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Wealth, Income, and Optimal Insurance

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Abstract

This article considers the decision to purchase insurance against possible losses of a property or wealth. The decision involves a standard economic trade-off between the benefit of protection against loss and the cost of insurance premium. The premium is paid out of the income and decreases the consumption of other goods and services, rather than out of wealth and decreases the property or wealth. The demand for insurance depends mainly on the income and preferences. As a result, unlike in the standard model, a fair premium is neither necessary nor sufficient for the optimality of full coverage insurance. Rather, the individuals with higher incomes purchase full coverage insurance even at unfair prices of insurance while the individuals with lower income purchase partial coverage insurance at a fair price.

Introduction

A large body of literature has studied optimal insurance (e.g., Mossin, 1968; Smith, 1968; Schlesinger, 2000).[1] The standard model considers a risk-averse individual who wishes to insure her property against a possible loss. The analyses concern the level of insurance coverage that maximizes her expected utility of wealth, a random variable. Her wealth equals the initial value of the property minus the insurance premium when no loss occurs, and includes additionally the loss and the indemnity when a loss occurs. The standard model predicts that full coverage insurance is optimal with a fair price, and partial coverage insurance is optimal otherwise. This conclusion holds regardless of the preferences or income or wealth. An implication of the standard prediction is that everyone switches from full coverage insurance to partial coverage insurance when the loading factor increases slightly from zero to a small positive number, a dramatic but unrealistic event.

The key assumption in the literature is that utility depends only on wealth. This assumption simplifies the analyses and enables one to obtain clear-cut results. However, this assumption appears unrealistic in one important respect. The insurance premium is in practice paid out of one's income, rather than from wealth or property. Stated differently, the insurance premium reduces the consumption of other goods and services,

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[1] Dionne (2000) contains a number of articles on optimal insurance.

not the property. To reflect this realism, the article assumes that utility depends on income (or composite good consumption) and wealth, more precisely the consumption of service flows generated by wealth. For instance, one cares about composite good consumption and housing consumption, as in the housing economics literature (Papageorgiou and Pines, 1999, Ch. 6; Zabel, 2004).

In this model, the purchase of an additional insurance coverage involves a standard economic trade-off. The coverage provides an additional protection against a possible loss when it occurs, but it increases the premium and decreases the income available for the consumption of other goods and services. The decision to purchase an additional coverage essentially depends on the valuation of the consumption of service flows generated by the property relative to the valuation of composite good consumption, or depends on the valuation of wealth relative to that of income. As a result, higher income individuals tend to purchase more insurance if both goods are normal. The reason is that as income increases, the composite good becomes less important, given the value of the property, and it increases the willingness to pay a larger premium and hence to buy more insurance. Thus, an individual may purchase full coverage insurance at unfair prices while she may purchase partial coverage insurance at a fair price, depending on her income and preferences. This conclusion appears to conform with real world observations (Beenstock, Dickinson, and Khajuria, 1988; Grace, Klein, and Kleindorfer, 2000).[2]

Making use of the model with income and wealth, the article reexamines a number of important propositions in the insurance economics literature: (i) full coverage insurance is optimal at a fair price while partial coverage is optimal at unfair prices (Mossin, 1968); (ii) no deductible is optimal at a fair price while a positive deductible is optimal at unfair prices (Mossin, 1968; Smith, 1968); (iii) with noninsurable loss, more than full coverage is optimal when the insurable loss and the noninsurable loss are positively correlated while less than full coverage is optimal when they are negatively correlated (Doherty and Schlesinger, 1983); and (iv) with risk-averse insurers, no deductible is optimal if the provision of insurance involves no deadweight loss and a deductible is optimal otherwise (Raviv, 1979). In the present model, the decision to purchase insurance depends largely on the income and preferences, regardless of the magnitude of the loading factor, or of the correlation between the insurable loss and the noninsurable loss, or of the deadweight loss. For example, a fair premium is neither necessary nor sufficient for full coverage insurance to be optimal. As a consequence, even if the loading factor increases slightly from zero, it does not affect most individuals and the dramatic switch from full to partial coverage insurance for everyone does not occur.

The next section models preferences with income and wealth, and section Optimality of Full Coverage Insurance considers an optimal insurance problem. Section Other Optimal Insurance Decisions with Income and Wealth extends the analyses to a number

[2] Using a set of international data, Beenstock, Dickinson, and Khajuria (1988) estimate that the elasticity of the property insurance premium with respect to per capita GNP is about 1.34, implying that property insurance is a super normal good. Grace, Klein, and Kleindorfer (2000) report an empirical result that noncatastrophe coverage is a superior good, based on a set of Florida state homeowners' insurance data.

of other known results on optimal insurance decisions. The last section offers a conclusion.

PREFERENCES

An individual has preferences toward two goods, composite good, c, and service flow, s, generated by a property such as a house or a car. The preferences are represented by an increasing and strictly concave utility function $U(c, s)$. The service per period may be written as $s = h(w)$ with w denoting the property. To simplify the presentation, let $s = h(w) = w$. This simplification has no effect on the subsequent analyses. We assume that (i) both goods, c and s, are normal, (ii) the utility function satisfies the Inada conditions, and (iii) the indifference curves do not cut the axes. Letting $\text{MRS}\,(c,s) \equiv U_s(c,s)/U_c(c,s)$ denote the marginal rate of substitution between the two goods, these assumptions can be stated as

$$\begin{aligned} \frac{\partial}{\partial c}\text{MRS}(c,s) > 0; \quad & \frac{\partial}{\partial s}\text{MRS}(c,s) < 0 \\ \lim_{c\to 0} U_c(c,s) = \infty; \quad & \lim_{c\to\infty} U_c(c,s) = 0 \\ \lim_{c\to 0} \text{MRS}(c,s) = 0; \quad & \lim_{c\to\infty} \text{MRS}(c,s) = \infty, \end{aligned} \tag{1}$$

where subscripts in the utility function denote partial derivatives such as $U_c(\cdot) = \partial U(\cdot)/\partial c$.

The utility function above is standard in the housing economics literature (e.g., Papageorgiou and Pines, 1999, Ch. 6; Zabel, 2004). In that literature, individuals enjoy the consumption of the composite good and service flows generated by housing stock. Housing is perhaps the single largest insurable property that an average individual owns, and the specification of the utility function here is appropriate for the study of property insurance. The implication of the utility function for the study of property insurance is that premiums and losses-indemnities affect different arguments of the utility function. The insurance premium is paid from income and decreases composite good consumption while a loss decreases (and indemnity increases) wealth and hence service flows.[3] In making an insurance purchasing decision, an individual attempts to equate the marginal utility of composite good consumption with that of service flows. Thus, the individual cannot transfer resources between composite good and service flows dollar for dollar. As demonstrated below, this turns out to imply that an individual does not necessarily purchase full coverage insurance even at a fair price.

The utility function here also has an intertemporal interpretation.[4] Rewrite $U(c, s) = u(c) + \delta v(s)$ with δ denoting the discount factor. An individual enjoys composite good consumption in the first period while enjoying wealth in the second period. The second period wealth can be thought of as all future composite good consumption generated

[3] For example, if a fire breaks out, it reduces the value of the property and service flows, not the owner's income or her composite good consumption. As another example, if an individual remodels her home, the cost of remodeling reduces her composite good consumption while it increases service flow (Helms, 2003).

[4] A referee provided this interpretation.

by wealth. An individual makes the insurance decision and pays the premium in the first period. The second-period utility depends on whether a loss occurs. An individual who maximizes her lifetime utility attempts to equate the marginal utility of the first-period consumption with that of the second-period wealth. The individual thus does not transfer between the first-period consumption and the second-period wealth dollar for dollar. This again turns out to imply that the individual does not necessarily purchase full coverage insurance at a fair price.

OPTIMALITY OF FULL COVERAGE INSURANCE

Consider an individual endowed with a property w and an income y. This property is subject to a given loss $x \leq w$ with probability π. The individual purchases insurance coverage or indemnity $I \leq x$ for the premium $P = (1 + \lambda)\pi I$ with $\lambda \geq 0$ denoting the loading factor. Assume $(1 + \lambda)\pi x < y$ so that the individual has income y enough to be able to purchase full coverage insurance. Normalizing the price of the composite good to unity, its consumption is the same in both states and equals $y - P = y - (1 + \lambda)\pi I$. The service flow generated by this property if the loss occurs and that if it does not are, respectively, $w - x + I$ and w.[5]

The individual chooses an insurance coverage I to maximize expected utility

$$E \equiv \pi U(y - P, w - x + I) + (1 - \pi)U(y - P, w).$$

The main question concerns if the individual purchases full coverage insurance, one of the most important propositions in the insurance economics literature (Mossin, 1968). To examine this question, we differentiate E and evaluate it at $I = x$, giving

$$\begin{aligned} \left.\frac{dE}{dI}\right|_{I=x} &= \pi U_s(y - P, w - x + I) - \pi^2(1 + \lambda)U_c(y - P, w - x + I) \\ &\quad - (1 - \pi)\pi(1 + \lambda)U_c(y - P, w) \\ &= \pi U_s(y - P, w) - \pi(1 + \lambda)U_c(y - P, w) \end{aligned} \tag{2}$$

with $P = (1 + \lambda)\pi x$. The last equality follows because with full coverage insurance $I = x$, and the two states coincide. In the standard model, there is no distinction between wealth and income, and the utility function is written as $U(c + s)$. In this case, $U_s = U_c$, and $(dE/dI)|_{I=x} = -\pi\lambda U_c$. Thus, full coverage is optimal at a fair price while partial coverage is optimal with a positive loading regardless of income or preferences.

The first-order approach can be justified if the maximization problem E is concave in I. To check concavity of E, it suffices to differentiate E twice to obtain

[5] As noted above, the key to the expressions of c and s is that the insurance premium is paid out of income and reduces c while a loss (indemnity) reduces (increases) wealth and hence s.

$$\begin{aligned}\frac{d^2E}{dI^2} &= \pi U_{ss}(y-P,w-x+I) - 2\pi^2(1+\lambda)U_{cs}(y-P,w-x+I)\\ &\quad + \pi^3(1+\lambda)^2U_{cc}(y-P,w-x+I) + (1-\pi)\pi^2(1+\lambda)^2U_{cc}(y-P,w)\\ &= \pi\big[U_{ss}(y-P,w-x+I) - 2\pi(1+\lambda)U_{cs}(y-P,w-x+I)\\ &\quad + \pi^2(1+\lambda)^2U_{cc}(y-P,w-x+I)\big] + (1-\pi)\pi^2(1+\lambda)^2U_{cc}(y-P,w) < 0.\end{aligned}$$

The inequality follows because the terms inside the squared bracket are negative due to the concavity of $U(c, s)$, and because $U_{cc}(y - P, w) < 0$ in the last term, again due to the concavity of $U(c, s)$. Thus, the maximization problem is indeed concave in I.

In the present model, the sign of (2) depends additionally on the magnitudes of U_s and U_c, which in turn depend on incomes and preferences. To see this, rewrite (2) as

$$\left.\frac{dE}{dI}\right|_{I=x} = \pi U_c(y-P,w)[\mathrm{MRS}(y-P,w) - (1+\lambda)] \equiv \pi U_c(y-P,w)\phi(y:\lambda,x,w) \qquad (3)$$

with $P = (1 + \lambda)\pi x$. Since $\pi U_c(y - P, w) > 0$, the sign of $(dE/dI)|_{I=x}$ coincides with that of $\phi(y:\cdot)$. Given the assumptions about the preferences in (1), a few properties of $\phi(y:\cdot)$ can be observed: (i) $\lim_{y\to P}\ \phi(y:\cdot) = \lim_{c\to 0}\ [\mathrm{MRS}\ (c, s) - (1 + \lambda)] = -\ (1 + \lambda) < 0$, (ii) $\phi'\ (y:\cdot) = \partial\ \mathrm{MRS}\ (c, s)/\partial c > 0$, and (iii) $\lim_{y\to\infty}\ \phi(y:\cdot) = \lim_{\,c\to\infty}\ \mathrm{MRS}\ (c, s) = \infty$. These properties imply that there is a critical value of y, denoted $y^* > P$, such that $\phi(y:\cdot) < (=, >)\, 0$ for $y < (=, >)\, y^*$. As a consequence, $(dE/dI)|_{I=x} < (=, >)\, 0$ for $y < (=, >)\, y^*$. This result means that the individuals with income less than a critical level, y^*, purchase partial coverage insurance while those with income greater than or equal to the critical level purchase full coverage insurance when limiting the coverage to not exceed the full coverage. Thus, a fair price is neither sufficient nor necessary for full coverage insurance to be optimal.

The result above has a simple intuition. The model views the purchase of insurance as a standard consumer-choice problem in a two-good framework with composite good c and service flow from wealth s. Since the price of insurance is $(1 + \lambda)$, an individual pays $(1 + \lambda)$ and hence decreases her composite good consumption by $(1 + \lambda)$ in order to increase service flow by one unit. If MRS $(c, s) < 1 + \lambda$ with $I = x$ for an individual, the individual can increase her utility by reducing her coverage and buys less than full coverage. If the inequality is reversed, the individual purchases the full coverage. Given that MRS $(\cdot)$ increases with income, lower income individuals purchase less than full coverage while higher income individuals purchase the full coverage.

To see the role of the loading factor, note that $\phi(y^* : \lambda, w, x) = 0$, and totally differentiate it to obtain

$$\frac{dy^*}{d\lambda} = \frac{1 + x(\partial\mathrm{MRS}(c,s)/\partial c)}{\partial\mathrm{MRS}(c,s)/\partial c} > 0,$$

where $c = y^* - P = y^* - (1 + \lambda)\ \pi x$ and $s = w$. Thus, as λ increases to λ', those individuals with income between $y^*(\lambda)$ and $y^*(\lambda')$ switch from full coverage to partial coverage insurance. However, those individuals with income below $y^*(\lambda)$ (above

$y^*(\lambda')$) continue to purchase partial (full) coverage insurance. The standard model predicts that everyone switches from full coverage insurance to partial coverage insurance as λ increases slightly from zero to a small positive number, λ'. By contrast, the present model predicts that such an increase in λ has no effect on the decision to purchase full coverage insurance except in those individuals with income between $y^*(0)$ and $y^*(\lambda')$.[6]

Casting the insurance problem in an intertemporal setting noted above, expected utility becomes $E = u(y - P) + \delta\,[\pi v(w - x + I) + (1 - \pi)v(w)]$. Equation (3) becomes

$$\left.\frac{dE}{dI}\right|_{I=x} = \pi\,[\delta v'(w) - (1+\lambda)u'(y - (1+\lambda)\pi x)].$$

An illuminating result is obtained in a simple case with $\delta = 1$, $v(\cdot) = u(\cdot)$ and $\lambda = 0$. In this case, given that $u(\cdot)$ is increasing and concave, $(dE/dI)|_{I=x} < (=, >)\ 0$ for $y < (=, >)\ y^* = w + \pi x$. Thus, even at a fair price, those individuals with $y < w + \pi x$ purchase partial coverage insurance. In a more general case, y^* cannot be solved explicitly, but it is clear that $(dE/dI)|_{I=x} < (=, >)\ 0$ for $y < (=, >)\ y^*$, and the same interpretation as (3) applies.

OTHER OPTIMAL INSURANCE DECISIONS WITH INCOME AND WEALTH

This section discusses how the consideration of income and wealth modifies a number of other known results on optimal insurance decisions.

Optimal Level of a Deductible

Loss x is a random variable and distributed over $[0, w]$ according to a density function $f(x)$. With deductible $\bar{x} \geq 0$, the amount the insurer pays when a loss x occurs is

$$I(x) = \begin{cases} 0 & \text{if } x \leq \bar{x} \\ x - \bar{x} & \text{if } x > \bar{x}. \end{cases}$$

The premium will then be

$$P = (1+\lambda)E(I) = (1+\lambda)\int_{\bar{x}}^{w} (x - \bar{x}) f(x)\,dx.$$

For future reference, differentiating P with respect to $\bar{x}$, it leads to

$$\frac{dP}{d\bar{x}} = -(1+\lambda)\int_{\bar{x}}^{w} f(x)\,dx < 0, \qquad (4)$$

and the premium decreases with the deductible.

[6] To see the role of preferences, write the utility as $U(c, s:\theta)$ with θ denoting a preference parameter. Assume a single-crossing condition so that MRS $(c, s:\theta)$ increases with θ (Mussa and Rosen, 1978). The individuals with higher θ's then purchase more insurance. As a result, the individuals with higher θ's may purchase full coverage insurance even at unfair prices while the individuals with lower θ's may purchase partial coverage even at a fair price.

The individual's problem is to choose $\bar{x}$ to maximize expected utility

$$E = \int_0^{\bar{x}} U(y - P, w - x) f(x)\, dx + U(y - P, w - \bar{x}) \int_{\bar{x}}^{w} f(x)\, dx.$$

The main question concerns if a positive amount of deductible is optimal, $\bar{x} > 0$, another important question in the literature (Mossin, 1968; Smith, 1968). To see this, differentiate E and evaluate it at $\bar{x} = 0$. Using (4), the resulting equation becomes

$$\begin{aligned}
\left.\frac{dE}{d\bar{x}}\right|_{\bar{x}=0} &= \left[\int_{\bar{x}}^{w} f(x)\, dx\right]\left[(1+\lambda)\int_0^{\bar{x}} U_c(y - P, w - x) f(x)\, dx - U_s(y - P, w - \bar{x})\right] \\
&\quad + \left[\int_{\bar{x}}^{w} f(x)\, dx\right]^2 (1+\lambda) U_c(y - P, w - \bar{x}) \\
&= \int_0^{w} f(x)\, dx \left[U_c(y - P, w)(1+\lambda)\int_0^{w} f(x)\, dx - U_s(y - P, w)\right] \\
&= (1+\lambda) U_c(y - P, w) - U_s(y - P, w).
\end{aligned} \tag{5}$$

The first equality in (5) follows by substituting $\bar{x} = 0$, and the second one makes use of $\int_0^w f(x)\, dx = 1$. The premium in (5) becomes $P = (1+\lambda)E(x)$ with $E(x) \equiv \int_0^w x f(x)\, dx$ denoting the expected loss.

In a manner analogous to (3), rewrite (5) as

$$\left.\frac{dE}{d\bar{x}}\right|_{\bar{x}=0} = U_c(y - P, w)[(1+\lambda) - \text{MRS}(y - P, w)] \tag{6}$$

with $P = (1+\lambda)E(x)$. Inspection of (3) and (6) reveals that they are the same except opposite signs and the premium being $(1+\lambda)E(x)$ in (6) instead of $(1+\lambda)x$. It can then be shown that there is a critical value of income, denoted y^{**}, such that $(dE/d\bar{x})|_{\bar{x}=0} > (=, <)\ 0$ for $y < (=, >)\, y^{**}$. A positive amount of deductible is thus optimal for the individuals with income less than a critical level while no deductible is optimal for those with income greater than or equal to the critical level. Consequently, a fair price is neither sufficient nor necessary for no deductible to be optimal.

Background Risk and Optimal Insurance

Let A denote the sum of insurable and noninsurable wealth, (x, n) denote the magnitudes of the insurable and noninsurable losses, and $(\pi_x, \pi_n, \pi_{n|x})$ denote the respective probabilities of the insurable loss, of noninsurable loss, and the conditional probability of a noninsurable loss, given that an insurable loss occurs. The individual chooses I to maximize

$$\begin{aligned}
E \equiv\ & (1 - \pi_n - \pi_x + \pi_x \pi_{n|x}) U(y - P, A) \\
& + (\pi_x - \pi_x \pi_{n|x}) U(y - P, A - x + I) \\
& + (\pi_n - \pi_x \pi_{n|x}) U(y - P, A - n) \\
& + \pi_x \pi_{n|x} U(y - P, A - x - n + I),
\end{aligned} \tag{7}$$

with $P = (1+\lambda)\pi_x I$. The four probabilities in (7) represent, respectively, the probability of no loss, the probability of insurable loss only, the probability of noninsurable loss only, and the probability of both insurable and noninsurable losses.

To see if full coverage is optimal, we differentiate E and evaluate it at $I = x$, yielding

$$\left.\frac{dE}{dI}\right|_{I=x} = (\pi_x - \pi_x\pi_{n\,|\,x})U_c(y-P,A)\left[\mathrm{MRS}(y-P,A) - (1+\lambda)(1-\pi_n)\frac{1}{(1-\pi_{n\,|\,x})}\right] + \pi_x\pi_{n\,|\,x}U_c(y-P,A-n)\left[\mathrm{MRS}(y-P,A-n) - (1+\lambda)\pi_n\frac{1}{\pi_{n\,|\,x}}\right]. \quad (8)$$

Let $\phi_1(y:\lambda, A, x)$ and $\phi_2(y:\lambda, A, x)$ denote the terms inside the first squared bracket of (8) and those inside the second one, respectively. Both $\phi_1(y:\cdot)$ and $\phi_2(y:\cdot)$ have the same properties as $\phi(y:\cdot)$ in section Optimality of Full Coverage Insurance, and there are y_1^* and y_2^* with $\phi_1(y_1^*:\cdot) = 0$ and $\phi_2(y_2^*:\cdot) = 0$. Since $y_2^* < y_1^*$, (i) $\phi_1(y:\cdot) < 0$ and $\phi_2(y:\cdot) < 0$ when $y < y_2^*$, and (ii) $\phi_1(y:\cdot) > 0$ and $\phi_2(y:\cdot) > 0$ when $y > y_1^*$.[7] As a result, $(dE/dI)|_{I=x} < 0$ for $y < y_2^*$ and $(dE/dI)|_{I=x} > 0$ for $y > y_1^*$, meaning that the individuals with income less than y_2^* purchase less than full coverage while the individuals with income greater than y_1^* purchase more than full coverage.[8] This result stands in contrast with the literature (Doherty and Schlesinger, 1983) in which more or less than full coverage is optimal, depending on whether insurable loss and noninsurable loss are positively or negatively correlated.[9]

Pareto Optimal Deductible

Assuming that both the insured and the insurer are risk averse, this section considers a Pareto optimal deductible policy (Raviv, 1979; Blazenko, 1985). The cost of insurance or the deadweight loss associated with insurance provision is $C(I)$ with $C'(I) \geq 0$ and $C(0) = 0$. Letting $V(\cdot)$ denote the insurer's utility function and Y his initial income, the problem is to choose the premium P and the indemnity schedule $I(x) \geq 0$ to maximize

$$\int_0^w U(y-P, w-x+I(x))\, f(x)\, dx$$

subject to

$$\int_0^w V(Y+P-I(x)-C(I(x)))\, f(x)\, dx \geq V(Y). \quad (9)$$

[7] Suppose that insurable loss and noninsurable loss are positively correlated and $\pi_n < \pi_{n|x}$. Since MRS $(y-P, A-n) >$ MRS $(y-P, A)$ by (1), $\phi_2(y:\cdot) > \phi_1(y:\cdot)$. Given that both $\phi_2(y:\cdot)$ and $\phi_1(y:\cdot)$ increase in y, $y_2^* < y_1^*$. If they are negatively correlated and $\pi_n > \pi_{n|x}$, either $y_2^* > y_1^*$ or $y_2^* < y_1^*$. However, the same result holds, because $(dE/dI)|_{I=x} > (<)\, 0$ for $y > \max\{y_1^*, y_2^*\}$ $(y < \min\{y_1^*, y_2^*\})$, and because higher (lower) income individuals buy more (less) than full coverage.

[8] The individuals with income between y_2^* and y_1^* may purchase less than or more than full coverage, because $\phi_2(y:\cdot) > 0$ and $\phi_1(y:\cdot) < 0$ for them, and Equation (8) cannot be signed.

[9] If insurable loss and noninsurable loss are positively correlated, the purchase of insurance against insurable loss also serves as insurance against noninsurable loss. It is desirable to buy more than full coverage. An analogous intuition applies to the negatively correlated case.

Setting up the Hamiltonian, the first-order condition with respect to $I(x)$ is

$$U_s(y-P, w-x+I^*) - \mu V'(Y+P-I^*-C(I^*))(1+C'(I^*)) = 0, \tag{10}$$

where μ is a multiplier.

With a deductible $\bar{x}$, the insured's expected utility is

$$u(\bar{x}) \equiv \int_o^{\bar{x}} U(y-P(\bar{x}), w-x) f(x)\, dx + \int_{\bar{x}}^{w} U(y-P(\bar{x}), w-x+I^*(x,\bar{x})) f(x)\, dx,$$

and the insurer's expected utility is

$$v(\bar{x}) \equiv \int_o^{\bar{x}} V(Y+P(\bar{x})) f(x)\, dx + \int_{\bar{x}}^{w} V(Y+P(\bar{x})-I^*(x,\bar{x})-C(I^*)) f(x) dx.$$

To complete the maximization problem in (9), $u(\bar{x})+\mu v(\bar{x})$ is to be maximized with respect to $\bar{x}$. The differentiation of this objective with respect to $\bar{x}$ and the evaluation of the resulting equation at $\bar{x}=0$ gives

$$\begin{aligned}
&\frac{d}{d\bar{x}}[u(\bar{x})+\mu v(\bar{x})]\bigg|_{\bar{x}=0} \\
&\quad = -P' \int_o^{w} \left[U_c(y-P, w-x+I^*) - \mu V'(Y+P-I^*-C(I^*))\right] f(x)\, dx \\
&\qquad + \int_o^{w} \left[U_s(y-P, w-x+I^*) - \mu V'(Y+P-I^*-C(I^*))(1+C')\right] \frac{\partial I^*}{\partial \bar{x}} f(x)\, dx \\
&\quad = P' \int_o^{w} \left\{ \frac{1}{1+C'(I^*)} U_c(y-P, w-x+I^*)[\mathrm{MRS}(y-P, w-x+I^*) \right. \\
&\qquad \left. -(1+C'(I^*))] \right\} f(x)\, dx.
\end{aligned} \tag{11}$$

The last equality uses (10). Let $\phi(y : C'(I^*), I^*, w)$ denote the terms inside the squared bracket of (11). $\phi(y : \cdot) > (<)\ 0$ for higher (lower) income individuals, and (11) becomes negative (positive) and a Pareto optimal policy involves no deductible (deductible) if $P' < 0$.[10] This result contrasts with the literature in which a Pareto optimal policy involves a deductible or no deductible depending on whether $C'(\cdot) > 0$ or $C'(\cdot) = 0$.

CONCLUSION

The article reexamines a number of established results on optimal insurance by assuming that the utility of an individual depends on composite good consumption

[10] Making use of Equation (10), the relationship between P and $\bar{x}$ satisfies $U_s(y-P, w-\bar{x}) - \mu V'(Y+P)(1+C'(0)) = 0$. Differentiating it, we have

$$P' = \frac{dP}{d\bar{x}} = \frac{-U_{ss}(,)}{U_{sc}(,) + U_s(,)[V''(Y+P)/V'(Y+P)'']},$$

where utility functions are evaluated at $(y-P, w-\bar{x})$. In the standard model, $P' < 0$, because $U_{sc}(\cdot) = U_{cc}(\cdot) = U_{ss}(\cdot) < 0$. In the present model, P' cannot be signed.

and service flows generated by a property. This simple and realistic modification of the utility function produces results markedly different from standard predictions. With this utility function, the decision to purchase insurance depends mainly on the valuation of wealth or property relative to the valuation of income, which in turn depend on the income and preferences. As a result, the individuals with higher incomes or higher marginal rates of substitution purchase full coverage insurance while the individuals with lower incomes or lower marginal rates of substitution purchase partial coverage insurance. A fair premium is thus neither necessary nor sufficient for optimality of full coverage insurance.

The article considers a few well-known propositions in the insurance literature. However, the model presented in this article would certainly apply to other results in insurance economics as well as to various issues in uncertainty and risk in general. The application of the model to other areas may produce useful insights and more realistic results.

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