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Precautionary Insurance Demand With State-Dependent Background Risk

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Abstract

This article considers a zero-mean background risk that is uncorrelated with insurable losses, but is not necessarily statistically independent. In particular, the size of the background risk can vary in different insurable-loss states. We show how a prudent individual will buy either more insurance or less insurance than with no background risk, depending on the relative size of the background risk in the loss states vis-á-vis the no-loss states. If we consider two individuals, with one more risk averse than the other, we need to compare the intensities of their precautionary motives, in addition to their measures of risk aversion, before we can determine who buys more insurance coverage in the presence of the state dependent background risk.

Introduction

Many models have been developed to examine insurance decisions in the presence of other risks. When one of the risks is exogenous and unhedgeable, we typically refer to it as a "background risk" and much research in the past 20 years has examined how the presence of this background risk affects decisions regarding the insurable risk.[1] Most of these studies focus on the case where the background risk is statistically independent from the endogenous risk. To our knowledge, Eeckhoudt and Kimball (1992) are the first to provide sufficient conditions on the utility function, such that the presence of an independent background risk makes people act in a more risk-averse manner toward the purchase of insurance. Conditions that are both necessary and sufficient were examined by Gollier and Pratt (1996). A few articles, including Doherty and Schlesinger (1983), Eeckhoudt and Kimball (1992), Briys and Viala (1995),

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[1] Doherty and Schlesinger (1983) and Mayers and Smith (1983) first examined this issue. A good summary of most of the key results can be found in Gollier (2001).

and Gollier (1996) also examine cases for which the background risk is statistically dependent on the insurable risk.

In addition to research examining the effects of background risk on an individual's behavior, another line of research has looked at whether or not interpersonal comparisons of insurance-purchasing behavior will continue to hold in the presence of a background risk. Kihlstrom, Romer, and Williams (1981) and Nachman (1982) were the first to show that a more risk-averse individual might not continue to purchase more insurance in the presence of an independent background risk. Ross (1981) looked at a similar question but assumed that the insurable risk and the background risk had a zero correlation, which is weaker than the assumption of independence.

In this article, we study the effect of a state-dependent, zero-mean background risk on the demand for insurance. We use a simple model with only two loss events: loss or no loss. We assume that the conditional mean of the background risk, given any realization of insurable losses, is always zero. Since such a background risk has a zero correlation with insurable loses, there is no possibility of using insurance for purposes of cross-hedging, as in Schlesinger and Doherty (1985).

The fact that the background risk can vary in different loss states leads to a precautionary effect of insurance. Even though the background risk is not cross-hedgeable via insurance, the consumer can adjust wealth levels in different loss states in order to better cope with the background risk. For a prudent consumer, changes in insurance can shift more wealth to states for which the background risk is relatively high, thus mitigating the untoward effects of the background risk.

One can also consider stochastic changes in the background risk. This has been done by Eeckhoudt, Gollier, and Schlesinger (1996) and by Meyer and Meyer (1998) for the case of an independent background risk. In this article, we examine stochastic changes in the background risk but where the change takes place only in the event of a loss or only in the event of no loss. The conditions under which such a change will always lead to more (or always lead to less) insurance are much simpler than they are for models with an independent background risk.

Our article is similar in spirit to that of Eeckhoudt, Gollier, and Schlesinger (1991), who consider localized increases in risk within a model of deductible insurance. Although they consider a change in the distribution of the insurable risk, the localized nature of their risk changes has effects similar to those we obtain in a setting of localized background risks, as we explain in the text.

We first set up a model with localized background risk and compare it to one with an independent background risk. We then show how prudence plays a crucial role in determining whether or not insurance demand increases in the presence of background risk. If the background risk in the different loss states varies only by a size factor (i.e., a scaling effect), we show how changes in relative scaling affect the demand for insurance. We next consider two particular stochastic changes in the background risk and determine conditions under which these changes lead to an increase in insurance demand. Finally, we consider two consumers, with one being more risk averse than the other. We examine conditions under which the more risk averse individual will demand more insurance in the presence of the background risk and we show how their comparative levels of prudence play a role in determining their comparative demands for insurance coverage.

THE MODEL

We use the simplest model of insurable risk, in which there are only two loss events, "loss" and "no loss," which partition the state space. We will refer to these partitioned states of the world as the "loss states" and the "no-loss states," respectively.[2] Since we only consider two loss events and since we assume the indemnity must be zero when there is no insurable loss, we cannot distinguish between various forms of indemnification. Several articles focus on the optimal form of the indemnity schedule, in particular whether or not it will be of the deductible form.[3] Our focus here is only on the level of insurance coverage demanded.

An insured with an initial wealth level W also has an insurable risk $\tilde{x}$. We assume that the loss $\tilde{x}$ has a value of $L \leq W$ with probability p and it has a value of zero with probability $1 - p$. We assume that the insured is strictly risk averse, with her utility function u satisfying the conditions $u' > 0$ and $u'' < 0$. We also assume that u is at least four times differentiable. An insurance contract is available that charges a premium $P(\alpha) = (1 + m)\alpha pL$ and pays an indemnity αL in the event of a loss, where α is the rate of coinsurance and $m \geq 0$ is the so-called "loading factor" for the premium. We require that $(1 + m)p < 1$, since if this was not the case, the insurance premium would be higher than the indemnity that was paid for any loss and no insurance would be sold. In the absence of any background risk, the insured chooses the optimal amount of insurance to maximize her expected utility as follows:

$$\max_{\alpha} EU \equiv pu(W - P(\alpha) - L + \alpha L) + (1 - p)u(W - P(\alpha)). \tag{1}$$

The optimal rate of coinsurance α^* satisfies the necessary first-order condition

$$[p(1 - p - mp)L]u'(y_L) - [p(1 - p - mp + m)L]u'(y_N) = 0, \tag{2}$$

where $y_L \equiv W - P(\alpha) - L + \alpha L$ and $y_N \equiv W - P(\alpha)$ are notations for wealth in the loss and no-loss states, respectively. Straightforward calculations show that EU as defined in (1) is concave in α, so that second-order conditions for a maximum also hold. For the sake of simplicity, we assume that $\alpha^* > 0$.[4]

If $m = 0$, it follows trivially from (2) that $y_L = y_N$, so that full insurance is optimal, $\alpha^* = 1$. If $m > 0$, it also follows easily from (2) that partial insurance is preferred, $\alpha^* < 1$. These two results are generally attributed to Mossin (1968).

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[2] A word on the terminology here is appropriate. Since there is only one size of loss in our model, the literature often ignores other attributes and only considers the world as having two states: the loss state and the no-loss state. However, since we also consider background risks, we prefer to label the set consisting of all of the states in which the insurable loss occurs as the "loss states" and its complement as the set of "no-loss states."

[3] See, for example, Doherty and Schlesinger (1983), Briys and Viala (1995), and Gollier (1996).

[4] All of the results can easily be extended to allow for $\alpha^* < 0$ or for a corner solution at $\alpha^* = 0$. However, such results do not add any new insights and only complicate the mathematics.

Suppose we now consider another consumer with utility $v(y)$, who is uniformly more risk averse than the consumer with utility $u(y)$. For ease of exposition, we refer to these individuals as v and u, respectively. Following Pratt (1964), it follows that for any $m > 0$, the optimal level of insurance for consumer v will be higher than that of consumer u, *ceteris paribus*, i.e., $\alpha_v^* > \alpha_u^*$.

We assume that any background risk for the individual takes the form $\tilde{\theta}_L$ in the states of the world for which $x = L$ and the form $\tilde{\theta}_N$ in the states of the world for which $x = 0$. We also assume that $E\tilde{\theta}_L = E\tilde{\theta}_N = 0$. If $var(\tilde{\theta}_L) > 0$ and $var(\tilde{\theta}_N) = 0$, then we have a case in which background risk only occurs if there has been a loss, and there is no background risk in the no-loss states. If $var(\tilde{\theta}_L) = 0$ and $var(\tilde{\theta}_N) > 0$, then we have just the opposite: a case in which background risk only occurs if there is no insurable loss.

Since this setting is a bit too general, we simplify it to allow only for the relative size of the background risk to vary in the loss states versus the no-loss states. To this end, let $\tilde{\varepsilon}$ be a zero-mean random variable with a variance that is positive (i.e., $\tilde{\varepsilon}$ is not identically zero). We assume that $\tilde{\varepsilon}$ is not directly hedgeable and further assume that $\tilde{\varepsilon}$ and $\tilde{x}$ have a zero correlation. Thus, one cannot indirectly hedge against the $\tilde{\varepsilon}$-risk via the purchase of additional insurance against $\tilde{x}$. We let β be a scalar $0 \leq \beta \leq 1$, and define $\tilde{\theta}_L = \beta\tilde{\varepsilon}$ and $\tilde{\theta}_N = (1 - \beta)\tilde{\varepsilon}$. If $\beta = 1$, we have a background risk only in the loss states. If $\beta = 0$, we have a background risk only in the no-loss states. If $\beta = \frac{1}{2}$, then $\tilde{\theta}_L = \tilde{\theta}_N = \frac{1}{2}\tilde{\varepsilon}$, which corresponds to the case of an independent background risk. Obviously, as β varies, we can tell other stories. For instance, if β is very small, then we have a background risk in the loss states, but there is still a very small amount of background risk in the no-loss states.

We next examine the basic results above to see how they are affected by the addition of a background risk.

BACKGROUND RISK AND INSURANCE DEMAND

The consumer's objective in the presence of the background risk now can be written as follows:

$$\max_{\alpha} EU \equiv pEu(y_L + \beta\tilde{\varepsilon}) + (1 - p)Eu(y_N + (1 - \beta)\tilde{\varepsilon}). \tag{3}$$

Let y_L^* and y_N^* denote the consumer's optimal wealth levels for the events "loss" and "no loss," without a background risk, i.e., $y_L^* \equiv W - P(\alpha^*) - L + \alpha^* L$ and $y_N^* \equiv W - P(\alpha^*)$, where α^* is the solution to (1).

For the case where $\beta = \frac{1}{2}$ and with $m > 0$, we know from Gollier and Pratt (1996), that α^* always will increase in the presence of the background risk if and only if preferences are "risk vulnerable." Since risk vulnerability is a difficult property to characterize, they also provide us with several sufficient conditions, the most well-known being "standard risk aversion," as defined by Kimball (1993).[5] For the case where insurance

[5] The sufficiency of standard risk aversion was first shown directly by Eeckhoudt and Kimball (1992). Standard risk aversion can be characterized as having a utility function that exhibits both decreasing absolute risk aversion and decreasing absolute pudence.

is actuarially fair, $m = 0$, it follows easily that we still require $y_L^* = y_N^*$, so that full coverage remains optimal.

Consider now the case where $\beta = 1$. In this case, a background risk manifests itself only when a loss occurs. For example, we might have other ancillary expenses and benefits associated with a loss. We might need to miss a work opportunity in order to take the time to apply for insurance benefits, or relatives might send us money to help us deal with our loss. In this case, standard risk aversion is stronger than necessary to guarantee an increase in the level of insurance coverage. Indeed, we do not even require the weaker condition of decreasing absolute risk aversion. Rather, the property of "prudence," $u''' > 0$, is both necessary and sufficient to guarantee an increase in insurance.

Since EU is still concave in α in the presence of a background risk, we can determine the effect of background risk by evaluating the sign of $dEU/d\alpha$, evaluated at the optimal level of coinsurance without background risk α^*,

$$\left.\frac{dEU}{d\alpha}\right|_{\alpha^*} = [p(1-p-mp)L]Eu'\big(y_L^* + \tilde{\varepsilon}\big) - [p(1-p-mp+m)L]u'\big(y_N^*\big). \qquad (4)$$

Comparing (4) with the first-order condition without background risk (2), it follows easily from Jensen's inequality that $dEU/d\alpha > 0$ if and only if $Eu'(y_L^* + \tilde{\varepsilon}) > u'(y_L^*)$. From Jensen's inequality, this in turn will follow for any arbitrary zero-mean risk $\tilde{\varepsilon}$ and for any arbitrary starting wealth W if and only if u' is convex in wealth; i.e., consumer u is prudent. Since we assume that u is thrice differentiable, this is equivalent to $u''' \geq 0$ everywhere, with $u''' > 0$ on a set of positive probability measure on the subset of loss states. Thus, a necessary and sufficient condition for insurance to increase in the presence of background risk only in the loss states is that the consumer is prudent. Note that this result will hold even with fair insurance, $m = 0$. That is, with fair insurance, the consumer will buy more than full coverage, $\alpha^* > 1$.

The intuition for this result follows from Kimball (1990), who analyzes the demand for precautionary savings. Here we do not have two periods but rather a partition of the states of nature into those with and without the occurrence of the insurable loss. Equation (2) shows how insurance is chosen to balance the marginal net utility benefit in the loss states from an increase in α, with the marginal net utility cost in the no-loss states, stemming from the associated higher premium. When we now make wealth in the loss states risky by adding a background risk, the prudent consumer can mitigate the loss of utility by shifting some more wealth to the loss states.

The logic is the same as for precautionary savings, in which a prudent investor shifts more wealth to a later period (i.e., saves more) to better cope with the introduction of a risk in future labor income. In the present case, the consumer uses the insurance contract not only to directly hedge against the loss $\tilde{x}$, but also in a precautionary sense to shift wealth into the states with the background risk. For example, if $m = 0$, the consumer not only buys enough insurance to fully hedge the $\tilde{x}$-risk but buys a bit more. By purchasing this excess insurance, the consumer shifts a little bit more wealth from the no-loss states into the loss states, which lowers the "pain" caused by the background risk $\tilde{\varepsilon}$, where "pain" here is measured as the loss of utility.[6]

If $\beta = 0$, background risk occurs only in the no-loss states. For instance, suppose that initial wealth is composed of cash in the amount of $W - L$, plus a noncash asset with a monetary value that is not known with perfect certainty. The value of the noncash asset is given by $L + \tilde{\varepsilon}$. The realized value of $\tilde{\varepsilon}$ is not observable until the asset is sold, which we assume is after any insurance decision is made. If the asset is stolen or destroyed, the individual has only a wealth of $W - L$. Insurance can only be based on the mean value L. As a result, when there is no insurance, wealth in the loss states is $W - L$ and wealth in the no-loss states is $W + \tilde{\varepsilon}$.[7]

The purchase of insurance in this setting can be compared to the case of no background risk by once again examining the sign of $dEU/d\alpha$, evaluated at the optimal level of coinsurance without background risk α^*.

$$\left.\frac{dEU}{d\alpha}\right|_{\alpha^*} = [p(1 - p - mp)L]u'\left(y_L^*\right) - [p(1 - p - mp + m)L]Eu'\left(y_N^* + \tilde{\varepsilon}\right). \tag{5}$$

Unlike for the case where $\beta = 1$, in this case a prudent consumer actually would purchase less insurance. Indeed, when $\beta = 0$, insurance would necessarily increase if and only if u' is concave, i.e., preferences are imprudent with $u''' < 0$. In the case where insurance is fair, $m = 0$, the prudent consumer (with $u''' > 0$) purchases less than full coverage. Again the reasoning is precautionary. By choosing partial coverage, $\alpha < 1$, the consumer can save on some of her premium expenditure, thus effectively shifting some of her wealth to the no-loss states so as to mitigate the "pain" from the background risk.

Of course for the well known case of independent risks, where $\beta = \frac{1}{2}$ in our model, we need to compare the precautionary effects in the loss states and in the no-loss states, which is not so simple. Assume that we have prudence. Then $Eu'(y_i^* + \frac{1}{2}\tilde{\varepsilon}) > u'(y_i^*)$ for both $i = N$ and $i = L$. From (2), it follows that if $\alpha^* < 1$, the optimal level of insurance in the presence of the background risk will be higher for any choice of parameters if and only if $Eu'(y + \frac{1}{2}\tilde{\varepsilon})/u'(y)$ is decreasing in wealth y. But this is itself is easily shown to be equivalent to $\frac{-Eu''(y+\frac{1}{2}\tilde{\varepsilon})}{Eu'(y+\frac{1}{2}\tilde{\varepsilon})} > \frac{-u''(y)}{u'(y)}, \forall y$. From here, we can apply the known result of Gollier and Pratt (1996): that more insurance is always purchased in the presence of an independent background risk if and only if preferences exhibit "risk vulnerability." Or, since risk vulnerability is difficult to characterize, we can

[6] This notion of increasing wealth to stem the pain follows from Eeckhoudt and Schlesinger (2006), who use the utility premium of Friedman and Savage (1948) as a measure of "pain."

[7] This example is similar in spirit to Gollier's (1996) case of an imperfectly observable loss. Since we only have an all or nothing loss, we assume that the value of wealth is observable following a full loss. However, the exact value of wealth is initially uncertain, since we do not initially observe the exact cash value of the asset. For example, the asset might be a rare painting or some gold bullion for which we have an appraisal, but for which we are unsure about the exact amount of cash it would trade for. In such a case, it is not uncommon for an insurance contract to be written based only upon the appraised value.

obtain the result of Eeckhoudt and Kimball (1992), that the combination of decreasing absolute risk aversion and decreasing absolute prudence is sufficient to yield a higher level of insurance.

If β is close to zero or close to one, we might expect that behavior is similar to that where the background risk only occurs in the loss states or only in the no-loss states. But, of course, how “close” would β need to be? As it turns out, assuming a prudent consumer, the optimal level of insurance coverage for a fixed loading m is strictly increasing in β: as β increases so does the optimal level of insurance α^*. This follows from simply differentiating the first-order condition for the general case in (3):

$$\left.\frac{\partial^2 EU}{\partial\alpha\partial\beta}\right|_{\alpha^*} = [p(1-p-mp)L]E\big[u''\big(y_L^* + \beta\tilde{\varepsilon}\big)\tilde{\varepsilon}\big] + [p(1-p-mp+m)L]E\big[u''\big(y_N^* + (1-\beta)\tilde{\varepsilon}\big)\tilde{\varepsilon}\big]. \tag{6}$$

It turns out that both $E[u''(y_L^* + \beta\tilde{\varepsilon})\tilde{\varepsilon}]$ and $E[u''(y_N^* + (1-\beta)\tilde{\varepsilon})\tilde{\varepsilon}]$ are positive, as we show in the Appendix. Hence, the derivative in (6) is positive and thus a higher β begets a higher level of insurance.

The above result also implies that there exists some critical level $\hat{\beta} \in (0, 1)$ such that the optimal level of insurance is the same with background risk as without background risk for this critical value of β. This follows trivially from the fact that the optimal level of insurance coverage is strictly increasing in β, together with the fact that the optimal levels of coverage with background risk are lower than the no-background-risk optimum α^* for the case with $\beta = 0$ and higher than the no-background-risk optimum α^* for this case with $\beta = 1$. It thus also follows that the optimal level of insurance in the presence of the background risk is always higher [lower] than the no-background-risk optimum α^* whenever $\beta > \hat{\beta}$ [$\beta < \hat{\beta}$].

We summarize the above results in the following Proposition:

Proposition 1: *Consider a risk-averse and prudent consumer facing the background risk $\beta\tilde{\varepsilon}$ in the loss states and $(1-\beta)\tilde{\varepsilon}$ in the no-loss states.*

(i) *The optimal level of insurance is an increasing function of β.*
(ii) *There exists a critical value $\hat{\beta} \in (0, 1)$ such that the optimal level of insurance is the same with background risk as without background risk.*
(iii) *The optimal level of insurance is always higher [lower] than the no-background-risk level of coverage for $\beta > \hat{\beta}$ [$\beta < \hat{\beta}$].*

In the case where preferences are “risk vulnerable,” as described by Gollier and Pratt (1996), we know that insurance is always higher in the presence of the zero-mean background risk for the case where $\beta = \frac{1}{2}$. Since risk vulnerability requires positive prudence, it follows from the above Proposition that, for risk-vulnerable preferences, $\hat{\beta} \leq \frac{1}{2}$.

CHANGES IN BACKGROUND RISK

In the above sections, we restricted background risk for the individual to take the form $\tilde{\theta}_L = \beta\tilde{\varepsilon}$ in those states of the world for which $x = L$ and the form $\tilde{\theta}_N = (1-\beta)\tilde{\varepsilon}$ in the states of the world for which $x = 0$. The case in which $\beta = \frac{1}{2}$ (equivalent to the case of an independent background risk) and where $\tilde{\varepsilon}$ becomes riskier via a mean-preserving increase in risk was examined by Eeckhoudt, Gollier, and Schlesinger (1996), who derive fairly complicated necessary and sufficient conditions on preferences to ensure that insurance demand increases in the face of this increase in background risk. Meyer and Meyer (1998) examine several stronger conditions on changes in risk that, together with somewhat weaker conditions on preferences, are sufficient for an increase in insurance demand.

In this section we examine a stochastic change in the background risk, but only in the loss states of the world or only in the no-loss states of the world. In this setting, the effect of stochastic change in the background risk are much easier to ascertain.

To this end, let $\alpha^{**} > 0$ denote the optimal level of coinsurance for the case of insurance under the background risk $\tilde{\theta}_L = \tilde{\theta}_N = \frac{1}{2}\tilde{\varepsilon}$. We assume that $m > 0$, so that $\alpha^{**} < 1$. Here we do not compare the level of insurance with versus without the background risk; rather, we consider a stochastic change in either $\tilde{\theta}_L$ or $\tilde{\theta}_N$.

$$\left.\frac{dEU}{d\alpha}\right|_{\alpha^{**}} = [p(1-p-mp)L]Eu'\left(y_L^* + \frac{1}{2}\tilde{\varepsilon}\right) - [p(1-p-mp+m)L]Eu'\left(y_N^* + \frac{1}{2}\tilde{\varepsilon}\right) = 0. \tag{7}$$

If we change $\tilde{\theta}_L$ to be riskier in the sense of Rothschild and Stiglitz (1970), we know from their results that $Eu'(y_L^* + \tilde{\theta}_L) \geq Eu'(y_L^* + \frac{1}{2}\tilde{\varepsilon})$ for any arbitrary increase in risk if and only if $u'(\cdot)$ is a convex function, i.e., whenever the individual is prudent. In a similar manner, it follows that an increase in the risk of $\tilde{\theta}_N$ alone will actually decrease coverage for a prudent individual.

Note that an increase in β, as we examined via Equation (6) is a special case of an increase in the risk of $\tilde{\theta}_L$, paired with a decrease in the risk of $\tilde{\theta}_N$. Moreover, it is easy to extend the above analysis to higher order risk changes. For example, if we change $\tilde{\theta}_L$ via an increase in the downside risk of the background risk $\frac{1}{2}\tilde{\varepsilon}$, as defined by Menezes, Geiss, and Tressler (1980), then insurance coverage will increase whenever the first derivative of $u'(\cdot)$ is a concave function. Assuming sufficient differentiability of u, this condition is equivalent to $u'''' \leq 0$. The condition is commonly referred to as "temperance."[8] These results are summarized as follows:

Proposition 2: *Let α^{**} denote the optimal level of coinsurance for the case of insurance under the background risk $\tilde{\theta}_L = \tilde{\theta}_N = \frac{1}{2}\tilde{\varepsilon}$.*

[8] The terminology "temperance" was coined by Kimball (1992) and is discussed more fully in Gollier (2001) and in Eeckhoudt and Schlesinger (2006). A downside increase in risk essentially consists of mean-preserving spreads in the distribution of $\tilde{\varepsilon}$ at lower ε levels, each paired a mean-preserving contraction at a higher ε level. To be an increase in downside risk, these transformations of the distribution (the pairs of mean-preserving spreads and contractions) are each restricted to also preserve the variance. See Menezes, Geiss, and Tressler (1980).

(i) *Any mean-preserving increase in risk for $\tilde{\theta}_L$ [mean-preserving decrease in risk for $\tilde{\theta}_N$] will cause the level of insurance purchased to rise if and only if the individual is prudent.*

(ii) *Any mean-and-variance-preserving increase downside risk for $\tilde{\theta}_L$[mean-and-variance-preserving decrease in downside risk for $\tilde{\theta}_N$] will cause the level of insurance purchased to rise if and only if the individual is temperate.*

It is perhaps not surprising that localized risk changes have much more predictable effects than do global changes. Eeckhoudt, Gollier, and Schlesinger (1991) examine localized changes in the loss distribution for a case of deductible insurance. One of their observations, for example, is that an increase in risk that is limited to that part of the loss-distribution for which losses are less than the deductible has no effect on the insurance premium. However, it does make wealth riskier in states of the world with small losses. Therefore, a prudent individual would increase the deductible on the insurance contract, for the express purpose of saving some of the premium dollars to better handle these small losses. In their result, the key is that the change in risk affects only states of the world in which there is no insurance indemnity, only a premium cost.[9] In our proposition above, it is the fact that the localized change is background risk affects only the loss states, or only the no-loss states, that drives the results.

INTERPERSONAL COMPARISONS OF INSURANCE DEMAND

Kihlstrom, Romer, and Williams (1981) show that if consumer v is more risk averse than consumer u, so that consumer v would purchase more insurance when $m > 0$ in the absence of any background risk, it might not follow that consumer v will continue purchase more insurance than consumer u in the presence of an independent background risk. However, consumer v will continue to purchase more insurance than consumer u in the presence of the background risk if either v or u exhibits nonincreasing absolute risk aversion.[10] In our model, this is the case where $\beta = \frac{1}{2}$. But what about the cases where $\beta \neq \frac{1}{2}$?

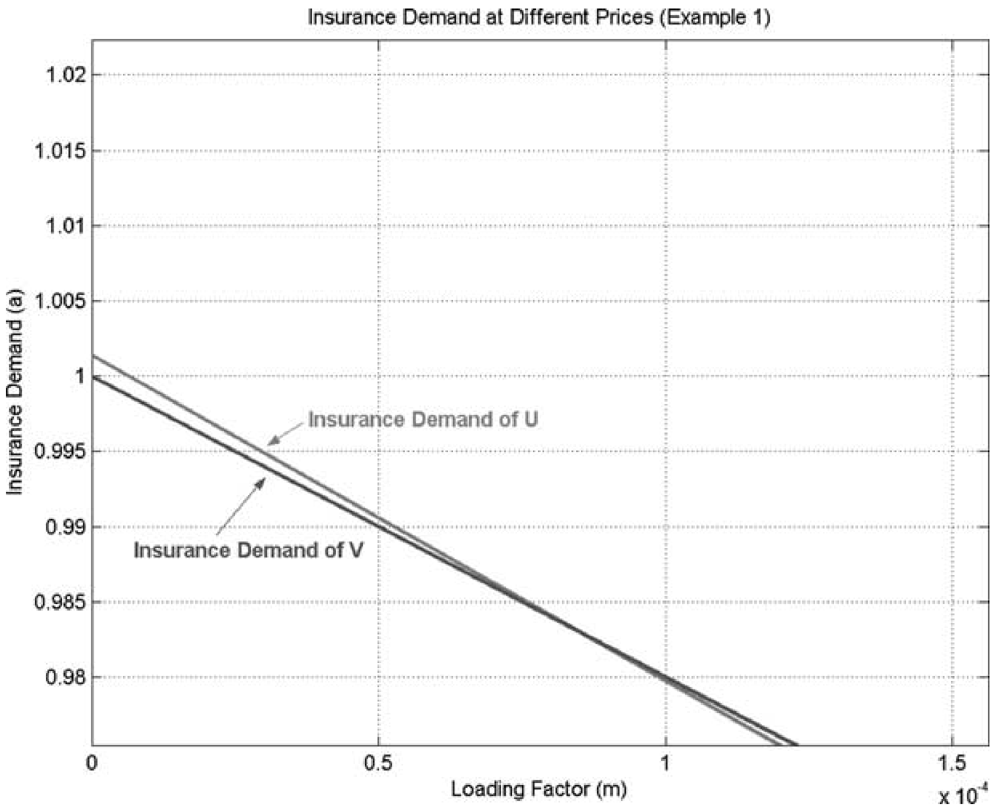
It turns out that nonincreasing absolute risk aversion by one of the two consumers is no longer sufficient for v to purchase more insurance in cases where $\beta \neq \frac{1}{2}$. We illustrate this for the two cases where $\beta = 1$ and $\beta = 0$ by way of examples. In both examples below, we set $W = 1$, $L = 0.5$, $p = 0.01$ and the random variable $\tilde{\varepsilon}$ takes on the values ± 0.3, each with a 50 percent chance of occurrence.

Example 1: Let $\beta = 1$, so that a background risk occurs only in the loss states. We define $u(w) = -(w + 160)^{-0.5}$ and $v(w) = -(w - 100)^2$ for $0 < w < 100$. Thus absolute risk aversion is easily calculated as $A_u(w) = 1.5(w + 160)^{-1} < 0.0094$ for all w and $A_v(w) = (100 - w)^{-1} > 0.010$ for all w. Thus, v is everywhere more risk averse than u. Moreover, risk aversion is decreasing in wealth for consumer u. The optimal level of coinsurance is shown as a function of the loading factor m in Figure 1. In this figure we see that the

[9] See Eeckhoudt, Gollier, and Schlesinger (1991), Proposition 2.

[10] This is a sufficient condition. Nachman (1982) introduces a more general condition: if $A_u(w)$ denotes the measure of absolute risk aversion as a function of wealth for consumer u, and $A_v(w)$ denotes the same measure for v, then v will continue to purchase more insurance than u in the presence of the independent background risk if there exists a nonincreasing function $A(w)$, such that $A_v(w) \geq A(w) \geq A_u(w)$ for all w.

FIGURE 1
$\beta = 1$

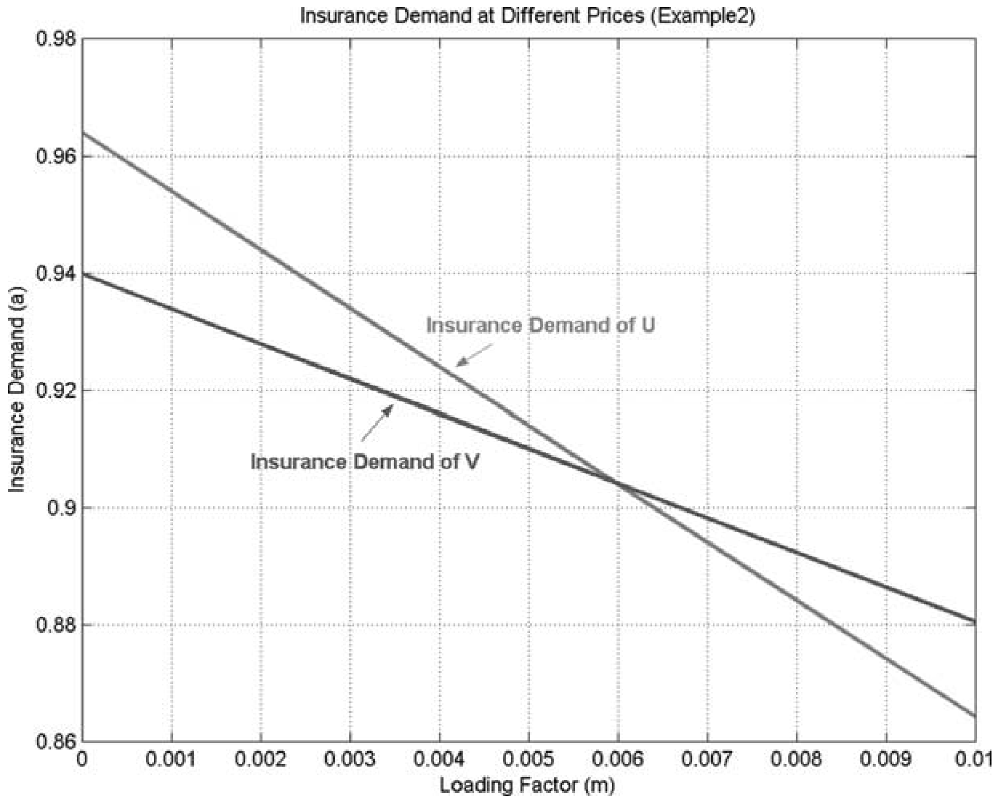


demand for consumer v is sometimes higher and sometimes lower than for person u, even though we know that for each m consumer v would demand more insurance than consumer u in the absence of any background risk. This example also illustrates how for the prudent consumer u, more than full coverage is demanded at a fair price, due to a precautionary effect. On the other hand, since $v''' = 0$ everywhere, there is no precautionary effect for consumer v.

Example 2: Let $\beta = 0$, so that a background risk occurs only in the no-loss states. We define $u(w) = \ln(w + 4)$ and $v(w) = \ln(w + 2)$. Thus, absolute risk aversion is easily calculated as $A_u(w) = (w + 4)^{-1}$ and $A_v(w) = (w + 2)^{-1}$. Hence, consumer v is once again everywhere more risk averse than consumer u. In this example, risk aversion for both consumers is decreasing in wealth. Since both consumers are prudent, we see in Figure 2 that only partial coverage is demanded, even at a fair price with $m = 0$. This is due to the precautionary effect when $\beta = 0$. But we also see in Figure 2 how the demand for consumer v is not everywhere higher than for the less risk averse consumer u.

Since prudence played a role in determining the demand for insurance, it is not too surprising that comparative measures of prudence between consumers v and u play a role in determining the effects of background risk on their relative insurance demand.

FIGURE 2
$\beta = 0$



This is easily shown using Kimball's (1990) "precautionary premium." For any thrice differentiable utility u and any zero-mean risk $\tilde{\varepsilon}$, define the precautionary premium $\varphi_u(y, \tilde{\varepsilon})$ implicitly via $u(y - \varphi_u(y, \tilde{\varepsilon})) \equiv Eu(y + \tilde{\varepsilon})$. Kimball shows that, for any arbitrary wealth y and arbitrary risk $\tilde{\varepsilon}, \varphi_v(y, \tilde{\varepsilon}) > \varphi_u(y, \tilde{\varepsilon})$ if and only if consumer v is more prudent than consumer u.

We consider first the case where the background risk manifests itself only in the loss state, $\beta = 1$.

Proposition 3: *Let $\beta = 1$. Given any $m \geq 0$, consumer v will always purchase more insurance than consumer u for every arbitrary W, L, and $\tilde{\varepsilon}$, if and only if v is both more risk averse and more prudent than consumer u.*

Proof: The first-order condition for consumer u, (4), can be rewritten as

$$\left.\frac{dEU}{d\alpha}\right|_{\alpha^*} = [p(1 - p - mp)L]u'\left(y_L^* - \varphi_u\right) - [p(1 - p - mp + m)L]u'\left(y_N^*\right) = 0, \quad (8)$$

where α^* denotes the optimal level of insurance coverage for consumer u. Since utility is unique only up to an affine transformation, assume without losing generality that $v'(y_N^*) = u'(y_N^*)$.[11] Now

$$\left.\frac{dEV}{d\alpha}\right|_{\alpha^*} = [p(1-p-mp)L]v'\left(y_L^* - \varphi_v\right) - [p(1-p-mp+m)L]u'\left(y_N^*\right)$$
$$> [p(1-p-mp)L]v'\left(y_L^* - \varphi_u\right) - [p(1-p-mp+m)L]u'\left(y_N^*\right)$$
$$> [p(1-p-mp)L]u'\left(y_L^* - \varphi_u\right) - [p(1-p-mp+m)L]u'\left(y_N^*\right). \qquad (9)$$

The first inequality in (9) stems for the fact that $\varphi_v > \varphi_u$ and that v' is decreasing. The second inequality stems from Theorem 1 in Pratt (1964), together with the assumption that $v'(y_N^*) = u'(y_N^*)$. Comparing (9) with (8), it follows that $\frac{dEV}{d\alpha}|_{\alpha^*} > 0$, so that the optimal level of insurance coverage for consumer v must be higher than α^*.

This proves sufficiency. The necessity of higher risk aversion for consumer v is seen most readily by letting the epsilon risk get very small. Likewise, necessity for higher prudence follows by letting the size of the loss, L, approach zero. □

The two scenarios described for necessity in the proof above help with the intuition of what is involved. Obviously, if the background risk is negligible, we are reduced to the well known result that a propensity for more insurance is equivalent to higher degree of risk aversion. Similarly, if the loss size L is negligible, then a "loss" is only important due to the $\tilde{\varepsilon}$ risk that accompanies it. In this case, insurance has only a precautionary value, as described by Kimball (1990).

Proposition 4: *Let $\beta = 0$. Given any $m \geq 0$, consumer v will always purchase more insurance than consumer u for every arbitrary W, L, and $\tilde{\varepsilon}$, if and only if v is both more risk averse than consumer u, but less prudent than consumer u.*

The proof is almost identical to Proposition 3 and is omitted. It is important to note that in Proposition 4 we require that v be less prudent than u. The intuition here should be obvious from the previous section. For the consumer who is prudent, the prudence effect is to reduce the level of insurance. This allows the consumer to save some of the premium as a precaution against the $\tilde{\varepsilon}$ risk, which occurs only in the no-loss states. If consumer v is less prudent, she has a smaller precautionary effect to reinforce her stronger effect of higher risk aversion. If consumer v is both more risk averse and more prudent than consumer u, then we cannot say who will purchase more insurance, *a priori*.

As we remarked earlier, the case where $\beta = \frac{1}{2}$ corresponds to an independent background risk and was essentially examined by Kihlstrom, Romer, and Williams (1981) and Nachman (1982). However, a similar framework was also examined by Ross (1981). Ross examined the case where the risky wealth always had the same conditional mean, regardless of the realized value of the background risk. In particular, for our setting, if random wealth is denoted as $\tilde{y}$ and the background risk is $\tilde{\theta}$, then $E(\tilde{y} \mid \tilde{\theta} = c) = E\tilde{y}$ (the unconditional mean) for every c in the support of the random

[11] In other words, simply multiply the original utility v by the positive constant $u'(y_N^*)/v'(y_N^*)$, which represents the same risk preferences as the original utility v.

variable $\tilde{\theta}$. This is clearly true when $\beta = \frac{1}{2}$, but clearly not true for $\beta \neq \frac{1}{2}$. Since $\tilde{\theta}$ is defined conditionally as either $\beta\tilde{\varepsilon}$ or $(1-\beta)\tilde{\varepsilon}$, if $\beta = 0$, then any value of $\tilde{\theta} \neq 0$ will signal that we are in the states of the world in which no loss has occurred. So, for example, if $\tilde{\varepsilon}$ has a continuous distribution, then a value of $\tilde{\theta} = 0$ will signal that a loss has occurred.[12] Thus, we see in this case that $E(\tilde{y}\,|\,\tilde{\theta} = 0) = y_L$, whereas $E(\tilde{y}\,|\,\tilde{\theta} \neq 0) = y_N$.

In terms of risk aversion, Ross (1981) proposed a stronger measure of risk aversion for which v is more risk averse than u in the strong sense of Ross if $\exists\lambda > 0$, such that

$$\frac{v''(x)}{u''(x)} \geq \lambda \geq \frac{v'(y)}{u'(y)} \quad \text{for all } x \text{ and all } y. \tag{10}$$

Clearly this definition is stronger than the usual (Arrow-Pratt) definition of more risk averse. Ross also shows how the inequality in (10) above, implies the existence of a real-valued function G, with $G' < 0$ and $G'' < 0$, such that $v(y) = u(y) + G(y)$ for all y.

Turning to insurance, let α^* denote the optimal level of insurance for consumer u. It follows easily that for $\beta = \frac{1}{2}$ and $m > 0$, for consumer v we have

$$\begin{aligned}\left.\frac{dEV}{d\alpha}\right|_{\alpha^*} &= [p(1-p-mp)L]E\left[u'\left(y_L^* + \frac{1}{2}\tilde{\varepsilon}\right) + G'\left(y_L^* + \frac{1}{2}\tilde{\varepsilon}\right)\right] \\ &\quad -[p(1-p-mp+m)L]E\left[u'\left(y_N^* + \frac{1}{2}\tilde{\varepsilon}\right) + G'\left(y_N^* + \frac{1}{2}\tilde{\varepsilon}\right)\right] \\ &= [p(1-p-mp)L]EG'\left(y_L^* + \frac{1}{2}\tilde{\varepsilon}\right) \\ &\quad -[p(1-p-mp+m)L]EG'\left(y_N^* + \frac{1}{2}\tilde{\varepsilon}\right) > 0.\end{aligned} \tag{11}$$

The last inequality follows since $y_L^* < y_N^*$ and G' is negative and decreasing. Thus, for every ε, $|G'(y_L^* + \frac{1}{2}\varepsilon)| < |G'(y_N^* + \frac{1}{2}\varepsilon)|$; i.e., the first term is less negative than the second term. As a consequence, the optimal level of insurance will be higher for the more Ross-risk-averse consumer v.

Since $\frac{dEV}{d\alpha}$ is continuous in β, the strict inequality in (11) must hold for β close enough to $\frac{1}{2}$. However, in general, for $\beta \neq \frac{1}{2}$, the above analysis need not hold. This can be seen in our two previous examples, with $\beta = 1$ and $\beta = 0$.

In Example 1, with $\beta = 1$, straightforward calculations show that $\frac{v''(w)}{u''(w)} = \frac{8}{3}(w + 150)^{\frac{5}{2}} \geq 863,510$, whereas $\frac{v'(w)}{u'(w)} = 4(100-w)(w+160)^{\frac{3}{2}} \leq 809,540$, for all w such that $0 \leq w \leq 100$. Thus, consumer v is more risk averse than consumer u in the strong sense

[12] This follows since, with a continuous distribution, the probability that $\tilde{\varepsilon} = 0$ is itself zero. So that observing $\tilde{\theta} = 0$ guarantees that we must be observing $\beta\tilde{\varepsilon}$ (since $\beta = 0$). Thus, we must be in a state of the world in which a loss has occured.

of Ross (1981) over all relevant wealth values. Yet we see in Example 1 that consumer v sometimes buys more insurance and sometimes buys less insurance than consumer u.

Similarly, in Example 2, one can show that $\frac{v''(w)}{u''(w)} = (\frac{w+4}{w+2})^2 \geq \frac{9}{4}$, whereas $\frac{v'(w)}{u'(w)} = \frac{w+4}{w+2} \leq 2$, for all w in the relevant wealth range, which here is $0 \leq w \leq 2$. Thus, we once again have consumer v is more risk averse than consumer u in the strong sense of Ross (1981). Yet, as in Example 1, consumer v sometimes buys more insurance and sometimes buys less insurance than consumer u.

CONCLUDING REMARKS

This article considered a zero-mean background risk that is uncorrelated with insurable losses but is not necessarily statistically independent. We studied the effect of such a background risk on the demand for insurance. If there is substantially more background risk in the states of the world with an insurable loss, the effect will be to increase the demand for insurance for any prudent person. In the case of a fair premium, the consumer will demand more than full insurance, contrary to Mossin's Theorem (1968). The rationale for this extra demand is a precautionary motive for insurance. Although the extra insurance in no way hedges the background risk, the extra wealth is on hand for precautionary purposes to help the individual in the event that the realized background risk turns out to be negative. This is identical to the motive for precautionary savings against future income risk, as described by Kimball (1990).

If the background risk is substantially larger in states where no insurable loss occurs, then the precautionary motive is to purchase less insurance. This is to reduce the premium and thus have more money on hand to handle the potential adverse effects of the background risk in these no-loss states. The exact extent of what we mean by background risk being "substantially larger" in one set of loss states is made more precise in the text.

The above reasoning shows that both risk aversion and prudence provide us with information about total insurance demand. Risk aversion is important in gauging the hedging demand (i.e., the usual reason for insurance purchases), whereas prudence gauges the precautionary demand. Similarly, in comparing two individuals facing identical risky choices, both comparative risk aversion and comparative prudence are required. Risk aversion alone, even in the stronger sense of Ross (1981), is not sufficient to guarantee that the more risk averse individual necessarily buys more insurance.

Such state-dependent types of background risk would seem to make sense in many risk-taking situations. Our modeling here is not normative and we make no claim about values of β that might be relevant. If a reader thinks that β close to one makes sense in one situation, but β closer to one-half seems more reasonable in another setting, so be it. Our point is simply that, for any given weighting of such background risk in loss versus no-loss states of the world, precautionary effects should not be ignored.

Obviously the model set up in this article is rather simple. More realistic loss distributions and more intricate state-dependencies for the background risk are bound to make the analysis very complicated. Still, we hope that some of the issues addressed here help with setting the "building blocks" for more complicated scenarios.

APPENDIX

We show here that $E[u''(y_L^* + \beta\tilde{\varepsilon})\tilde{\varepsilon}] > 0$. The proof for $E[u''(y_N^* + (1-\beta)\tilde{\varepsilon})\tilde{\varepsilon}] > 0$ is essentially the same, so that $\partial^2 EU/\partial\alpha\partial\beta$ in (6) is positive as claimed. Let F denote the distribution function for $\tilde{\varepsilon}$. Assuming that we have prudence, $u''' > 0$, then for any $\beta > 0$ we have

$$\begin{aligned}
E\left[u''\left(y_L^* + \beta\tilde{\varepsilon}\right)\tilde{\varepsilon}\right] &= \int_{-\infty}^{+\infty} u''\left(y_L^* + \beta\varepsilon\right)\varepsilon \, dF(\varepsilon) \\
&= \int_{-\infty}^{0} u''\left(y_L^* + \beta\varepsilon\right)\varepsilon \, dF(\varepsilon) + \int_{0}^{+\infty} u''\left(y_L^* + \beta\varepsilon\right)\varepsilon \, dF(\varepsilon) \\
&> \int_{-\infty}^{0} u''\left(y_L^*\right)\varepsilon \, dF(\varepsilon) + \int_{0}^{+\infty} u''\left(y_L^*\right)\varepsilon \, dF(\varepsilon) = u''\left(y_L^*\right)\int_{-\infty}^{+\infty} \varepsilon \, dF(\varepsilon) = 0.
\end{aligned}$$

We should point out that the inequality above reverts to an equality when $\beta = 0$. This is simply due to the fact that risk aversion is a second-order effect, so that the first infinitesimal of $\tilde{\varepsilon}$ risk has no effect of decision making. But in this case, $1 - \beta > 0$, so that a strict inequality will hold for $E[u''(y_N^* + (1-\beta)\tilde{\varepsilon})\tilde{\varepsilon}] > 0$ and $\partial^2 EU/\partial\alpha\partial\beta$ in (6) is thus strictly positive.

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