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A Remark on "A Shortcut Way of Pricing Default Risk Through Zero-Utility Principle"

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Abstract

This remark studies Tibiletti's bargaining condition and shows that, for risk neutral buyers or the default loss are small relative to the buyer's size, there exists a more shortcut bargaining condition.

Introduction

In her 2006 *Journal of Risk and Insurance* article, Luisa Tibiletti considers an individual (the insured or the "seller") would be willing to pay to get rid of an infinitesimal chance p of undergoing a default risk

$$X = \begin{cases} -w, & p \\ 0, & 1-p, \end{cases} \tag{1}$$

where $w > 0$. On the other hand, the counterpart (the insurer or the "buyer") would be willing to accept the risk of undergoing a possible loss against collecting a certain amount.

In the framework of zero-utility principle, Tibiletti gives a shortcut necessary condition for risk exchange:

$$\frac{\dfrac{v(x_0)}{v'(x_0)}\,\dfrac{v(x_0)-v(x_0-w)}{v(x_0)}}{\dfrac{u(w_0)}{u'(w_0)}\,\dfrac{u(w_0)-u(w_0-w)}{u(w_0)}}\,\frac{q}{p} \le 1, \tag{2}$$

where u is the seller utility function, w_0 (with $w_0 > w$) is the initial wealth of seller, p is the seller's evaluation of the default probability, v is the buyer utility function,

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x_0 (with $x_0 > w$) is the initial wealth of buyer, and q is the buyer's evaluation of the default probability.

This bargaining condition shows that the default risk pricing is proportional to (1) the sensitivity "in the small and in the large" in risk aversion, where the sensitivity "in small" is measured by fear of ruin coefficient and (2) the individual evaluation of probability of default.

This remark shows that, for risk-neutral buyers and risk-averse sellers, (1) if $q \leq p$, then the bargaining condition (2) holds; (2) if $q > p$, then bargaining condition depends on $\frac{q}{p}$ and the ratio of the small change rate to the large change rate of seller's utility; and (3) the greater, w, the default loss, the easier the bargaining. Moreover, if buyer's wealth is highly greater than the default loss, no matter is the buyer's risk profile (risk lover/neutral or adverse), the same above-mentioned sufficient conditions are achieved.

RISK-NEUTRAL BUYER AND RISK-AVERSE SELLER

In this section, we assume $u(0) = 0$, $u'(x) > 0$, $u''(x) < 0$ and $v(x) = x$. This risk-neutral assumption for buyer is not unreasonable for some situations. For example, an insurance company operates in a competitive environment and that insurer can be treated as risk neutral, or an insurance company has so many risky things that she on average tends to cancel each other out.

The bargaining condition (2) can be rewritten as:

$$\frac{u'(w_0)}{\frac{u(w_0) - u(w_0 - w)}{w}} \frac{q}{p} \leq 1. \tag{3}$$

We are going to show:

$$\frac{u'(w_0)}{\frac{u(w_0) - u(w_0 - w)}{w}} < 1 \quad \text{for} \quad 0 < w < w_0. \tag{4}$$

Denote

$$F(w) = \frac{u'(w_0)}{\frac{u(w_0) - u(w_0 - w)}{w}}. \tag{5}$$

We notice that $u'(w_0)$ is the small change rate of seller's utility at w_0 and $\frac{u(w_0) - u(w_0 - w)}{w}$ is the large change rate of seller's utility at w_0. $F(w)$ is ratio of the small change rate of seller's utility to the large change rate of seller's utility.

We have $\lim_{w \to 0} F(w) = 1$ and

$$F'(w) = \frac{u'(w_0)}{[u(w_0) - u(w_0 - w)]^2}[u(w_0) - u(w_0 - w) - wu'(w_0 - w)]. \tag{6}$$

Let $f(w) = u(w_0) - u(w_0 - w) - wu'(w_0 - w)$, we have $f(0) = 0$ and

$$f'(w) = wu''(w_0 - w) < 0. \tag{7}$$

Hence, $f(w) < 0$ for $0 < w \le w_0$, and

$$F'(w) < 0 \quad \text{and} \quad F(w) < 1 \quad \text{for} \quad 0 < w \le w_0. \tag{8}$$

Condition (3) and (8) imply that: (1) if $q \le p$, then the bargaining condition (3) holds; (2) if $q > p$, then bargaining condition (3) depends on $\frac{q}{p}$ and the $F(w)$, the ratio of the small change rate to the large change rate of seller's utility; and (3)the greater, w the default loss, the easier the bargaining.

THE RISKS ARE SMALL RELATIVE TO THE BUYER'S SIZE

In this section, we assume that $u(0) = 0$, $u'(x) > 0$, $u'(x) < 0$, $v'(x)$ exists, and $x_0 \gg w_0 > w$. The assumption, $x_0 \gg w_0 > w$, implies that the default loss are small relative to the buyer's size. Here, no hypothesis on buyer's risk profile (risk lover/neutral or adverse) are posed (we impose no restriction on v).

Let us consider the bargaining condition (2):

$$\frac{\dfrac{v(x_0)}{v'(x_0)}}{\dfrac{u(w_0)}{u'(w_0)}} \frac{\dfrac{v(x_0) - v(x_0 - w)}{v(x_0)}}{\dfrac{u(w_0) - u(w_0 - w)}{u(w_0)}} \frac{q}{p} \le 1 \Leftrightarrow \frac{u'(w_0)}{v'(x_0)} \frac{\dfrac{v(x_0) - v(x_0 - w)}{w}}{\dfrac{u(w_0) - u(w_0 - w)}{w}} \frac{q}{p} \le 1. \tag{9}$$

For $x_0 \gg w_0 > w$, we have $\frac{v(x_0) - v(x_0 - w)}{w} \approx v'(x_0)$. Then (9) can be rewritten as

$$\frac{u'(w_0)}{v'(x_0)} \frac{\dfrac{v(x_0) - v(x_0 - w)}{w}}{\dfrac{u(w_0) - u(w_0 - w)}{w}} \frac{q}{p} \le 1 \approx \frac{u'(w_0)}{\dfrac{u(w_0) - u(w_0 - w)}{w}} \frac{q}{p} = F(w)\frac{q}{p} \le 1. \tag{10}$$

By the analysis in the previous section, we know that $F(w) < 1$ and $F'(w) < 0$. Thus we get same results for bargaining condition as in that section.

CONCLUSION

This remark shows that for risk-neutral buyers and risk-averse sellers, (1) if $q \le p$, then the bargaining condition (2) holds; (2) if $q > p$, then bargaining condition depends on $\frac{q}{p}$ and the ratio of the small change rate of to the large change rate of seller's utility; (3) smaller the default loss with respect to the buyer's wealth, the easier the bargaining. Moreover, if default loss are small relative to the buyer's size, no matter the buyer's inclination toward the risk, the same above-mentioned sufficient conditions are achieved.

REFERENCE

Tibiletti, L., 2006, A Shortcut Way of Pricing Default Risk through Zero-Utility Principle, *Journal of Risk and Insurance*, 73: 303-308.