversion is allowed to rise initially for (26) to hold as long as A is sufficiently small.

Before leaving this section, note that an asymmetric bell-shaped density function seems suitable for the case of health risk because an insured often incurs some small to medium-sized medical expenses every year. On the other hand, it seems reasonable that with a large realized loss, a rise in medical cost leading to a further reduction in wealth is likely to make an insured more risk averse, which is in favor of DARA. The same reasoning, however, may not be appealing when the realized loss is small, meaning that DARA is possibly violated. Corollaries 5 and 6 are thus important, especially because many commonly encountered distribution functions, such as Weibull distribution and truncated normal with positive mean prior to truncation, have asymmetric-bell-shaped density functions. Other distributions, such as beta, gamma, lognormal, and chi-square distributions are asymmetric bell-shaped under some reasonable parameter values. Moreover, all the above-mentioned distributions, except for beta distribution, always satisfy condition (21) as $\tilde{L}$ tends to infinity.

CONCLUSION

This article has derived the necessary and sufficient conditions under which a coinsurance-type insurance policy covering a particular risk is inferior and under which it is Giffen. Although these conditions are complicated, they do have some important implications. For instance, the necessary condition for insurance to be Giffen can be made less restrictive than that proposed by Hoy and Robson (1981) and Briys, Dionne, and Eeckhoudt (1989). More importantly, it is possible to take advantage of one's knowledge about the distribution of a particular risk to loosen the requirements on an insured's utility function for a coinsurance-type insurance policy to be inferior or to be Giffen.

The knowledge about loss distributions allows one to explain at least some seemingly controversial empirical findings on the uniform DARA hypothesis. Particularly, it may be consistent for an individual to treat a particular insurance policy as inferior and another policy as normal, and yet the same individual may not show uniform DARA under some hypothetical gambling or investment situations. Using one's knowledge about the distribution of a risk to derive weaker conditions on an individual's utility function for different types of insurance policies (such as deductible insurance and upper limit insurance) to be inferior or to be Giffen is a research agenda that is worth pursuing in the future.

APPENDIX

Proof of Proposition 1' (necessity): The necessity is proved by construction and contradiction. The proof proceeds by first constructing utility function U satisfying condition (C). Then it will be checked that such a U allows first-order condition (3) to be satisfied at some parameter values. Finally, it will be shown that with all these conditions satisfied, a violation of (7) gives rise to a violation of (4).

To simplify the exposition, let $\rho = 1 - \alpha^*$ and $\bar{W} = W_0 - \alpha^* \lambda E(\tilde{L})$ such that the support of the individual's random wealth equals $[\bar{W} - \rho \bar{L}, \bar{W}]$. Construct utility function U as follows:

$$U = \begin{cases} U_0 + \xi W - \frac{1}{2} \eta (\bar{W} - W)^2 + \int_{\hat{W}}^{W_s} \Phi(k) dk + \Phi(\hat{W})(W - \hat{W}), & W \in [\bar{W} - \rho \bar{L}, \hat{W}], \\ U_0 + \xi W - \frac{1}{2} \eta (\bar{W} - W)^2 + \int_W^{W_s} \Phi(k) dk, & W \in (\hat{W}, W_s), \\ U_0 + \xi W - \frac{1}{2} \eta (\bar{W} - W)^2, & [W_s, \bar{W}], \end{cases}$$

where $\bar{W} - \rho \bar{L} < \hat{W} < W_s < \bar{W}$; $0 < U_0 < \infty$ and $0 < \xi < \infty$ are sufficiently large positive constants such that $U > 0$ and $U' > 0$; η is a constant to be defined later; $W_s = \bar{W} - \rho L_s$ and $\hat{W} = \bar{W} - \rho \hat{L}$ where L_s and $\hat{L}$ are constants to be defined later; and Φ satisfies the following:

$$\Phi(W) = -s \cdot \left\{ \left(\frac{W_s - \hat{W}}{2\rho} \right)^2 \left(\frac{W - \hat{W}}{\rho} \right) - \frac{1}{3} \left(\frac{W_s - \hat{W}}{2\rho} \right)^3 - \frac{1}{3} \left[\left(\frac{W - \hat{W}}{\rho} \right) - \left(\frac{W_s - \hat{W}}{2\rho} \right) \right]^3 \right\},$$

where $s > 0$ is a constant to be chosen later, or equivalently,

$$\Phi(W) = -s \cdot \left\{ \left(\frac{\hat{L} - L_s}{2} \right)^2 (\hat{L} - L) - \frac{1}{3} \left[(\hat{L} - L) - \left(\frac{\hat{L} - L_s}{2} \right) \right]^3 - \frac{1}{3} \left(\frac{\hat{L} - L_s}{2} \right)^3 \right\}$$

such that

$$\Phi'(W) = -\rho^{-1} s \cdot \left\{ \left(\frac{\hat{L} - L_s}{2} \right)^2 - \left[(L - L_s) - \left(\frac{\hat{L} - L_s}{2} \right) \right]^2 \right\}$$

as $dL/dW = -\rho^{-1}$. By differentiation,

$$U' = \begin{cases} \xi + \eta(\bar{W} - W) + \Phi(\hat{W}), & W \in [\bar{W} - \rho\bar{L}, \hat{W}] \\ \xi + \eta(\bar{W} - W) + \Phi(W), & W \in (\hat{W}, W_s) \\ \xi + \eta(\bar{W} - W), & W \in [W_s, \bar{W}] \end{cases}.$$

U' is certainly differentiable on $[0, \bar{L}]$. A second differentiation yields

$$U'' = \begin{cases} -\eta + \Phi'(W), & W \in (\hat{W}, W_s) \\ -\eta, & \text{otherwise} \end{cases}.$$

It can be checked that U is continuous. Also, Φ is continuous on $(\hat{W}, W_s)$ and $\Phi(\hat{W}) = 0$ so that U' is continuous on $[\bar{W} - \bar{L}, \bar{W}]$. Now, Φ' is continuous on $[\bar{W} - \bar{L}, W_s)$ and $\Phi'(W_s) = 0 = \Phi'(\hat{W})$, implying that U'' is continuous on $[\bar{W} - \bar{L}, \bar{W}]$. In addition, $\Phi' < 0$ on $(\hat{W}, W_s)$ implies $U'' < 0$ on $[\bar{W} - \bar{L}, \bar{W}]$. Next, Φ' is differentiable on $(\hat{W}, W_s)$ with $\Phi'' = -2\rho^{-2}s[L - (\bar{L} + L_s)/2]$ on $(\hat{W}, W_s)$ such that U'' is piecewise differentiable. Finally, it can be checked that

$$A(W) = \begin{cases} \frac{\eta}{\xi + \eta(\bar{W} - W) + \Phi(\hat{W})}, & W \in [\bar{W} - \rho\bar{L}, \hat{W}] \\ \frac{\eta - \Phi'(W)}{\xi + \eta(\bar{W} - W) + \Phi(W)}, & W \in (\hat{W}, W_s) \\ \frac{\eta}{\xi + \eta(\bar{W} - W)}, & W \in [W_s, \bar{W}] \end{cases}$$

is bounded regardless of the actual value of s to be chosen for Φ .

To save space, let

$$T(L) = \int_L^{\bar{L}} (t - \lambda E(\bar{L})) dF(t).$$

Substituting the above into first-order condition (3) gives

$$0 = \xi T(0) + \int_0^L \eta(1 - \alpha^*)T(L) dL - \int_{L_s}^{\bar{L}} \Phi'(W(L))(1 - \alpha^*)T(L) dL. \quad (A1)$$

To simplify the exposition, let

$$\Psi_0 = -\xi(1 - \alpha^*)^{-1}T(0); \quad \Psi_1 = \int_0^L T(L) dL; \quad \Psi_2 = \int_{L_s}^{\bar{L}} \Phi'(W(L))T(L) dL.$$

Assume for the moment that $\Psi_1 \neq 0$. The case with $\Psi_1 = 0$ given the above specification of U will be handled later. Collecting terms with η in (A1) suggests that one must set

$$\eta = \frac{\Psi_0 + \Psi_2}{\Psi_1} \quad (\text{A2})$$

in order for first-order condition (3) to hold. This finalizes the construction of the utility function.

The remaining task is to show that given the constructed utility function, a violation of (7) gives rise to a violation of (4). Denote

$$T^A(L) = \int_L^{\bar{L}} A(W(t))(t - \lambda E(\tilde{L})) dF(t)$$

and

$$\Psi_0^A = -\xi(1 - \alpha^*)^{-1} T^A(0); \quad \Psi_1^A = \int_0^{\bar{L}} T^A(L) dL; \quad \Psi_2^A = \int_{L_s}^{\hat{L}} \Phi'(W(L)) T^A(L) dL.$$

Under this notation, using (A2) gives

$$\text{Sgn } H_{\alpha W_0}(\alpha; W_0, \lambda) = \text{Sgn}(\Psi_0^A - \eta \Psi_1^A + \Psi_2^A) = \text{Sgn} \left[-\frac{\Psi_1^A}{\Psi_1}(\Psi_0 + \Psi_2) + (\Psi_0^A + \Psi_2^A) \right] \quad (\text{A3})$$

Assume by contradiction that (7) does not hold. Then for each $\sigma \in \mathfrak{R}$, there exists $L \in [0, \bar{L}]$ such that

$$T^A(L) < \sigma T(L) \quad (\text{A4})$$

Since T^A and T are both continuous, the inequality in (A4) holds for a nontrivial interval $I(\sigma) \subset (0, \bar{L})$. For a particular σ to be chosen later, choose $\hat{L}$ and L_s such that $(L_s, \hat{L}) \subset I(\sigma)$ together with (A4) and $\Phi'(W) < 0$ on $(\hat{W}, W_s)$ gives

$$\Psi_2^A = \int_{L_s}^{\hat{L}} \Phi'(W(L)) T^A(L) dL > \sigma \int_{L_s}^{\hat{L}} \Phi'(W(L)) T(L) dL = \sigma \Psi_2. \quad (\text{A5})$$

Since $T(L)$ and $T^A(L)$ are bounded, one can choose s sufficiently large such that Ψ_2 dominates Ψ_0 and Ψ_2^A dominates Ψ_0^A so that (A5) gives rise to

$$\Psi_0^A + \Psi_2^A > \sigma(\Psi_0 + \Psi_2), \quad \forall \sigma \in \mathfrak{R}. \quad (\text{A6})$$

The last inequality in (A6) implies that

$$-\frac{\Psi_1^A}{\Psi_1}(\Psi_0 + \Psi_2) + (\Psi_0^A + \Psi_2^A) > \left(\sigma - \frac{\Psi_1^A}{\Psi_1} \right) (\Psi_0 + \Psi_2). \quad (\text{A7})$$

Since $\Psi_1 \neq 0$ and the expression $\Psi_1^A - \sigma \Psi_1$ is linear in σ , there exist nonempty sets

$$S^- = \{\sigma \mid \Psi_1^A - \sigma \Psi_1 < 0, \quad \sigma \in \Re\}$$

$$\text{and } S^+ = \{\sigma \mid \Psi_1^A - \sigma \Psi_1 \geq 0, \quad \sigma \in \Re\}.$$

It can be checked that one can always choose σ to make sure that

$$\left(\sigma - \frac{\Psi_1^A}{\Psi_1}\right)(\Psi_0 + \Psi_2) \geq 0. \quad (\text{A8})$$

The following are four possible cases to be considered:

1. If $\Psi_1 > 0$ and $\Psi_0 + \Psi_2 \geq 0$, then choose $\sigma \in S^-$ such that $\sigma - (\Psi_1^A/\Psi_1) \geq 0$.
2. If $\Psi_1 > 0$ and $\Psi_0 + \Psi_2 < 0$, then choose $\sigma \in S^+$ such that $\sigma - (\Psi_1^A/\Psi_1) \leq 0$.
3. If $\Psi_1 < 0$ and $\Psi_0 + \Psi_2 \geq 0$, then choose $\sigma \in S^+$ such that $\sigma - (\Psi_1^A/\Psi_1) \geq 0$.
4. If $\Psi_1 < 0$ and $\Psi_0 + \Psi_2 < 0$, then choose $\sigma \in S^-$ such that $\sigma - (\Psi_1^A/\Psi_1) \leq 0$.

Choosing σ in the above cases accordingly gives rise to (A8). Substituting (A8) into (A7) and then (A3) gives $H_{\alpha W_0}(\alpha; W_0, \lambda) > 0$. A contradiction!

Finally, suppose $\Psi_1 = 0$ under the constructed utility U . Then simply replace the $-\frac{1}{2}\eta(\bar{W} - W)^2$ term in U by $-\frac{1}{2}\eta(\bar{W} - W - \kappa)^2$, where $\kappa \neq 0$. Now, (A1) becomes

$$0 = -\xi T(0) + \eta \kappa T(0) + \int_{L_s}^L \Phi'(W(L))(1 - \alpha^*)T(L)dL. \quad (\text{A9})$$

It suffices to set $\Psi_1 = \kappa T(0)$ such that (A2) still holds. Next, set $\Psi_1^A = \kappa T^A(0)$. The proof goes through exactly as before. $\square$

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