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Heavy-tailed Behavior of Commodity Price Distribution and Optimal Hedging Demand

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Abstract

This study explores the importance of imposing a correct distributional hypothesis in a risk management strategy, by comparing hedge ratios under the restrictive normality assumption to those under the generalized stable distribution. Concepts are illustrated for the case of a representative Pennsylvania dairy farm manager who purchases corn as a feed input. The results show that time processes of corn prices and basis risk in five Pennsylvania regions do not correspond to the normal distribution, and they more correctly correspond to one of the stable distribution set. The estimated hedge ratios under the stable distribution are typically larger than those under the normal distribution. The difference would be a bias from imposing a wrong distributional assumption.

Introduction

The subject of this study is the comparison of hedge ratios under a restricted distributional assumption for price changes with those under a generalized distribution. In particular, *restricted* refers to limitations in describing significantly large price changes, that is, heavy-tail, or fat-tail, behaviors of price distribution. In this study, we choose the conventional *normal distribution* as a restricted case and *stable distribution* as a generalized case. At the outset, we test whether there are heavy-tail behaviors in commodity price changes and whether such movements are explained better by stable distribution or normal distribution. Futures hedging demand are presented with respect to a representative hedger who needs a long hedging to avoid risks from price changes, and the effects of the normality restriction and its relaxation on the estimated hedge ratios will be emphasized.

Our application is to Pennsylvania (PA) dairy farm managers who purchase corn as a feed input. Although many PA dairy farm managers grow their own, they still need to purchase corn and other crops as feed inputs. Commodity prices of interest are local corn cash prices in five farming regions of PA and corn futures prices in the

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Chicago Board of Trade (CBOT). Demand for futures hedging is estimated for each local region, under different levels of risk aversion.

Farmers and agribusiness managers forecast commodity price in the next period using the best techniques available to them, but price in the next period may be extreme or irregular, deviating from their expectation. Agricultural commodity prices are affected by many factors. Among them, shocks to supply are major sources and their distributions are not regular, implying that there are substantial chances of abrupt, large changes in their processes. In practice, mild price variation may not cause serious economic impacts on producers' or buyers' profits, but unexpected large changes would be a significant risk. The most important reason for hedging in futures markets or having insurance contracts is to avoid risks from unexpected large price changes. The large changes from the long-run average may correspond to the tails of a price distribution. Accordingly, an effective capture of the tail behaviors would help a risk management tool perform better. However, such extreme events do not receive much weight under the conventional normal distribution, which screens out the outliers so that the distribution is not appropriate for describing the tail behavior of financial and economic data. Therefore, when commodity prices show distinct heavy-tailed behaviors, at a substantive level, it would be interesting to test how optimal hedge ratios under the normality restriction differ from those under a distributional model that captures tail behaviors of price movements more efficiently.

Related to agricultural commodity markets, two recent studies considered nonnormal behaviors of futures or cash prices. Chambers and Bailey (1996) suggest that commodity cash prices do not follow the normality and show that the time processes of commodity cash prices have considerable numbers of large spikes—periods when the price jumps to a very high or low level relative to its long-run average. This property corresponds to the fat-tailed behavior of economic and financial data, called *leptokurticity*. Corazza, Malliaris, and Nardelli (1997) examine abnormal (heavy tails and higher peakedness) behaviors of six agricultural futures price series and find that the series are characterized better by the stable distribution.

Commodity and financial time series have been traditionally assumed to follow the normal distribution. The normal distribution is easy to apply to economic analyses, and many of its properties have been revealed to economists. For these reasons, the distributional hypothesis has been embraced in economic and financial analyses, despite the fact that empirical evidences show distinct anomalies from the normal distribution. Empirical studies have found that financial and economic data have distributions with fatter tails and higher peaks than the normal distribution in many cases, suggesting the need for finding a distributional model that fits the real data better. Refer, for example, to Leitch and Paulson (1975), Fielitz and Rozelle (1983), Hall, Brorsen, and Irwin (1989), McCulloch (1996), and Corazza, Malliaris, and Nardelli (1997). A class of studies has tried to discover a more generalized distributional model that would include a broad range of different distributions, and the stable distribution,[1] which

[1] The word *stable* is used because the shape of the distribution is stable or unchanged under the sums of same-type processes. That is, if two independent random variables with the same type of the distribution are combined linearly, then the resulting distribution is also a random variable with the same type. There are several different names for the stable distribution:

explains various abnormal behaviors of financial and economic data relatively well, is considered promising (cf. Fama and Roll, 1971; Cornew, Town, and Crowson, 1984; Nolan and Panorska, 1997; Nolan, 1999). With specific values of its parameters, the stable distribution corresponds to a specific distribution with a different character. The normal distribution is a special case in the stable distribution set.

This study extends the literature in two directions. First, it tests heavy-tailed behaviors of selected commodity prices (corn cash and futures prices) and compares the performance of the stable distribution with that of the normal distribution in capturing the abnormal processes. Second, as an empirical application, it assesses the extent to which a different distributional assumption may lead to different estimation of optimal hedge ratios.

The empirical procedure of this article to analyze the main questions is as follows: First, a hedging demand model is constructed, and optimal hedge ratios are estimated by restricting the price movements to follow the normal distribution. Second, we test whether the cash and futures price series follow the normal distribution.[2] If the prices show abnormal behaviors, hedge ratios under the normality restriction could be nonoptimal, and therefore imposing the normality condition might cause a bias in the estimation of optimal hedge ratios. Third, we estimate the stable distribution fitted to the commodity prices. Lastly, we compare the hedge ratios under the normality assumption with those under the stable distribution, which is a more generalized distributional model where asset price changes are not restricted by the narrow class of normal distribution but are specified by a specific distribution more close to their data-generating processes.

The remainder of the study proceeds as follows. A hedging demand model is constructed in the second section. Data used in this study are explained in the third section and optimal hedge ratios under the normality restriction are presented in the fourth section. The stable distribution is fitted to our price series in the fifth section and the optimal hedge ratios are reestimated under the generalized distribution in the sixth section. Concluding remarks are presented in the last section.

A HEDGING DEMAND MODEL

Assume that there is a representative hedger who needs to buy a commodity product as an input in the local cash market and the hedger maximizes a von Neumann–Morgenstern (1944) expected utility and derives a Marshallian-type demand system for hedging. In this model, hedging product is a *good* and hedging cost is the *price* of the good. Here, only a futures contract is considered, but an extension to multiple goods will be straightforward. Hedging cost may include transaction costs, brokers' fees, opportunity costs, and learning costs. Consider two regions producing a same good. Region 0 is a central marketplace that contains a futures market and a cash market, and the markets comprise risk-averse hedgers and risk-neutral speculators.

Pareto–Levy distribution, stable Paretian distribution, stable family distribution, and stable distribution.

[2] Most hedging studies typically assume the normal distribution of cash and futures prices changes (cf. Hirshleifer, 1988; Kroner and Sultan, 1993; Simman, 1993; Lence, 1995; Locke and Venkatesh, 1997; Haigh and Holt, 2000).

Region 1 is a local place with only a cash market and hedgers there are risk-averse. For simplicity, it is assumed that there is only one risk management tool: the futures contract. That is, local hedgers in region 1 can access only the central futures market to avoid risk from cash price changes. It is also assumed that there are only two time periods: time t and time $t+1$. Hedgers place hedging at time t and lift or revise it at time $t+1$. The time can be any unit from minutes to months. Assume that there is no multiperiod trading, that is, hedgers buy contracts that are the closest to hedgers' desired hedging dates. The hedger's purpose is only to reduce risk from price variation from time t to time $t+1$.

The representative hedger needs to buy the good in the local cash market at period $t+1$. His profits expected on the hedged portfolio at time t can be written as follows:

$$E_t[\Pi_{t+1}] = -x \cdot \tilde{p}_{1\cdot t+1} + x_f(\tilde{f}_{t+1} - f_t - \tau), \tag{1}$$

where E_t denotes the mathematical expectation operator for time $t+1$, x represents the amount of input, x_f denotes the amount hedged in the futures market, $p_{1,t+1}$ represents cash price in the local market at time $t+1$, f_t denotes futures price at time t in the central market, and τ represents the *price* of hedging in the futures market, that is, hedging cost faced by the hedger. The hedging cost is assumed to be constant; thus τ is both marginal and average hedging cost. Prices denoted with a t subscript are known at time t, whereas prices denoted with a $t+1$ subscript are unknown, and thus $p_{1,t+1}$ and f_{t+1} are treated as random variables and denoted by the $\sim$ symbol.

The first term on the right-hand side is the total cost for purchasing grain in the local market at time $t+1$; therefore, a negative sign is added to the term. Since our objective is to estimate an optimal hedge ratio for a given amount of the input, x can be fixed to a specific level. The second term is the return from engaging in the commodity futures market. The representative producer needs a long hedging; thus, the sign of the second term is positive. The hedger lifts the futures contracts at time $t+1$. If $\tilde{f}_{t+1} > f_t + \tau$, the trader would make money on the transaction. Alternatively, if $\tilde{f}_{t+1} < f_t + \tau$, the trader would lose on the transaction.

If we divide Equation (1) by the total amount of grain needed, x, then the objective of the hedger is changed to maximize *per unit* expected profit, and the choice variable is hedge ratio, h $(=x_f/x)$, rather than quantity hedged, x_f, as follows:

$$E_t[\pi_{t+1}] = -\tilde{p}_{1,t+1} + h(\tilde{f}_{t+1} - f_t - \tau), \tag{2}$$

The hedger's objective is to maximize expected utility with respect to h. The state variables are p_t, f_t, and τ, and a distribution function, $f(\cdot)$, can also be considered a state variable. Assume that the utility function $U(\cdot)$ is continuous, monotonic increasing, and strictly concave, where the utility is a function of expected profits. Our concern is to derive the demand function for hedging $h = h(\tau, r\,|\,p_{1,t}, f_t, f(\cdot), U(\cdot))$, where r represents the coefficient of absolute risk aversion, $r = -U''(x)/U'(x)$. The objective function of the hedger is

$$\underset{h}{\text{Max}}\; E[U(\pi_{t+1})], \tag{3}$$

where $U(\cdot)$ denotes a von Neumann–Morgenstern utility function, and $E[U(\cdot)]$ represents expected value of $U(\cdot)$ at time $t+1$, based on information available at time t. The first-order condition with respect to h is

$$E[U'(\pi_{t+1}) \cdot (\tilde{f}_{t+1} - f_t - \tau)] = 0. \tag{4}$$

The second-order condition is satisfied by the assumption of strict concavity of $U(\cdot)$.

To characterize the solution of Equation (4), it is necessary to be specific about the relationship between local cash price and futures price. Following Lapan, Moschini, and Hanson (1991) and Frechette (2001), the futures price is written as a linear function of the cash price:

$$\tilde{f} = a + b\tilde{p}_1 + \tilde{\nu}, \tag{5}$$

where $\tilde{\nu}$ is a random variable, independent to $\tilde{p}_1$, and $E[\tilde{\nu}] = 0$. In this case, the first-order condition is rewritten as follows:

$$E[U'(\pi_{t+1}) \cdot (a + b\ \tilde{p}_{1,t+1} + \tilde{\nu}_{t+1} - f_t - \tau)] = 0. \tag{6}$$

In the first-order condition, random variables are $\tilde{p}_{1,t+1}$ and $\tilde{\nu}_{t+1}$. It can be computed through a numerical double-integral over $\tilde{p}_{1,t+1}$ and $\tilde{\nu}_{t+1}$, and the expected utility can be maximized with respect to h.

DATA

The data set consists of weekly corn cash prices for the five regions of PA collected by the PA Department of Agriculture (PDA) and the nearby corn futures price in the CBOT. This empirical analysis therefore implicitly assumes that decisions by the producers for purchasing input are made weekly. The corn cash prices were collected through surveys and phone calls and reported by PDA on Monday mornings, and the futures price is the previous Friday's settlement price for the nearby futures. The data start in January 1990 and end in December 1998. The unit for the prices is *cents* per bushel.

Sample variances are calculated by the equation

$$\mathrm{Var}(y) = \left(\frac{1}{n-1}\right) \cdot \sum_{i=1}^{n} (y_{t+1} - E_t y_{t+1})^2, \tag{7}$$

where y can be a series of cash price or futures price, and the variances are assumed to be constant over the sample period. It is also assumed that hedgers know the variances before the sample period began. Hedgers are not homogeneous; thus, their demands and valuations will differ by the sample period and information available to them.

Assume that markets are unbiased in this model. Therefore, f_t is the unbiased futures price. The expected values of futures and cash prices are calculated through an adaptive expectation model. Here, an autoregressive moving average (ARMA) is used for the adaptive expectation. The lags, p and q, in the ARMA model were chosen by the

TABLE 1
Descriptive Statistic for PA Data Set

Region	Mean of p_1	SD of p_1	Mean of ν	SD of ν
Southeast PA	293.01	5.31	0.001	2.85
Central PA	293.86	2.16	−0.012	0.81
South Central PA	290.21	3.12	0.013	4.67
Lehigh Valley	274.70	5.53	0.002	0.69
Western PA	281.01	4.09	−0.001	6.31

Notes: SD denotes standard deviation. The mean of Chicago corn futures price is 262.65 and the standard deviation of the futures price is 7.24.

AIC and SIC statistics. Equation (5) was fitted to the expected cash and futures prices and the basis risk, $\tilde{\nu}$, was estimated. Finally, the variances of cash price and basis risk were calculated using Equation (7). The summary statistics for the cash prices and basis risk set are presented in Table 1.

EMPIRICAL HEDGE RATIOS

The hedging demand model is applied to dairy producers in the five regions of PA. The local markets, region 1, are Southeastern, Central, South Central, the Lehigh Valley, and Western PA. The central marketplace, region 0, is Chicago. The dairy producers buy corn as an input in the local cash market at time $t+1$. They would need a long hedging to avoid risk from price variation in purchasing corn. Although the problem is static, solving the non linearity of the integrand is troublesome. The function $h(\cdot)$ solves the utility maximization problem and can be represented numerically under general conditions. Note that the distribution function, $\xi(\cdot)$, does not depend on the hedge ratio, h. Thus, the first-order condition becomes:

$$\int\int (a + b\,\tilde{p}_{1,t+1} + \tilde{\nu}_{t+1} - f_t - \tau) \times U'(\cdot)\xi(\nu_{t+1} \mid \cdot)\xi(p_{1,t+1} \mid \cdot)\, dp_{1,t+1}\, d\nu_{t+1} = 0, \quad (8)$$

where $\xi(p_{1,t+1} \mid p_{1,t}, \sigma_p^2)$ is the probability distribution function (*pdf*) for $p_{1,t+1}$, conditional on $p_{1,t}$ and σ_p^2; $\xi(\nu_{t+1} \mid \nu_t, \sigma_\nu^2)$ is the *pdf* for ν_{t+1}, conditional on ν_t and σ_ν^2; σ_p^2 denotes the variance of local cash price, p_1; and σ_ν^2 is the variance of ν. Since $\tilde{\nu}$ is orthogonal to $\tilde{p}_1$, the two *pdf*s are independent.

If a numerical root-finding algorithm is used, the first-order condition, Equation (8), can be solved quickly. To illustrate how an explicit solution for Equation (8) can be obtained, assume a special case of the distribution function, $\xi(\cdot)$, say, the uniform distribution over 0 to S, and a constant coefficient of absolute risk aversion, r. Begin with $U(x) = k \cdot e^{-rx}$, where $k < 0$, and break the integral into two pieces, one piece from 0 to S and the other from S to infinity. Then, the piece from S to infinity can be solved directly and the remaining piece from 0 to S can be solved by integration by parts, producing an inverse demand for the long hedging. That is, the expected utility problem can be computed by a numerical double-integral over $\tilde{p}_1$ and $\tilde{\nu}$, and it is maximized with respect to h. Here, the trapezoidal rule, specifically the

Newton–Cotes formula (cf. Ueberhuber, 1997), is used to calculate the integrals, and the simplex method is used for the optimization.[3]

One needs to set the coefficient of absolute risk aversion r to cover a range of possible risk preferences. Lapan and Moschini (1994), Lence (1995), and Frechette (2000) are used as a guide to select values for the coefficient. Here, three different values of absolute risk aversion are selected: 2.0 for high risk aversion, 0.2 for moderate risk aversion, and 0.02 for low risk aversion. In this study, hedging cost is assumed to be 0.5 and 1.0. Frechette (2000, 2001) states that a reasonable value for the hedging cost could be \$25 per contract of 5,000 bushels of corn. Thus, the costs equal to 0.5 and 1.0 can be considered a medium range of hedging cost for corn futures contracts in the CBOT. A sophisticated hedger may pay a hedging cost lower than 0.5, while the costs for a novice may be higher than 1.0.

From the first-order condition, Equation (6), an implicit form of demand function for long hedging, is derived. The shape and location of the demand curve for hedging depends on the coefficient of risk aversion (r), the hedging cost in futures markets (τ), the variances of cash price (σ_p^2), the variance of basis risk (σ_ν^2), and futures price at time t (f_t). The relationship between the hedging cost, τ, and the optimal hedge ratio, h^*, is expected to be negative. As hedging cost increases, the optimal hedge ratio decreases. The relationship between the coefficient of risk aversion, r, and h^* is expected to be positive. If the value of the coefficient of risk aversion increases, the slope of the demand curve for hedging goes to infinity, implying that as the hedger's risk aversion increases, the optimal hedge ratio increases and the demand for hedging becomes increasingly inelastic with respect to hedging cost. The relationship between the standard deviation of cash prices and h^* is expected to be positive, implying that as the volatility of commodity cash prices is higher, hedgers may want more hedging to avoid the risk from the changes in cash prices.

Table 2 summarizes the empirical results of hedging demand under the normal distribution restriction. The optimal hedge ratios change with different hedging costs. As hedging costs increase, optimal hedge ratios decrease. This suggests that the hedging cost is at play in changing optimal hedging positions. When hedging cost equals 0.5 and the risk-aversion coefficient equals 2.0, that is, $\tau = 0.5$ and $r = 2.0$, the optimal hedge ratios, h^*, are 0.6409, 0.4303, 0.3268, 0.5202, and 0.2169 for Southeastern, Central, South Central, the Lehigh Valley, and Western PA, respectively. If we change hedging cost, τ, to 1.0, while fixing r at 2.0, h^* falls slightly to 0.6366, 0.4192, 0.3216, 0.5179, and 0.2110. This suggests that although these differences are typically small, the hedging cost cannot be neglected for hedgers in these regions. This is also true for the cases of $r = 0.2$ and $r = 0.02$.

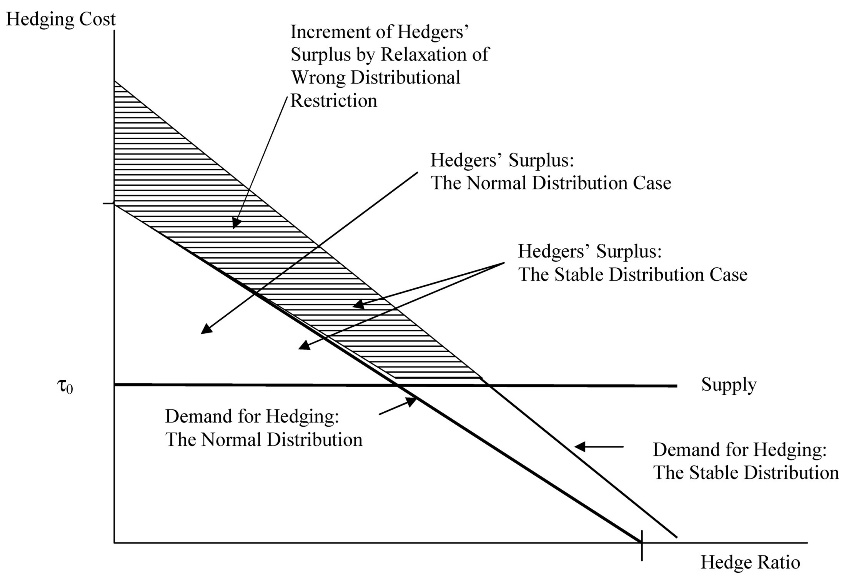
[3] To calculate a numerical double integral with respect to the normal or stable distribution, it is necessary to calculate the *pdfs* of cash price and basis risk for the normal and stable distribution. Note that Equation (5) suggests that the basis risk, ν, is orthogonal to cash price, p_1, that is, they are independent, which implies that the two *pdf*s are independent. Therefore, the univariate normal and stable *pdfs* were calculated. To calculate the normal *pdfs*, the mean and standard deviation in Table 1 were used, and to calculate the *pdfs* for the stable distribution, the estimated parameters in Table 3 were used.

TABLE 2
Hedge Ratios Under the Normal Distribution

	$r = 2.00$		$r = 0.2$		$r = 0.02$	
Region	$\tau = 0.5$	$\tau = 1.0$	$\tau = 0.5$	$\tau = 1.0$	$\tau = 0.5$	$\tau = 1.0$
Southeast PA	0.6409	0.6366	0.5995	0.5567	0.1919	0
Central PA	0.4303	0.4192	0.2585	0.1470	0	0
South Central PA	0.3268	0.3216	0.2769	0.2255	0	0
Lehigh Valley	0.5202	0.5179	0.4984	0.4760	0.2807	0.0560
Western PA	0.2169	0.2110	0.1601	0.1114	0	0

Notes: r denotes the coefficient of risk aversion. τ denotes the hedging cost, and it is assumed constant.

FIGURE 1
Hedge Ratios Under the Normal and Stable Distribution



Since we set the hedging cost as the *price* of the hedging goods, the curve can be interpreted as a Marshallian-type demand curve. The supply curve for hedging is represented by a horizontal line where hedging cost equals transaction cost in Figure 1. The hedging demand curve and supply curve intersect at a point, the equilibrium point. Without the central futures market, producers in the five geographical regions may not manage price risk, and this situation corresponds to when h^* is zero. The incremental value of the hedging opportunity embodied by the central futures market is the hedgers' surplus, the area of the smaller triangle above the supply curve as illustrated in Figure 1.

Up to now, it is assumed that the movements of $\tilde{p}_1$ and $\tilde{v}$ follow the normal distribution. However, we did not perform a formal test for the normality of the time processes. If the time series do not follow a narrow set of the normal distribution, the results may be biased and the hedge ratios may not be an optimal.

FITTING THE STABLE DISTRIBUTION TO CORN CASH AND FUTURES PRICES

The stable distribution has been used as a generalized distributional model that can explain the distributions of asset returns significantly better than the conventional normal distribution. Using the stability-under-addition test,[4] for example, Cornew, Town, and Crowson (1984) and So (1987) confirm that a better correspondence for futures price changes is usually obtained when using the stable distribution; that is, the distribution more adequately describes futures price changes (in particular, heavy tails) than does the normal distribution.

The stable distribution has four parameters, which can be estimated through the characteristic function of the distribution.[5] The estimates of the parameters provide information about the behavior of a time series: peakedness, skewness, location, and scale of the distribution. The estimates also help to confirm what type of distributional form a random variable follows. There are three special cases where one can write down closed form expressions for the density of the stable distribution and verify directly that they are stable: normal, Cauchy, and Levy distributions.[6] Other than these three distributions, there are no known closed-form expressions for the stable densities. Instead, the stable distribution can be characterized by its characteristic functional form as follows:

$$F(t) = E[\exp^{iyt}] = \exp[i\delta t - \gamma|t|^{\alpha}(1 + i\beta(t/|t|)w(t,\alpha))], \tag{9}$$

where y is a random variable, i is $\sqrt{-1}$, t is any real number, and $w(t, \alpha)$ is $(2/\pi)$ $(\tan|t|)$ if $\alpha = 1$; otherwise $w(t, \alpha)$ is $\tan(\alpha\pi/2)$.

The function has four parameters: α, β, δ, and γ. The first parameter, $\alpha \in (0, 2]$, is the characteristic exponent that accounts for the relative importance of the tails. It measures the peakedness of the distribution as well as the fatness of the tails. If α is equal to 2, then the distribution corresponds to the normal distribution with finite

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4 Fama (1963) suggests, using the fact that a stable distribution is invariant under addition, that the distribution of sums of the stable distribution is also a stable distribution with the same values of α, which is the most important parameter in the stable distribution and is called the characteristic exponent. Thus, the estimated values of α must be equal across the sums, provided that the underlying leptokurtic distribution is stable. Whether or not the stable distribution effectively explains the leptokurticity has been extensively tested using the stability-under-addition test (cf. Fama and Roll, 1971; Fielitz and Rozelle, 1983).

5 The characteristic function is the Fourier transform of a random variable, y, that is, $E[\exp(i \cdot y \cdot t)]$ for t, where t is any real number, i denotes the square root of -1, $E[\cdot]$ is a mathematical expectation operator, and $\exp(\cdot)$ denotes the exponential function. The Fourier transform is used to define the stable distribution, since with certain exceptions the probability distribution of the stable processes cannot be specified explicitly.

6 Setting $\alpha = 2$, $\beta = 0$, $\delta = \mu$, and $\gamma = \sigma^2/2$ in Equation (1) yields the characteristic function of the normal distribution. Cauchy distribution is the case with $\alpha = 1$, $\beta = 0$, $\gamma \in (0, +\infty)$, and $\delta \in (-\infty, +\infty)$. Levy distribution is the case with $\alpha = 1/2$, $\beta = 1$, $\gamma \in (0, +\infty)$, and $\delta \in (-\infty, +\infty)$.

mean and variance. When α is in $(1, 2]$, the random variable has finite mean. The second parameter, $\beta \in [-1, 1]$, is the skewness parameter. In particular, when β is equal to 0, the distribution is symmetric. When β corresponds to $+1$ (-1), the distribution is fat-tailed to the right (left), or skewed to the right (left). The degree of right skewness increases as β approaches $+1$, and vice versa. As α approaches 2, β loses its effect and the distribution approaches the symmetric normal distribution regardless of the value of β. The third parameter, $\gamma \in (0, +\infty)$, is the scale parameter. It compresses or extends the distribution from the point of its location parameter. The last parameter, $\delta \in (-\infty, +\infty)$, is the location parameter. It shifts the distribution to the left or right.

If tails of a distribution are heavier than those of a distribution that has exponentially decreasing tails, it is said that the distribution has heavy tails, or fat tails. A statistical result of the fat tails is that some of the moments may not be defined. When a distribution has sufficiently long tails, the first few moments will not characterize the distribution because they diverge. Under the fat tails, mean and variance may be undefined or infinite and therefore unsuited as effective descriptions of the distribution. However, when the traditional first and second moments, mean and variance, are not definable under a specific stable distribution, the location and scale parameters of the stable distribution, γ and δ, can be suitable alternatives.

The Jarque–Bera normality test was performed on the series to check the normality. The test results clearly indicate that all series are nonnormal. When the same test was applied to the expected values of futures and cash prices, the null of normality was rejected at all conventional significance levels. This indicates that one needs to relax the normality restriction to effectively capture the behaviors of the data. We proceed with the stable distribution test on the series. To fit the stable distribution to a time series, one needs the initial values of the four parameters of the characteristic function. This study uses McCulloch's (1986) quantile method to estimate the initial values. With the quantile estimators as initial approximations to the parameters, a constrained quasi-Newton method is used to maximize the log-likelihood function for an i.i.d. stable sample of a random variable, $X_1, X_2, \ldots, X_n$, as follows:

$$l(\omega) = \sum_i \log f(X_i \mid \omega), \tag{11}$$

where $f(X_i \mid \omega)$ is the density function of the stable distribution, ω denotes the parameter vector by $\omega \equiv (\alpha, \beta, \gamma, \delta)$ in a parameter space $\Omega = (0, 2] \times [-1, 1] \times (0, +\infty) \times (-\infty, +\infty)$, and the quasi-Newton method is constrained by the parameter space.[7]

There have been several efforts to compute the stable density, such as Holt and Crow (1973), Panton (1992), and Nolan (1997). Among them, Nolan provides an efficient

[7] DuMouchel (1973) showed that when the parameter vector, ω, is on the interior of the parameter space, Ω, the maximum-likelihood estimator follows the standard theory so that it is consistent and asymptotically normal. If ω is near the boundary of Ω, the finite-sample behavior of the estimator is not precisely known, because the distribution of the estimator may be skewed away from the boundary. Furthermore, in this case, the asymptotic normal distribution of the estimator tends to be a degenerate distribution at the boundary point.

TABLE 3
The Results of Fitting the Stable Distribution to PA Data Set

	Cash Price p_1		Basis Risk ν	
Region	α	β	α	β
Southeast PA	1.600	0.998	1.636	−0.182
Central PA	1.647	0.992	1.647	−0.063
South Central PA	1.673	0.993	1.703	0.036
Lehigh Valley	2.000	0.000	1.900	0.999
Western PA	1.663	0.992	1.659	−0.058

Notes: α is the characteristic exponent and β is the skewness parameter of the stable distribution. The α and β of CBOT corn futures prices are 1.403 and 0.985, respectively.

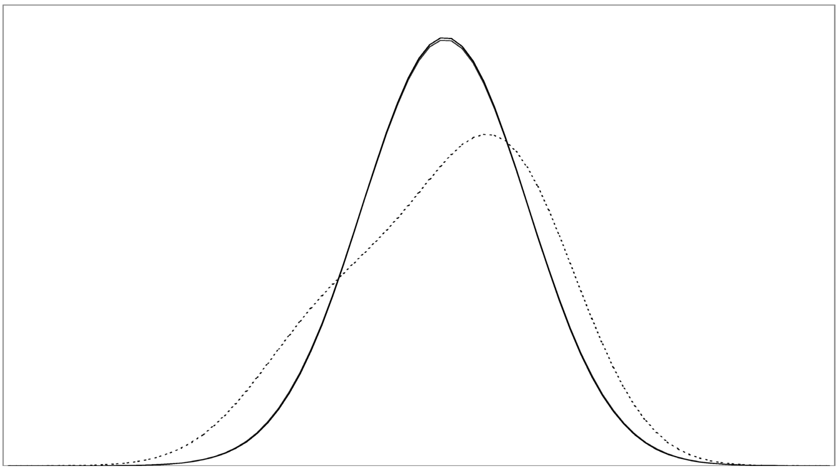
numerical computation method, and this study adopts Nolan's methodology.[8] Table 3 displays the estimated parameters of the characteristic exponent, α, and the skewness parameter, β, of the stable distribution, which are the most important parameters that direct the data to a specific distribution in the stable set. $\hat{\alpha}$ for the cash prices are 1.600, 1.647, 1.673, 2.0, and 1.663 in Southeast, Central, South Central, Lehigh Valley, and Western PA, respectively, indicating that cash price processes in the regions, except for Lehigh Valley, are relatively closer to the stable distribution set than to the normal distribution. These values also suggest that there are substantial numbers of outliers in the tails of the data distribution, that is, significantly large changes exist. When $\alpha < 2.0$, the distribution is naturally more spread out than the normal case where $\alpha = 2.0$. $\hat{\alpha}$ for basis risk are similar to those of cash price series: 1.636, 1.647, 1.703, 1.900, and 1.659. A time process with estimated characteristic and skewness parameter corresponds to a stable distribution characterized with $\hat{\alpha}$, $\hat{\beta}$, $\gamma \in (0, +\infty)$, and $\delta \in (-\infty, +\infty)$. For example, cash price of Southeast PA follows one of the stable distribution set with $\alpha = 1.600$, $\beta = 0.998$, $\gamma \in (0, +\infty)$, and $\delta \in (-\infty, +\infty)$.

In contrast, $\hat{\alpha}$ for cash price of Lehigh Valley region is 2.0, suggesting that the cash price series may follow the normal distribution. However, note that when observations are not enough to estimate the stable characteristic function, it may produce a value of $\hat{\alpha}$ close to 2.0. Since the Jarque–Bera normality test clearly rejects the normal distribution for the cash price series of Lehigh Valley, we doubt the normality, although the estimated characteristic parameter indicates it.

The stable distribution has been used in literature, but testing whether the distribution fits real data better than the restrictive normal distribution has been elusive. One method of performing such a test is to plot a smoothed density of the data and compare it with the fitted stable density alongside the fitted normal density (Nolan, 1999). The Gaussian kernel can be used to obtain the smoothed density of the data (cf. Pagan and Hong, 1990; Campbell, Lo, and MacKinlay, 1997). If fitted stable density matches the

[8] We thank Dr. J. P. Nolan for providing the estimator software for the stable distribution used by Nolan (1997, 1999).

FIGURE 2
Plots of Data Density, Stable Fitted Density, and Normal Fitted Density: Cash Prices in Lehigh Valley



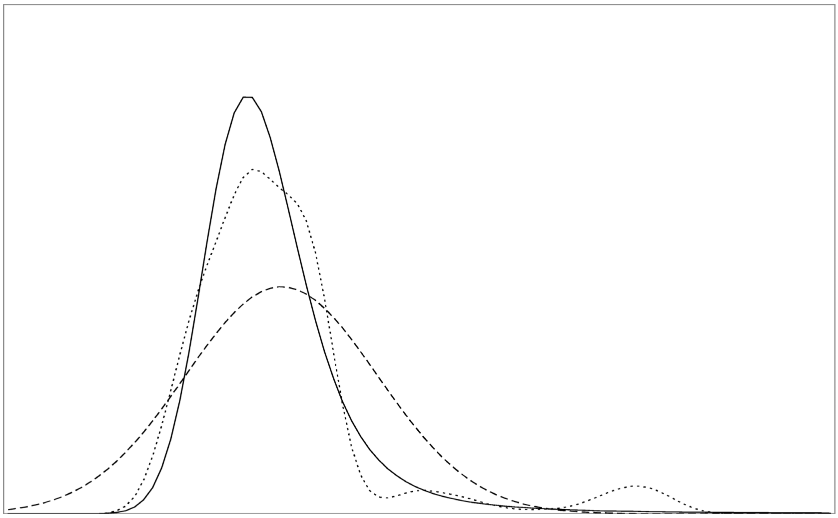
Note: y-axis is *pdf* and *x*-axis is observation for each density. Solid line denotes the stable fitted density, dashed line denotes the normal fitted density, and dotted line denotes the smoothed data density. In this figure, the normal fitted and stable fitted densities exactly coincide so that the two lines are overlapped each other.

data density better than the fitted normal density, it indicates that the movements of the data are better described by the stable density than by the normal density.

The smoothed data density, fitted normal density, and fitted stable density were estimated for each series of cash price and basis risk.[9] Figure 2 displays the density plots of cash price of Lehigh Valley. The stable density (solid line) and the normal density (dashed line) exactly coincide, which shows that the two densities are the same. Note that $\hat{\alpha}$ for the cash price is 2.0, that is, the stable distribution corresponds to the normal distribution. The figure shows that either the stable density or the normal density has a low power to capture the behavior of the series. Figure 3 presents the density plots of cash prices of Southeast region, which shows that the data behaviors around the mode and tails, especially the right-hand side tail, are explained significantly better by the stable density. The normal distribution does not effectively describe the skewed large events. Note that the value of $\hat{\beta}$ for the cash prices is 0.998, indicating positive skewness of the data distribution. This implies that there are a substantial number of large upward changes in the cash price process of the region, which may create significant risk for the input buyers. Figure 4 presents the density plots of basis risk of the same region, which shows that the data behavior is not efficiently captured by the

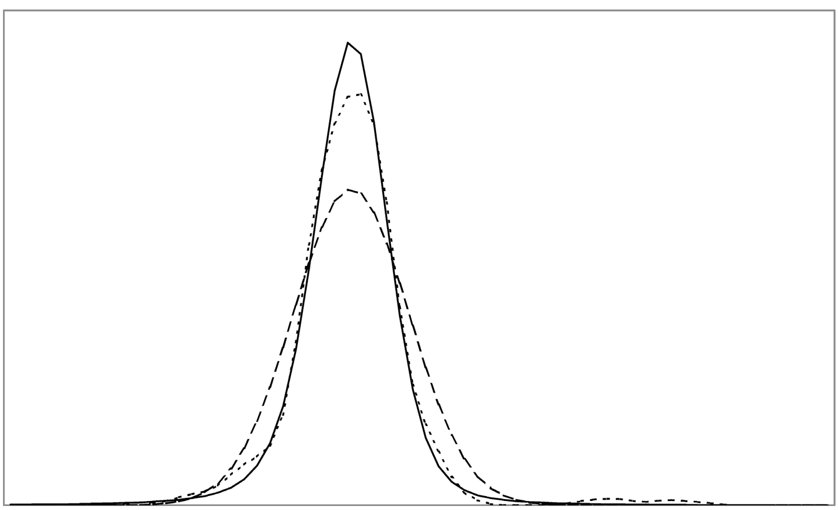
[9] The data are smoothed with a Gaussian kernel, with standard deviation given by a width parameter. The Gaussian kernel provides information about how much weight each frequency should be given. A difficulty in the kernel smoothing is placed on choosing the width parameter. This study uses the cross-valuation method (cf. Hamilton, 1994) for choosing the parameter.

FIGURE 3
Plots of Data Density, Stable Fitted Density, and Normal Fitted Density: Cash Prices in Southeast PA



Note: y-axis is *pdf* and *x*-axis is observation for each density. Solid line denotes the stable fitted density, dashed line denotes the normal fitted density, and dotted line denotes the smoothed data density.

FIGURE 4
Plots of Data Density, Stable Fitted Density, and Normal Fitted Density: ν in Southeast PA



Note: y-axis is *pdf* and *x*-axis is observation for each density. Solid line denotes the stable fitted density, dashed line denotes the normal fitted density, and dotted line denotes the smoothed data density.

normal density but is comparatively well explained by the stable density. The results of the density plots of the other three regions, Central, South Central, and Western PA, are similar to those of Southeast region.[10]

For a more concrete statistical inference, we performed a formal test of whether the data density is identical to the normal fitted density or to the stable fitted density, using the Wilcoxon rank-sum test in the one-way nonparametric analysis of SAS. The null hypothesis of the test is that the two distributions in the analysis are statistically identical. In particular, there are two null hypotheses: null 1—no difference between the data density and the normal fitted density— and null 2—no difference between the data density and the stable fitted density. Both the null 1 and null 2 were rejected for cash price of Lehigh Valley at the 5 percent significance level. The null 1 was rejected, but the null 2 was not rejected for all other time series at the 5 percent level. These support the results from the density plots.[11]

Overall, the estimated parameters suggest that the normality restriction is misleading and, thus, imposing the normality may cause a bias in the estimation of optimal hedge ratios. The density plots suggest that the stable set is relatively effective in capturing large changes in price movements, while the normal distribution screens out the outliers, neglecting a real source of risks from price variation or basis variation.

Hedge Ratios Under the Generalized Distribution

The optimal hedge ratios are reestimated under the generalized stable distribution. The utility maximization problem remains the same, with the exception of the different distributional assumption. Now the normality assumption is relaxed, and each time process corresponds to one of the stable set with the specific values of the characteristic and skewness parameters. Since Equation (8) is in a numerical double-integral form, if we use different densities for price and basis variations, different hedge ratios are obtained. Therefore, using a more efficient density to describe real data distributions will provide more reliable hedge ratios. The *pdf* of the stable distribution is calculated

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[10] They are not displayed here because of the reason of the space, but they are obtainable from the authors on request.

[11] We obtained 100 values of the smoothed data density, the normal fitted density, and the stable fitted density, respectively, by using Nolan's software (used in Nolan, 1997, 1999). The null hypothesis of the test, H_0, is that the two distributions, say A and B, in the analysis are statistically identical, that is, their shapes and locations are the same. For the alternative hypothesis, we may write as H_a: $A > B$ (or H_a: $A < B$), that is, the distribution of A is shifted to the right (or left) of distribution of B. When there is no strong prior reason for expecting a shift of a distribution in a particular direction, we write as H_a: $A \neq B$. Not rejecting the null hypothesis implies that the two distributions have the same shapes and locations, while rejecting the null and accepting one-sided alternative, e.g., $A > B$, means that the distribution of A is shifted to the right of distribution of B. To perform the Wilcoxon test for the null hypothesis 1, we combined the two samples of smoothed data density and the normal fitted density and sorted the combined data set into ascending order and ranked each value. For the two-sided test, that is, testing H_a: $A \neq B$, a rank sum that is either too big or too small provides evidence against H_0.

TABLE 4
Hedge Ratios Under the Stable Distribution

	$r = 2.00$		$r = 0.2$		$r = 0.02$	
Region	$\tau = 0.5$	$\tau = 1.0$	$\tau = 0.5$	$\tau = 1.0$	$\tau = 0.5$	$\tau = 1.0$
Southeast PA	0.7436	0.7266	0.6134	0.5793	0.2343	0.1053
Central PA	0.5624	0.5471	0.3356	0.3068	0.0341	0
South Central PA	0.3325	0.3070	0.2851	0.2691	0	0
Lehigh Valley	0.5194	0.5124	0.4973	0.4732	0.2782	0.0502
Western PA	0.3246	0.3105	0.2779	0.2183	0.0124	0

Notes: r denotes the coefficient of risk aversion. τ denotes the hedging cost, and it is assumed constant.

using Nolan's (1997) algorithm and the estimated values of α and β in Table 3. As a location parameter for the processes, the mean in Table 1 were used.[12]

The variance is one measure of spread; the scale parameter γ of the stable distribution is another. Variance is finite and stable only if α equals 2.0; that is, sample variance is efficient information only when the distribution corresponds to the normal distribution. If α is less than 2.0, the sample variances may not be a good measure of data dispersion.[13] Under the stable distribution, the estimated scale parameter, $\hat{\gamma}$, could be a better dispersion measure than the standard deviation. The results in Table 3 show that $\hat{\alpha}$ are less than 2.0, except for Lehigh Valley. Therefore, if we use the standard deviation in Table 1 for estimating optimal hedge ratios under the stable distribution, the results may still be biased even though we relax the restrictive normality assumption. Cornew et al. argue that using the standard deviation might yield erratic results when $\hat{\alpha}$ indicates a departure from normality, and they propose the scale parameter, γ, as a proper dispersion measure under the stable distribution. Therefore, we estimate optimal hedge ratios under the stable distribution with the estimated scale parameter, $\hat{\gamma}$.

The results of optimal hedge ratios under the stable distribution are displayed in Table 4. The shape of the hedging demand curve is the same as the case of the normal distribution since the optimization problem is not changed, except for the distributional assumption. One noticeable result is that the estimated hedge ratios are larger than those under the normal distribution. This suggests that the demand curves lie

[12] If the characteristic parameter is between 1.0 and 2.0, a stable mean exists in the distribution. If the parameter is less than 1.0, the mean may not be defined. The results in Table 3 show that the estimated characteristic exponents of all $\tilde{p}_1$ and $\tilde{v}$ series range between 2.0 and 1.0. Thus, the mean can be used as a location parameter.

[13] When we take a sample variance under the Gaussian assumption, we consider it as an estimate of the unknown population variance. Thus, if it is not Gaussian, that is, $\alpha < 2.0$, the sample variance may not tell something about the population variance, because population variances are not defined in a given time horizon. In this case, sample variance would be expected to be unstable and not to tend to any value, even as the sample size increases. If $\alpha \leq 1.0$, the same goes for the mean.

farther from the line $h^* = 0$ than those under the normal distribution. The differences would be a bias from imposing a wrong distributional assumption.

Results from the stable distribution case show higher hedge ratios than those obtained under the normal distribution. Although there is no guiding principle yet in the literature to explain empirical implications of higher hedge ratios under the stable distribution, this is intuitively plausible because the stable distribution describes "large" price variations better than the normal distribution, as noticed in the density plots in Figures 2 through 4. This implies that under the normality restriction, real risk may be underestimated, which will affect performance of a risk management tool. The stable distribution would help to reduce the bias. The incremental value of the hedging embodied by a proper distributional assumption is the difference (the shaded area) between the smaller triangle and the larger triangle above the supply curve in Figure 1. The difference would be the value of using a proper distribution to the stochastic processes.[14]

A noticeable result is that h^* for Lehigh Valley under the stable distribution with the scale parameter, γ, are not bigger than those under the normal distribution. As noted before, $\hat{\alpha}$ for $\tilde{p}_1$ of the region is 2.0 and that of $\tilde{v}$ is close to 2.0, which corresponds to the normal distribution. Therefore, results under the normality assumption should be similar to those under the stable distribution.

In this study, the coefficient of absolute risk aversion, r, is set equal to 2.0, 0.2, and 0.02 per each cent. Changing the value of risk aversion might result in changes in the optimal hedge ratios. We proceeded further to expand the empirical analysis to other risk aversions. When the value of the risk aversion coefficient increases, higher hedge ratios are obtained, and vice versa.

Primarily this article argues that the conventional normal distribution is not an appropriate distribution for the specific data set and should be replaced by a stable distribution. In that way one moves from a light-tailed world to a heavy-tailed setting. In this context, one can use a semi-heavy-tailed model for a comparison with the cases of both the normal distribution and stable distribution. We estimated the hedge ratios using the generalized hyperbolic model (Barndorff-Nielsen and Blaesild, 1983), which is one of semi-heavy-tailed distributional models. Using a package,[15] the data set was fitted to the hyperbolic distribution and the fitted hyperbolic density function was generated using estimated parameters. Finally, we estimated hedge ratios under the hyperbolic distribution.[16] The results show that more than half of the hedge ratios are larger than those under the normal distribution and smaller than under the stable distribution. However, the results do not show an unclear effect from using the semi-heavy-tailed model. Hedge ratios under the hyperbolic model do not exhibit

[14] If one can calculate the exact value of the hedger's surplus, it will be the dollar cost of the incremental value. However, we could not achieve the calculation because of the nonlinearity of the curve. Estimation of the values may involve a myriad of difficult measurement issues and guesswork, and therefore it is likely to be regarded as significant contribution as a stand-alone paper.

[15] The package is "the HyperbolicDist package-ver 0.0-1," provided by David Scott.

[16] They are not presented here because of lack of space but can be obtained from the authors on request.

any single pattern, for example, all larger than those under the normal distribution or smaller than under the stable distribution. It suggests that the results under the hyperbolic distributional setting do not provide pellucid implications for the comparison. Finding a principle (reasoning) for the results under the hyperbolic distribution is beyond the limit of this study and it will be interesting follow-up work.

SUMMARY AND CONCLUSION

The subject of this study is to compare hedge ratios under a restricted distributional scheme with those under a generalized distribution. The contribution of this article comes from its exploration of the importance of imposing a correct distributional assumption and measuring the difference in the hedge ratios under the restrictive normality assumption with those under the generalized stable distribution. The concept is illustrated for PA dairy farm managers, who need to purchase corn as a feed input.

The results show that time processes of corn prices and basis risk in five PA regions have heavy tails and do not correspond to the normal distribution. They are more correctly fitted to the stable distribution set. While the fit of the stable distribution to the data set is not perfect either, the stable model captures an important part of the price variations that is left by a Gaussian model. This suggests that the stable model is useful to better calculate the correct hedge ratios. The estimated hedge ratios under the stable distribution are larger than those under the normal distribution. The difference could be considered a downward bias from imposing the restricted distributional assumption for capturing large price changes.

The overall results indicate that both hedging costs and distributional assumption are at play in changing optimal hedging positions. If we consider a large-size farm, small changes in the optimal hedge ratio could result in large differences in the amount hedged and hedging performance. This suggests that the role of the distributional assumption (as well as the hedging cost) should not be ignored in the estimation of optimal hedge ratios.

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