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Stock Options and the Corporate Demand for Insurance

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Abstract

This article shows that a corporate manager compensated in stock options makes corporate decisions to maximize stock option value. Overinvestment is a consequence if risk increases with investment. Facing the choice of hedging corporate risk with forward contracts on a stock market index fund and insuring pure risks the manager will choose the latter. Hedging with forwards reduces weight in both tails of corporate payoff distribution and thus reduces option value. Insuring pure risks reduces the weight in the left tail where the options are out-of-the-money and increases the weight in the right tail where the options are in-the-money; the effect is an increase in the option value. Insurance reduces the overinvestment problem but no level of insurance coverage can reduce investment to that which maximizes the shareholder value.

Most of the existing literature on the corporate demand for insurance rests either implicitly or explicitly on the notion that the decisions on corporate account are made to maximize the current shareholder value. Mayers and Smith (1982) began a discussion of the determinants of the demand for corporate insurance by noting that the "... corporate form provides an effective hedge since stockholders can eliminate insurable risk through diversification." Equivalently, the value of the insured firm is equal to that of the uninsured firm and insurance plays no role in the management of corporate risk.[1] The role insurance plays in managing corporate risk has since been clarified by Main (1983), MacMinn (1987), Mayers and Smith (1987), MacMinn and

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[1] This is a corollary of the Modigliani–Miller (1958) theorem, which shows that the value of the levered firm equals that of the unlevered firm and implies that capital structure decisions are irrelevant.

Han (1990), Garven and MacMinn (1993), and Han (1996). MacMinn (1987) shows that the risk-shifting problem can be solved with an insurance contract and so increase the value of the corporation;[2] similarly Mayers and Smith (1987) and Garven and MacMinn (1993) show that the underinvestment problem can be solved with insurance.[3] These results were derived assuming the maximization of current shareholder value. Stock option grants, however, have become an increasingly important component of executive compensation in the last two decades of the twentieth century (Murphy, 1998; Murphy, 1999). Stock options are supposed to align the incentives of management and shareholder since the options give management the incentive to increase the share price. The deductive foundation for this conventional wisdom has not been provided in the literature. Hence, the objective here is first to provide the link between the executive compensation scheme and corporate decision making, second to examine the impact of the executive compensation on the corporate demand for insurance, and third to examine the role insurance plays in aligning the interest of executives and shareholders.

The literature on the demand for corporate insurance is one thread of the broader literature on risk management. In the risk management literature, Smith and Stulz (1985) consider a managerial motive that provides a linkage between compensation and corporate decision making. They show that the risk-averse manager compensated with stock will use forward contracts to hedge risk; they also show that when the compensation is stock options, the options will ultimately eliminate the incentive to hedge.[4] There is some empirical support for the managerial theory in Tufano (1996).[5] The Smith and Stulz model differs from that here because they do not allow the corporate executive to hold a portfolio on personal account or diversify that portfolio. The managerial analysis is reframed here and the corporate objective function is explicitly derived for the manager paid in stock options; then the analysis shows that the stock options eliminate the incentive to hedge with forward contracts.[6] This might suggest that neither will the manager use insurance to manage risk but this model shows that not all risk management tools are created equal. The forward contract reduces risk by eliminating weight from the tails of the corporate earnings distribution and this reduces the value of the stock options. The liability insurance considered here requires a premium *now* for coverage *then*; it transfers cash from out-of-the-money states to in-the-money states, increases the value of the stock options and alters the incentives of the manager.

There is a large and growing empirical literature on executive compensation and corporate performance. DeFusco, Johnson, and Zorn (1990) provide an early example

[2] The risk-shifting problem was originally solved by Green (1984). See MacMinn (1993) for an alternative proof.

[3] These articles are apparently the first demonstration of a solution to the underinvestment problem.

[4] Also see Carpenter (2000) for the effects of a convex compensation scheme on the behavior of a risk-averse manager.

[5] Tufano studies the risk management practices in the gold mining industry and finds that managers who own more stock options manage gold price risk less using forward sales, gold loans, options, and other hedging activities as measures of risk management.

[6] The objective function could also be derived for the manager paid in stock to show that such a manager would not have the incentive to use forward contracts.

that the use of stock options is consistent with increasing corporate risk. Main (1991) studied a sample of Britain's largest industrial firms in an attempt to estimate the link between executive pay and shareholder wealth; he found some evidence that executive compensation policy aligns executive and shareholder interests. Lewellen et al. (1992) studied a sample of large industrial firms to determine the relationship between top executive compensation and firm performance; they find a positive relation and conclude that executive compensation is designed to reduce agency costs. Mayers and Smith (1992) considered differences between compensation in mutual versus stock insurance companies; they found the compensation in mutual insurance companies to be less responsive to firm performance than the stock insurance companies and found that result consistent with the difference in investment opportunity sets between the two firm types. Some of the more recent studies have tested the management entrenchment hypothesis by examining firms that have reset the option exercise prices (Acharya, 1992; Acharya, John, and Sundaram, 2000; Bebchuk, Fried, and Walker, 2002; Chidambaran and Prabhala, 2003); the evidence is mixed.

While it may be too early to expect definitive empirical results on the incentive effects of stock options,[7] it is possible to provide a theoretical foundation for the effects. In the next section, a financial market model is constructed, which allows the linkage between executive compensation and corporate performance to be established. The standard economic assumption of self-interested behavior is made here and the analysis shows that if a risk-averse corporate manager is paid in stock options then the manager makes decisions on corporate account to maximize the value of the stock option package, i.e., independent of preferences for consumption *now* versus *then*. In this model, the manager makes investment and financing decisions on behalf of the corporation. The analysis first shows that the corporate manager overinvests relative to the investment level that maximizes the current shareholder value. The analysis shows that the stock options fail to align the manager's interests with those of the shareholders; equivalently, the use of stock options in the compensation package creates an agency problem for the corporation by separating the interest of management and shareholders. This analysis also shows that the manager paid in stock options has no incentive to hedge with forward contracts. In the penultimate section, insurance is introduced and management makes an insurance decision as well as the investment and financing decisions. The analysis shows when insurance allows for a transfer of cash from the out-of-the-money states to the in-the-money states the manager has an incentive to purchase insurance. Finally, the analysis shows that the insurance allows a better alignment between the interests of shareholders and management. The final section provides concluding remarks.

THE UNINSURED CORPORATION

To introduce the first version of the model, consider a competitive economy operating between the dates *now* and *then*. The economy is composed of many individuals and firms. All choices are made *now* and the uncertain consequences of those choices are experienced *then*. The risk-averse individuals make choices for consumption *now*

[7] It should be noted that Tufano has provided empirical support for the hedging theorem provided here in addition to the support he claims for the Smith and Stulz managerial theory. It would be useful to find a test that can distinguish between the two models.

and *then*; in a financial market setting this means that consumers select a portfolio of financial assets, i.e., saving, that determines consumption *now* and *then*. Firms make investment and financing choices that payoff *then*. The financial markets are assumed to be competitive.[8] The analysis will focus on the behavior of a firm that is controlled by the decisions of its risk-averse chief executive officer (CEO); the standard assumption that agents act in their own self-interests is employed here for the firm. The CEO is faced with two sets of decisions. The first set consists of the portfolio decisions on personal account and the second is the set that consists of the investment and financing decisions made on corporate account. The analysis here will show that a Fisher separation result holds; acting in the pursuit of self-interests yields a well-defined objective function for the firm which does not depend on intertemporal consumption preferences or on risk aversion. The analysis will also show how the CEO's compensation scheme influences the corporate decisions.

Stock options have become an important component of executive compensation schemes and are used here to motivate the CEO. The conventional wisdom has been that the stock options provide the manger with an incentive to increase the corporate share price and so align the interests of management with that of shareholders. To capture the pure stock option effects, we will suppose that the CEO is paid with stock options alone.[9] The manager is granted the stock options *now* and the options vest *then*. This framework allows the conventional wisdom to be tested. The CEO then makes all decisions on personal and corporate account to maximize expected utility subject to a budget constraint on personal account and a financing constraint on corporate account. The budget constraint holds the risk-adjusted present value of the consumption plan to that of the income stream. The financial constraint specifies that the value of the debt issue covers the investment expenditure; the financial constraint could include a new stock issue as well as a debt issue but MacMinn and Page (2006) show that if the manager is paid in stock options *then*, that manager will finance first with retained earning if any exist, second with debt, and finally with equity. The MacMinn–Page theorem is a pecking order theorem similar to that of Myers and Majluf (1984), but it is motivated by stock options rather than asymmetric information.[10] The MacMinn–Page theorem shows the manager selects the

[8] To make the uncertainty operational suppose that the consumer can transfer dollars from *now* to *then* by purchasing one or more risky assets; each basis asset in a complete market model is a promise to pay one dollar if state of nature ω occurs and zero otherwise. Let $p(\omega)$ be the price of an asset that yields one dollar in state ω and zero otherwise.

[9] A fixed salary could also be added without affecting any of the results. Including bonuses or stock (restricted or not) would alter the results if given sufficient weight in the compensation scheme and deserve a separate focus. A compensation scheme that included just salary and stock would provide an incentive for the manager to maximize the current shareholder value of the corporation or equivalently would align interests with current shareholders. A compensation scheme that included salary, stock, and stock options would not align incentives just as the stock option scheme here shows; the stock in this scheme would dilute part of the incentive to increase risk that is provided by the stock options. See MacMinn (2005). Bonuses can be specified in so many different ways that it is difficult to generalize; some bonus packages can mimic stock or stock options and so yield similar results. For bonus scheme incentives see Brander and Poitevin (1992). Also see MacMinn's *Lecture on Brander and Poitevin* (1992).

[10] Unlike Myers–Majluf, the MacMinn–Page theorem also explicitly shows that retained earnings is preferred to safe debt, which in turn is preferred to stock. There was not difference between retained earnings and safe debt in the Mayers–Majluf model.

financing instrument that generates the least dilution in the manager's ownership stake in the corporation *then* when the stock options vest. No retained earnings are included in this model and so debt finance is selected due to the MacMinn–Page theorem.

The following notations are used in the model here:

ω	state of nature
$\Omega = (0, \zeta)$	set of states of nature
$\Psi(\omega)$	probability distribution for the states
$\Pi(I, \omega)$	payoff function for the corporation where $\frac{\partial \Pi}{\partial I} > 0, \frac{\partial^2 \Pi}{\partial^2 I} < 0$
$L(I, \omega)$	loss function for the corporation where $\frac{\partial L}{\partial I} > 0$
$(c_{i0}, c_{i1}(\omega))$	consumption *now* and *then* in state ω by individual i
$(m_{i0}, m_{i1}(\omega))$	income *now* in state ω by individual i
$p(\omega)$	basis stock price
$P(\omega)$	sum of the basis stock prices up to ω, i.e., $P(\omega) = \int_0^{\omega} \mathrm{d}P(t)$
P	corporate share price
m	number of stock options
N	number of shares of common stock *now*
b	promised repayment on a zero coupon bond issued to finance the corporate investment and insurance
a	manager's stake given exercise; $a \equiv \frac{m}{N+m}$
e	stock option exercise price *then*
W^j	warrant value, $j = u, i$, where u and i correspond to the uninsured and insured cases, respectively
V^j	corporate value, $j = u, i$.

In this financial market setting the payoff on the firm's zero coupon bond issue is $\max\{0, \min\{\Pi(I, \omega) - L(I, \omega), b\}\}$ and so value of the firm's debt issue is $D(I, b)$ where

$$D(I, b) = \int_0^{\zeta} \max\{0, \min\{\Pi(I,\omega) - L(I,\omega), b\}\} \, \mathrm{d}P(\omega)$$
$$= \int_{\gamma}^{\zeta} \min\{\Pi(I,\omega) - L(I,\omega), b\} \, \mathrm{d}P(\omega) \tag{1}$$

where γ is the boundary of the insolvency event or equivalently the state at which the net corporate payoff equals the promised debt payment. Suppose the manager is paid in stock options *now*. Let each option give the manager the right to purchase one share of corporate stock *then* at an exercise price of e dollars. Suppose the manager is paid with m options *now*. The corporation has N shares outstanding *now* and issues an additional m shares *then* if the manager exercises the options. Without loss of generality, suppose the firm issues bonds *now* to cover its investment expenditure. The manager selects a consumption pair, equivalently portfolio on personal account and an investment level on corporate account to solve the following constrained maximization problem[11]

$$\max \int_\Omega u_i(c_{i0}, c_{i1}(\omega))\, \mathrm{d}\Psi_i(\omega)$$

$$\begin{aligned}
\text{s.t.}\quad & c_{i0} + \int_\Omega c_{i1}(\omega)\, \mathrm{d}P(\omega) \\
&= m_{i0} + \int_\Omega m_{i1}(\omega)\, \mathrm{d}P(\omega) \\
&\quad + \int_\Omega \max\left\{0, \frac{m}{N+m}(\Pi(I,\omega) - L(I,\omega) + em - b) - em\right\} \mathrm{d}P(\omega) \\
&\text{and} \quad \int_\Omega \max\{0,\ \min\{\Pi(I,\omega) - L(I,\omega), b\}\}\, \mathrm{d}P(\omega) = I.
\end{aligned} \tag{2}$$

The last expression in the problem is the financing constraint; it determines the promised payment b on the debt issued *now* to raise I dollars for investment. The problem may also be expressed in reduced form by noting that the financing constraint implicitly defines a function $b(I)$.[12] Substituting this function into the budget constraint and simplifying yields the constrained maximization problem in the following reduced form:

$$\begin{aligned}
&\max \int_\Omega u_i(c_{i0}, c_{i1}(\omega))\, \mathrm{d}\Psi_i(\omega) \\
&\text{s.t.}\quad c_{i0} + \int_\Omega c_{i1}(\omega)\, \mathrm{d}P(\omega) = m_{i0} + \int_\Omega m_{i1}(\omega)\, \mathrm{d}P(\omega) + W(I)
\end{aligned} \tag{3}$$

where W represents the value of the stock option package, or equivalently, the warrant value.[13] The warrant value may be equivalently expressed as

$$\begin{aligned}
W(I) &= \int_\Omega \max\left\{0, \frac{m}{N+m}(\Pi(I,\omega) - L(I,\omega) + e\,m - b) - e\,m\right\} \mathrm{d}P(\omega) \\
&= \int_\Omega \max\{0, a(\Pi(I,\omega) - L(I,\omega) - b) - (1-a)E\}\, \mathrm{d}P
\end{aligned} \tag{4}$$

where $E = e$ m is the gross exercise value. In this form, it is clear that the manager makes decisions on corporate account to maximize the warrant value. Hence, we have a *Fisher Separation* result for the *publicly held and traded corporation*. This result specifies the corporate objective function and is noted in the next proposition.

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[11] This constrained maximization problem can be expressed in several forms and may include investments in other corporate assets without altering the conclusions reached here.

[12] See Lemma 1 for a derivation of this function in the more general case.

[13] A warrant usually refers to an option that is issued by a corporation and so becomes part of its capital structure whereas a call option trades against the same underlying asset but is not issued by the corporation and is not part of its capital structure. See Merton (1973).

Fisher Separation Theorem:[14] The manager paid in stock options *now* that vest *then* makes decisions on corporate account that are independent of the manager's preferences for consumption *now* versus *then*; the decisions on corporate account are made to maximize the warrant value W(I).

There are several versions of Fisher separation. While each depends on how the corporate manager is compensated, all versions show that the individual's decisions about portfolios on personal account can be separated from the decisions on corporate account; of course, the portfolio decisions on personal account depend upon the individual's measure of risk aversion but not the operating and financing decisions on corporate account. If the manager is paid in shares of corporate stock, then the same method, used in the appendix to prove this theorem, shows that the manager makes decisions on corporate account to maximize the stock value.

The manager makes the investment decision on corporate account to maximize the value of the objective function determined by the compensation scheme. The warrant scheme puts the payoff in the right tail of the distribution and motivates the manager to increase the probability weight there. How that motivation affects the investment decision depends on how the scale of the investment affects the payoff distribution. We will suppose in what follows that the principle of increasing uncertainty (PIU) (Leland, 1972) applies to the net payoff $\Pi - L$ so that the following derivative properties hold:

$$\frac{\partial(\Pi - L)}{\partial I} > 0; \quad \frac{\partial^2(\Pi - L)}{\partial\omega\partial I} > 0. \tag{5}$$

The PIU yields an increase in risk in the Rothschild–Stiglitz sense after correcting for a change in the mean of the payoff distribution (MacMinn and Holtmann, 1983); equivalently, increasing the scale of the investment will increase the weight in the tails of the payoff distribution.

The next theorem compares the optimal investment choices of the manager paid in stock options versus stock. The I^W and I^S denote the optimal choices of the managers paid in stock options and stock, respectively.

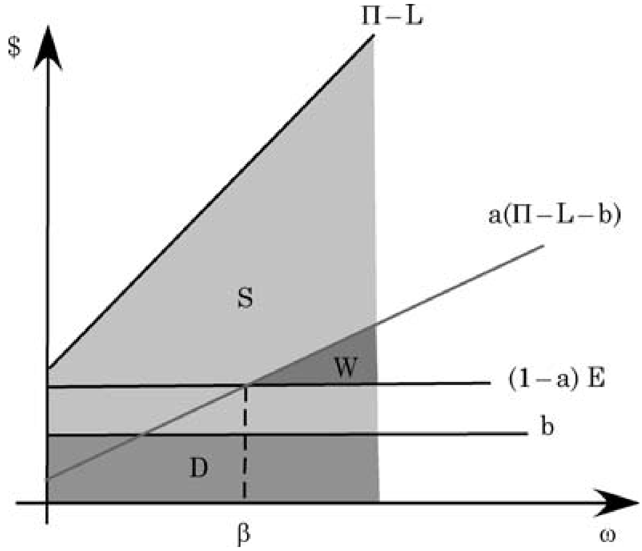
Theorem 1: *The manager compensated with stock options selects a larger investment than that same manager compensated with an equivalent equity stake, $I^W > I^S$.*

Proof: The stock value in the absence of stock options is

$$\begin{aligned} S(I) &= \int_{\Omega} \max\{0, \Pi(I,\omega) - L(I,\omega) - b(I)\}\, \mathrm{d}P(\omega) \\ &= \int_0^{\zeta} (\Pi(I,\omega) - L(I,\omega) - b(I))\, \mathrm{d}P(\omega) \end{aligned} \tag{6}$$

[14] The proof of this version of the Fisher separation result is provided in the Appendix. It, as all other versions, flows from the more is preferred to less assumption.

FIGURE 1
Values



where $b(I)$ is implicitly defined by the financing condition $D(b) = I$. The manager paid in stock selects the investment level to maximize $S(I)$.[15] The manager paid in stock options selects the investment level to maximize $W(I)$ and that investment level is larger if $\partial W/\partial I > 0$ when evaluated at I^S. Letting β denote the boundary of the exercise event as shown in Figure 1, note that

$$\begin{aligned}
\left.\frac{\partial W}{\partial I}\right|_{I=I^S} &= a\int_{\beta}^{\zeta}\left(\frac{\partial\Pi}{\partial I}-\frac{\partial L}{\partial I}-\frac{\partial b}{\partial I}\right)\mathrm{d}P \\
&= a\left(\int_{0}^{\zeta}\left(\frac{\partial\Pi}{\partial I}-\frac{\partial L}{\partial I}-\frac{\partial b}{\partial I}\right)\mathrm{d}P-\int_{0}^{\beta}\left(\frac{\partial\Pi}{\partial I}-\frac{\partial L}{\partial I}-\frac{\partial b}{\partial I}\right)\mathrm{d}P\right) \\
&= -a\int_{0}^{\beta}\left(\frac{\partial\Pi}{\partial I}-\frac{\partial L}{\partial I}-\frac{\partial b}{\partial I}\right)\mathrm{d}P \\
&> 0.
\end{aligned} \tag{7}$$

The first term after the second inequality is the first order condition for maximizing the stock value in (6) and so vanishes when evaluated at I^S. Since $\partial b/\partial I$ is a constant, the PIU suffices to show that

$$\frac{\partial}{\partial\omega}\left(\frac{\partial\Pi}{\partial I}-\frac{\partial L}{\partial I}-\frac{\partial b}{\partial I}\right) > 0$$

[15] Suppose the manager is paid in stock that amounts to a percent of the firm's equity. The manager will select the investment level that maximizes a $S(I)$, which will yield the same first order condition as that maximizes $S(I)$.

and so $\partial\Pi/\partial I - (\partial L/\partial I) - (\partial b/\partial I)$ goes from negative to positive monotonically. Hence (7) is positive when evaluated at $I = I^S$. Since (7) is positive when evaluated at I^S, it follows that $I^W > I^S$.

Theorem 1 establishes the benchmark. The CEO makes decisions to maximize the value of the warrants and in doing so selects an investment level that does not maximize the stock value of the corporation; the manager selects a larger investment that puts more weight in the tails of the payoff distribution and increases the risk. The risk is pushed too far from the shareholder perspective but the manager does not share in the downside risk with the shareholders. The division of value is represented in Figure 1. The risk-adjusted values of the areas depicted by S, W, and D represent the stock, warrant, and debt values, respectively, of the corporation.

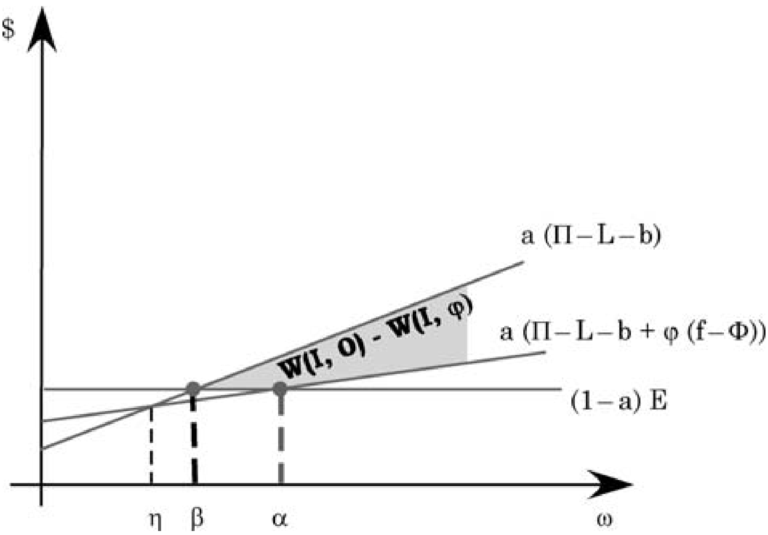
An insurance contract is one of a number of contracts that the firm can use in managing corporate risk. It will be considered in the next section. It is, however, also possible to use forward and futures contracts to manage the same risk. To the extent that both insurance and forward contracts can be used to hedge risk, it is natural to claim that the two contracts are substitutes. If they are perfect substitutes, then it is only necessary to use one; similarly, if the firm chooses not to use one, then it will not use the other. Smith and Stulz (1985) show that the manager compensated with stock options will have less incentive to hedge with forward contracts.

> ... if the manager's end-of-period utility is a convex function of the end-of-period firm value, Jensen's Inequality implies that the manager's end-of-period utility has a higher expected value if the firm is not hedged at all. Bonus or stock option provisions of compensation plans can make the manager's expected utility a convex function of the value of the firm. If the manager's expected utility is a convex function of the value of the firm, the manager will behave like a risk-seeker even though his expected utility function is a concave function of his end-of-period wealth.

Smith and Stulz consider a risk-averse manager with no ability to diversify on personal account. The model here provides for the diversification on personal account and shows that it yields a corporate objective function which eliminates risk aversion in corporate decision making. The distinction is significant because the manager in the Smith and Stulz model who is paid in stock has the incentive to hedge risk but that is due to the risk aversion; the motivation is altered by stock options only because sufficient options make the objective function used by the manager convex rather than concave.[16] The distinction is also important because the manager paid in stock in this model has no incentive to hedge and that motivation is not qualitatively altered by stock options. Since such a result on stock options and hedging does not exist in the literature, it is provided in the next theorem.

[16] The objective function is the expected value of the composition of functions; one is the utility function and the other is the compensation, which is a function of the hedge. If the compensation is stock then the second function is linear and the composite function is concave. If the compensation is stock options then the second function is convex and the composite function may be convex.

FIGURE 2
Hedging with Forwards



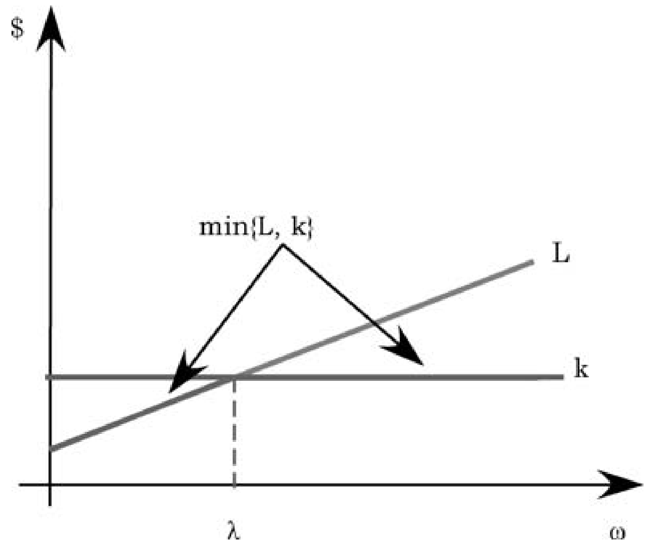
Suppose there are no traded forwards either on the firm's operating payoff or on the firm's insurable loss; but there are forwards on a stock market index traded in the financial market. Let Φ denote the unit payoff on the stock index and suppose the payoff increases in state. Further, suppose that forward contracts are traded with a unit gain or loss *then* of $(f - \Phi(\omega))$, where f is the forward price. The manager chooses the number of forward contracts, φ. The corporate payoff becomes $\Pi - L + \varphi(f - \Phi) - b$ as shown in Figure 2. By hedging, the manager may increase the weight in the left tail of the corporate payoff and decrease the weight in the right tail as shown in Figure 2. Letting η be the state in which there is no capital gain or loss on the forward position, β be the boundary of the in-the-money event for the unhedged firm and α be the boundary of the in-the-money event for the hedged firm, the figure depicts the clearest case of a loss in option value due to the hedge.

Hedging Theorem:[17] Let the payoff on the stock index Φ be increasing in state. *Ceteris paribus*, the manager paid in stock options does not have an incentive to hedge in forward contracts written on the stock index.

This theorem provides an alternative explanation for why managers paid in stock options do not hedge. Hedging in forward contracts reduces the weight in the right tail of the corporate payoff distribution and this has the effect of reducing the value of the warrants. The result is similar to that in Smith and Stulz (1985) but the model is not. In Smith and Stulz, the manager considers hedging due to risk aversion and there is no separation between decision on personal and corporate account; there the result followed when the convexity introduced by the stock option package was enough to counteract the concavity of the utility function, i.e., the risk aversion. Tufano (1996)

[17] See the Appendix for additional discussion and the proof of this theorem.

FIGURE 3
Insurance Payoff



notes ". . . the data bear out Smith and Stulz's (1985) prediction that firms whose managers own more stock options manage less gold price risk, and those whose managers have more wealth invested in common stock manage more gold price risk."[18] There are other theories to explain this. With the Fisher separation, here the result follows more generally because a hedge in forward contracts reduces the value of the warrants. It does not have to be risk aversion that is driving it. A manager paid in stock options wants more weight in the right tail because the manager maximizes the warrant value in the compensation package. Indeed it is not apparent that Tufano's empirical results can distinguish between the two models.

THE INSURED CORPORATION

In the last section, the manager chose to overinvest and given the ability to manage risk with forward contracts chose not to manage the risk. An insurance contract is introduced here and first we consider whether the manager has an incentive to purchase insurance on corporate account and if so why. Second, we consider how insurance changes the investment incentives.

To explicate the points made here suppose that the losses are liability losses and that the contract provides indemnity up to a cap of k dollars. Then the insurance contract payoff is $\min\{L, k\}$. Recall that the corporate payoff net of insurable losses, i.e., $\Pi - L$, is assumed to be an increasing function of state. This can be a result of many combinations of the payoff function and loss function. Two cases might be considered. The first case is provided in Figure 3. It is supposed there that the loss increases in state and incorporates the notion that losses increase with economic activity. The second case of a decreasing loss function is provided in the Appendix. Let λ denote the state

[18] See p. 1099 in Tufano (1996).

where the loss is equal to the cap; λ is implicitly defined by the condition $L(I, \lambda) - k = 0$. Given the increasing loss, the insured corporation gets k dollars from the insurer as indemnity and absorbs the loss over and beyond k in states greater than λ. Letting $i(k)$ denote the market value of the insurance contract, we may note that

$$\begin{aligned} i(I,k) &= \int_0^\zeta \min\{L(I,\omega),k\}\,\mathrm{d}P \\ &= \int_0^\lambda L(I,\,\omega)\,\mathrm{d}P + \int_\lambda^\zeta k\,\mathrm{d}P. \end{aligned} \tag{8}$$

Observe that the corporate value of the uninsured corporation is V^u, where

$$V^u = \int_0^\zeta (\Pi - L)\,\mathrm{d}P \tag{9}$$

while the value of the insured corporation is V^i where

$$\begin{aligned} V^i &= -i + \int_o^\zeta (\Pi - L + \min\{L,k\})\,\mathrm{d}P \\ &= V^u. \end{aligned} \tag{10}$$

The second equality in (10) follows by (8) and provides a familiar corollary to the 1958 Modigliani–Miller theorem (Modigliani and Miller, 1958); for examples in the literature see MacMinn (1987). Equation (10) provides a clear corollary to the theorem and shows that insurance *ceteris paribus* does not add value. Equivalently, it may be shown that the stock market value increases equal to that of the increase in the insurance premium so that again no value is added. A gain or loss in value will follow, however, if the investment level is affected by the insurance decision.

Now consider the value of the manager's warrant given the possible insurance coverage. The warrant payoff becomes

$$\begin{aligned} &\max\{0,\ a(\Pi - L + \min\{L,k\} + e\,m - b) - e\,m\} \\ &\quad = \max\{0,\ a(\Pi - L + \min\{L,k\} - b) - (1-a)\,E\}. \end{aligned} \tag{11}$$

The warrant value becomes $W(I, k)$ where

$$W(I,\,k) = \int_0^\zeta \max\{0, a(\Pi - L + \min\{L,\,k\} - b) - (1-a)\,E\}\,\mathrm{d}P. \tag{12}$$

The financing condition in the constrained maximization problem must be altered to raise enough now to cover the insurance premium as well as the investment level; the financing constraint becomes

$$D(b) \equiv \int_0^\zeta b\,\mathrm{d}P = I + i(I,k) \tag{13}$$

but it continues to implicitly define the function $b(I, k)$. As previously, suppose that any debt issue is safe. Then let $F(I, k, b) = 0$ implicitly define $b(I, k)$ where

$$\begin{aligned} F(I, k, b) &= D(b) - I - i(I,k) \\ &= \int_0^{\zeta} b \, \mathrm{d}P - I - \left(\int_0^{\lambda} L(I, \omega) \, \mathrm{d}P + \int_{\lambda}^{\zeta} k \, \mathrm{d}P \right). \end{aligned} \tag{14}$$

The derivative properties of the function $b(I, k)$ are provided in the following lemma.

Lemma 1: *The promised payment on the debt issue increases in the investment I and the cap k.*

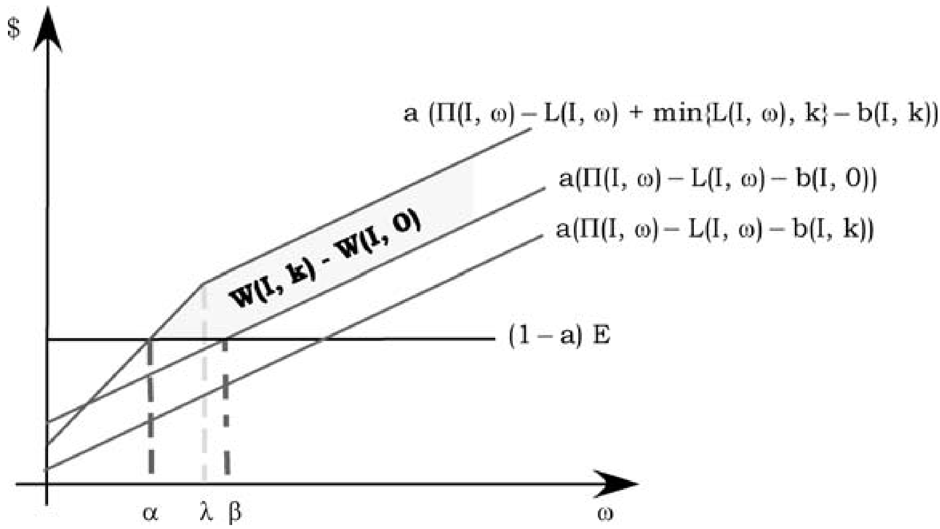
Proof: By the Implicit Function Theorem,

$$\begin{aligned} \frac{\partial b}{\partial I} &= -\frac{\frac{\partial F}{\partial I}}{\frac{\partial F}{\partial b}} \\ &= \frac{1 + \int_0^{\lambda} \frac{\partial L}{\partial I} \, \mathrm{d}P + (L(I, \lambda) - k) p(\lambda) \frac{\partial \lambda}{\partial I}}{\int_0^{\zeta} \mathrm{d}P} \\ &= \frac{1 + \int_0^{\lambda} \frac{\partial L}{\partial I} \, \mathrm{d}P}{\int_0^{\zeta} \mathrm{d}P} \\ &> 0 \end{aligned} \tag{15}$$

and

$$\begin{aligned} \frac{\partial b}{\partial k} &= -\frac{\frac{\partial F}{\partial k}}{\frac{\partial F}{\partial b}} \\ &= \frac{\int_{\lambda}^{\zeta} \mathrm{d}P + (L(I, \lambda) - k) p(\lambda) \frac{\partial \lambda}{\partial k}}{\int_0^{\zeta} \mathrm{d}P} \\ &= \frac{\int_{\lambda}^{\zeta} \mathrm{d}P}{\int_0^{\zeta} \mathrm{d}P} \\ &> 0. \end{aligned} \tag{16}$$

FIGURE 4
The Difference in Warrant Values



Now consider the manager's insurance decision in isolation. Maintain the assumption that the manager is compensated with stock options. Then the manager will choose to insure if $W(I, k) > W(I, 0)$ for $k > 0$.

To motivate the comparison of warrant values consider Figure 4.[19] The line labeled with $a(\Pi(I,\omega) - L(I,\omega) - b(I,0))$ represents the gross warrant payoff in the uninsured case. The line labeled with $a(\Pi(I, \omega) - L(I, \omega) - b(I, k))$ is the payoffs prior to the indemnity from the insurer. Note that $b(I, k)$ is the promised debt payment required to pay for the investment and insurance. The line labeled $a(\Pi(I, \omega) - L(I, \omega) + \min\{L(I,\omega), k\} - b(I,k))$ represents the gross warrant payoff after receipt of insurance indemnity. The gross insured warrant payoff falls below the gross uninsured warrant payoff for some states but above for others. The gross insured warrant payoff is greater than the uninsured counterpart for the larger states and the manager benefits because those are the in-the-money states for the options; the gross uninsured payoff is greater than the insured counterpart for smaller states but those correspond more to the out-of-the-money event and so do not adversely affect the manager. To motivate this comparison of gross warrant payoffs, note that the insurance decision yields more weight in the right tail of the payoff distribution if

$$\Pi(I,\omega) - L(I,\omega) + \min\{L(I,\omega),k\} - b(I,k) > \Pi(I,\omega) - L(I,\omega) - b(I,0) \qquad (17)$$

for states $\omega > \alpha$. For states $\omega > \lambda$, (17) becomes

[19] The relative positions of α, β, and λ in Figure 4 are just for illustration and do not affect the result in Theorem 2a.

$$\Pi(I,\omega) - L(I,\omega) + k - b(I,k) > \Pi(I,\omega) - L(I,\omega) - b(I,0) \tag{18}$$

or equivalently

$$k > b(I,k) - b(I,0). \tag{19}$$

Given the safe debt issue, the financing condition in (13) yields $b(I,k) = (I + i(I,k))/\int_0^\zeta \mathrm{d}P$ and so (19) may be equivalently expressed as

$$\begin{aligned} k &> \frac{i(I,k)}{\int_0^\zeta \mathrm{d}P} \\ \Leftrightarrow \int_0^\zeta k\,\mathrm{d}P &> \int_0^\lambda L(I,\,\omega)\,\mathrm{d}P + \int_\lambda^\zeta k\,\mathrm{d}P \\ \Leftrightarrow \int_0^\lambda k\,\mathrm{d}P &> \int_0^\lambda L(I,\,\omega)\,\mathrm{d}P. \end{aligned} \tag{20}$$

The last inequality in (20) clearly holds and so establishes (18) for all $\omega > \lambda$ and for some $\omega < \lambda$ by continuity. Hence, the insurance increases the payoff in the right tail of the corporate payoff distribution and reduces it in the left tail as shown in Figure 4.

The warrant payoffs are shown in Figure 4. The shaded area in the figure represents $W(I, k) - W(I, 0)$ and shows that the manager has a motivation to insure. The positive intercepts simply indicate that the debt is safe. As long as the manager can optimally select an investment and finance that investment with safe debt, the motivation to insure exists. The incentive to insure also follows from the first order condition and is expressed in the following theorem.

Theorem 2a: *Let Π, L, and $\Pi - L$ be increasing in state.*[20] Ceteris paribus, *the manager paid in stock options and financing the corporate investment with safe debt has an incentive to insure the liability losses.*

Proof: The manager selects k to maximize the warrants' value. Consider two cases.

Case I: For small values of the cap $0 < \lambda \leq \alpha$ the warrant value is

$$W(I,k) = \int_\alpha^\zeta [a(\Pi - (L-k) - b) - (1-a)E]\,\mathrm{d}P \tag{21}$$

and the first order condition for k is

[20] This theorem demonstrates what we consider the more common case that as the corporate payoff increases so does the liability loss. The case in which the corporate payoff Π increases and the liability losses decrease is in the Appendix.

$$
\begin{aligned}
\frac{\partial W}{\partial k} &= a\int_{\alpha}^{\zeta}\left(1-\frac{\partial b}{\partial k}\right)\mathrm{d}P \\
&= a\left\{\int_{\alpha}^{\zeta}\mathrm{d}P - \frac{\displaystyle\int_{\lambda}^{\zeta}\mathrm{d}P}{\displaystyle\int_{0}^{\zeta}\mathrm{d}P}\int_{\alpha}^{\zeta}\mathrm{d}P\right\} \\
&= a\int_{\alpha}^{\zeta}\mathrm{d}P\left\{1-\frac{\displaystyle\int_{\lambda}^{\zeta}\mathrm{d}P}{\displaystyle\int_{0}^{\zeta}\mathrm{d}P}\right\} \\
&> 0.
\end{aligned} \tag{22}
$$

Case II: For larger values of the cap, $0 < \alpha < \lambda$ the warrant value is

$$
W(I,k) = \int_{\alpha}^{\lambda}[a(\Pi - b) - (1-a)E]\,\mathrm{d}P + \int_{\lambda}^{\zeta}[a(\Pi-(L-k)-b)-(1-a)E]\,\mathrm{d}P \tag{23}
$$

and the first order condition for k is

$$
\begin{aligned}
\frac{\partial W}{\partial k} &= a\left(\int_{\alpha}^{\lambda}\left(-\frac{\partial b}{\partial k}\right)\mathrm{d}P + \int_{\lambda}^{\zeta}\left(1-\frac{\partial b}{\partial k}\right)\mathrm{d}P\right) \\
&= a\left\{\int_{\lambda}^{\zeta}\mathrm{d}P - \frac{\partial b}{\partial k}\int_{\alpha}^{\zeta}\mathrm{d}P\right\} \\
&= a\left\{\int_{\lambda}^{\zeta}\mathrm{d}P - \frac{\displaystyle\int_{\lambda}^{\zeta}\mathrm{d}P}{\displaystyle\int_{0}^{\zeta}\mathrm{d}P}\int_{\alpha}^{\zeta}\mathrm{d}P\right\} \\
&= a\int_{\lambda}^{\zeta}\mathrm{d}P\left\{1-\frac{\displaystyle\int_{\alpha}^{\zeta}\mathrm{d}P}{\displaystyle\int_{0}^{\zeta}\mathrm{d}P}\right\} \\
&> 0.
\end{aligned} \tag{24}
$$

The warrant value is shown to increase monotonically although not continuously in k subject to the boundary condition set by the safe debt assumption, i.e., $\Pi - L - b(I,k^{+}) + \min\{L,k^{+}\} = 0$, where k^{+} is the cap that satisfies that boundary condition. The safe debt assumption is made to avoid the effect of insolvency risk on insurance decisions, which is well established in the literature.

Since the insurance does not change the corporate value and the debt issue is appropriately priced, it follows that there is a transfer of value from shareholders to the manager who is compensated with stock options. Note that the manager paid with stock options has the incentive to insure if the optimal investment level can be selected and financed with safe debt.[21] The incentive to insure exists because the insurance places more weight in the right tail of the corporate earnings distribution and hence increases the stock option value for the manager. The result does depend directly on the compensation scheme and liability insurance contract used in the analysis.

In conjunction with the hedging theorem, Theorem 2a shows that not all hedging instruments are created equally. The forward contracts do reduce risk but do it by moving weight from the tails of the earnings distribution; since weight is removed from the right tail the manager paid in stock options has no incentive to hedge with forward contracts but does have the incentive to hedge with insurance contracts.

Theorem 2b in the appendix shows the opposite scenario in which insurable losses decrease in state. The result is the opposite of that reported in Theorem 2a, i.e., the manager will not purchase liability insurance on corporate account. The result follows because liability insurance in this case shifts weight to the left tail of the earnings distribution and so to the out-of-the-money event for the manager.

Next, consider the second objective, i.e., what impact does the insurance have on the manager's investment choice? Does the investment choice become a function of the cap on the insurance? To answer the first question, consider the impact that insurance has on the manager's investment decision by comparing the optimal investment in the uninsured and insured cases. The manager with insurance coverage selects the investment level to maximize the warrant value expressed in (12). Let I^{W^i} and I^{W^u} denote the manager's optimal investment choices in the insured and uninsured cases, respectively. The next theorem demonstrates the relation between these investment choices.

Theorem 3: *Let Π, L and $\Pi - L$ be increasing in state and let the PIU hold. Insurance reduces the manager's incentive to overinvest, i.e., $I^{W^u} > I^{W^i}$.*

Proof: The insurance cap, k, creates a kink in the payoff where the warrant is not differentiable and so the analysis must consider the cases where the cap is not binding and where it is binding.

Case I: When k is small, $\lambda < \alpha < \beta$, where, as previously, α and β denote the boundaries of the option exercise events in the insured and uninsured cases, respectively. The warrant value in this case is stated in Equation (21). The first order condition for the investment level of the insured is

$$\frac{\partial W(I,k)}{\partial I} = \int_{\alpha}^{\zeta} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I} \right) \mathrm{d}P = 0. \tag{25}$$

[21] As noted, the safe debt assumption is used more for convenience than necessity. The manager paid in stock options will always select debt finance over equity finance. If the debt finance is not safe then the promised payment or equivalently the implied rate of return must be increased to compensate for the insolvency risk and that will, *ceteris paribus*, reduce but not eliminate the incentive to insure for a sufficiently small probability of insolvency. This claim does, of course, require proof.

The warrant value in the uninsured case is stated in Equation (4). The first order condition in the uninsured case is

$$\frac{\partial W(I,0)}{\partial I} = \int_{\beta}^{\zeta} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,0)}{\partial I} \right) \mathrm{d}P = 0. \tag{26}$$

From (25) and (26), the difference in first order conditions is

$$\begin{aligned}
&\left.\frac{\partial W(I,k)}{\partial I} - \frac{\partial W(I,0)}{\partial I}\right|_{I=I^{W^u}} \\
&= \int_{\alpha}^{\zeta} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I} \right) \mathrm{d}P - \int_{\beta}^{\zeta} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,0)}{\partial I} \right) \mathrm{d}P \\
&= \int_{\alpha}^{\beta} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b}{\partial I} \right) \mathrm{d}P + \int_{\beta}^{\zeta} a \left(\frac{\partial b(I,0)}{\partial I} - \frac{\partial b(I,k)}{\partial I} \right) \mathrm{d}P \qquad (27) \\
&= \int_{\alpha}^{\beta} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b}{\partial I} \right) \mathrm{d}P - a \frac{\int_0^{\lambda} \frac{\partial L}{\partial I} \mathrm{d}P}{\int_0^{\zeta} \mathrm{d}P} \int_{\beta}^{\zeta} \mathrm{d}P \\
&< 0.
\end{aligned}$$

The first term on the right-hand side of the third equality in (27) is negative by the PIU; the second term on the right-hand side of the third equality in (27) follows by (15) and direct calculation and is negative since the loss is increasing in dollars invested. Hence, the uninsured investment level exceeds the insured level, i.e., $I^{W^u} > I^{W^i}$.

Case II: As k increases, $\alpha < \lambda < \beta$ as depicted in Figure 4. The warrant value for the insured firm in this case is given in Equation (23). The first order condition in the insured case may be expressed as follows:

$$\frac{\partial W(I,k)}{\partial I} = \int_{\alpha}^{\lambda} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial b(I,k)}{\partial I} \right) \mathrm{d}P + \int_{\lambda}^{\zeta} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I} \right) \mathrm{d}P = 0. \tag{28}$$

Similarly, the first order condition in the uninsured case is in Equation (26). From (26) and (28), the difference in the derivatives evaluated at the optimal uninsured investment level is

$$\begin{aligned}
&\left.\frac{\partial W(I,k)}{\partial I} - \frac{\partial W(I,0)}{\partial I}\right|_{I=I^{W^u}} \\
&= \int_{\alpha}^{\lambda} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial b(I,k)}{\partial I} \right) \mathrm{d}P + \int_{\lambda}^{\zeta} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I} \right) \mathrm{d}P \\
&\quad - \int_{\beta}^{\zeta} a \left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,0)}{\partial I} \right) \mathrm{d}P
\end{aligned}$$

$$\begin{aligned}&= \int_{\alpha}^{\lambda} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) \mathrm{d}P + \int_{\lambda}^{\beta} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) \mathrm{d}P \\ &\quad + \int_{\beta}^{\zeta} a\left(\frac{\partial b(I,0)}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) \mathrm{d}P \\ &< 0.\end{aligned} \tag{29}$$

First observe that the third term on the right-hand side of the second equality in (29) is negative just as in the previous case. Second recall that the PIU shows that $((\partial\Pi/\partial I) - (\partial L/\partial I) - (\partial b/\partial I))$ is increasing; since this derivative is negative for some ω and monotone increasing, when we eliminate the integrand from β to ζ we have eliminated part of the positive component and left the negative. Hence the difference in derivatives in (29) is negative by the PIU. It follows that the uninsured investment level exceeds the insured level, i.e., $I^{W^u} > I^{W^i}$.

Case III: As k increases further, $\alpha < \beta < \lambda$. The same method used to establish Cases I and II applies here and shows that $I^{W^u} > I^{W^i}$.

Theorem 2 shows that there is an incentive to insure. Theorem 3 shows that the insurance reduces the incentive to overinvest.

Some questions remain. Can insurance be used to effectively eliminate the overinvestment problem? Does the optimal investment get smaller as the insurance coverage is increased? The next theorem shows how increasing insurance coverage affects the manager's investment decision.

Theorem 4: *Let Π, L, and $\Pi - L$ be increasing in state. The optimal investment increases with added insurance coverage if $\alpha < \lambda$ and decreases with added insurance coverage if $\alpha > \lambda$.*

Proof: Let $I(k)$ denote the manager's optimal investment decision given the insurance cap k. Since the optimal investment is implicitly defined by

$$\frac{\partial W(I,\, k)}{\partial I} = 0 \tag{30}$$

it follows by implicit differentiation that

$$\frac{\mathrm{d}I}{\mathrm{d}k} = -\frac{\dfrac{\partial^2 W}{\partial k\, \partial I}}{\dfrac{\partial^2 W}{\partial^2 I}}. \tag{31}$$

Hence, the optimal investment will increase or decrease as the cross partial in the numerator of (31) is positive or negative, respectively.

Case I: For k such that $\alpha > \lambda$, the warrant value is given in (21). The marginal warrant value for this case is expressed in Equation (25) and differentiating again with respect to coverage yields

$$
\begin{aligned}
&\frac{\partial^2 W(I,k)}{\partial k\,\partial I} \\
&= \frac{\partial}{\partial k}\left(\int_\alpha^\zeta a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right)\mathrm{d}P\right) \\
&= -a\left(\frac{\partial \Pi(I,\alpha)}{\partial I} - \frac{\partial L(I,\alpha)}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right)p(\alpha)\frac{\partial \alpha}{\partial k} + \int_\alpha^\zeta a\left(-\frac{\partial^2 b(I,k)}{\partial k\,\partial I}\right)\mathrm{d}P \\
&< 0.
\end{aligned}
\tag{32}
$$

The inequality in (32) follows since

$$
\begin{aligned}
\frac{\partial^2 b}{\partial k\,\partial I} &= \frac{\displaystyle\int_0^\zeta \mathrm{d}P\,\frac{\partial}{\partial k}\left(\int_0^\lambda \frac{\partial L}{\partial I}\,\mathrm{d}P\right)}{\left(\displaystyle\int_0^\zeta \mathrm{d}P\right)^2} \\
&= \frac{\left(\dfrac{\partial L(I,\lambda)}{\partial I}\,p(\lambda)\,\dfrac{\mathrm{d}\lambda}{\mathrm{d}k}\right)}{\displaystyle\int_0^\zeta \mathrm{d}P} \\
&> 0.
\end{aligned}
$$

α is a decreasing function of the cap k and the marginal equity payoff in parentheses is negative at α.[22] Hence, the optimal investment decreases as the cap increases.

Case II: For k such that $\alpha < \lambda$, the warrant value is expressed in (23) and the marginal warrant value is expressed in (28). Differentiating (28) with respect to k yields

$$
\begin{aligned}
\frac{\partial^2 W(I,k)}{\partial k\,\partial I} &= a\frac{\partial L(I,\lambda)}{\partial I}\,p(\lambda)\frac{\mathrm{d}\lambda}{\mathrm{d}k} + \int_\alpha^\zeta a\left(-\frac{\partial^2 b(I,k)}{\partial k\,\partial I}\right)\mathrm{d}P \\
&= a\frac{\partial L(I,\lambda)}{\partial I}\,p(\lambda)\frac{\mathrm{d}\lambda}{\mathrm{d}k} - a\,\frac{\left(\dfrac{\partial L(I,\lambda)}{\partial I}\,p(\lambda)\,\dfrac{\mathrm{d}\lambda}{\mathrm{d}k}\right)}{\displaystyle\int_0^\zeta \mathrm{d}P}\int_\alpha^\zeta \mathrm{d}P
\end{aligned}
$$

[22] The negative sign of the marginal equity payoff follows due to the PIU. Note that the marginal equity payoff is monotone increasing by the PIU and so takes negative values followed by positive values.

$$= a\frac{\partial L(I,\lambda)}{\partial I}\,p(\lambda)\frac{d\lambda}{dk}\left(1-\frac{\displaystyle\int_{\alpha}^{\zeta} dP}{\displaystyle\int_{0}^{\zeta} dP}\right) \tag{33}$$

$$> 0.$$

The inequality in (33) follows since λ is an increasing function of k, the loss increases in the investment level and $\alpha > 0$. Hence, in this case, the optimal investment increases as the cap increases.

Finally, consider the case of a cap k^0 such that $\alpha = \lambda$. Compare the derivatives in the previous two cases when evaluated at k^0. Equation (32) becomes

$$\left.\frac{\partial^2 W(I,k)}{\partial k\,\partial I}\right|_{k\leftarrow k^0}$$

$$= -a\left(\frac{\partial\Pi(I,\lambda)}{\partial I}-\frac{\partial L(I,\lambda)}{\partial I}-\frac{\partial b(I,k)}{\partial I}\right)p(\lambda)\,\frac{\partial\lambda}{\partial k}\;-a\,\frac{\left(\dfrac{\partial L(I,\lambda)}{\partial I}\,p(\lambda)\,\dfrac{d\lambda}{dk}\right)}{\displaystyle\int_0^{\zeta} dP}\int_{\lambda}^{\zeta} dP$$

$$= -a\left(\frac{\partial\Pi(I,\lambda)}{\partial I}-\frac{\partial b(I,k)}{\partial I}\right)p(\lambda)\,\frac{\partial\lambda}{\partial k}+a\,\frac{\partial L(I,\lambda)}{\partial I}\,p(\lambda)\,\frac{\partial\lambda}{\partial k}\left(1-\frac{\displaystyle\int_{\lambda}^{\zeta} dP}{\displaystyle\int_{0}^{\zeta} dP}\right). \tag{34}$$

Similarly, (33) becomes

$$\left.\frac{\partial^2 W(I,k)}{\partial k\,\partial I}\right|_{k\rightarrow k^0} = a\,\frac{\partial L(I,\lambda)}{\partial I}\,p(\lambda)\frac{d\lambda}{dk}\left(1-\frac{\displaystyle\int_{\lambda}^{\zeta} dP}{\displaystyle\int_{0}^{\zeta} dP}\right). \tag{35}$$

Observe that the derivative in (34) represents the cross partial obtained by approaching the k such that $\alpha = \lambda$ from the right-hand side while (35) represents the cross partial obtained by approaching the same k from the left-hand side.

Note that Case I results when the insurance cap is sufficiently small. Without violating the Case I condition, an increase in the cap reduces the set in which the warrant is out-of-the-money, thus increasing the warrant value. As a result, the manager has less incentive to resort to overinvestment as a tool to raise the warrant value. Case II follows when the cap is sufficiently large so that the "at-the-money" state α is smaller than the state λ in which the loss is equal to the cap. This is the case depicted in Figure 4. Since premium increases with k, state α also increases in k. Two effects are at work here. As state α increases, out-of-the-money states increase and so does the insurance premium; thus more cash is available to be transferred from out-of-the-money states to in-the-money states. As Equation (34) shows, the effect of the increase in α dominates the effect of more transfer of cash; i.e., more investment is needed to increase the warrant value. Thus, when the Case II condition holds, an increase in k is accompanied with an increase in the investment.

Theorem 4 clearly shows that the CEO's investment choice is influenced by the insurance coverage. The second case in Theorem 4 shows that the insurance may either increase or decrease the investment incentive but may well be moot since Theorem 2 shows that the warrant value can be increased with insurance coverage. In combination, Theorems 2 and 4 show that insurance can be an effective tool in reducing the adverse incentives created by the stock option compensation scheme. Theorem 4 suggests that the overinvestment problem is minimized when k is set so that $\alpha = \lambda$. If the manager's compensation is set only based on stock value, the manager's decision on corporate account cannot be consistent with shareholders' interests. This has been shown in Han (1996). To minimize overinvestment, the manager's compensation needs to be based on both operating results and risk management results; i.e., the compensation package should consist of both the exercise price and the insurance cap.

So far the results have been explained with a given exercise price. Suppose the manager can sway the setting of the exercise price. Theorem 4 can be interpreted as suggesting that a sufficiently large exercise price in conjunction with insurance reduces the overinvestment.

CONCLUDING REMARKS

With few exceptions, the literature on the demand for corporate insurance either implicitly or explicitly assumes that firm decisions are made to maximize the stock value. The connection between management compensation and corporate decision making has not been adequately explored.[23] Stock options have become an increasingly important component of compensation packages in part due to the conventional but flawed wisdom that they align management and shareholder incentives. This analysis has shown that the flaw becomes evident in the objective function provided by the separation theorem.

Smith and Stulz (1985) show that managers paid in stock options will ultimately not hedge corporate risk. The manager in their analysis maximizes the expected utility of the end of period wealth due to the compensation with no ability to diversify on personal account; this eliminates the possibility of a separation result. This model allows the manager to make decisions on corporate account and personal account. The model allows the endogenous derivation of the objective function used by the manager in making decisions on corporate account. Given managerial compensation

[23] There is an empirical literature that attempts to make this connection but the results are ambiguous or weak; the ambiguity may well arise because the incentives arising from a bonus scheme may be different from those arising from a restricted stock or a stock option scheme. Those differences are not typically considered in the empirical literature and have not been adequately explored in the theoretical literature. On bonuses see Brander and Poitevin (1992) and MacMinn (1992).

in stock options the separation result yields an objective function, which shows that the manager will overinvest relative to the investment that maximizes current shareholder value or equivalently net present value and that the manager will not manage the corporate risk with forward contracts. Risk aversion drove the hedging decision in the Smith and Stulz analysis. In this setting, the manager paid in stock would be on the razor's edge of indifference and the manager paid in stock options would see a reduction in the option value with any hedge in forwards and so would eschew that risk management tool.

Hedging with forwards creates a symmetrical reduction in volatility. Insurance, however, does not reduce volatility in the same way. Insurance involves a risk transfer from the insured to the insurer for a price. The loss and thus indemnity can be higher, lower, or equal to the future value of the insurance premium. Insurance can transfer cash from states with small losses to states with large losses. If the transfer moves cash from the out-of-the-money states to the in-the-money states then the manager benefits and thus has an incentive to insure. Hence, not all risk management tools are created equally. The analysis in Theorem 2 shows a case in which the manager will insure despite having no incentive to hedge with forward contracts. The result in Theorem 2 depends critically on the form of compensation; it should continue to hold in a property insurance setting with deductible contracts for essentially the same reason that it holds here. It is also possible to conjecture that the demand for insurance would continue to hold even if the contract structure was changed to coinsurance; if this conjecture holds then it emphasizes the more important role played by the form of the compensation scheme.

Finally, this analysis shows that managers paid in stock options overinvest to maximize their stock option values at the expense of shareholders and that insurance is irrelevant to the value of the corporation, holding investment constant. The manager's gain from insuring is the shareholders' loss *ceteris paribus*. However, when managers insure on corporate account, the overinvestment problem induced by stock option compensation is reduced. Hence, while the shareholder will lose from the manager's insuring on corporate account, that loss will be offset by the closer-to-optimal investment that the manager is motivated to make.

This analysis has focused on stock options and the consequent motivation to purchase corporate insurance. The separation theorem provided the basis for an analysis of the incentives. A similar analysis would be appropriate using other forms of compensation. A compensation scheme that included salary and stock, phantom stock, or restricted stock should provide an incentive for the manager to maximize the current shareholder value of the corporation or equivalently would align interests with current shareholders (MacMinn, 2005); the salary and stock compensation scheme yields maximization of current shareholder value as the objective and so would put the manager on the razor's edge when it comes to making an insurance decision for the corporation. There are notable exceptions in which the manager maximizing current shareholder value does purchase insurance to solve an agency problem, e.g., see Garven and MacMinn (1993). A compensation scheme that included salary, stock, and stock options would not align incentives as the stock option scheme here shows; the stock in this scheme would dilute part but not all of the incentive to increase risk that is provided by the stock options. Hence, part of the motivation to purchase liability insurance should remain with this compensation scheme. Bonuses would alter the results if given sufficient weight in the compensation scheme and deserve a separate focus. Bonuses can be and are specified in so many different ways that it is difficult to generalize; some bonus packages can mimic stock or stock options and so yield similar results. Suppose the bonus scheme is set so that a bonus is paid if the manager exceeds a specified earnings target and, as in this analysis, the PIU holds then it may be shown that the investment choice is an increasing function of the earnings target, e.g., see Brander and Poitevin (1992) and MacMinn (1992). In this case, the bonus may motivate the insurance purchase as it does here but more analysis is required.

This study treats stock options' exercise price as given. Bebchuk et al. (2002) argue that setting the exercise price reflects the agency problem. Theorem 4 supports this view to the extent that it shows that a sufficiently large exercise price in conjunction with insurance reduces the overinvestment. Research taking into consideration the interaction among exercise price, corporate investment, and insurance decisions can further add to our understanding of the effects of stock option compensation on corporate decisions.

APPENDIX

Fisher Separation Theorem: *The manager paid in stock options* now *that vest* then *makes decisions on corporate account that are independent of the manager's preferences for consumption* now *versus* then; *the decisions are made to maximize the warrant value W(I).*

Proof: Let n denote the number of states of nature and $c_1 = (c_1(\omega_1),\ c_1(\omega_2),\ K, \ldots, c_1(\omega_n))$ denote the consumption *then* vector. Let $L(c_0, c_1, I, \theta)$ be the Lagrange function corresponding to the constrained maximization problem in (3) where θ is the Lagrange multiplier.

$$L(c_0, c_1, I, \theta) = \int_\Omega u(c_0, c_1(\omega))\, \mathrm{d}\Psi(\omega) - \theta\left(c_0 + \int_\Omega c_1(\omega)\, \mathrm{d}P(\omega) - m_0 - \int_\Omega m_1(\omega)\, \mathrm{d}P(\omega) - W(I)\right). \tag{A1}$$

The first order conditions for a maximum are:

$$\frac{\partial L}{\partial c_0} = \int_\Omega \frac{\partial u}{\partial c_0}\, \mathrm{d}\Psi(\omega) - \theta = 0 \tag{A2}$$

$$\frac{\partial L}{\partial c_1(\omega_j)} = \frac{\partial u}{\partial c_1(\omega_j)} \psi(\omega_j) - \theta\ p(\omega_j) = 0, \quad j = 1, \ldots, n \tag{A3}$$

$$\frac{\partial L}{\partial I} = -\theta\, \frac{\mathrm{d}W}{\mathrm{d}I} = 0 \tag{A4}$$

and

$$\frac{\partial L}{\partial \theta} = -\left(c_0 + \int_\Omega c_1(\omega)\,\mathrm{d}P(\omega) - m_0 - \int_\Omega m_1(\omega)\,\mathrm{d}P(\omega) - W(I)\right) = 0. \tag{A5}$$

Equations (A2) and (A3) express the conditions for the optimal consumption pair while (A4) expresses the condition for an optimal investment. Hence, the Fisher separation result follows since (A4) is independent of preferences for consumption *now* and *then*; in this uncertainty setting, it should also be noted that (A4) is independent of the manager's measure of risk aversion and probability beliefs. Clearly, the objective function that the manager uses to make the investment decision on corporate account is the value of the stock option package.

$$\begin{aligned} W(I,\varphi) &= \int_0^\zeta \max\{0, a(\Pi - L + \varphi(f - \Phi) - b) - (1-a)E\}\,\mathrm{d}P \\ &= \int_\alpha^\zeta (a(\Pi - L + \varphi(f - \Phi) - b) - (1-a)E)\,\mathrm{d}P. \end{aligned} \tag{A6}$$

Next consider the corporate hedging decision. The payoff on the forward market position taken by the corporation is shown in Figure A1. Let Φ denote the unit payoff on a stock index forward contract and let f denote the forward price. Suppose that the unit payoff is increasing in state. The payoff on the forward position is $\varphi(f - \Phi(\xi))$, where φ is the position taken by the firm in forwards. Let η be the economic state implicitly defined by the condition $f - \Phi(\eta) = 0$ as shown in Figure A1. The payoff depicted is sometimes referred to as a short position in the futures contract. If the corporation selects φ units of the hedge then the corporate payoff is $\Pi - L + \varphi(f - \Phi) - b$ as shown in Figure 2. The following hedging theorem follows easily.

Hedging Theorem: *Let the payoff on the stock index Φ be increasing in state.* Ceteris paribus, *the manager paid in stock options does not have an incentive to hedge in forward contracts written on the stock index.*

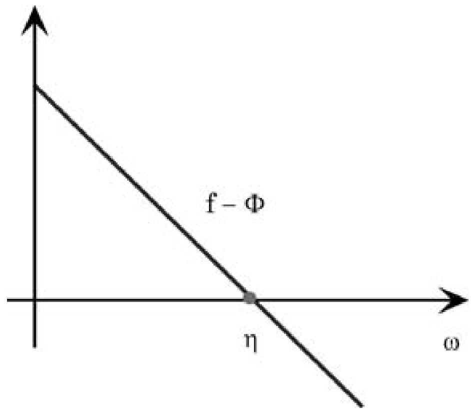
Proof: The manager's objective function for decision made on corporate account is the warrant value $W(I, \varphi)$ where

$$\begin{aligned} W(I,\varphi) &= \int_0^\zeta \max\{0, a(\Pi - L + \varphi(f - \Phi) - b) - (1-a)E\}\,\mathrm{d}P \\ &= \int_\alpha^\zeta (a(\Pi - L + \varphi(f - \Phi) - b) - (1-a)E)\,\mathrm{d}P. \end{aligned} \tag{A7}$$

The first order condition for the forward contracts is

$$\frac{\partial W}{\partial \varphi} = a\int_\alpha^\zeta (f - \Phi)\,\mathrm{d}P < 0. \tag{A8}$$

FIGURE A1
Forward Payoff



The inequality follows since letting $H(\alpha) = \int_\alpha^\zeta (f - \Phi)\,\mathrm{d}P$ it may be observed that the no arbitrage condition for pricing the futures contract is $H(0) = 0$ and H is a decreasing function.

Next consider the case of a liability loss that is decreasing in state. The specification of the insurance premium changes as may be seen in Figure A2. The insurance payoff is now

$$\begin{aligned} i(I,k) &= \int_0^\zeta \min\{L(I,\omega), k\}\,\mathrm{d}P \\ &= \int_0^\lambda k\,\mathrm{d}P + \int_\lambda^\zeta L(I,\omega)\,\mathrm{d}P. \end{aligned} \tag{A9}$$

Hence, the promised payment on debt as a function of the investment must be implicitly specified as

$$\begin{aligned} G(I,k,b) &= D(b) - I - i(I,k) \\ &= \int_0^\zeta b\,\mathrm{d}P - I - \left(\int_0^\lambda k\,\mathrm{d}P + \int_\lambda^\zeta L(I,\omega)\,\mathrm{d}P\right). \end{aligned} \tag{A10}$$

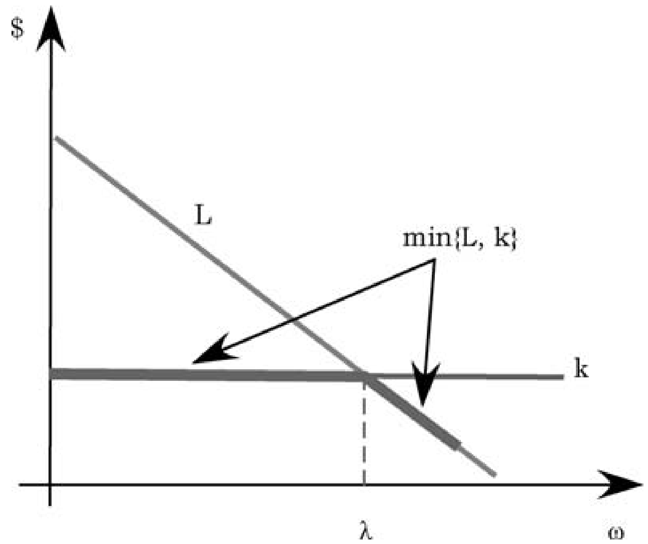
The next lemma replaces Lemma 1 in this case.

Lemma 2: *Given the loss that decreases in state, the promised payment on the debt issue increases in the investment I and the cap k.*

Proof: By the Implicit Function Theorem

$$\frac{\partial b}{\partial I} = -\frac{\dfrac{\partial G}{\partial I}}{\dfrac{\partial G}{\partial b}} = \frac{1 + \displaystyle\int_\lambda^\zeta \frac{\partial L}{\partial I}\,\mathrm{d}P}{\displaystyle\int_0^\zeta \mathrm{d}P} > 0. \tag{A11}$$

FIGURE A2
Insurance Payoff



and

$$\frac{\partial b}{\partial k} = -\frac{\dfrac{\partial G}{\partial k}}{\dfrac{\partial G}{\partial b}} = \frac{\displaystyle\int_0^{\lambda} \mathrm{d}P}{\displaystyle\int_0^{\zeta} \mathrm{d}P} > 0. \tag{A12}$$

Theorem 2b: *Let Π and $\Pi - L$ be increasing in state; let L be decreasing in state.* Ceteris paribus, *the manager paid in stock options has an incentive not to insure the liability losses.*

Proof: Consider the case in which the losses are decreasing in state as shown in Figure A2. The corporate payoff *then* is

$$\Pi - L + \min\{L, k\} = \begin{cases} \Pi - (L - k) & \omega \le \lambda \\ \Pi & \omega > \lambda. \end{cases} \tag{A13}$$

The following two cases must be considered.

Case I: $\alpha < \lambda$. The manager's warrant value is

$$W(I, k) = \int_{\alpha}^{\lambda} [a(\Pi - (L - k) - b) - (1 - a)E]\,\mathrm{d}P + \int_{\lambda}^{\zeta} [a(\Pi - b) - (1 - a)E]\,\mathrm{d}P \tag{A14}$$

and the first order condition for insurance coverage is

$$
\begin{aligned}
\frac{\partial W}{\partial k} &= a\int_{\alpha}^{\lambda}\left(1-\frac{\partial b}{\partial k}\right)\mathrm{d}P - a\int_{\lambda}^{\zeta}\frac{\partial b}{\partial k}\,\mathrm{d}P \\
&= a\left(\int_{\alpha}^{\lambda}\mathrm{d}P - \frac{\partial b}{\partial k}\int_{\alpha}^{\zeta}\mathrm{d}P\right) \\
&= a\left(\int_{\alpha}^{\lambda}\mathrm{d}P - \frac{\int_{0}^{\lambda}\mathrm{d}P}{\int_{0}^{\zeta}\mathrm{d}P}\left(\int_{\alpha}^{\lambda}\mathrm{d}P + \int_{\lambda}^{\zeta}\mathrm{d}P\right)\right) \\
&= a\left(\int_{\alpha}^{\lambda}\mathrm{d}P\left(1 - \frac{\int_{0}^{\lambda}\mathrm{d}P}{\int_{0}^{\zeta}\mathrm{d}P}\right) - \frac{\int_{0}^{\lambda}\mathrm{d}P}{\int_{0}^{\zeta}\mathrm{d}P}\int_{\lambda}^{\zeta}\mathrm{d}P\right) \\
&= a\left(\frac{\int_{\alpha}^{\lambda}\mathrm{d}P}{\int_{0}^{\zeta}\mathrm{d}P}\left(\int_{0}^{\zeta}\mathrm{d}P - \int_{0}^{\lambda}\mathrm{d}P - \frac{\int_{0}^{\lambda}\mathrm{d}P}{\int_{\alpha}^{\lambda}\mathrm{d}P}\int_{\lambda}^{\zeta}\mathrm{d}P\right)\right) \\
&= a\left(\frac{\int_{\alpha}^{\lambda}\mathrm{d}P}{\int_{0}^{\zeta}\mathrm{d}P}\int_{\lambda}^{\zeta}\mathrm{d}P\left(\frac{\int_{\alpha}^{\lambda}\mathrm{d}P}{\int_{\alpha}^{\lambda}\mathrm{d}P} - \frac{\int_{0}^{\lambda}\mathrm{d}P}{\int_{\alpha}^{\lambda}\mathrm{d}P}\right)\right) \\
&= -a\left(\frac{\int_{\alpha}^{\lambda}\mathrm{d}P}{\int_{0}^{\zeta}\mathrm{d}P}\int_{\lambda}^{\zeta}\mathrm{d}P\left(\frac{\int_{0}^{\lambda}\mathrm{d}P}{\int_{\alpha}^{\lambda}\mathrm{d}P} - \frac{\int_{\alpha}^{\lambda}\mathrm{d}P}{\int_{\alpha}^{\lambda}\mathrm{d}P}\right)\right) \\
&= -a\left(\frac{\int_{\alpha}^{\lambda}\mathrm{d}P}{\int_{0}^{\zeta}\mathrm{d}P}\int_{\lambda}^{\zeta}\mathrm{d}P\left(\frac{\int_{0}^{\alpha}\mathrm{d}P}{\int_{\alpha}^{\lambda}\mathrm{d}P}\right)\right) \\
&= -a\left(\frac{\int_{0}^{\alpha}\mathrm{d}P}{\int_{0}^{\zeta}\mathrm{d}P}\int_{\lambda}^{\zeta}\mathrm{d}P\right) \\
&< 0.
\end{aligned}
\tag{A15}
$$

Case II: $\alpha > \lambda$. Here the warrant value is

$$
W(I, k) = \int_{\alpha}^{\zeta}[a(\Pi - b) - (1 - a)E]\,\mathrm{d}P \tag{A16}
$$

and the first order condition is

$$\frac{\partial W}{\partial k} = -a \frac{\partial b}{\partial k} \int_{\alpha}^{\zeta} \mathrm{d}P < 0. \tag{A17}$$

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